# Model-based and model-free characterization of epidemic outbreaks — Technical notes on Dehning et al., Science, 2020

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changes.  2. Model-free methods produce delayed estimates that are difficult to interpret  3. R estimates depend on the assumed			After the initial release of our manuscript "Is change points in the spread of COVID-19 reveals to tiveness of interventions" in Science [1], we have	

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ng ec- $^{\mathrm{ed}}$ 60 many constructive comments and interesting questions, 61 and have also faced some recurring misunderstandings. 62 This technical note is intended to answer the most impor-63 tant of these questions, to give additional background for 64 understanding our results, and to also discuss the robust-65 ness and performance of our model in the light of newly 66 available data, in particular data based on on symptom 67 onset times.

The inspiration and comments we received can be 7 broadly categorized into four topics:

> 1. Remarks on apparent discrepancies between the values for the reproductive number R as reported by the Robert Koch Institute (RKI) and the corresponding spreading rate resulting from our published analysis. We will explain below how this apparent discrepancy arises from the comparison of model-free estimates to those from a differentialequation based modeling of disease dynamics. We show how the model-free approach may substantially underestimate the reproductive number R immediately after a sudden drop in R has occurred. From the comments we received it seems that this very important fact related to estimating R is largely unknown, and also counterintutive to most readers. This effect, however, fully explains the apparent discrepancies between the RKI reports and our study. We therefore derive and demonstrate it in detail here.

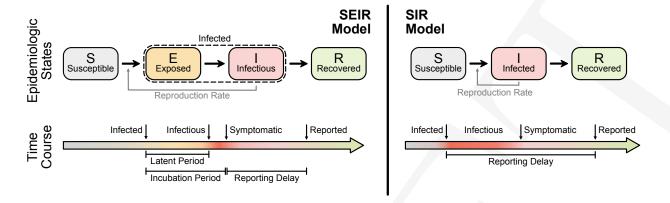


FIG. 1. Illustration of two basic compartmental models in epidemiology. The SEIR model (left) captures the basic steps that every infection passes through: A healthy person becomes infected (leaves S, enters E) but not infectious; after some time the person becomes infectious (leaves E, enters I) but symptoms only show after some incubation period; after some time the person is no longer infectious (leaves I, enters R), which can have several reasons including isolation, conventional recovery, or death. During a outbreak that authorities try to control, the dominant transition from I to R should be by isolation after some reporting delay. The SIR model (right) is the most basic compartmental model and does not distinguish between infectious and infected: A healthy person becomes infected (leaves S, enters I) but only shows symptoms with some delay; after some time the person "recovers" (leaves I, enters R), which again includes isolation, recovery, or death.

2. Questions revolving around the philosophy and inter- 121 based estimation of epidemiological parameters. Next, we tion. As we will explain below, our approach selects 127 question of testing. the most plausible of multiple causal explanations of the observed data, but does not establish strict interventional causality.

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- 3. New data have been released in the time since our analyses were completed. Most prominently, data 128 on the exact times of symptom onsets (epi curve) 129 are now available and supersede the case report data as the best data source for modeling the outbreak. As we will show below, our conclusions remain un- 130 changed when updating our model to the new data.
- 4. Questions on how changes in testing capacity may 131 have influenced our results. Given the data that have become available on the weekly (daily) number of performed tests, test capacity, and on delays between symptom onset, test and case report, we reanalyze in great detail the disease and testing dynamics, especially with respect to the timing of the peak in new symptom onsets. We conclude that all symptom onsets that are relevant for the main conclusions of our previous publication have been tested at a time when testing had sufficient capacity and was sufficiently constant.

117 fundamental difference between model-free and model- 147 are:

pretation of our modeling approach that combines a 122 will discuss philosophy and interpretation of model-based differential equation model of the disease outbreak, 123 estimation in the Bayesian framework and the causality Bayesian parameter inference and Bayesian model 124 question. We then show how our original analyses can be comparison. Most frequently we were asked if and 125 evolved to incorporate new data, in particular on sympin what sense our results have a causal interpreta- 126 tom onset (epi curve). Last we turn to the important

# ESTIMATING THE REPRODUCTIVE NUMBER

# Basic SIR dynamics

Before we define the reproductive number R, we briefly 132 recapitulate the basic SIR dynamics we consider (Fig. 1). 133 In principle, the course on an infection can be described 134 as follows: A susceptible person (not infected and not 135 immune) becomes infected but is initially not infectious; 136 after some time the person starts to be infectious but 137 symptoms only show after the incubation period; eventu-138 ally, the person is no longer infectious because it is either isolated, it recovers, or it dies. The idea of compartmen-140 tal models is to group the population into compartments,  $_{141}$  in the most simple but established SIR model these are 142 susceptible (S), infected (I), and recovered (R). Assuming 143 a well-mixed population (a mean-field approximation of We will in the following address the issues revolving 144 everybody interacting with everybody), one can formulate around the reproductive number R first, also introduc- 145 differential equations that describe the time development 119 ing the basic terminology of disease spreading and the 146 of these compartments, in the case of SIR model these

$$\frac{dS}{dt} = -\lambda \frac{SI}{N} \tag{1}$$

$$\frac{dI}{dt} = \lambda \frac{SI}{N} - \mu I \tag{2}$$

$$\frac{dR}{dt} = \mu I \tag{3}$$

$$\frac{dI}{dt} = \lambda \frac{SI}{N} - \mu I \tag{2}$$

$$\frac{dR}{dt} = \mu I \tag{3}$$

This assumes a spreading rate  $\lambda$  for infected people to infect susceptible people, which they meet randomly, and 150 a recovery rate  $\mu$  for infected people to recover. These 151 differential equations can be extended to include various different compartments, in order to better resolve the temporal course of the disease, but typically keep the mean-field assumption of a well-mixed population unless evaluated on some (typically unknown) network. In this case, additional compartments reflect spatial information.

Observed case numbers are always delayed from the true infection date (Fig. 2). When a person becomes infected, the onset of symptoms is usually delayed by the incubation period. Upon symptom onset, it typically takes a few days until the person undergoes a test and the case is reported. However, for the modeling, we are usually interested in the actual time when a person becomes infected, but, because this information is not directly available, we have to work with reported case numbers. In the best case scenario, we can use reported case numbers to reconstruct symptomatic case numbers 168 (e.g. by nowcasting), which are still delayed from the true 169 infection. We link the observed cases — symptomatic or 170 reported — to the model dynamics by considering them as a convolution of infected cases with a distribution of incubation periods or reporting delays, respectively.

#### Model-free estimation of reproduction number 173 174

# Definition of R.

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The reproductive number R quantifies how many susceptible persons S are on average infected by one infected person. Estimating the reproductive number R in principle can be done in two manners, either by inferring it from observed case numbers, or by following infection chains step by step. If one infers it from observed case numbers, there are a number of possible approaches. Some <sup>183</sup> approaches are summarized in Fig. 4 and detailed below. All these approaches are applied to the observed case 185 numbers (day of symptom onset, i.e., epi curve, or day 186 of reporting). In the following we assume that they are applied to the epi curve.

number assumes a reproductive process with offspring 197 the new reported cases. generation, such as a branching process [2]. For this, one 198  $_{192}$  can generate offspring infections. In the simplest case,  $_{200}$  consider  $R_t$  to characterize the number of future infections 193 one could consider that offspring infections occur exactly 201 that are caused by infections at time t (left-edge conven-194 after one generation time q to define

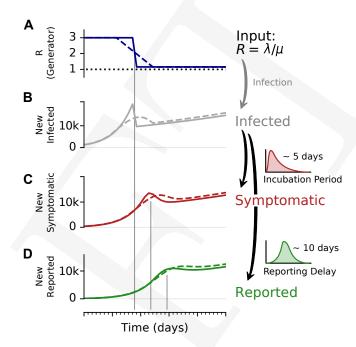


FIG. 2. Observed case numbers are delayed from the true infection date. To illustrate the effect of a change point, we consider an SIR model to generate synthetic case numbers. A The case numbers derive from a time-dependent reproductive number R that is provided to the generator. It exhibits a change point from R=3 to R=1.15, with a duration of either 1 day (solid) or 9 days (dashed). B The number of new infections can show an intermediate decrease in response to a change point in R, even though the underlying dynamics are still in the exponentially growing regime of R > 1. C The number of new symptomatic cases is here illustrated by the convolution of new infected with a log-normal incubation period and shows the same transient decrease. **D** The number of new reported cases is here illustrated by another convolution of new infected with a (longer) log-normal reporting delay and shows the same transient decrease upon close inspection. Note that the delays in C and D shift and smooth the counterintuitive effects of transient decreases due to the non-linear changes in the SIR dynamics.

$$\hat{R}_t = \frac{\text{number of newly infected at time } t + g}{\text{number of newly infected at time } t}$$
 (4)

$$=\frac{C_{t+g}}{C_t}. (5)$$

195 In reality, these newly infected case numbers  $C_t$  have to The most straight-forward definition of the reproductive 196 be approximated, e.g., by new symptomatic cases or by

Note that there are two different conventions for the assumes a generation time q in which an infectious person 199 timing of the reproductive number (Fig. 3). Above, we 202 tion). Alternatively, one can consider  $R_t$  to characterize

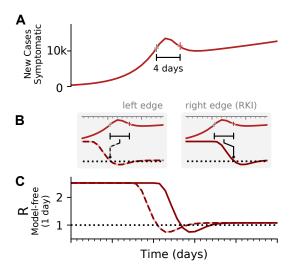


FIG. 3. Different conventions to define the reproductive number: Infections in the future or infections from the past. A Synthetic data for new symptomatic cases. The marked interval indicates an assumed generation time of 4 days. B The basic reproductive number can be defined either on the left edge of the generation interval (left, dashed line), describing the average number of future infections that are cause by today, or on the right edge of the interval (right, solid line), describing the average number of infections today that were caused by the past. C Depending on the convention, the resulting curve of R is shifted by the generation time. Notice that in both cases the transient decrease in A due to a non-linear reproductive rate (but with R > 1 all the time) is inferred as a transient period of sub exponential growth (R < 1). See Fig. 4 for a more details.

the number of infections at time t that were caused by the past pool of infected (right-edge convention), defined

$$\hat{R}_t = \frac{\text{number of newly infected at time } t}{\text{number of newly infected at time } t - g} = \frac{C_t}{C_{t-g}}$$
(6

The results for R are exactly equivalent, apart from a 207 shift in time by q.

R as calculated by the RKI. Real-world data are often noisy, and therefore averaging over a certain time window can help to smooth the estimate. This procedure is used in two variants by the RKI, smoothing over four days or over seven days[3]. In both cases, they assume a constant serial interval (generation time) of g=4 days (Fig. 4). The four-day smoothing has the advantage that it reacts a bit faster, the seven-day smoothing has the advantage that the tate of testing numbers.

The general equation then reads as follows:

$$R_{t} = \frac{\sum_{j=t-w}^{t} C_{j}}{\sum_{k=t-g-w}^{t-g} C_{k}}$$

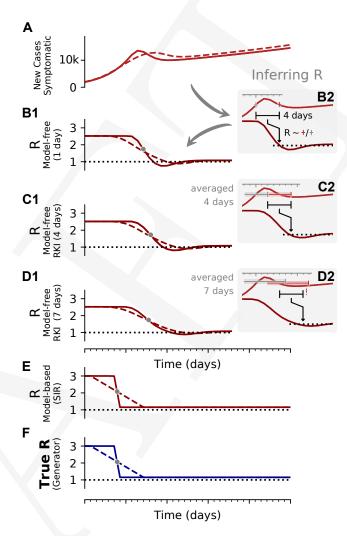
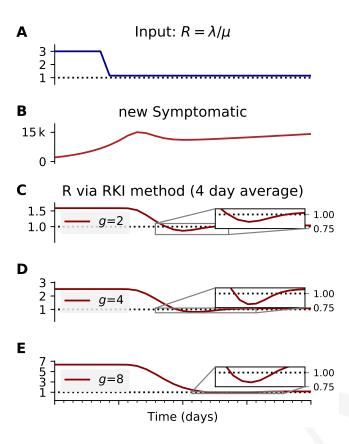


FIG. 4. Inferred reproductive number depends on the inference method. A Synthetic data for new symptomatic cases generated with SIR dynamics from an underlying time-dependent R > 1 (F) with one change point of duration 1 day (solid) or 9 days (dashed). B Model-free inference of R (B1) based on a single day in the past and the presence using the right-edge convention (**B2**, cf. Fig. 3). C Model-free inference of R (C1) following the Robert Koch Institut convention averaging over 4 days with right-edge convention (C2). D Same as C but averaging over 7 days. Notice that the model-free methods (B-C) can suffer from transient R < 1 periods that are not present in the true R (**F**). **E** Inferred R using change point detection with underlying dynamic model (SIR) does not suffer from transient erroneous R < 1 periods. If the underlying dynamic model corresponds to the true dynamics, then this approach reproduces the true  $R(\mathbf{F})$  that was used to generate the data  $(\mathbf{A})$ .

 $^{219}$  w is the window. The Robert Koch Institute chooses a  $^{220}$  window of 4 or 7 days.



Inferred reproductive number depends on the assumed generation duration. We generate synthetic data using SIR dynamics with time-dependent R including a 1-day change point (A, cf. Fig. 4) that yields new symptomatic cases with transient decrease (B) despite all R > 1. Using the RKI convention to infer R (4 day average, right-edge convention), we demonstrate how generation times (g) result in different Rcurves (C-E). In particular, we find different initial levels of R(left plateau), differently long crossover duration (time from left plateau to right plateau), and differently deep transients of R < 1 (see insets).

# Model-free methods versus model-based methods to infer reproductive number.

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In order to demonstrate potential issues when inferring  $_{224}$  the reproductive number R, we systematically compare model-free methods and model-based methods on synthetic data from an SIR model (Fig. 2). With model-free methods, we refer to inference methods for R, which do 228 no explicitly incorporate disease dynamics (SIR). The 229 three methods we presented above belong to this group. These methods to estimate R are straight forward and 231 easy to implement. However, they might lead to biased 234  $_{232}$  estimates when R is changing rapidly.

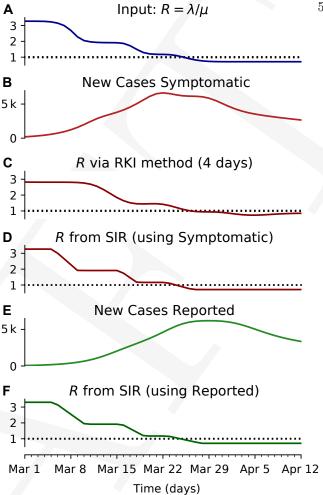


FIG. 6. The change-point detection methodology yields consistent results irrespective of whether it is applied to the new reported cases or the new symptomatic cases (e.g. obtained by nowcasting). A Time-dependent reproductive number as inferred from case numbers in Germany [1]. B Synthetic data for new symptomatic cases generated with SIR dynamics from the underlying time-dependent R (see A) C Inferred R from new symptomatic cases using RKI method (4 days generation time, right-edge convention) would reproduce step-like behavior (no noise present) but prematurely drops below R=1(dashed line).  $\mathbf{D}$  Inferred R from new symptomatic cases using change-point detection with dynamic model (SIR) correctly reproduces the input (A). E Synthetic data for new reported cases generate with SIR dynamics as in B (cf. Fig. 2). F Inferred R from new reported cases (**E**) using change-point detection with dynamic model (SIR) also correctly reproduces the input (A). Note that both **D** and **F** show sharper steps because of the assumed piece-wise linear change points in the model, and that they perform so well because they employ the true dynamic model that is used for the synthetic data. Both are model assumptions that need to be justified in our approach.

### Model-free methods may smooth out fast changes.

In Fig. 4, the R that is inferred by model-free methods 235 undergoes a smoother change than was truly generated.

236 The smoothing has two reasons: First, when using the 291 (Fig. 4E). sliding-window of four or seven days (RKI methods),  $_{238}$  multiple days are combined to obtain an R value for one day. Second, R has to be calculated from the daily new symptomatic or reported cases (Fig. 2 C, D) because the dates of infection (Fig. 2 B) are not directly accessible 242 in real-world data. As discussed before, symptom onset to-case, these two curves are smoothed out compared to the infection curve (In other words, the smoothing originates from the variance in incubation period and reporting delay, see later Fig. 10 in the section about testing). Hence, if smoothing is not explicitly incorporated in the inference of R, fast changes appear slower than they truly are, and successive fast changes may appear as a long transient.

# Model-free methods produce delayed estimates that are difficult to interpret

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In our example in Fig. 4, we estimated R based on the 255 number of new symptomatic cases from synthetic data. The estimated R of all three model-free methods is shifted  $^{309}$ in time compared to the true R (Fig. 4F).

How does one interpret the shift and where does it come from? To interpret the shift and compare between 311 averaging further causes a non-trivial shifting of the start- 325 line). 275 and end-points of the change. In combination, multiple 326

280 a post-hoc shift of the R-curve cannot reproduce the true 331 true exponential decay (R < 1). The model-free meth-288 account for this and other potential biases. When us-339 one infers R in a model-free manner, by computing ratios 289 ing a good model, such a model-based approach returns 340 of case numbers, then the local minimum leads to an <sub>290</sub> the correct R with the correct steepness and time point <sub>341</sub> erroneous estimate of R < 1 (Fig. 4B,C.D).

292 3. R estimates depend on the assumed generation duration.

In Fig. 5 we illustrate the impact of the assumed generaand reporting date are delayed from the infection date. 294 tion duration. Focusing on the model-free method used by Because the delays vary from patient-to-patient and case-  $^{295}$  the RKI (4 day average), we vary the generation duration  $_{296}$  that is assumed for the calculation of R. (Note that the <sup>297</sup> same dependence persists in model-based inference.) Dur-298 ing the exponential phase, the inferred R increases with 299 the assumed generation duration. At the onset,  $R \simeq 1.6$ for q=2, and  $R\simeq 6.5$  for q=8. The larger R-value when assuming longer generation-durations is caused by the 302 larger increase of cases in-between successive generations. One should however point out that this dependence is 304 less pronounced near a reproductive number of 1. Due 305 to parameter-dependencies, such as here demonstrated 306 for the generation duration, the comparability of the re-307 productive number derived from different models and 308 inference methods is limited.

# Model-free methods may return erroneous transient periods of R < 1 at change points.

In our examples (Figs. 4 and 5), we consider that Rthe different methods, we focus on the time point where  $_{312}$  changes rapidly from  $R_0 = 3$  to  $R_1 = 1.15$  within one half of the steep step in R has been detected (gray dots). 313 day. Such a sudden change leads to a transient decrease This shift has multiple sources, which are not trivial to  $_{314}$  in new case numbers — despite R > 1 always. How disentangle. One source for the shift is the incubation 315 can there be decrease in new cases although R > 1? period (in our example 5 days), which explains the 4-5 316 The transient decrease results from the pool of infected day shift of the one-day method (Fig. 4B). Because the 317 suddenly infecting considerably less people. This decrease incubation period is not constant and typically comes from 318 in infections causes the sharp peak and a sudden drop in a non-Gaussian distribution, there is an additional shift 319 new infections (Fig. 2B, solid line). It then carries over to towards either direction, depending on the shape of the 320 the number of new symptomatic and new reported cases, actual distribution. Another source for the shift comes 321 with the respective delay and smoothing (Fig. 2C,D]). from the time average, which explains the additional 322 This transient decrease depends on the duration of the (approximate) 1-day shift in the four-day and seven-day 323 change point: While it is strongest for steep changes, it methods employed by the RKI (Fig. 4C,D). This time 324 also occurs for a nine-day change point (Fig. 2, dashed

Naively, a transient decrease might be interpreted as sources cause shifts that can point into opposite directions.  $_{327}$  a transient R < 1, but that is not the case here. A While the sources can be identified conceptually, the 328 model-free method cannot distinguish between different combined effect cannot be disentangled or compensated. 329 causes for transient decreases in case numbers, being it Due to multiple sources of shifts and smoothing, even 330 due to transient non-linear effects (Fig. 2) or due to a change point. For example, a shift of Fig. 4D by 5 days 332 ods in our example (Figs. 4 and 5) correspondingly yield would suggest a start of the change point before it starts in 333 non-negligible periods of R < 1, even though the underreality (Fig. 4F). This fact has led to multiple prominent  $_{334}$  lying model dynamics have R > 1 always. Model-based misunderstandings in relation to the RKI data and the 335 approaches, on the other hand, can account for transient effects of governmental interventions. Instead of shifting 336 non-linear effects if included in the model, e.g., as change curves to partially correct for one or another potential 337 points, and — if the model is correct — even reproduce delay, an inference of R using model-based methods can 338 the true underlying dynamics (Fig. 4E). To conclude, if

## Well chosen model-based methods can reconstruct complex disease dynamics

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When the chosen model describes the true disease dv-344 namics well, robust inference of the true underlying reproduction number (and other parameters) is possible. To demonstrate the robustness of model-based inference, we generate synthetic data using an SIR-model as inferred from case numbers in Germany between March 2 and April 21 [1] (Fig. 6). The Bayesian model inference can recover the reproductive rate (Fig. 6D,F), whereas with the model-free method, the recovered R is slightly biased (Fig. 6C). Note, however, that the model has to match at least approximately the disease dynamics, to allow a good inference. This is why we used different models to assess the robustness of our results in Ref. [1] (SIR: Fig. 3, 358 Fig. S4).

# WHAT CONCLUSIONS CAN ONE DRAW FROM A BAYESIAN ANALYSIS?

# Modeling background

When the Coronavirus-pandemic arrived in Germany we set out to model the spread of the disease as rapidly as possible. Thus, our model from the start was aimed at giving estimates with their corresponding error bounds based on the data available at that time. To this end we decided to use a Bayesian strategy as it allowed formulating welldocumented assumptions on those aspects not available from data at that time. Within the Bayesian framework these assumptions can and should be replaced by data as soon as these become available, and we implement such an improvement below for the case of information on symptom onset times that have become available in the meantime. Given such new data it will also be interesting to evaluate post-hoc the assumptions and the performance of our model. This will also give some guidance as to whether to employ a model of this kind again in a new scenario (another disease outbreak or pandemic) where some relevant data will also not be available immediately. We note that taking these steps is the intended development in Bayesian inference.

We also note that all statistical procedures come with their own assumptions, e.g. on distribution of the data, models of measurements and random errors. Bayesian analysis is no exception to this rule; in our view the only difference is that modeling assumptions are not taken for granted based on the long-established used of a method (say, a t-test) but need to be formulated anew for each case. The fact that the assumptions are hand-tailored to the application case may seem subjective sometimes; yet, similar assumptions are being made, more tacitly perhaps, in other frameworks, as well. This said, it is neverthe- 441 i.e. taking such a decision entails accounting for a-priori

395 the modeling assumptions. As far as space restrictions allowed we have discussed our assumptions already in the main manuscript [1], but we here give a much deeper and broader and more educational treatment.

# Bayesian inference as reasoning under uncertainty, bound to be updated

The results of a Bayesian analysis at some publication  $_{402}$  time point T represent what we should believe in at that 403 time point T, given the knowledge available at T (causes and data known at T). These results represent something that we should be able to agree on given the knowledge at  $_{406}$  T (and some practical constraints, see below), but these 407 results may change given more information at a later time 357 SEIR-like: Fig. S3, SIR without weekend modulation: 408  $T + \Delta_T$ . Changing ones mind with the availability of 409 additional information is designed into Bayesian inference 410 as "the logic of science" (E.T. Jaynes) from the start. 411 In other words, scientific inference and the associated 412 models are bound to be updated - just like the relativity 413 theory and quantum theory in physics overrode their former model counterparts. The important question is thus not whether a model is correct in absolute terms, but whether it was possible to agree on the model (and the inference provided by it) at time T, and also if the inference provided at T was robust, for example in the sense that the credible intervals for the model parameters 420 at T comprise those obtained at  $T + \Delta_T$ .

From this perspective it is obvious that now, more than 422 a month after finalization of our published analyses on 423 April 21st, new data have become available and that the 424 model can, and should, be improved accordingly. Impor-425 tant data in this respect are the reliable data on putative 426 infection dates which at present take about 7 days to 427 come in for at last 80% of the cases (Fig. 10), and which 428 where only published more recently than our internal analysis cut-off. We present results obtained using these 430 data below and compare them to our published results.

# Conditions for plausible alternative models entering model comparison

A frequent, and important misunderstanding around 434 Bayesian model comparison is that one is allowed to formulate very many models at random and then let the data decide on the best model via the Bayesian model 437 evidence (or the LOO-scores). This notion fails to notice 438 that the model evidence  $p(D|M_i)$  is only one part of the 439 decision on the preferred model. The formal equation for 440 deciding between models i and j would be:

$$\frac{p(M_i|D)}{p(M_j|D)} = \frac{p(D|M_i)}{p(D|M_j)} \frac{p(M_i)}{p(M_j)} , \qquad (7)$$

less important to question and discuss (our) modeling 442 plausibility of different models, i.e.  $p(M_i)$  and  $p(M_i)$ . 394 assumptions and to test the sensitivity of our results to 443 While it is customary to assign equal a-priori plausibility 444 to all the models being considered, this does not mean that just any model qualifies for use in this decision procedure. Rather, each model subjected to a model comparison needs to be well justified. This is one of the reasons why we did not consider for example models of sustained, constant drifts in the effective spreading rate  $\lambda^*$  (or, equivalently the reproductive number R), as we did not come up with plausible explanations for such a behaviour (except perhaps arguments based on herd-immunity, which seem implausible now, in the light of second waves of infections and a recent rise in  $\lambda^*$  from its all-time low, and also in the light of country to country comparisons, Fig. 7).

On a practical note, useful modeling also has to reflect certain limits on model complexity in relation to the available data, and also computational resources. Known phenomena, that can nevertheless not be modeled must therefore often be integrated into noise terms that are designed accordingly (as was done with the modeling of observation noise in our case, instead of using full 463 stochastic differential equations). The best that can be done then is to investigate the sensitivity of results with respect to the simplifying assumptions that have been made. 466

It is also in order to explain in simple terms how results 467 statements about the world, given our present knowledge. 501 most plausible model. inconsistent with the information we have. In this sense, 506 or not. these credible intervals may form the basis for decisions we have to take.

### Models as competing causal explanations of data

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Last, we note that the notion of causality resides only in the construction of the models – with different models incorporating different possible causal explanations (e.g. in the form of differential equations for the disease dynamics) of the data. Performing model comparisons then selects more plausible over less plausible explanations, but does not provide a proof of causality in the strict sense advocated for example by Judea Pearl [4] or by Ay and Polani [5]. Yet, fulfilling the formal criteria for causality in this strict sense would need multiple replications of 520 497 of our Bayesian analysis must be seen as a search for 528 on due to a lack of data, and often cannot inform deci-

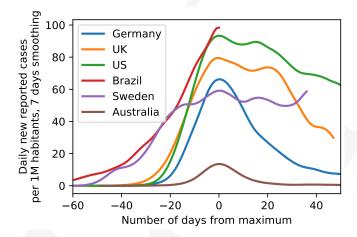


FIG. 7. Comparison of the case numbers per one million inhabitants of exemplary countries as illustration of the range of possible case numbers developments. Note how both the peak height as well as peak width of some countries are considerably larger than for Germany, providing evidence against saturation effects ('herd immunity') in Germany (Data until June 3, 2020).

of a Bayesian analysis may be interpreted: In the Bayesian 499 data available at the time of analysis, and as providing framework probabilities are measures of the plausibility of 500 credible ranges of the parameter values relative to this

Thus, the results of Bayesian parameter inference for 502 Later, discussions (such as the one presented here) of example indicate credible (plausible) ranges in which we 503 the selected models and the inferred parameter ranges should assume the unknown parameters to be. Assuming 504 should then investigate and update modeling assumptions, them to be elsewhere with high probability would be 505 and reason whether the causal model can be maintained,

> When analyzing improved data that reflect the dates 508 of symptom onset rather than case reports to improve 509 our modeling we find that both the preference for a three 510 change point model as well as the inferred parameter 511 ranges do not change drastically, and we maintain our 512 original interpretation of the pandemic process and the effectiveness of governmental interventions.

> Last, alternative models assuming herd immunity as a 515 reason for the sustained observed drop in infection rates 516 still do not seem plausible to us in the light of rapidly 517 surging second waves or sustained high levels of new 518 infections (such as in Sweden, see Figure 7).

# MODEL EVOLUTION

Modeling efforts at the beginning of an epidemic outthe pandemic process, each time with different settings of 521 break are aimed at providing a rough but timely and the relevant variables, such as interventions. Even when 522 robust description of the disease outbreak, making use treating the SARS-CoV-2 outbreaks in different countries 523 of whichever data are available at that time. Later modaround the world, with their different interventions (or 524 eling efforts in contrast make use of more detailed data lack thereof), as replications establishiung formal casu- 525 and provide deeper insights into how the outbreak unality may remain an elusive goal due to multiple other 526 folded. While these latter models are useful for a better variations from country to country. In sum, the results 527 understanding after the fact, they cannot be applied early 498 the most plausible causal model of the data, given the 529 sions fast enough. However, a comparison of early and

550 later models can provide important insights about the 585 and three change points, based both on SIR and SEIR to the later ones (here usefulness means that the early 587 are shown). models describe the epidemiological parameters and their 588 symptom onset and testing/reporting is still considerable 599 than the curve reflecting reporting date (see Fig. 9C). (see Fig. 13). In particular, this means that reliable epi curve data for April  $21^{st}$ , our analysis cut-off date in [1], were not available until much later. Now that these data 548 are available, however, we can compare models based on 549 data organized by reporting date, modeling the reporting 550 delay and incubation period, and models based on the <sup>551</sup> epi curve, modeling the incubation period only.

#### Model updates based on time of symptom onset and comparison to previous results based on time of 553 reporting 554

Ideally modeling of an epidemic outbreak should rely on data organized by infection date - yet, such data are rarely available outside of the analysis of individual, wellconfined infection chains. The next best option then are Naturally, symptom onset precedes the test and report in time. Thus, the epi curve is only available after a certain delay, which can be substantial. Furthermore, the time of symptom onset may remain unknown for a significant fraction of reported cases. If so, then reconstructing the epi curve requires data imputation and further modeling 604 (e.g. nowcasting [6, 7]), which may further delay the availability of this curve. At the initial stages of an outbreak one may therefore decide to analyze data organized by 606 delay distribution.

584 available (see Figs. 17 and 19), using models with one, two 622 16). The differences are minor.

robustness and usefulness of the early models with respect 586 dynamics (only figures for the three change points models

These new results do not change our main conclusions uncertainties well enough to inform decisions). For the 589 presented in [1]. Specifically, model comparison still favors case of the COVD-19 outbreak in Germany, the initially  $_{590}$  the three change point models over their simpler counteravailable data were sorted based on date of reporting, 591 parts (see table I), and only the third change point leads where the reporting occurred after an unknown delay  $_{592}$  to a value of the spreading rate  $\lambda^*$  that is clearly below between symptom onset and report. Only later, data 593 zero. At the quantitative level, however, we see some organized by time of symptom onset, the so-called epi  $_{594}$  evidence for a larger drop introduced by the first change curve, became available. Even after their initial release, 595 point when using the epi curve data, and smaller drops these data were still updated and refined (see Fig. 8); 596 induced by the second and third change point, especially also note that data for symptom onsets still take some 597 when using an SEIR model (see Fig. 19). These quantitatime to arrive and be compiled, i.e. the delay between 598 tive changes are driven by the epi curve dropping faster

TABLE I. Model comparison: Using leave-one-out (LOO) cross-validation, we compare the SIR and SEIR model variants using the epi curve as data (Figs. 17 and 19). Lower LOO-scores represent a better match between model and data (pLOO is the effective number of parameters).

Model	# c-pts.	LOO-score	pLOO
SIR main	0	$900 \pm 13$	6.36
SIR main	1	$774 \pm 14$	12.72
SIR main	2	$755 \pm 13$	12.17
SIR main	3	$725 \pm 15$	19.66
SEIR-like	0	$900 \pm 14$	6.65
SEIR-like	1	$749 \pm 12$	8.05
SEIR-like	2	$739 \pm 13$	10.28
$\operatorname{SEIR}$ -like	3	$726 \pm 14$	14.04

In sum, we conclude that the original model based on data organized by date of symptom onset - the epi curve. 601 data organized by reporting date was useful to understand 602 disease dynamics in the absence of the epi curve and 603 robust in the sense that its main results still hold.

#### $\mathbf{B}.$ Differences between results based on RKI versus JHU data sources

At the beginning of the out break data were made availreporting data. For a comparison of analyses it is impor- 607 able on a daily basis both by John Hopkins University tant to understand how the curve of reporting dates and 608 (JHU) and the German Robert Koch Institute (RKI). the epi curve are linked. Both curves originate from the  $_{609}$  Both sources initially provided only reported cases, with curve of initial infections by a convolution (see Fig. 2). 610 the JHU resources providing data faster and with a bet-The epi curve is the curve of initial infections convolved 611 ter interface for automated analyses. The RKI resources by the distribution of incubation periods, while the curve 612 were updated only a few days later, as information albased on reporting date is the curve of true infections 613 ways has to be transmitted from regional agencies to the convolved by the (less well known) distribution of delays 614 RKI, whereas the JHU data for Germany are gathered between infection data and reporting date. Technically, a 615 from a few reputed online media (Berliner Morgenpost, report can happen before symptom onset, albeit this may 616 Taggesspiegel and Zeit Online [8]). However the JHU be rare. Therefore, the curve of reporting dates is not 617 resources have been partially criticised for lacking quality exactly a convolution of the epi curve with an additional 618 control (see issues section on the Github page [9]). We therefore compared the JHU data used in [1] to the official We have reanalyzed the initial stages of the outbreak 620 RKI count (Fig. 14) and have rerun the analysis using until April 21<sup>st</sup> based on the epi curve that has become 621 the RKI reported cases (the "Meldedatum", Fig. 15 and

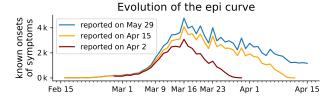


FIG. 8. The numbers of known onsets of symptoms per day as reported at different dates in the past. As testing confirms onset of symptoms in the past with varying delay, the epi curve not only grows at its tail, but over a wide time period with each new publication. Known onsets are reproduced from the RKI's daily situation reports and the publicly available RKI-database. Unknown onsets of symptoms, which account for 40% of total 670 curves from the RKI (imputation and Nowcasting), as reported on a past date, are not publicly available for the month of April, hence the focus on the numbers of known onsets here.

#### IMPACT OF TESTING

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introduces additional sources of uncertainty. While we 684 unchanged.

the actual disease dynamics.

643 first wave, i.e. strong exponential growth in new cases, 698 of week 12 (Fig. 9 B). A more in depth analysis of Fig. 9 the available data, even if changes in testing are taken 701 And when did it start to decline? into account.

650 on a given day or in a given week and ii) the fraction of 705 cases reported, and an accompanying day-to-day decrease translates to a confirmed case.

one of these quantities changes, whereas the other one 709 measure the underlying epidemiological dynamics in this remains constant. In the case that a constant number of 710 period. tests is performed day-over-day and we observe a growing 711 fraction of positive test results, this corresponds to an 712 cations that during the epidemic outbreak, a growth in increase of the underlying case numbers. Conversely, if 713 case numbers was indeed present, as well as a decline. 659 the number of tests is increased and we find a constant 714

660 fraction of positive tests, this implies the same, an increase of underlying cases. The second case only holds with additional assumptions: i) the fraction of positive tests is larger than the prevalence and ii) tests are not performed randomly, both of which were met in Germany.

Fig. 9 A,B shows that in Germany in early March both, 666 the number of tests as well as the fraction of positives 667 increased simultaneously. This simultaneous increase in-668 dicates a significant growth in new case numbers.

## Strong growth until week 12

Focusing on testing in weeks 10 & 11 in Fig. 9 A and B, number of cases, are not considered. The estimated total epi 671 we can clearly deduce a strong growth in daily new cases, 672 as both the fraction of positives as well as the number 673 of performed tests rise, matching the combination of the 674 two scenarios described earlier. The rise in the fraction 675 of positive tests is apparent in the daily values, especially 676 as the daily number of tests can be taken as constant 677 throughout the week, see Fig. 8 in [10]. For weeks 14 678 onward, the number of performed tests stays constant Our modeling depends on reported case numbers, which 679 and thus, the fraction of positive tests correlates with in turn depend on testing. Throughout the COVID-19 680 the number of reported cases, exhibiting a decline in spread, test availability, test requirements and known case 681 underlying case numbers. A similar direct comparison numbers changed continuously over time, see Fig. 8. Such 682 for weeks 10-12 is unfortunately not that simple, as the an inconsistent and fluctuating data-acquisition obviously 683 number of tests changed week-to-week in that time period.

For a better understanding of the following part of decided to exclude the effects of testing in previous mod- 685 the analysis, we recall an important fact on exponential els, concerns about results derived from data that stem 686 growth: In each doubling period the same number of new from inconsistent testing should be taken seriously. Thus, 687 infections occurs as in all preceding periods combined. As we analyze possible distortions in more detail. As we 688 the number of tests approximately doubles every week will demonstrate below, our major conclusions remain 689 until week 12 and the fraction of positive tests increases 690 to week 13, the doubling period of new infections has to Please also note that at the time of writing of the initial 691 be shorter than 1 week. In a time frame of less than a manuscript, only very preliminary data and statistics on 692 week, more new infections occur than in the period since testing was available. Now, with better data, we come to 693 the onset of the outbreak. If we assume constant testing the conclusion that reported case numbers, although they 694 over the span of one week, a difference in the fraction of might derive from variable testing, are still useful to infer 695 positive tests on each day during that week should be 696 observable, and this is indeed what we see for testing in In particular, evidence for the key characteristics of the 697 weeks 10 and 11 and to a lesser extent from start to end change in transmission dynamics over a limited time pe- 699 is attached in Sec. VD. The important questions that riod and slow exponential decline, can be derived from 700 remain are: When did the number of new infections peak?

Deferring the first question to Sec. VB, we answer the We start our analysis by considering two central 703 second: From week 14 on, there is an approximately conquantities: i) the number of tests that are performed, say, 704 stant high level of testing, but a decline in the number of the performed tests that are positive — a positive tests 706 in the fraction of positive test results. These observations 707 are consistent with an exponential decline in the number Let us consider two simple limiting cases, in which only 708 of new infections, confirming that testing can properly

Summing up the above analysis so far, we have indi-

Hypothetical Scenario: If we were to reject the

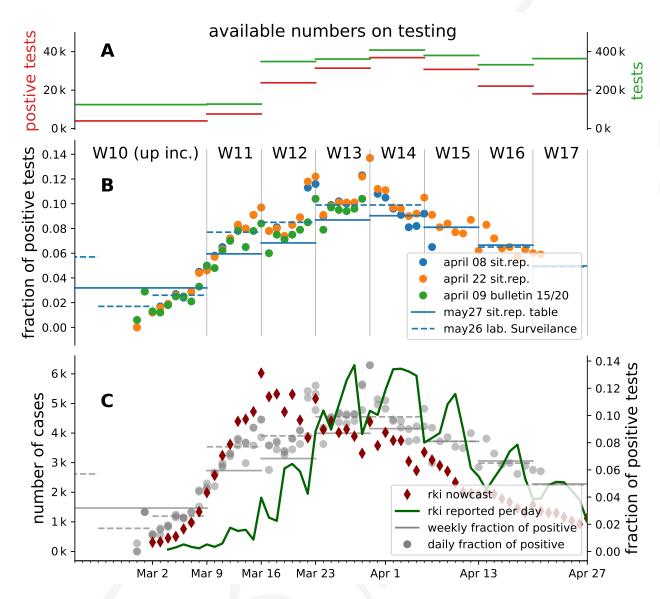


FIG. 9. The evolution of the fraction of positive tests in weeks 10 to 17. Weeks 10 to 12 strong exponential growth in in the number of new cases, which was not limited by the early testing capacity. A Comparison of number of positive test results with the number of tests performed for each week. Reproduced from Table 5 in [10]. Note: Numbers for week 10 and earlier are represented by a single data point. B Mid-term changes in the fraction of positive tests is more obvious in the daily data (points) than in the weekly (bars), especially in early March. Daily values are taken from situation reports [10–12] (full dataset) and the epi bulletin [13, 14] (ARS dataset). Weekly values, represented as horizontal lines, are taken from a situation report table and a weekly lab surveillance report (ARS dataset). Note: the latter represents a subset of all tests. Compared to the situation report, the ARS dataset lists weeks 8 to 10 individually. C Overlay of Panel 2 with the number of cases reported per day by the RKI and the estimated epi curve (imputation and Nowcasting, as described in [7]). The fraction of positive tests correlates with the number of reported cases from week 13 onward, as the total number of tests reaches a constant level.

716 flect growing numbers of infections, there is one alter-725 the same time at which the amount of tests was increased 717 native scenario to explain the observed trend, which we, 726 from weeks 11 to 12, coincidentally the effectiveness of 718 however, deem highly implausible. As this scenario has 727 the testing could have increased, so that the unobserved 721 lying assumption in this scenario is that the few tests that 730 symptoms and risk of exposition) that were required from 722 were performed during the initial outbreak until week 731 patients in order to qualify for one of the early tests, we 723 11 missed most of the actual cases, i.e. a large pool of 732 deem this scenario of an unobserved and constant pool

715 above simple explanation that growing case numbers re- 724 infected persons would have existed unobserved. Then, at frequently occurred in the public debate on the spread of 728 pool (of constant size!) is identified and, thus, apparent COVID-19 in Germany, we discuss it briefly. The under- 729 case numbers rise. Given the rigorous criteria (based on 733 to be quite unlikely. Especially so because the fraction of 734 positive tests stayed below 10% during the entire time.

#### Locating the peak position

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In other words, we are interested in the peak position in the curve of onsets of symptoms, see again Fig. 9, C, red. The day of the peak is constrained by the initial simultaneous increase in tests and fraction of positive tests to occur no earlier than week 11, as the peak would indicate the end of the growth. In this section we're focusing on how it can be reliably identified from the stable period of testing: From week 12 and onward, the number of tests remained on an almost constant, high level  $\sim 400$ k and changes in the daily new cases reported are directly reflected by the fraction of positive tests.

To understand this in more detail, we introduce the 747 following important rule of thumb here: Tests of week idescribe well what happened in week i-1.

The key is the connection between the date of symptoms onset (when symptoms first show), the testing (when the symptom onset is confirmed or an asymptomatic case is uncovered), and the reporting date (when a positive test-result is registered).

Any reported case must inherently be preceded by test and according to the RKI, positive test results are reported within 24 hours to the responsible health department. The remaining task then is to reveal the connection between symptom onset and reporting date, i.e. the reporting delay for each individual case. The date of testing is taken as the day before reporting in the rest of the analysis, the testing delay is one day shorter than the reporting delay. 763

In Fig. 10 we detail the reporting delay by plotting distributions of how many days after the symptom onset case is reported. For example, if each and every infected person would receive a test result (become a reported case) exactly three days after they showed symptoms, then the plotted distributions would have only one entry: a delta-peak at three days. However, we see that most reports arrive 1–7 days after symptom onset, where the details of the (lognormal) distribution depend on the week of onset of symptoms. Until and including week 12, the distributions have heavy tails. After week 12, the distributions have lighter tails. This provides some intuition of the distributions and the meaning of the heavy tails: most of the symptom onsets are reported within the first week but *some* will be reported much later, so that shape of the distribution still keeps changing. If the test level is low, more cases will be reported later and the tails of the distribution are heavier. This is latter 795 785 available. To rephrase based on Fig. 11: Half of the onsets 799 of onsets of illness peak at the end of week 11 or the

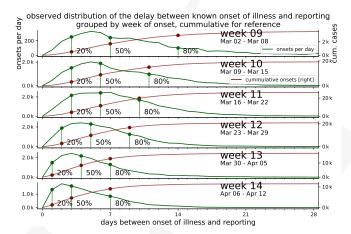


FIG. 10. The onsets of symptoms are confirmed by testing at later point in time, which accounts for most of the delay till all or the main fraction of known onset of symptoms (IstErkrankungsbeginn in RKI-database) are reported. From the RKI data, the number of cases per delay between onset of illness and reporting (i.e. RefDatum and Meldedatum) for cases with known onset of symptoms (IstErkrankungsbeginn) are counted for each week. The fraction of reported cases out of the total onsets up to a delay are highlighted for 20%, 50% and 80%. The cumulative number of cases reported up to each delay is displayed for reference.

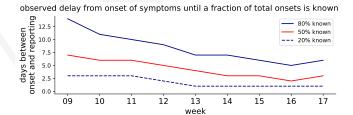


FIG. 11. Overview of the delay between onset of symptoms and the reporting of a fraction of total known onsets for a day changes with time. The 50% fraction represents the median reporting delay between onsets of symptoms and reporting. Derived from Fig. 10

788 onsets on Wednesday get tested until Sunday, the other 789 half in the following weeks for every following day of the 790 week the fraction of test performed in the next week rises. 791 Without explicitly working out the details, it's fair to 792 declare the initial rule of thumb valid. A more thorough 793 analysis based on actual per case testing-delays instead 794 of reporting delay distributions is conducted in Sec. VC.

Let's turn back to Fig. 11 A. The onsets in week 11, effect is what we see for the onsets during the first weeks 796 the estimated position of the peak, should be robustly until 11; due to limited testing capacities, many cases 797 measured by the testing in week 12, with a high number of are only reported weeks later — when more testing was 798 total tests. From Fig. 11 C, we can see, that the number of symptoms in week 11 are reported within 5 days, 80% beginning of week 12. This time point doesn't suffer from within 9 days. The crucial example here is: Half the 801 lower testing numbers in week 11.

# decomposition of the epicurve into weeks of testing

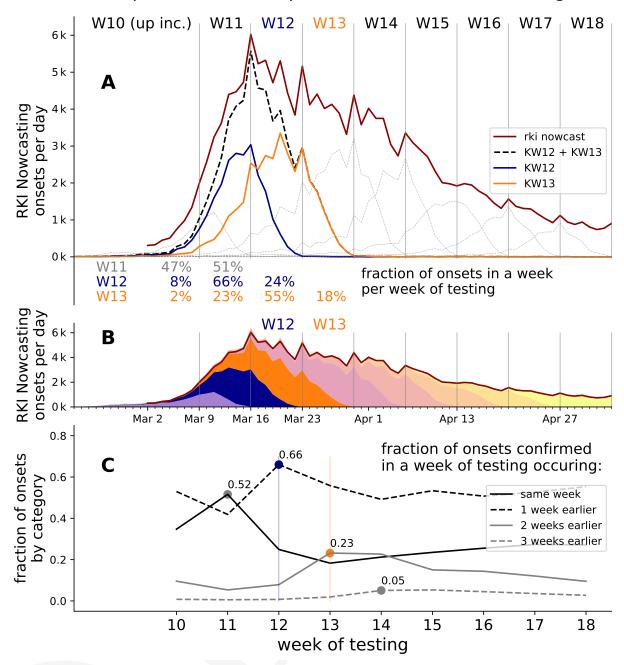


FIG. 12. Testing in one week confirms onsets of symptoms that occur up to 4 weeks earlier. The extend of this effect is analyzed based on the RKI database through decomposition by allocation of onsets of symptoms to weeks of testing. It is assumed that the delay between the time of testing and *Meldedatum* is 1 day. Tue-Mon *Meldedatum* is taken as a proxy for Mon-Sun testing. A Onsets of symptoms per day curves allocated to weeks of testing, weeks 12 and 13 are highlighted. Most known onsets around the peak of the epi curve in week 11 are confirmed by the testing in weeks 12 and 13. B stacked decomposition of the epi curve into weeks of testing. C To reveal crucial information about week-to-week change in the number of total onsets based on one week of testing, the shape of the distributions of onsets of symptoms confirmed by that week of testing is characterized. The fraction of onsets in the same week and each preceding week out of the total onsets confirmed by the week of testing is calculated. This indicates, the portion of a week's positive tests confirming onsets in the same week or in preceding weeks (max. 3 weeks earlier). The evolution of these 4 values is plotted by the week of testing. The peak of the epi curve can be tracked through testing results of weeks 11 to 14 as a maximum in the same-week/n-weeks earlier fraction of onsets confirmed in those respective weeks: 52% of all cases confirmed through testing in week 11 had onset of symptoms in the same week. Even more notable: 66% of positive tests in week 12 are linked to onsets 1 week earlier: in week 11. For comparison, see Fig. 12

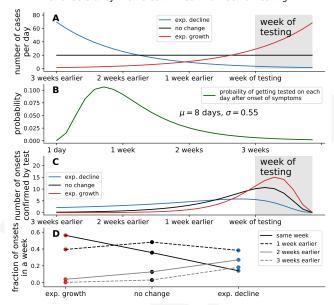
# Decomposing the epi curve into weeks of testing

Having established the delay between symptom onset and reporting, we can decompose the epi curve and identify parts of the curve that stem from certain weeks of testing. In more detail, for all the cases with a chosen testing period, we also know the respective date of onset of symptoms, if a complete dataset for those cases is available from the publicly available RKI database. This is the case for 60% of the datasets available. In Fig. 12 A,B we apply this method to collect all the symptom onsets that were found by testing in weeks 12 and 13. Through this allocation of "which part of the curve stems from which tests", we can thoroughly justify the connection that we made above, when we said that growth in weeks 11 and 12 stems from the tests in week 12 and 13. As we see, the peak on March 16 stems almost completely from tests of week 12 and 13; these weeks already featured the 819 high level of tests performed.

We can extend this method and compare the number of onsets in different weeks, that were confirmed by one week of testing. Think: distribution of onsets seen by the testing in one week. Some cases with onset of 824 symptoms on Monday will receive their positive result within the same week as the symptom onset itself, others get tested further away from their onset of symptoms. Seeing the same phenomenon from the week of testing, we can distinguish 4 categories which are sufficient for the available data: onsets 3 weeks, 2 weeks and 1 week earlier than the test and onset in the same week as testing. In Fig. 13 three different scenarios are considered and their effect on the fraction of cases in each week-category is worked out. All three scenarios show distinctive combination of fractions per week-category. Comparing the artificial result with Fig. 12 C, we find that in week 11 most of the tests (52%) found symptom onsets within the same week. This indicates weeks 10 and 9 had significantly less new onsets of illness. This is consistent with the exponential growth uncovered in sec. VA. In the extreme case that no tests were performed in week 10 and we were to observer that the number of onsets in week 10 were comparable or higher than in week 11, the backlog from week 10 would lead to higher fraction of 1-week-earlier onsets than same week onsets, for testing in week 11. As the fraction of 1-week-earlier onsets is lower than same-week for testing in week 11, we can see that the assumption, no tests and highter number of cases in week 10, cannot be valid. Reaffirming the observation 859 toms into week 11, constraining the peak in new onsets of indicate decline in onsets week over week.

858 cates strong exponential growth in new onsets of symp- 869 of week 12.

impact of evolution of cases on the shape of the distribution of onsets of symtoms confirmed in a week of testing



Changes in the number of onsets of symptoms from one week to the next can be estimated from the distribution of onsets of symptoms confirmed by testing in the latter week, if we group those onsets by week of onset. A Three different scenarios for the evolution of the number of cases are considered, whereby the number of onsets of symptoms per day is plotted. B Each case from A has a probability to be tested on every day. Half of the cases get tested within 8 days. The distribution is derived from observed data. C Number of onsets confirmed by the week of testing for each day of onset of symptoms. As a result of A and B. The shape of the distribution is characteristic for the change in cases and can be compared with Fig. 12, B. D As a last step the onsets confirmed by the testing in the highlighted week are summed up by week of onset and the group's respective fraction of the total number of positive tests in the highlighted week is computed. If no change in the number of cases occurs, more onsets in the week preceding testing are confirmed (45% of total) than from the same week as testing (35% of total). In case the number of cases rises, onsets from the same week as testing constitute the majority of onsets confirmed by tests in the week. If the number of cases declines, old onsets (older than 1 week) take over a significant fraction of total onsets tested in the week.

of growth from week 10 to week 11. Testing in week 12 860 illness to no earlier than March 9. The declining phase of shows a significant peak for onsets 1 week earlier. That 861 the wave is well documented. The exponential decline in indicates the number of new onsets is comparable in week 862 cases from week 13 onward is measured with consistent 11 and week 12 (see artificial result, Fig. 12). Note, that 863 high level in the number of tests. As testing in one week is a lower total number of tests in week 11 amplifies this 864 shown to uncover onsets of symptoms in the 3 preceding observation. Weeks 13 onward show distributions which 865 weeks, the alleged period of the peak in new onsets of 866 symptoms in week 11 is covered by robust testing from In Summary: Even though the number of total tests 867 weeks 12 and 13. Based on testing in weeks 12-13, the performed changed until week 12, the available data indi- 868 peak can be identified at the end of week 11 or beginning

### Available data on testing

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The epi bulletin [15] outlines the different networks that 872 the RKI uses to source information on testing: Voxco, 873 Resp Vir, the antibiotics-resistance-surveillance (ARS) [14] and lab-accociation queries. These sources are compiled into weekly data-sets with total number of tests and positive tests, which are published in the daily situation report once a week.

Data from the ARS contains daily number on testing and a separate weekly report is published on the RKI website. The ARS dataset covers 25-30% of the total number of tests reported by the RKI, as only 62 of 180+ labs participate. The ARS data-set shows a mean delay between 933 Additional information relevant to the discussion can be weeks 12 to 15, where the delay is 1.5 days, peaking in week 13 at 1.8 days.

An overview of all publicly available data on testing for 937 individual states exceeded 20% positive results. march 2020 is presented in Fig 9. The following observations along with additional comments are based on this presentation:

- From week 8 to week 12 the number of tests rises week to week by a factor greater than 2. 120k is a combined number for weeks up to 10. Individual numbers of tests for those weeks has to be estimated with help from the ARS-subset (Fig. 9 B may 26 lab. Surveilance). Assuming ARS is representative the number of test performed in week 10 should be around 60k, 30k in week 9 and 30k in all weeks up to and including 8, extending the exponential pattern.
- The number of tests remains on a high level from week 12 on. In the range of 340-430k.
- The number of positive test rises faster than the total number of tests until week 14.
- countries.
- March. Afterwards declining to less than 2% in 963 our present knowledge. week 20 (not shown in figure). The day-to-day rise 964 weekly average would suggest.
- the epi-curve is coincidental.

- The ARS data shows a steady day to day increase in positive fraction of test in weeks 10 and 11. Weekends show a higher fraction, while the total number of tests is lower (daily total number not shown in the figure). Deviating from the rise in the positive fraction, weeks up to 8 have a 3 times higher fraction of positive results than week 9.
- The maximum test-capacity per week as reported by the labs increased to 1M in week 19, showing strong growth till week 14. A week to week doubling in test capacity continues for two more weeks compared to growth in number of tests performed (not shown).

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sampling and testing between 1 and 1.2 days except for 934 found in the publications cited earlier. For the total 935 data-set, the fraction of positive tests varies from 1.5 to 936 7.2% for different states. Not a single day of testing for

# VI. SUMMARY & CONCLUSIONS

In these technical notes, we have comprehensively ad-940 dressed questions and comments regarding our recent 941 publication [1]. First, we compared direct, model-free es-942 timates of the reproduction number to the ones obtained 943 from dynamical modeling. To this end, we established 944 synthetic ground-truth data based on an SIR model and 945 subsequently inferred the reproduction number based on 946 various complementary approaches that are in practical 947 use. We reveal how sudden changes in the spreading 948 rate, as expected from the broad implementation of non-949 pharmaceutical interventions, can lead to counterintuitive 950 transient drops in new reported cases. Most importantly, 951 we find that only modeling of spreading dynamics can cor-952 rectly capture effects of sudden changes in the spreading

Second, we provided extensive background on our mod-955 eling rationale which combines differential-equation based • The fraction of positive tests per week peaks around 956 modeling of dynamics with Bayesian parameter infer-10%, relatively low compared with neighbouring 957 ence and formal model comparison. Within the Bayesian 958 framework, we argue that based on prior knowledge, the 959 most plausible models explaining the data can be system-• The fraction of positive tests per day varies with 960 atically identified and also updated as new information time from 2% around March 1 to around 10% in 961 becomes available. We also discuss why we do not think weeks 13 and 14, peaking at 14% at the end of 962 that models based on herd immunity are plausible given

Third, we analyzed additional data on the COVID-19 in week 10 and 11 is more pronounced than the 965 spread in Germany, which has become available since 966 the completion of the analysis presented in [1]. Most 967 importantly, we include data sets from the German Robert • The increase in the fraction of positive tests does 968 Koch Institute based on the reporting date as well as based not correlate to the rise in number of reported cases 969 on the onset of symptoms (epi curve). We analyzed the until week 13, but correlates with the decline in 970 data in the framework of SIR and SEIR models, and we reported cases from week 13 on, which is expected 971 also tested a broad range of varying prior assumptions. as the total number of tests fluctuates around 380k 972 We find our results to be robust across these varying tests per week on a high level. The correlation with 973 modeling assumptions and data sets, and to support the 974 conclusions drawn in [1]. In turn, this leads us to conclude

975 that under the conditions comparable to those in Germany, 984 test capacities, the crucial part of our analysis is based the epi curve becomes available — as long as the reporting 987 point is unaffected by testing. delay is properly modeled.

980 onset of the pandemic is presumably affected by a rise in 992 the central conclusions of our publication [1].

models based on reporting date are a viable alternative 985 on a regime of comparably stable testing. In particular, for analyzing the early stages of a disease outbreak, before 986 we find that the inference of the second and third change

Overall, the analysis here evaluates the robustness of Finally, we address the issue of changes in the testing 999 our previously reported results with respect to statistical capacities and procedures over the course of our analysis. 990 and dynamical modeling assumptions as well as comple-Most importantly, we find that while data from the initial 991 mentary data sources and provides additional support for

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# SUPPLEMENTARY INFORMATION: **FIGURES**

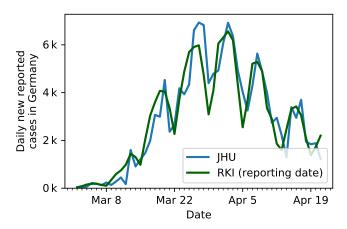


FIG. 14. Comparison of the German case numbers as published by the John Hopkins University (JHU) used in our previous publication [1], to the case number of the Robert Koch Institute (RKI). The difference is limited.

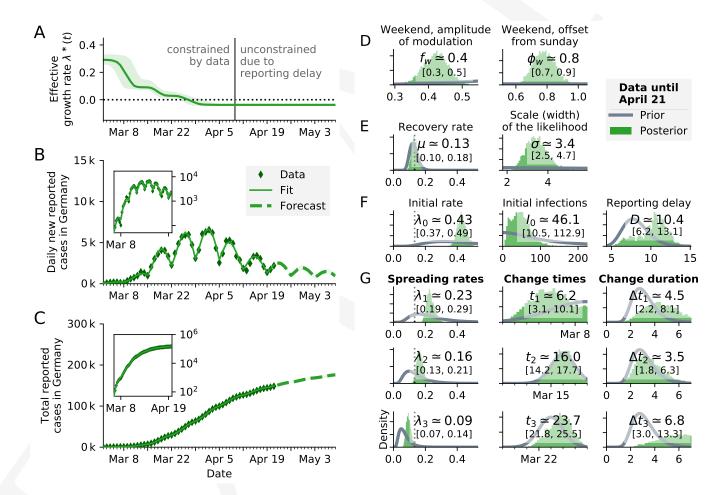


FIG. 15. SIR model (see Fig. 3 of [1]) using the reporting date (Meldedatum) of the RKI data for inference. A Time-dependent model estimate of the effective spreading rate  $\lambda^*(t)$ . B Comparison of daily new reported cases and the model (green solid line for median fit with 95% credible intervals, dashed line for median forecast with 95% CI); inset same data in log-lin scale. C: Comparison of total reported cases and the model (same representation as in B). D–G Priors (gray lines) and posteriors (green histograms) of all model parameters; inset values indicate the median and 95% credible intervals of the posteriors.

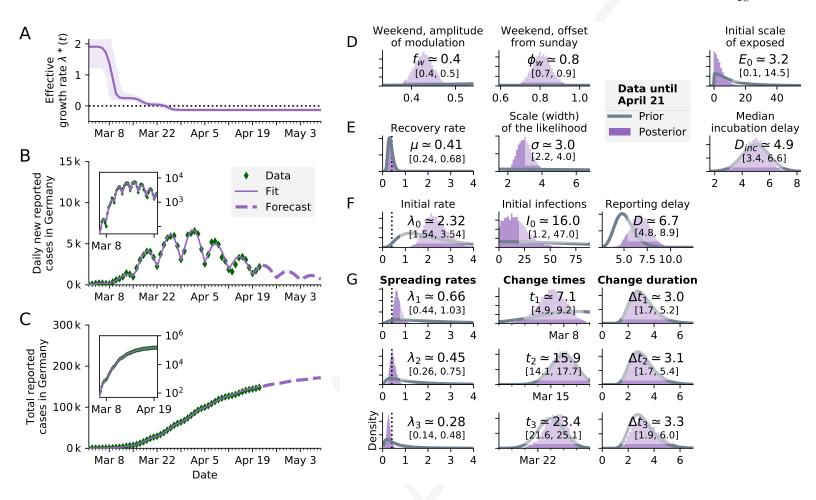


FIG. 16. SEIR-like model (see Fig. S3 in Supplementary Information of [1]) using the reporting date (Meldedatum) of the RKI data for inference. A Time-dependent model estimate of the effective spreading rate  $\lambda^*(t)$ . B Comparison of daily new reported cases and the model (purple solid line for median fit with 95% credible intervals, dashed line for median forecast with 95% CI); inset same data in log-lin scale. C Comparison of total reported cases and the model (same representation as in B). D-G Priors (gray lines) and posteriors (purple histograms) of all model parameters; inset values indicate the median and 95% credible intervals of the posteriors.

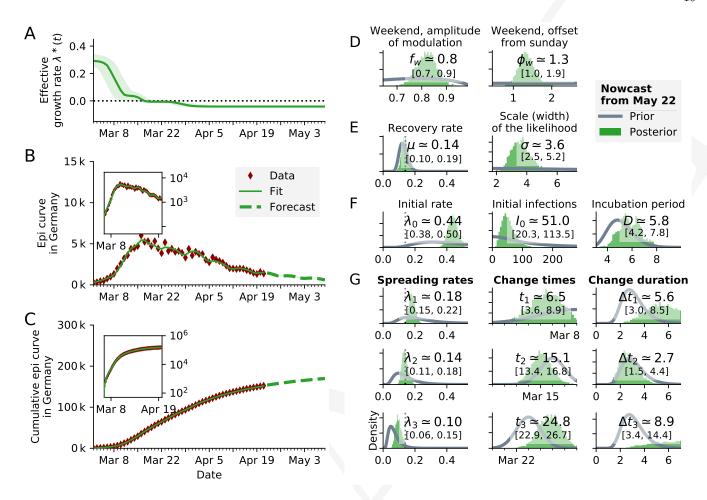


FIG. 17. SIR model using the onset of symptoms (nowcast from May 22) of the RKI data for inference. The median of the lognormal prior of the delay between infection and onset of symptoms has been set to 5 days (right-most panel F). A Time-dependent model estimate of the effective spreading rate  $\lambda^*(t)$ . B Comparison of daily new reported cases and the model (green solid line for median fit with 95% credible intervals, dashed line for median forecast with 95% CI); inset: same data in log-lin scale. C Comparison of total reported cases and the model (same representation as in B). D–G Priors (gray lines) and posteriors (green histograms) of all model parameters; inset values indicate the median and 95% credible intervals of the posteriors.

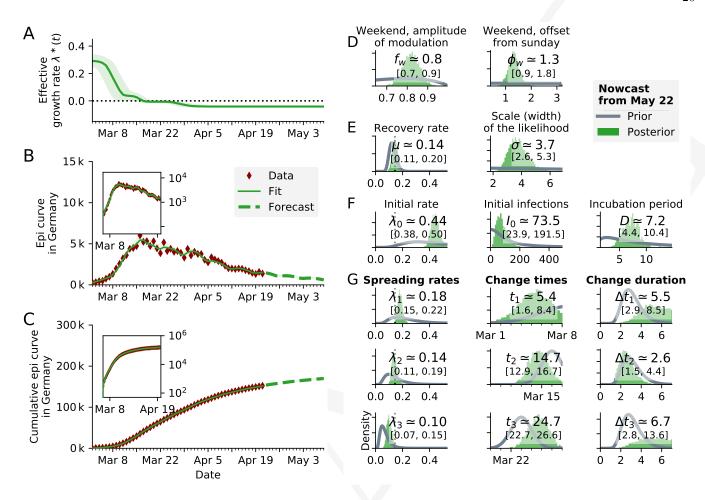


FIG. 18. SIR model using the onset of symptoms (nowcast from May 22) of the RKI data for inference. The median of the lognormal prior of the delay between infection and onset of symptoms has been set to a relatively uninformative prior (right-most panel F). The posterior of the delay has as median 7.2 days, which is close to the expected incubation period of 5 days. A Time-dependent model estimate of the effective spreading rate  $\lambda^*(t)$ . B Comparison of daily new reported cases and the model (green solid line for median fit with 95% credible intervals, dashed line for median forecast with 95% CI); inset same data in log-lin scale. C Comparison of total reported cases and the model (same representation as in B). D-G Priors (gray lines) and posteriors (green histograms) of all model parameters; inset values indicate the median and 95% credible intervals of the posteriors.

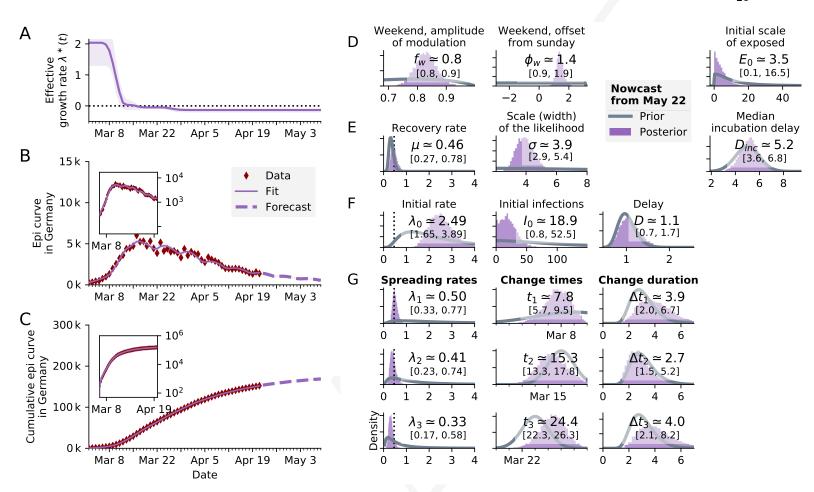


FIG. 19. **SEIR-like model using the onset of symptoms** (nowcast from May 22) of the RKI data for inference. The median of the lognormal prior of the delay between infectious and onset of symptoms has been set to 1 day (right-most panel F). **A** Time-dependent model estimate of the effective spreading rate  $\lambda^*(t)$ . **B** Comparison of daily new reported cases and the model (purple solid line for median fit with 95% credible intervals, dashed line for median forecast with 95% CI); **inset** same data in log-lin scale. **C** Comparison of total reported cases and the model (same representation as in B). **D**–**G** Priors (gray lines) and posteriors (purple histograms) of all model parameters; inset values indicate the median and 95% credible intervals of the posteriors.

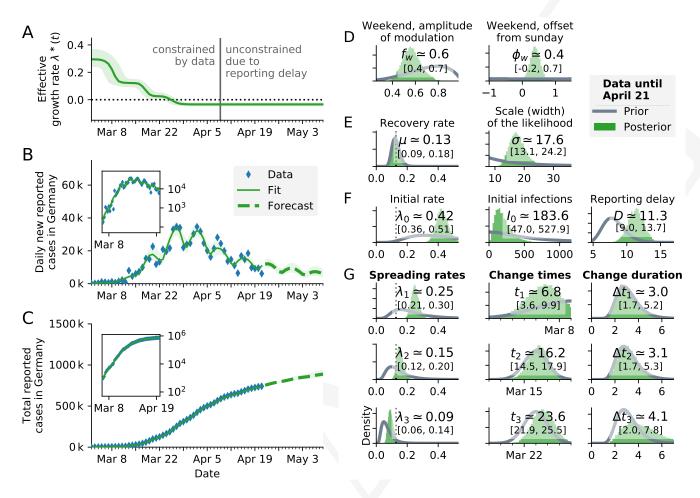


FIG. 20. SIR model with reported case number multiplied by 5, to account for an eventual factor five of unknown cases. Results are nearly identical to original non-multiplied plot (Fig 3. in [1]), showing that a constant underreporting has a negligible effect. The median inferred spreading rates  $\lambda$  are about 0.01 larger. A Time-dependent model estimate of the effective spreading rate  $\lambda^*(t)$ . B Comparison of daily new reported cases and the model (green solid line for median fit with 95% credible intervals, dashed line for median forecast with 95% CI); inset same data in log-lin scale. C Comparison of total reported cases and the model (same representation as in B). D–G Priors (gray lines) and posteriors (green histograms) of all model parameters; inset values indicate the median and 95% credible intervals of the posteriors.

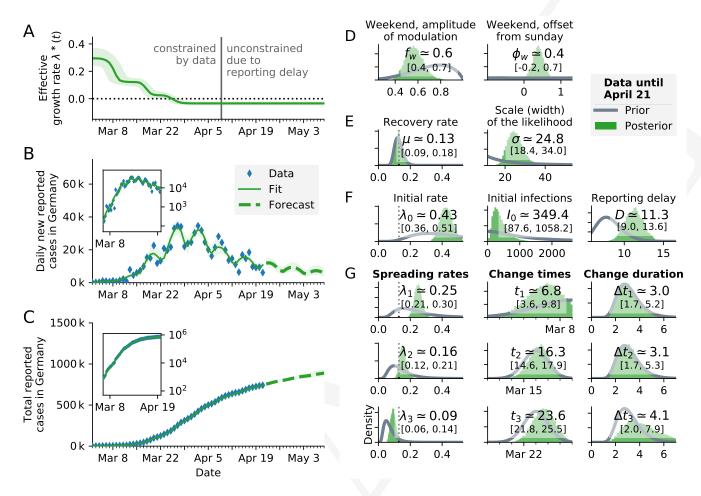


FIG. 21. SIR model with reported case number multiplied by 10, to account for an eventual factor 10 of unknown cases. Results are nearly identical to original non-multiplied plot (Fig 3. in [1]), showing that a constant under-reporting has a negligible effect, similar to Fig. 20. The median inferred spreading rates  $\lambda$  are 0.01-0.02 larger. A Time-dependent model estimate of the effective spreading rate  $\lambda^*(t)$ . B Comparison of daily new reported cases and the model (green solid line for median fit with 95% credible intervals, dashed line for median forecast with 95% CI); inset same data in log-lin scale. C Comparison of total reported cases and the model (same representation as in B). D–G Priors (gray lines) and posteriors (green histograms) of all model parameters; inset values indicate the median and 95% credible intervals of the posteriors.