1. (10 points)

   (a) (5 points) Give a DFA recognizing the following language. The alphabet is $\{0, 1\}$.

   $$\{w \mid w \text{ contains exactly two 0s and at least one 1}\}$$

   (b) (5 points) Give an NFA recognizing the language $0^*1^*0^+$ with four states (including a reject state). The alphabet is $\{0, 1\}$.

2. (20 points) For any string $w = w_1 w_2 \cdots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_n \cdots w_2 w_1$. For any language A, let $A^R = \{w^R | w \in A\}$. Show that if A is regular, so is A^R.

3. (20 points) Let A be any language. Define DROP-OUT(A) to be the language containing all strings that can be obtained by removing one symbol from a string in A. Thus, DROP-OUT(A) = $\{a | \exists y \in \Sigma, \exists x, z \in \Sigma^* : xyz \in A \bigwedge a = xz\}$. Show that the class of regular languages is closed under the DROP-OUT operation. In other words, if A is a regular language, so is DROP-OUT(A).

4. (20 points) For a language A, define an equivalence relation between two strings: $x \equiv y$ means $\forall z \in \Sigma^* : xz \in A \iff yz \in A$. This allows the set Σ^* to be divided into different equivalence classes.

   For example, if $A = \{w \mid w \text{ contains exactly two 0s and at least one } 1\}$, then $1 \equiv 1111$, $101 \equiv 110$, but $1 \not\equiv 011$.

   (a) (5 points) How many different equivalence classes are there for A in the above example?

   (b) (5 points) Prove that for every regular language A, Σ^* can be divided into a finite number of equivalence classes.

   (c) (5 points) Prove that if Σ^* can be divided into a finite number of equivalence classes for a language A, then A is a regular language.

   (d) (5 points) With the above conclusions, prove $A = \{0^n 1^n | n \in \mathbb{N}\}$ is not a regular language.

5. (10 points) Prove that the following languages are not regular.

   (a) (5 points) $\{0^m 1^n | \text{m and n are coprime}\}$.

   (b) (5 points) $\{w | w \in \{0, 1\}^* \text{ is not a palindrome}\}$. Here a palindrome is a string that reads the same forward and backward.

6. (20 points)

   Let the rotational closure of language A be $RC(A) = \{yx | xy \in A\}$.

   (a) (10 points) Show that for any language A, we have $RC(A) = RC(RC(A))$.

   (b) (10 points) Show that the class of regular languages is closed under rotational closure. In other words, if A is a regular language, so is $RC(A)$.