UNIVERSITA` DEGLI STUDI DI TRENTO

DOCTORAL SCHOOL OF CIVIL, ENVIRONMENTAL AND MECHANICAL ENGINEERING

Mathematical modeling and numerical simulation of water-heat coupled movements in 1D soil column

Niccolò Tubini, Riccardo Rigon, Vincenzo Casulli

Trento, May 14 2019

Outline

- Research issue: temperature and soil water flow
- Research background
- Mathematical models
- Numerical model
- Conclusions

1

Temperature and soil water flow

1.1 Temperature affects:

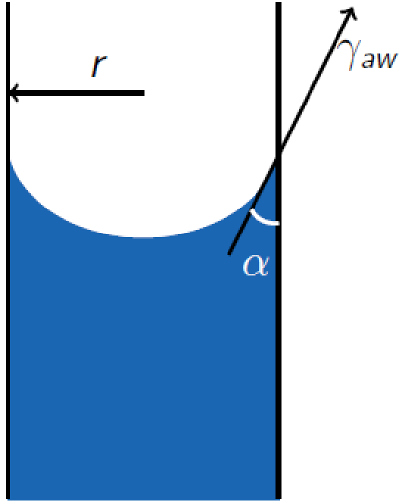
- Water contact angle and water surface tension
- Water phase change
- Water viscosity

1.2 Water contact angle and water surface tension

A review on recent developments in soil water retention theory: interfacial tension and temperature effects

Jörg Bachmann[1*] and Rienk R. van der Ploeg[1]

[1]Institut für Bodenkunde, Universität Hannover, Herrenhäuser Str. 2, D-30419 Hannover, Germany



$$\psi(T) = \psi_{Tr}\left(\frac{\beta_0 + T}{\beta_0 + T_r}\right)$$

$$\theta(\psi, T) = \theta_r + \frac{\theta_s - \theta_r}{\left((1 + (\alpha\psi(T))^n\right)^m}$$

1.3 Phase change water-ice


Biophysical Journal Volume 81 September 2001 1389–1397 1389

Intercellular Ice Propagation: Experimental Evidence for Ice Growth through Membrane Pores

Jason P. Acker,* Janet A. W. Elliott,† and Locksley E. McGann*

*Departments of Laboratory Medicine and Pathology and †Chemical and Materials Engineering, University of Alberta, Edmonton, Alberta, Canada

Freezing and thawing processes occur over a range of temperature due to capillary pressure and the presence of solutes.

The freezing point depression is defined as:

$$T - T_m = \frac{2\gamma_{aw} T_m \cos\alpha}{l r \rho_w} + \frac{\pi_w T_m}{\rho_w l}$$

1.4 Water viscosity

The Flow of Heterogeneous Fluids Through Porous Media

MORRIS MUSKAT AND MILAN W. MERES, *Gulf Research & Development Company, Pittsburgh, Pa.*

(Received July 22, 1936)

$$K_s = k\frac{\rho g}{\nu}$$

$$K_s(T) = K_s(T_r)\frac{\nu(T_r)}{\nu(T)}$$

1.5 Interim conclusions

These aspects require to:

- modify the parametrization of the SWRC and unsaturated hydraulic conductivity
- ‘extend’ the Richards’ equation to model freezing-thawing processes
- model the ground temperature.

2

Why study coupled groundwater flow and heat transfer?

2.1 Surface-subsurface exchange

Field study and simulation of diurnal temperature effects on infiltration and variably saturated flow beneath an ephemeral stream

Anne Dudek Ronan
Department of Civil and Environmental Engineering, San Jose State University, San Jose, California

David E. Prudic and Carl E. Thodal
U.S. Geological Survey, Carson City, Nevada

Jim Constantz
U.S. Geological Survey, Menlo Park, California

The importance of coupled modelling of variably saturated groundwater flow-heat transport for assessing river–aquifer interactions

I. Engeler[a], H.J. Hendricks Franssen[a,c], R. Müller[b], F. Stauffer[a,*]

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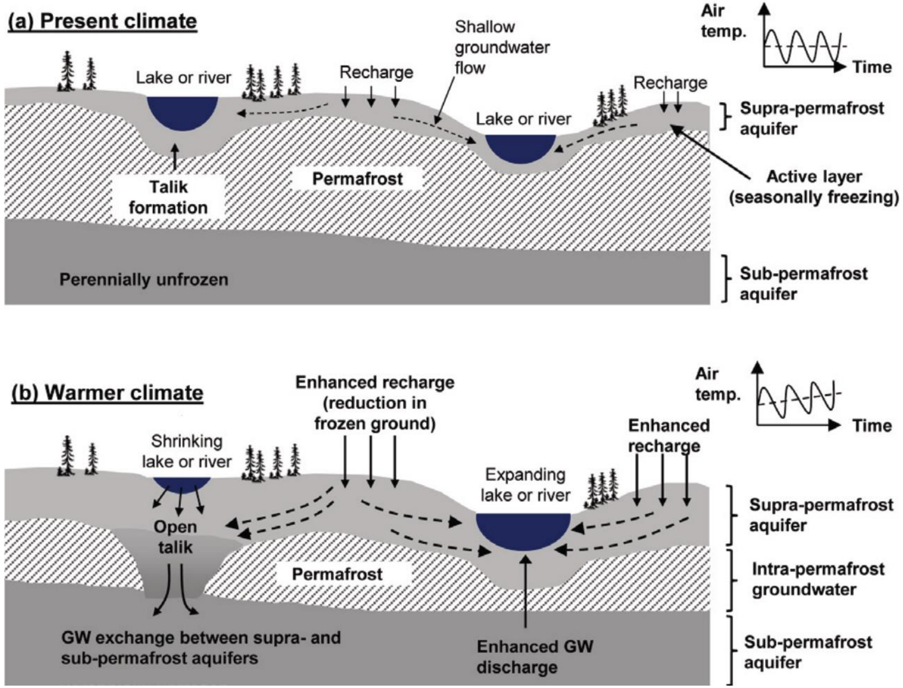
2.1 Surface-subsurface exchange



2.2 Heat conduction

Geotech Geol Eng (2015) 33:207–221
DOI 10.1007/s10706-015-9843-2

ORIGINAL PAPER

Critical Review of Thermal Conductivity Models for Unsaturated Soils

Yi Dong · John S. McCartney · Ning Lu

Soil is a multi-phase material consisting of soil particles, gas and/or liquid

- Thermal conductivity

$$\lambda_{air} < \lambda_{dry} < \lambda_w < \lambda_{sat} < \lambda_{mineral}$$

- Heat capacity

$$C_T = \rho_{sp} c_{sp} (1 - \theta_s) + \rho_w c_w \theta_w + \rho_i c_i \theta_i$$

2.3 Heat advection

Geotech Geol Eng (2015) 33:207–221
DOI 10.1007/s10706-015-9843-2

ORIGINAL PAPER

Critical Review of Thermal Conductivity Models for Unsaturated Soils

Yi Dong · John S. McCartney · Ning Lu

Identification of soil-cooling rains in southern France from soil temperature and soil moisture observations

Sibo Zhang[1,a], Catherine Meurey[1], and Jean-Christophe Calvet[1]

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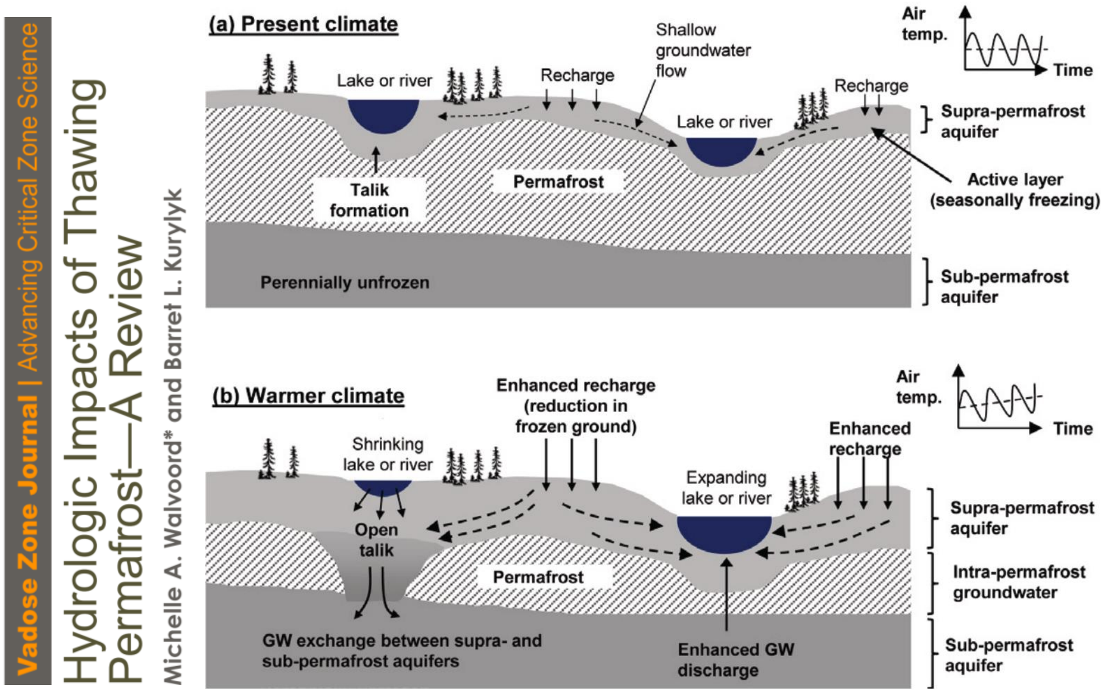
[a]now at: Qian Xuesen Laboratory of Space Technology, China Academy of Space Technology (CAST), Beijing, China

2.4 Ground temperature



- Water has a high heat capacity and latent heat of fusion
- Ice thermal conductivity is fourfold great than water, and its heat capacity is half.

3

Mathematical model

3.1 1D model

$$\begin{cases} \dfrac{\partial H}{\partial t} = 0 \\[2ex] \dfrac{\partial \theta}{\partial t} = \dfrac{\partial}{\partial z}\left[K\dfrac{\partial \psi}{\partial z} + K\right] \end{cases}$$

$$\begin{cases} \dfrac{\partial}{\partial t}(\rho_w c_w (T - T_r)) = 0 \\[2ex] \dfrac{\partial}{\partial t}\{[\rho_w c_w \theta + \rho_{sp} c_{sp}(1-\theta_s)](T - T_r)\} = \\ \qquad -\dfrac{\partial}{\partial z}\left(\rho_w c_w (T - T_r) J_\theta - \lambda \dfrac{\partial T}{\partial z}\right) \end{cases}$$

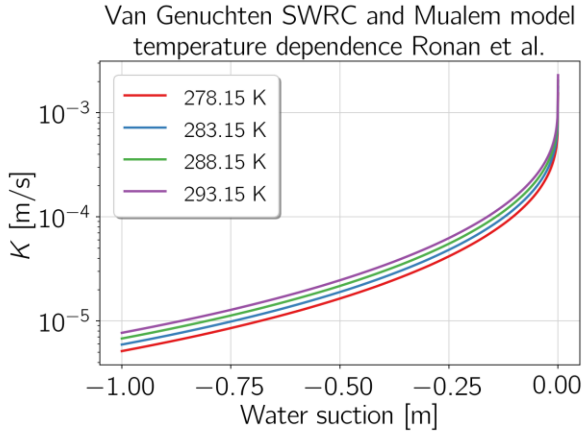
3.2 Temperature dependence of the soil hydraulic functions

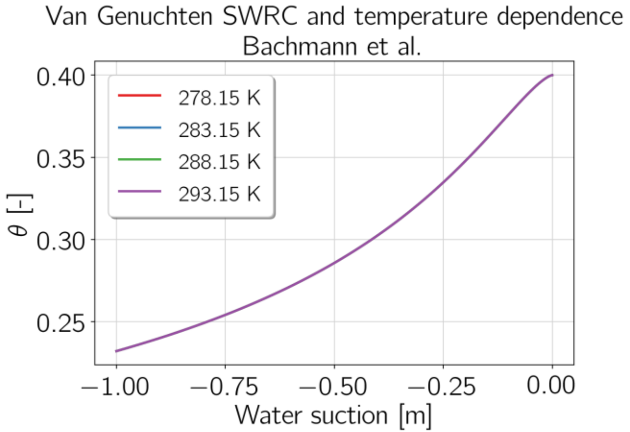
$$K_s(T) = K_s(T_r)\frac{\nu(T_r)}{\nu(T)}$$

$$\nu(T) = 0.00002414 \cdot 10^{247.8/(T-140)}$$

$$\psi(\theta, T) = \psi(\theta, T_r)\left(\frac{\beta_0 + T}{\beta_0 + T_r}\right)$$

$$\theta(\psi, T) = \theta_r + \frac{\theta_s - \theta_r}{\{1 + [\alpha\psi(T)]^n\}^m}$$





4 Numerical model

4.1 Numerical model - 1

$$K = K(\psi, T)$$

Richards' equation is solved with a semi-implicit method and then and then the energy equation.

4.2 Numerical model - 2

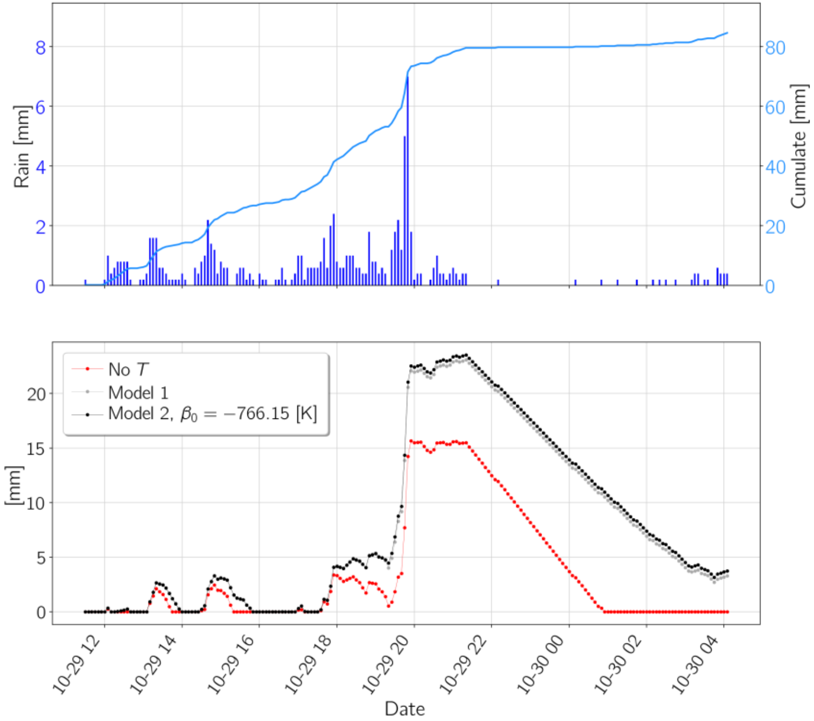
$$K = K(\psi, T) \quad \theta = \theta(\psi, T)$$

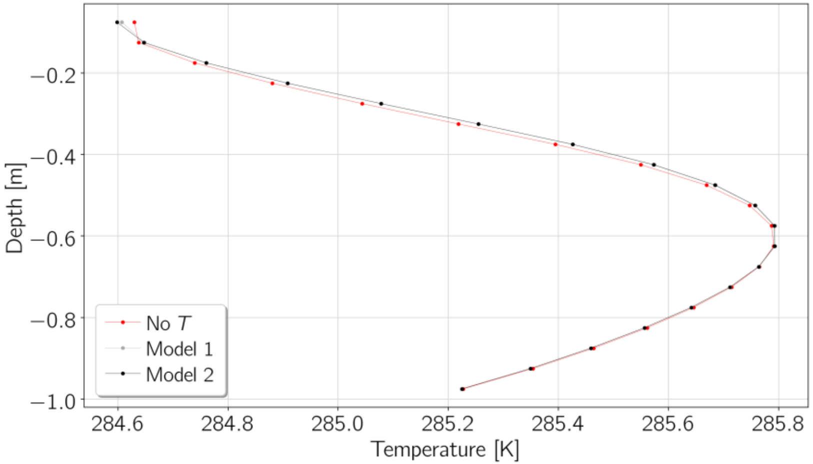
The mass and energy equation are fully coupled. To solve the system we adopt a *splitting method*. In the first half step the internal energy is updated with the conduction flux. In the second half step we solve the Richards' equation and update the internal energy with the advection flux in order to find the new temperature.

Numerical experiment

	α [m^{-1}]	n [-]	θ_s [-]	θ_r [-]	K_s [m/s]
Loam	3.6	1.56	0.43	0.078	2.8E-06
Sandy loam	7.5	1.89	0.41	0.065	1.23E-05
Silty loam	2	1.41	0.45	0.08	1.25E-06

Numerical experiment model





Tubini N., Rigon R., Casulli V.

5 Conclusions

5.1 Earth's critical zone


Hydrol. Earth Syst. Sci. Discuss., 6, 3417–3481, 2009
www.hydrol-earth-syst-sci-discuss.net/6/3417/2009/
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Hydrology and Earth System Sciences Discussions

Earth's Critical Zone and hydropedology: concepts, characteristics, and advances

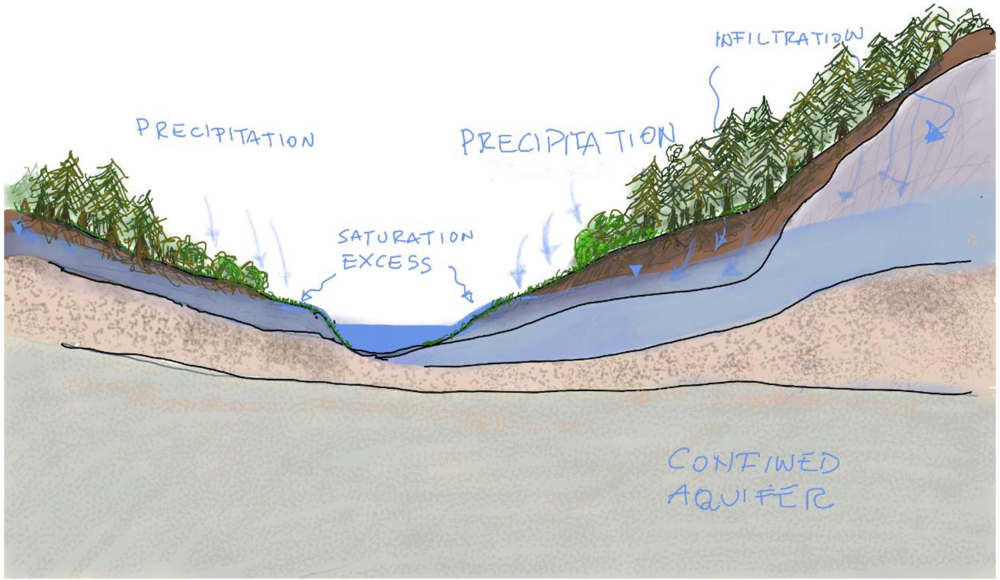
H. S. Lin

The aspects mentioned before can be related to the wider concept of Earth's critical zone.

'the heterogeneous, near surface environment in which complex interactions involving rock, soil, water, air and living organisms regulate the natural habitat and determine the availability of life-sustaining resources'

5.2 Earth's critical zone

Picture credit: Rigon Riccardo



Modelling water infiltration in a hillslope or a catchment we have to look to physical properties of capillary water but this is not enough: we have to look also to vegetation.

5.3 Future work

- Extend this model to the 2D and 3D case
- Include the water phase change

Thank you!

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