

$$\begin{aligned}
\frac{\partial \phi}{\partial t} &= D \nabla^2 \phi \\
\frac{\partial \phi}{\partial t} &= D \frac{\partial^2 \phi}{\partial x^2} \\
\int_{\Omega} \frac{\partial \phi}{\partial t} d\Omega &= D \int_{\Gamma} \frac{\partial \phi}{\partial x} \cdot \hat{n} d\Gamma \\
\int_{\Omega} \frac{\partial \phi}{\partial t} d\Omega &= D \int_W \frac{\partial \phi}{\partial x} (-1) d\Gamma_W + D \int_E \frac{\partial \phi}{\partial x} (1) d\Gamma_E \\
\int_{\Omega} \frac{\partial \phi}{\partial t} d\Omega &= D \left[\left(\frac{\partial \phi}{\partial x} \right)_E - \left(\frac{\partial \phi}{\partial x} \right)_W \right] \Delta y
\end{aligned}$$

Central diff $\frac{\partial \phi}{\partial x}$

$$\frac{\partial \phi}{\partial x} \approx \frac{\phi_{i+1} - \phi_i}{\Delta x}$$

$$\begin{aligned}
\int_{\Omega} \frac{\partial \phi}{\partial t} d\Omega &= D \left[\left(\frac{\phi_{i+1} - \phi_i}{\Delta x} \right)_E - \left(\frac{\phi_i - \phi_{i-1}}{\Delta x} \right)_W \right] \Delta y \\
\int_{\Omega} \frac{\partial \phi}{\partial t} d\Omega &= D \left[\frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{\Delta x} \right] \Delta y
\end{aligned}$$