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LISTA 4 GA 4

1) a) $x = 2$
$y = -1$

b) $x = 4$
$y = 3$
$z = 2$
$w = 1$

c) $x = 6$
$y = 3$
$z = 2 - w$

d) $x = -32$
$y = z + 2$

e) $x = 8 + 7w$
$y = 2 - 3w$
$z = -5 - w$

f) $x = -2 + 6y + 3t$
$y = ?$
$z = 7 - 4t$
$w = 8 - 5t$

2) a) $\left(\begin{array}{cc|c} 3 & -4 & 1 \\ 1 & 3 & 9 \end{array}\right)$ $L_1 \leftarrow L_2$ $L_2 \leftarrow (-3 \cdot L_1) + L_2$ $\left(\begin{array}{cc|c} 1 & 3 & 9 \\ 0 & -13 & -26 \end{array}\right)$ $L_2 \leftarrow L_2 / -13$ $L_1 \leftarrow L_1 + L_2 \cdot -3$

$\left(\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 1 \end{array}\right)$ $x = 6$ $y = 1$

b) $\left(\begin{array}{cc|c} 5 & 8 & 34 \\ 10 & 16 & 50 \end{array}\right)$ SISTEMA IMPOSSIVEL

c) $\left(\begin{array}{cc|c} 1 & 2 & 5 \\ 2 & -3 & -4 \end{array}\right)$ $L_2 \leftarrow L_1 \cdot (-2) + L_2$ $\left(\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & -7 & -14 \end{array}\right)$ $L_2 \leftarrow L_2 / 7$ $L_1 \leftarrow L_1 + -2 \cdot L_2$

$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right)$ $x = 1$ $y = 2$

d) $\left(\begin{array}{ccc|c} 3 & 2 & -5 & 8 \\ 2 & -4 & -2 & -4 \\ 1 & -2 & -3 & -4 \end{array}\right)$ $L_3 \leftarrow L_1$ $L_3 \leftarrow (L_1 \cdot -3) + L_3$ $L_2 \leftarrow L_1 \cdot -2 + L_2$ $\left(\begin{array}{ccc|c} 1 & -2 & -3 & -4 \\ 0 & 0 & 4 & 4 \\ 0 & 8 & 4 & 20 \end{array}\right)$ $L_2 \leftarrow L_2 / 4$ $L_3 \leftarrow L_3 - 4 \cdot L_2$

x y z

$\left(\begin{array}{ccc|c} 1 & -2 & -3 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 8 & 0 & 16 \end{array}\right)$ $z = 1$ $y = 2$ $x = 3$

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e)	2	-6	-4	$L_1 \leftarrow L_1/2$	1	-3	-2
	1	3	1	$L_3 \leftarrow L_3/2$	0	0	-2
	4	12	2	$L_2 \leftarrow L_2 - L_1$			

Sistema impossível

f)	1	2	-1	2	$L_2 \leftarrow L_2 - (L_1 \cdot 2)$	1	2 -1	2
	2	-1	3	9	$L_3 \leftarrow L_3 - (L_1 \cdot 3)$	0	-3 5	5
	3	3	-2	3		0	-3 -1	-3

$L_3 \leftarrow L_3 \cdot -1$

$L_3 \leftarrow L_3 + L_2$

	1	2	-1	2		1	2 0	5/2
	0	-3	5	5	$L_1 \leftarrow L_3 + L_1$	0	-3 5	5
	0	0	4	2	$L_3 \leftarrow L_3/4$	0	0 1	2

$z = 2$

$y = 5/3$

$x = 2/3$

g)	1	0	3	-8	$L_3 \leftarrow L_3 - L_1 \cdot 3$	1	0 3	-8
	2	-4	0	-4		2	-4 0	-4
	3	-2	-5	26		0	-2 -14	50

$L_3 \leftarrow L_2$

$L_2 \leftarrow L_2 - 2 \cdot L_1$

	1	0	3	-8	$L_2 \leftarrow L_2 + L_3$	1	0 3	-8
	0	-4	-6	20		0	-4 -6	20
	0	1	7	-25		0	-3 1	-5

$L_2 \leftarrow L_2 \cdot -3$

$L_3 \leftarrow L_3 \cdot +4$

	1	0	3	-8	$L_3 \leftarrow L_3 + L_2$	1	6 3	-8
	0	12	18	-60	$L_2 \leftarrow L_2/6$	0	2 3	-10
	0	-12	1	-20		0	0 19	80

h)	1	2	-3	10	$L_2 \leftarrow L_2 - L_3$	-1	0 0	-3
	3	4	6	23	$L_1 \leftarrow L_1 - L_3$	-1	2 3	10
	2	2	3	13		2	2 3	13

$L_2 \leftarrow L_1 + L_2$

$L_3 \leftarrow L_3 + 2 \cdot L_1$

	-1	0	0	-3	$L_2 \leftarrow L_2 - L_3$	1	0 0	3
	0	2	3	7	$L_1 \leftarrow L_1 \cdot (-1)$	0	0 0	0
	0	2	3	7		0	2 3	7

$x = 3$

$2y = 7 - 3z$

$y = 7/2 - 3z$

$z = z$

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i)
$$\left(\begin{array}{cccc|c} 1 & -3 & 4 & -1 & 2 \\ 2 & -1 & 3 & -2 & 19 \end{array}\right) \quad L_2 \leftarrow L_2 - 2 \cdot L_1$$
$$\left(\begin{array}{cccc|c} 1 & -3 & 4 & -1 & 2 \\ 0 & 1 & -1 & 0 & 3 \end{array}\right)$$
$$L_1 \leftarrow L_1 + 3 \cdot L_2$$
$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & -1 & -7 \\ 0 & 1 & -1 & 0 & 3 \end{array}\right)$$
$$\left(\begin{array}{cccc|c} 1 & -3 & 4 & -1 & 2 \\ 0 & 5 & -5 & 0 & 15 \end{array}\right) \quad L_2 \leftarrow L_2/5$$
$$x + z - w = -7$$
$$y + z = 3$$

3)
$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{array}\right) \quad \begin{array}{l} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - L_1 \end{array}$$
$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{array}\right) \quad \begin{array}{l} L_3 + 2 \cdot L_2 \to L_3 \\ L_3 \cdot -7 \to L_3 \\ L_1 - 3 \cdot L_3 \to L_1 \\ L_2 + 3 \cdot L_3 \to L_2 \end{array}$$
$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right) \quad L_1 \leftarrow L_1 - 2 \cdot L_2 \to L_1$$
$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right)$$
$$x = 1 - w$$
$$y = 2$$
$$z = 1$$

b)
$$\left(\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{array}\right) \quad \begin{array}{l} L_3 \leftarrow L_3 - L_1 \quad L_3 \leftarrow L_3 + L_2 \\ L_2 \leftarrow L_2/2 \quad L_3 \leftarrow -2 \cdot L_3 \\ L_1 \leftarrow L_1 - L_2 \end{array}$$
$$\left(\begin{array}{cccc|c} 1 & 0 & 2,5 & -1,5 & 1,5 \\ 0 & 1 & 0,5 & -1,5 & 1,5 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right)$$
$$L_1 \leftarrow L_1 - 2,5 \cdot L_3 \quad \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right)$$
$$L_2 \leftarrow L_2 - 0,5 \cdot L_3$$
$$x = 1 - w$$
$$y = 2 + w$$
$$z = -1 + w$$

c)
$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 3 & 3 & 0 \end{array}\right) \quad \begin{array}{l} L_3 \leftarrow L_3 - L_1 \\ L_4 \leftarrow L_4 - L_1 \\ L_2 \leftarrow L_2 - L_1 \\ L_1 \leftarrow L_1 - 3 \cdot L_3 \end{array}$$
$$\left(\begin{array}{ccc|c} 1 & 0 & -4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$
$$L_1 \leftarrow L_1 - 2 \cdot L_2 \quad x = 0$$
$$L_2 \leftarrow L_2 - 2 \cdot L_3 \quad y = 0$$
$$z = 0$$

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4) $\left(\begin{array}{ccc|c|c} 1 & -2 & 1 & 1 & 2 \\ 2 & -5 & 1 & -2 & -1 \\ 3 & -7 & 2 & -1 & 2 \end{array}\right)$ $L_2 \leftarrow L_2 - 2 \cdot L_1$ $L_3 \leftarrow L_3 - 3 \cdot L_1$ $-1 \cdot L_2 \rightarrow L_2$ $\left(\begin{array}{ccc|c|c} 1 & -2 & 1 & 1 & 2 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & -1 & -1 & -4 & -4 \end{array}\right)$ $L_3 \leftarrow L_3 + L_2$

$\left(\begin{array}{ccc|c|c} 1 & -2 & 1 & 1 & 2 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$

$x + 3z = 9 \quad e \quad 12$

$y + z = 4 \quad e \quad 5$

5) $A + 4I \rightarrow \left(\begin{array}{ccc} 5 & 0 & 5 \\ 1 & 5 & 1 \\ 0 & 1 & 0 \end{array}\right) \cdot X = \left(\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right)$ $L_2 \leftarrow L_2 - 5 \cdot L_1$ $1/5 \cdot L_2 \rightarrow L_2$ $L_3 - L_2 \rightarrow L_3$ $\left(\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$x = -z$ b) $\left(\begin{array}{ccc|c} -1 & 0 & 5 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & -6 & 0 \end{array}\right)$ $L_1 \leftarrow L_1 \cdot -1$ $L_2 \leftarrow L_2 - L_1$ $-1 \cdot L_2 \rightarrow L_2$ $L_3 \leftarrow L_3 - L_2$ $\left(\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$y = 0$

$z = -x$

$x = 5z$

$y = 6z$

$z = z \in \mathbb{R}$

6) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & z \\ 2 & 3 & 2 & 5 \\ 2 & 3 & a^2-1 & a+1 \end{array}\right)$ $L_2 - 2L_1 \rightarrow L_2$ $L_3 - 2L_1 \rightarrow L_3$ $L_3 - L_2 \rightarrow L_3$ $\left(\begin{array}{ccc|c} 1 & 1 & 1 & z \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-3 & a-4 \end{array}\right)$

Sem solução $= a = \pm\sqrt{3}$

Solução única $= a \neq \pm\sqrt{3}$

∞ = Impossível

b)

1	2	$L_3 \to L_3 + L_2$	
4	5		
-4	-4		

5	1	0
1	0	0
0	0	0

1	0	-5	0
0	1	-6	0
0	0	0	0

7) a)

2	-2	1	0	$1/2 \cdot L_1 \to L_1$	$1/4\, L_2 \to L_2$
3	1	0	1	$L_2 - 3 \cdot L_1 \to L_2$	$L_1 + L_2 \to L_1$

1	0	0,125 1/4
0	1	-0,375 1/4

b)

2	-2	0	1	0 0	$1/2 \cdot L_2 \to L_2$
1	2	1	0	1 0	$L_2 - L_1 \to L_2$
0	1	-1	0	0 1	$L_3 - 1/3 L_2 \to L_3$

1	0	0	3/8 1/4 1/4
0	1	0	-1/6 1/4 1/4
0	0	1	-1/6 1/4 -3/4

c)

3	5	1	0	$1/3\, L_1 \to L_1$	$3 \cdot L_2 \to L_2$
1	2	0	1	$L_2 - L_1 \to L_2$	$L_1 - 5/3 \cdot L_2 \to L_1$

1	0	2 -5
0	1	-1 3

d)

0	-1	1	1	0 0	$1/2\, L_1 \to L_1$	$L_3 \div L_2 \to L_3$
2	0	-1	0	1 0	$L_3 - L_1 \to L_3$	$2/3 \cdot L_3 \to L_3$
1	1	0	0	0 1	$-1 \cdot L_2 \to L_2$	

1	0	0	1/3 1/3 1/3
0	1	0	-1/3 -1/3 2/3
0	0	1	2/3 -1/3 2/3

e)

8) $x + 2y + 3z = 26$

$2x + 5y + 6z = 60$

$2x + 3y + 4z = 40$

$-2x - 4y - 6z = -52$

$0 + y + z = 8$

$y = 8$

$0x - y - 2z = -12$

$-8 - 2z = -12$

$2z = 4$

$z = 2$

$x + 16 + 6 = 26$

$x = 4$

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9) $3x = x + y$ $3x = 300$ $x = 100$ $6x + 6x + 10x = 2200$

$2x = y$ $y = 200$

BANANA = 160

CASCA = 300

SUNDAE = 200

10) $4\phi + 3\phi + 1\phi z = 700\phi$ $-2x - 4y - 8z = -1000$ $-5z = 940$

$2\phi x + 4\phi y + 3\phi y = 600\phi^{-2}$ $2x \quad 4y \quad 3z = 60$ $z = 94016$

$1\phi x + 7\phi y + 4\phi = 500\phi^{-2}$ $4x + 3y + z = 706$ $z = 188$

$-4x - 8y - 8z = 120$

$-5y - 4z = 580$ $y = 266$

$-5y = 580 + 4z$

$x = 784$

11) $2x + 3y + z = 8420$ $7y = 11270$ $y = 1610$

$x + 2y + 2z = 7940$ $4x + 3 \cdot 1610 = 8110 = 820$

$4x + 3y = 8100$ $b = 820$

$z = 8420 - 2(4920) - 3 \cdot (1610) = 1450$

$z - x = 1950 - 820 = 1130$