Initial Estimates in the Design of Rack-and-Pinion Steering Linkages

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Initial Estimates in the Design of Rack-and-Pinion Steering Linkages

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Based on recent results concerning the occurrence of function cognates in Watt II linkages, it is shown that only 3 geometric parameters are sufficient for defining the kinematic function of simplified planar rack-and-pinion steering linkages. The steering performances of the mechanisms are analytically expressed in terms of these parameters and, by employing an optimization-based synthesis method involving increasing the degree of freedom of the mechanism, the optimum domains are determined. The parameter sets corresponding to these minimum steering error domains are displayed in design charts. These charts aid the automotive engineer in the early stages of conceiving a new steering linkage by providing initial estimates of the basic geometry of the mechanism. They also provide information on two other characteristics of concern, i.e. the minimum pressure angle occurring in the joints and the rack stroke required for maximum turn of the wheels. [S1050-0472(00)00402-5]

Keywords: Rack-and-Pinion Steering Linkage, Steering Error Minimization, Parametric Design Charts

1 Introduction

Rack-and-pinion steering mechanisms are widely used in independent front suspension vehicles and are finding increasing use in rigid-axle tractors and the like, where the control comes from a transversally mounted hydraulic cylinder. There are two different embodiments of such steering mechanisms. One of these, with the central joints very close to each other (Fig. 1(*a*)), is called the outrigger or central take-off rack-and-pinion steering mechanism. It is associated with MacPherson strut damper front axles having high location steering knuckle arms [1]. The other (standard) arrangement (Fig. 1(*b*)) has the tie-rods connected to the ends of the actuating element-the steering rack or the hydraulic piston.

Real steering mechanisms are complex spatial linkages, because their kingpins are not parallel. Constraints upon the disposition of the steering gear in the engine compartment and any necessary kinematic correlation with the suspension, such as minimization of the cross-coupling effect between steering and suspension, may further prohibit the use of planar mechanisms. The three-dimensionality of the mechanism implies that its exact geometry must be expressed using an increased number of parameters and explains why most of the works published on this subject [2,3] are case studies. Zarak and Townsend [4] have attempted to generalize the approach to the standard rack-and-pinion steering linkage design by considering dimensionless link lengths in equivalent planar mechanisms. This normalization of the linear dimensions makes good sense because, neglecting the dynamic influence of the elasticity and side-slipping of the tires, the turning geometry of a vehicle is unaffected by change in scale. In addition, the kingpin inclination angles have only a small influence upon the steering transmission function and are usually chosen on other considerations such as maintaining the stability of the wheels for straight-ahead motion of the vehicle and assuring a self-aligning effect after cornering. Prior to Zarak and Townsend [4], Wolfe [5] and Ramachandra [6] used normalized link lengths and a single vehicle size parameter (wheel base vs. wheel track ratio) in the synthesis of the Ackermann linkage (a symmetric four-bar linkage, the kinematic function of which is defined by two parameters only). The latter author also provided a number of parametric charts and made design recommendations which help in choosing an adequate linkage geometry for a given vehicle.

In a recent paper, Simionescu and Smith [7] have shown that three parameters are sufficient for describing the transmission function of rack-and-pinion steering linkages and also that any standard configuration has an infinity of function cognates, including one of the central outrigger type with merged central joints. This implies that attempts at fine tuning by using a fourth parameter cannot bring about any improvement of the steering error beyond that achieved by varying the initial three. Zarak and Townsend [4] used four geometric parameters to describe the geometry of the mechanism, while the steering error was expressed unusually as the distance between the turning centres of the left and right wheels. Consequently, their conclusions are of limited practical value, concerning mainly the choice of various weightings in steering-error-based objective functions.

The purpose of the present paper is to provide generalized kinematic models for both central outrigger and standard rack-and-pinion steering linkages and, by searching for the minimum steering error domains, to provide a number of parametric charts useful in the automotive engineer's design activity.

The main requirement of the steering mechanism in a vehicle is to provide a correlated pivoting of the wheels such that their axis intersection point lies on the rear wheel axis, a condition known as the Ackermann law:

$$\theta_{OA}(\theta_I) = \arctan \frac{1}{ctg\,\theta_I + 1/(W_b/W_t)} \tag{1}$$

where θ_O and θ_I are the turning angle of the outer and inner wheels respectively measured from a reference position, and θ_{OA} is the theoretical imposed angle. In most real vehicles this geometric condition is seldom fulfilled [1], roll steer, compliance and wheel slip at high speed moving the instant center of rotation from the extension line of the rear axle towards the vehicle. However, the Ackermann law is still used in comparisons of various vehicle steering systems. In the above relation the front wheel track W_t is usually approximated to the kingpin track O_1O_2 (see for example Fig. 2), which has been used as the normalizing dimension in all

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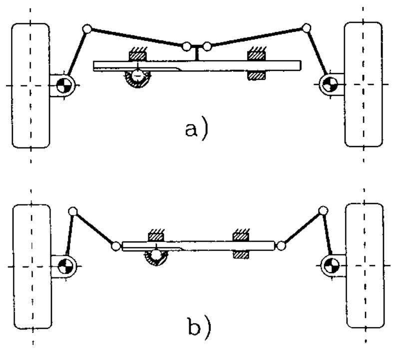

Fig. 1 The central outrigger (*a*) and standard (*b*) rack-and-pinion steering linkage

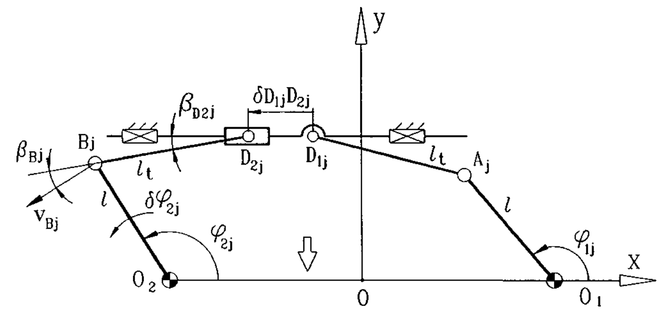

Fig. 3 2 DOF associated mechanism used in formulating the synthesis problem

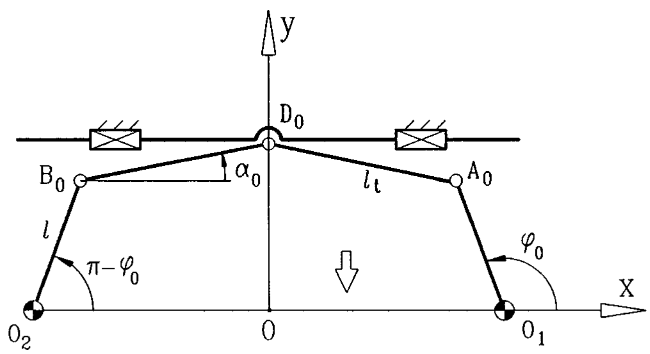

Fig. 2 Simplified planar model of central outrigger steering linkage

the analytical formulae. This implies that all the linear dimensions (l, l_t, y_D etc.) will be expressed per unit W_t or O_1O_2.

A measure of the steering performance of the steering linkage, which can be considered as a function generator, is the *steering error*, defined as the difference between the actual θ_O and theoretical turning angle θ_{OA}:

$$\delta\theta_O(\theta_I)=\theta_O(\theta_I)-\theta_{OA}(\theta_I). \tag{2}$$

It is logical to estimate this difference at the outer wheel, taking the inner wheel as the input element, since the value of the maximum pivoting angle $\theta_{I-\max}$ is usually restricted by the maximum operating angle of the universal joints of the axle shafts or, in the case of rear wheel drive vehicles, by possible wheel-axle or wheel-body interference.

2 Central Outrigger Rack-and-Pinion Linkage

The simplified planar mechanism shown in Fig. 2 is the result of merging the two central joints of the outrigger into a single triple joint. The parameters describing the geometry of the reference position are the normalized steering knuckle arm length l, and the angles φ_0 and α_0 (positive α_0 angles are measured counter-clockwise). The mechanism geometry is expressed in as many angular parameters as possible, since they are scaling invariants and because the loops of the mechanism will always close in the reference position. The tie rod length l_t and rack axis offset y_D are simply obtained as:

$$l_t=A_0B_0/(2\cdot\cos\alpha_0) \quad \text{and} \quad y_D=l\cdot\sin\varphi_0+l_t\cdot\sin\alpha_0 \tag{3}$$

where A_0B_0 is the distance between joints A and B in the straight-ahead reference position:

$$A_0B_0=1+2\cdot l\cdot\cos\varphi_0. \tag{4}$$

The usual approach to function generation synthesis problems using optimization techniques is to define an objective function equal to the mean square, or to the maximum norm of the output steering error $\delta\theta_O$, the design variables being the above-mentioned geometric parameters φ_0, α_0 and l. This requires establishing the input-output transmission function of the mechanism in an analytical form. The ''method of increasing the degree of freedom of the mechanism'' proposed by Simionescu and Alexandru [8] as an extension of the method of Suh and Mecklenburg [9] and Alizade et al. [10] has been adopted here. The associated 2 DOF mechanism required by this method is shown in Fig. 3, where the constituent joints of the central outrigger are now allowed to move independently along the straight line $y=y_D$ representing the rack axis. The variable distance $\delta D_{1j}D_{2j}$ measured between the joints D_1 and D_2 in any current design position will be:

$$\delta D_{1j}D_{2j}=x_{D1j}-x_{D2j} \tag{5}$$

where x_{D1j} and x_{D2j} are obtained by solving the following equations of constraint (the position problem in the RRT dyad):

$$\begin{aligned}(x_{D1j}-x_{Aj})^2+(y_D-y_{Aj})^2&=l_t^2\\(x_{D2j}-x_{Bj})^2+(y_D-y_{Bj})^2&=l_t^2\end{aligned} \tag{6}$$

with the solutions:

$$\begin{aligned}x_{D1j}&=x_{Aj}\pm\sqrt{l_t^2-(y_D-y_{Aj})^2}\\x_{D2j}&=x_{Bj}\pm\sqrt{l_t^2-(y_D-y_{Bj})^2}\end{aligned} \tag{7}$$

In Eq. (7) the appropriate signs of the square root term (*viz.* ''−'' for the former and ''+'' for the latter) are obtained such that the resulting positions of the pivot joints D_{1j} and D_{2j} are close to the Oy axis. The co-ordinates of the joints A and B relative to the reference frame OXY are given by the following relations:

$$\begin{aligned}x_{Aj}&=l\cdot\cos\varphi_{1j}+0.5; \quad y_{Aj}=l\cdot\sin\varphi_{1j}\\x_{Bj}&=l\cdot\cos\varphi_{2j}-0.5; \quad y_{Bj}=l\cdot\sin\varphi_{2j}.\end{aligned} \tag{8}$$

Following the approach of Suh and Mecklenburg [9] and Alizade et al. [10], a prime objective function can be defined with the aim of finding those parameters of the associated 2 DOF mechanism for which the distance $\delta D_{1j}D_{2j}$ varies the least:

$$f_1(l,\varphi_0,\alpha_0)=\max|\delta D_{1j}D_{2j}| \quad (j=1,\ldots,n), \tag{9}$$

employing the following expressions for the angles φ_{1j} and φ_{2j} in relations (8):

$$\varphi_{1j}=\varphi_0+\theta_{Ij} \quad \text{and} \quad \varphi_{2j}=\pi-\varphi_0+\theta_{OA}(\theta_{Ij}), \tag{10}$$

which correspond to a design position j for which the inner wheel turning angle is:

$$\theta_{Ij}=j\cdot\theta_{I-\max}/n. \tag{11}$$

The above objective function should be used with some caution because its global minimum is a degenerate mechanism with zero

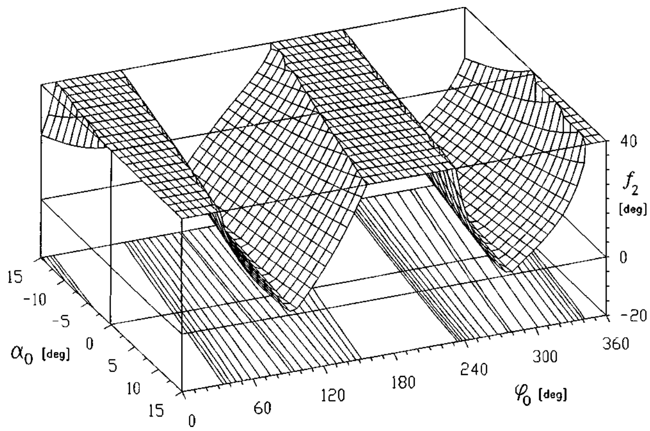

Fig. 4 3*D* plot of objective function f_2 for the case of $l=0.16$ and $W_b/W_t=1.9$

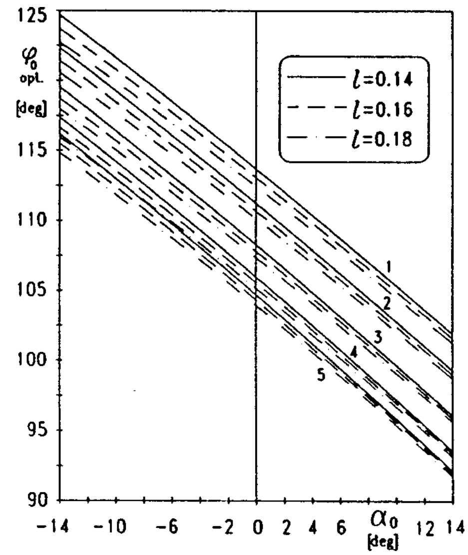

Fig. 5 Trailing central outrigger steering linkage design chart (1: $W_b/W_t=1.4$; 2: $W_b/W_t=1.6$; 3: $W_b/W_t=1.9$; 4: $W_b/W_t=2.2$; 5: $W_b/W_t=2.4$)

input and output members. Furthermore, it provides only a rough approximation of the output error $\delta\theta_{Oj}$ to be minimized.

A correction that gives an improved approximation of the steering error $\delta\theta_O$ given by Eq. (2) eliminates the degenerate global minimum and preserves the analytical simplicity of f_1 is:

$$f_2(l,\varphi_0,\alpha_0)=\max\left|\delta D_{1j}D_{2j}\cdot\frac{\cos\beta_{Dj}}{l\cdot\cos\beta_{Bj}}\right|\cong\max|\delta\theta_{Oj}| \quad (12)$$

derived from the Projection Theorem. According to this, the following relation holds between the velocities of points D_2 and B of the tie rod BD_2 when the steering knuckle arm O_1A is held fixed:

$$\frac{\delta D_{1j}D_{2j}}{\delta t}\cdot\cos\beta_{D2j}=v_{Bj}\cdot\cos\beta_{Bj}=\left(\frac{\delta\varphi_{2j}}{\delta t}l\cdot\cos\beta_{Bj}\right), \quad (13)$$

where the small displacement $\delta\varphi_{2j}$ measured on the output member O_2B is equivalent to the steering error $\delta\theta_{Oj}$ at the respective design point j. It may be seen that the closer to the minimum, where $\delta D_{1j}D_{2j}$ is small, the better is the approximation of the output error of the mechanism in the objective function f_2. The angles β_{D2j} and β_{Bj} appearing in the above relation are easily determined from the expression of the scalar product of vectors $\mathbf{B}_j\mathbf{D}_{2j}$ and $\mathbf{D}_{1j}\mathbf{D}_{2j}$, and $\mathbf{v}_{Bj}$ and $\mathbf{O_2B}_j$ respectively, where:

$$v_{2j}=\begin{bmatrix}dx_{Bj}\\ dy_{Bj}\end{bmatrix}\cdot\frac{1}{dt}=\begin{bmatrix}-l\cdot\sin\varphi_{2j}\\ l\cdot\cos\varphi_{2j}\end{bmatrix}\cdot\frac{d\varphi_{2j}}{dt}. \quad (14)$$

Furthermore, because interest is in the direction of vector $\mathbf{v}_{2j}$ only, the term $d\varphi_{2j}/dt$ can be neglected in evaluating the scalar product.

Both objective functions f_1 and f_2 must be penalized by assigning to them an arbitrary large value in cases when any of the square root factors in relations (8) become negative.

A maximum norm has been chosen because it ensures a close coincidence between the actual and imposed transmission function of the mechanism after minimization. However it has the disadvantage of not being differentiable, which is evident in the graphical representation of $f_2(\varphi_0,\alpha_0)$ in Fig. 4 for a vehicle with size ratio $W_b/W_t=1.9$ and a normalized steering knuckle arm $l=0.16$ (the values greater than 40° together with the occasional discontinuities occurring for $\beta_B=90°$ have been trimmed out). It can be seen that there is no well defined global optimum, but two minimum domains exist, one corresponding to the trailing link configuration (near $\varphi_0=90°$) and the other to the leading link configuration (around $\varphi_0=270°$).

Objective function f_2 has been minimized with respect to φ_0 only, using Brent's [11] algorithm, considering different values of α_0 and l and various vehicle size parameters W_b/W_t. The number of design points in relation (11) has been chosen as $n=60$ and the inner wheel maximum turning angle $\theta_{O-\max}$ as 40°. The optimum sets $(\alpha_0,\varphi_{0-\text{opt}})$ obtained from the optimization process have been used in plotting the parametric design charts of the central outrigger trailing linkage (Fig. 5) and leading linkage (Fig 7). The corresponding approximate maximum steering errors, (minimum values of function f_2) vary between 0.52883° and 0.82469° (compared with 0.52892° and 0.82460° for the exact calculated values) for the trailing link configuration, and between 0.23655° and 0.43375° (exact calculated values 0.23652° and 0.43408°) for the leading link configuration. The exact values were obtained by analyzing mechanisms with the optimum angles φ_0. The kinematic analysis thus determined the exact pressure angles occurring in the joints of the mechanism and the stroke $S_{\max}$ of the steering rack required for maximum turn of the wheels to the imposed angle—two important design features of the steering linkage.

Figs. 6 and 8 give values of the stroke $S_{\max}$ required to turn the wheels to $\theta_{I-\max}=40°$, the global maximum pressure angle $\beta_{\max}$ occurring at the joints A and B, and on both sides of the triple joint D, for the whole steering range $0°<\theta_I<\theta_{I-\max}$, in the normalized optimum mechanisms. When using the proposed design charts, favorable pressure angles around 60° or less must be sought to ensure a good force transmission efficiency throughout the linkage. Fortuitously, in a real vehicle the self-aligning effect of the wheels due to kingpin inclination, camber and castor angles diminishes the possibility of jamming the steering linkage during operation. It can be seen that the leading link configuration provides lower maximum pressure angles than the trailing one, but both give values greater than 60° for small W_b/W_t ratio vehicles.

When considering the steering rack maximum stroke, larger values of $S_{\max}$ should be sought since this implies a lower actuating force. For the optimum parameter sets used in the design charts, $S_{\max}$ lies between 0.068 and 0.102 for the trailing link configuration, and between 0.061 and 0.101 for the leading link configuration. The stroke of the steering rack can be increased most simply by lengthening the steering knuckle arm. Searching for lower pressure angles by varying the angle α_0 will also result in an increase of the steering rack stroke, but to a lesser extent.

In a real design the merged central joints must necessarily be offset. Simionescu and Smith [7] described a method to obtain the same kinematic function of the mechanism by scaling the link lengths and readjusting the kingpin track. An alternative solution would be to maintain the same length l and angles φ_0 and α_0 but shorten the tie-rods and correspondingly reduce the value of y_D,

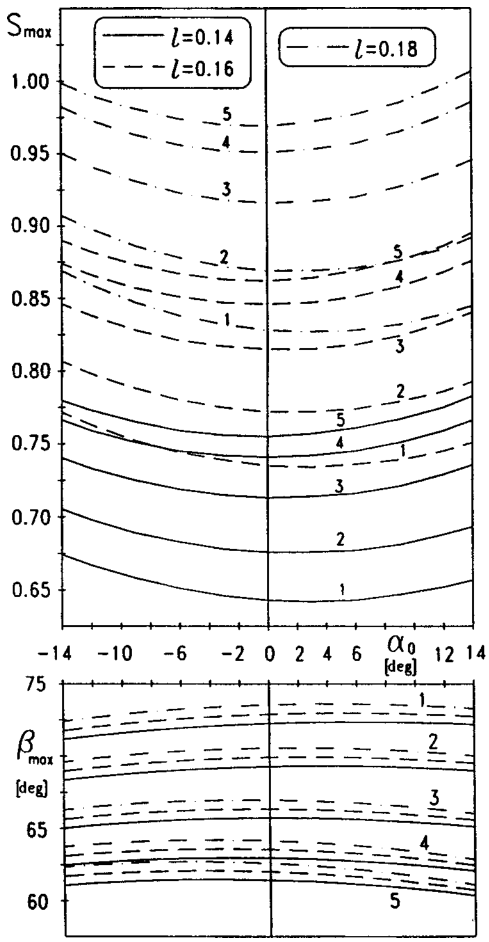

Fig. 6 Maximum stroke and pressure angles for trailing central outrigger actuating element

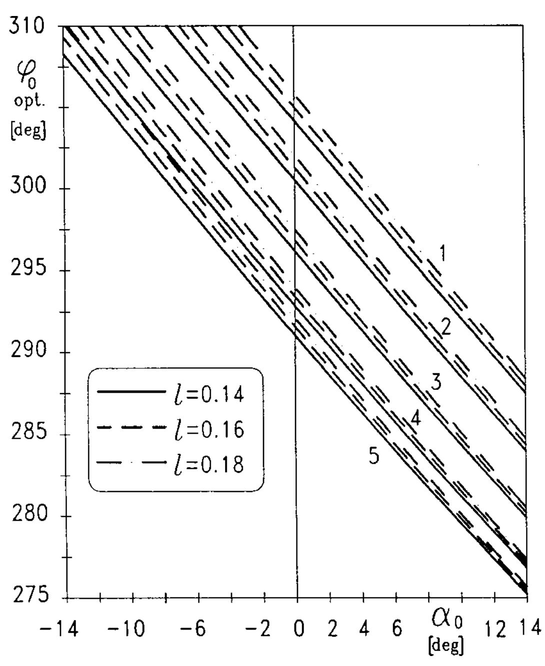

Fig. 7 Leading central outrigger steering linkage design chart (1: W_b/W_t=1.4; 2: W_b/W_t=1.6; 3: W_b/W_t=1.9; 4: W_b/W_t=2.2; 5: W_b/W_t=2.4)

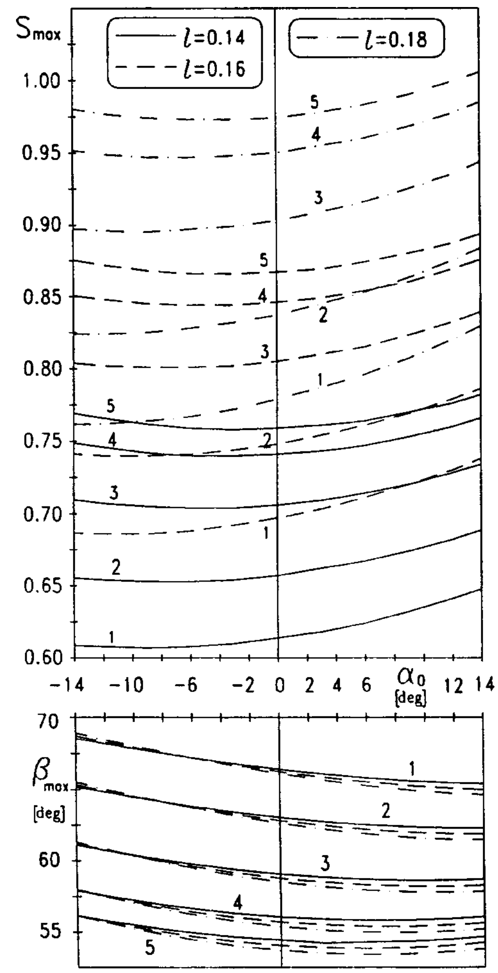

Fig. 8 Maximum stroke and pressure angles for leading central outrigger actuating element

so as to obtain the desired distance between the central joints. The transmission function of the modified mechanism will correspond to a vehicle with lower wheel track (increased W_b/W_t ratio). This will result in a larger positive steering error $\delta\theta_{O-\max}$ occurring for tight turns, which is indeed desirable (see Reimpell and Stoll [1]).

3 Standard Rack-and-Pinion Steering Linkage

This is a more frequently encountered configuration, characterized by widely separated central joints (Fig. 9). The number of geometric parameters defining the transmission function of the mechanism is again three and these are best chosen as the initial

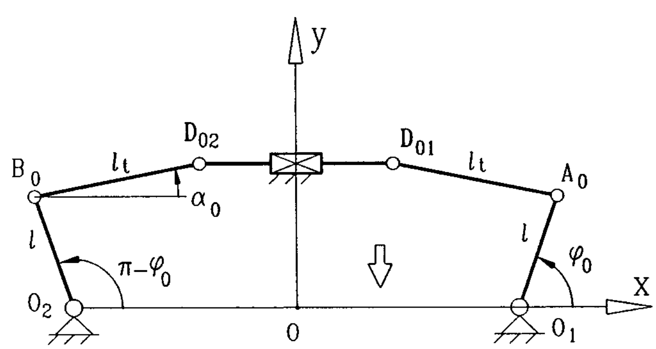

Fig. 9 Planar model of standard rack-and-pinion steering linkages

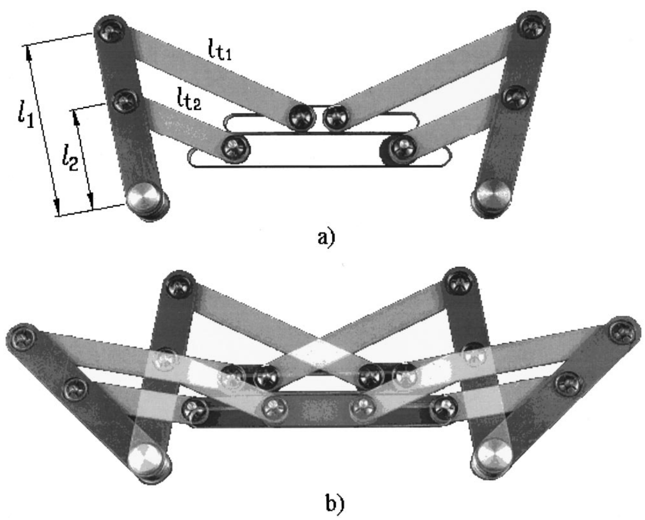

Fig. 10 (*a*) and (*b*) Working models of the symmetrical version of the 3RT3R Watt II overconstrained mechanism, showing the kinematic similarity of the central outrigger and standard rack-and-pinion steering linkages

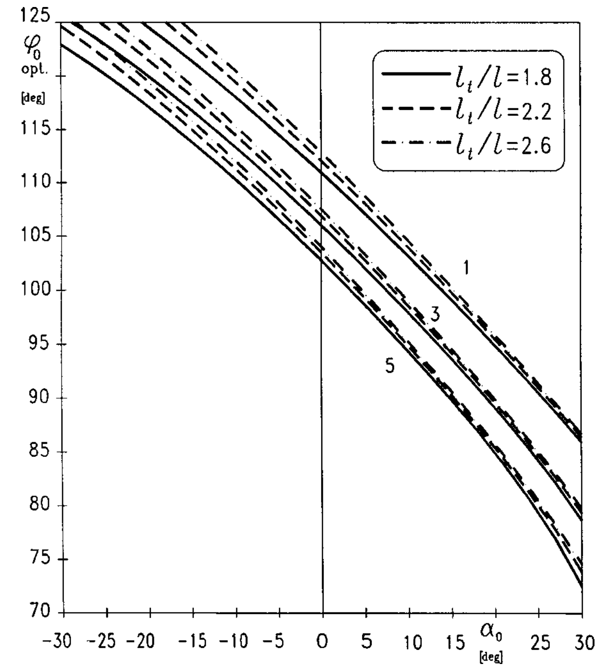

Fig. 11 Trailing rack-and-pinion steering linkage design chart (1: $W_b/W_t=1.4$; 3: $W_b/W_t=1.9$; 5: $W_b/W_t=2.4$)

angles φ_0 and α_0 and the tie rod vs. steering knuckle arm length ratio (l_t/l). The fact that the actual *length* of any of these elements does not have any independent influence upon the kinematic function of the mechanism is suggested by consideration of the overconstrained mechanism shown in Fig. 10. By scaling an initial configuration of the central outrigger type with steering knuckle arm length l_1 and tie-rod length l_{t1} (Fig. 10(*a*)) by a factor less than unity (resulting in lengths l_2 and l_{t2}) and readjusting the rack length in order to obtain the same kingpin track, an infinity of cognates can be found. For the mechanism in Fig. 10(*a*), if the following relation holds:

$$l_{t1}/l_1 = l_{t2}/l_2 \quad \text{and} \quad l_2/l_1 = l_{t2}/l_{t1}, \tag{15}$$

then the tie-rods remain parallel and the mechanism is mobile (Fig. 10(*b*)) despite the fact that Grübler's mobility formula gives zero DOF.

In the straight-ahead reference position, the distance between joints D_{01} and D_{02} (which is the rack length) is given by:

$$D_{01}D_{02} = 1 + 2\cdot(l\cdot\cos\varphi_0 + l_t\cdot\cos\alpha_0) \tag{16}$$

the rack axis position being:

$$y_D = l\cdot\sin\varphi_0 + l_t\cdot\sin\alpha_0. \tag{17}$$

Employing the method of increasing the degree of freedom of the mechanism as above, the objective function will be:

$$F_1(l_t/l, \varphi_0, \alpha_0) = \max|\delta D_{1j}D_{2j} - D_{01}D_{02}| \quad (j=1,\ldots,n) \tag{18}$$

where $\delta D_{1j}D_{2j}$ is calculated as for the objective function f_1 based on Fig. 3.

The improved form of the above function will be:

$$F_2(l_t/l, \varphi_0, \alpha_0) = \max\left|(\delta D_{1j}D_{2j} - D_{01}D_{02})\cdot\frac{\cos\beta_{D2j}}{l\cdot\cos\beta_{Bj}}\right|. \tag{19}$$

By plotting the function F_2 for the same fixed parameters $l_t/l = 2.2$, $W_b/W_t = 1.9$ and $\theta_{I\_\max} = 40°$, a figure closely resembling Fig. 4 was obtained, showing the similar kinematic behavior of the central outrigger and standard rack-and-pinion steering mechanisms. Again two optimum domains were identified, corresponding to a trailing and leading link configuration (φ_0 near 90° and 270° respectively).

Performing an identical procedure of searching for the minimum domains of φ_0 for regular values of α_0, produces the parametric charts in Figs. 11 and 13. For the points used in plotting the diagrams, the steering error varies between 0.49194° and 0.90879° (exact values 0.49229° and 0.90880°) for the trailing link configuration, and between 0.04796° and 0.39314° (exact values 0.04797° and 0.39301°) for the leading link configuration.

Figs. 12 and 14 show the steering rack stroke $S_{\max}$ and maximum pressure angles $\beta_{\max}$ occurring in the joints A and B over the

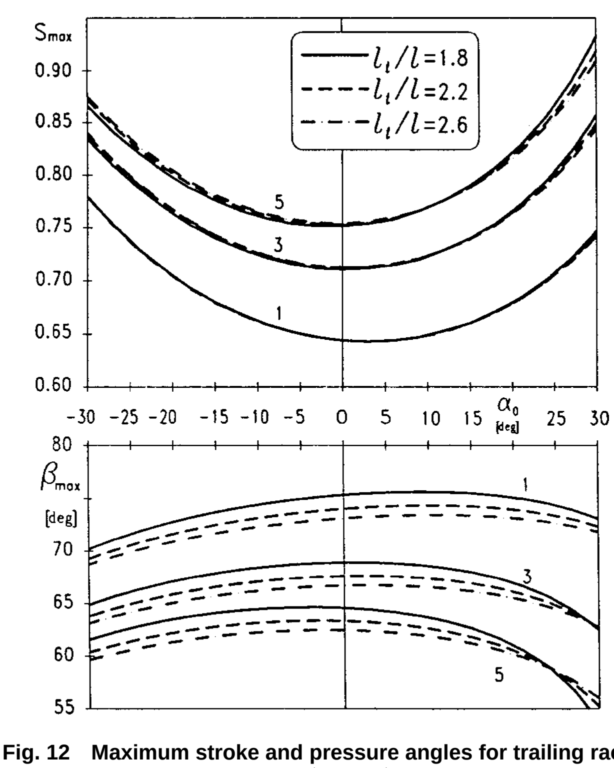

Fig. 12 Maximum stroke and pressure angles for trailing rack-and-pinion actuating element ($l=0.14$)

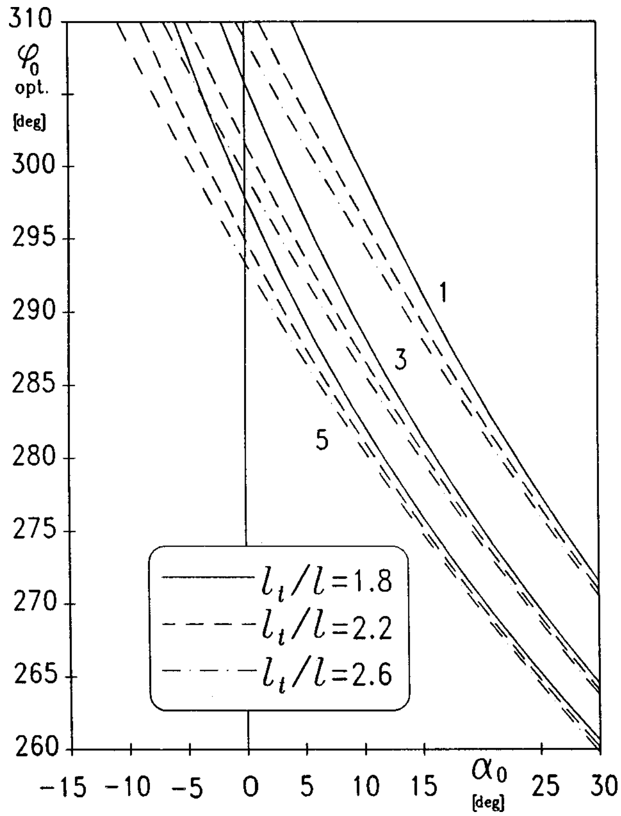

Fig. 13 Leading rack-and-pinion steering linkage design chart (1: W_b/W_t=1.4; 3: W_b/W_t=1.9; 5: W_b/W_t=2.4)

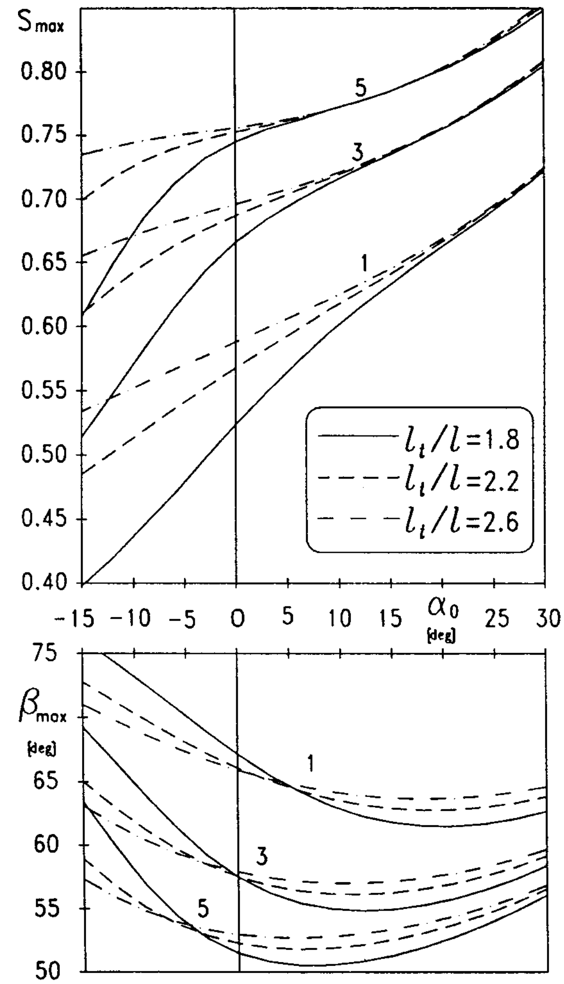

Fig. 14 Maximum stroke and pressure angles for leading rack-and-pinion actuating element (l=0.14)

whole working range ($0° < \theta_I < \theta_{I\_max}$). As in the previous case, similar conclusions may be drawn about the possible increase of steering rack stroke and corresponding reduction of the required control force by lengthening the steering knuckle arm. The rack stroke also depends significantly upon the initial angle α_0. The $S_{max}(\alpha_0)$ curves in Figs. 12 and 14 correspond to a maximum turning angle of the inner wheel $\theta_{I\_max}=40°$ and to a normalized steering knuckle arm length $l=0.14$. For larger values of the latter, l^*, the values taken from the diagrams should, following Eq. (16), be multiplied by the ratio l/l^* to obtain the actual maximum rack stroke S^*_{max}.

The curves for global maximum pressure angle $\beta_{max}(\alpha_0)$ show, as in the previous case, that the leading steering linkage ensures more favorable pressure angles than the trailing link configuration, but generally less than those provided by the central outrigger type.

Inspection of the design chart in Fig. 13 and the corresponding diagram in Fig. 14 leads to the conclusion that a leading rack-and-pinion steering linkage with negative α_0 is not a favorable design because of the widely divergent steering knuckle arms (angles φ_0 greater than 300°), unfavorable pressure angles and short stroke of the actuating member (steering rack or hydraulic piston).

4 Design Strategy Recommendations

When designing the steering mechanism of a particular vehicle, the steering rack stroke S_{max} and the corresponding maximum turning angle of the inner wheel θ_{I-max} will have imposed values. In this case, apart from providing an acceptable steering error and maximum pressure angles, the goal of the design should be to find a mechanism with the rack stroke close to the imposed S_{max} in which case a trial and error search must be performed using the design charts and the accompanying $S_{max}(\alpha_0)$ and $\beta_{max}(\alpha_0)$ diagrams.

As is evident from Figs. 6, 8, 12 and 14, there are many cases, mostly for low W_b/W_t ratios, in which the maximum pressure angles are larger than the usually accepted value $\beta_{max}=60°$, and they will be even larger for $\theta_{I\_max}>40°$. As a consequence, steering accuracy must be sacrificed in the interest of more favorable pressure angles. One solution is to accept an increased positive steering error for tight turns. This is in accordance with the recommendation of Reimpell and Stoll [1] of ensuring an oversteered outer wheel which will permit turning radii smaller than the theoretical minimum, and consequently better maneuverability of the vehicle in narrow spaces. (In the examples provided by these authors, a BMW car steering mechanism assures a maximum steering error $\delta\theta_{O\_max}=+2°$ and a that of a Mercedes a maximum steering error of about $\delta\theta_{O\_max}=+4°$.)

A simple method to obtain larger positive values of $\delta\theta_{O\_max}$ is to consider the vehicle to have a hypothetically increased wheelbase vs. wheel-track ratio, i.e. W_b/W_t multiplied by a coefficient k where $k>1$. Using Eq. (1) and neglecting the steering error of the mechanisms resulting from the proposed design charts, the following relation can be written:

$$\arctan\frac{1}{ctg\,\theta_{I\_max}+1/(W_b/W_t)}+\delta\theta_{O\_max} = \arctan\frac{1}{ctg\,\theta_{I\_max}+1/(k\cdot W_b/W_t)}. \tag{20}$$

This can be solved numerically and an appropriate value of the coefficient k obtained for a given W_b/W_t vehicle and for a chosen positive steering error at the end of the turn ($\delta\theta_{O\_\max}$).

In the case of a vehicle with a maximum turning angle $\theta_{O\_\max}$ larger than 40°, a fictitiously increased W_b/W_t ratio should be considered when choosing the parameters α_0 and φ_0 from the provided design charts. Refinements of the initial set of parameters thus obtained can be made by performing a manual search using mechanism analysis software.

6 Conclusions

Parametric charts have been produced for the design of both the central outrigger and the classical rack-and-pinion steering linkages. The values provided by these diagrams should however be considered as initial estimates in a design process. Whenever possible, the final tuning of the steering linkage dimensions should be carried out in an advanced design environment, employing an accurate model of the spatial mechanism. In this way the influence of the kingpin inclination angles upon the rack stroke and the values of the pressure angles can be properly taken into consideration, together with any possible collisions of the mobile elements with other neighboring parts of the vehicle.

In the case of independent front suspension vehicles, the steering linkage geometry must be correlated with that of the suspension mechanism so as to minimize the cross coupling effect between the steering and suspension. Dynamic simulation and real model testing are also necessary in order to determine the correlated effects of joint elasticity, springs and dampers upon the overall behavior of the car.

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