1. (12 points) Give context-free grammars that generate the following languages. In all parts the alphabet Σ is $\{0, 1\}$.
   (a) $\{w|w$ starts and ends with the same symbol$\}$.
   (b) $\{w|w$ is a palindrome$\}$.
2. (12 points) For any language A with alphabet Σ, let $\text{PREFIX}(A) = \{w|wv \in A$ for some $v \in \Sigma^*\}$. Show that the class of context-free languages is closed under the PREFIX operation.
3. (12 points)
   (a) Prove that the language $L = \{0^{2^n}|n \geq 0\}$ is not context free.
   (b) Let B be the language of all palindromes over $\{0, 1\}$ containing an equal number of 0s and 1s. Show that B is not context free.
4. (16 points) Define a two-dimensional Turing machine to be a TM where each of its tapes is an infinite grid (and the read/write head can move not only Left and Right but also Up and Down). Show that for every $T : \mathbb{N} \to \mathbb{N}$ and every Boolean function f, if f can be computed in time $T(n)$ using a two-dimensional TM, then it can also be computed by a standard (one-dimensional) TM in time $O(T(n)^2)$. (T may not be time constructible)
5. (16 points) Let $T = \{\langle M\rangle|M$ is a TM that accepts $\alpha^{\mathcal{R}}$ whenever it accepts $\alpha\}$. Show that T is undecidable. Note: $\alpha^{\mathcal{R}}$ is the reverse string of α. You may assume the alphabet is $\{0, 1\}$.
6. (16 points) Define the language

   $C_{TM} = \{\langle M_1, M_2\rangle|M_1, M_2$ are two Turing machines such that $L(M_1) \subseteq L(M_2).\}$

   Show that C_{TM} is undecidable.
7. (16 points) Show that single-tape TMs that cannot write on the tape recognize only regular languages. (Hint: You may use the conclusion from Exercise 4 in HW1, which is a simplified version of Myhill–Nerode theorem.)