Data: 
$$B, \mathcal{H}(), H, R, o$$
, etc.
Input:  $x^g, q^a$ : present guess/analysis in different grid spaces
Input:  $\hat{x}^s:=x^g-L^{-1}q^b, x$ -increment
Input:  $\hat{y}^s:=B\hat{x}^s, y$ -increment

1  $d^{\hat{x}}=0$ .
2  $d^{\hat{y}}=0$ .
3  $\hat{x}=0$ .
4  $\hat{f}=0$ .
5 do  $i=1,\ldots$ 
6  $g^{\hat{x}}=H^TR^{-1}(\mathcal{H}(x^g+\hat{x})-o)$ 
7  $g^{\hat{y}}=Bg^{\hat{x}}$ 
8  $g^{\hat{x}}=\hat{y}^s+g^{\hat{x}}$ 
9  $g^{\hat{y}}=\hat{x}^s+g^{\hat{y}}$ 
10  $\hat{f}=g^{\hat{y}}-\hat{f}$ 
11  $\beta=\hat{f}^Tg^{\hat{y}}/\hat{f}^Td^{\hat{x}};\,\hat{f}=g^{\hat{y}}$ 
12  $d^{\hat{x}}=-g^{\hat{y}}+\beta d^{\hat{x}}$ 
13  $d^{\hat{y}}=-g^{\hat{x}}+\beta d^{\hat{y}}$ 
14 minimize  $J(\hat{x}+\alpha d^{\hat{x}})$  for  $\alpha$ 
15  $\hat{x}=\hat{x}+\alpha d^{\hat{x}}$ 
16  $\hat{x}^s=\hat{x}^s+\alpha d^{\hat{x}}$ 
17  $\hat{y}^s=\hat{y}^s+\alpha d^{\hat{y}}$ 
18 enddo
19  $q^a=q^a+L\hat{x}$ 
Result:  $\hat{x}^s,\hat{y}^s,q^a$