$det = a_{11} \cdot (-1)^{i+j} \cdot D + \cdots$

LISTA 2 GA.

GUILHERME FALCUCCI
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1) a) $\begin{pmatrix} 1 & 2 \\ -4 & 3 \end{pmatrix}$ $det(a) = 1 \cdot (-1)^2 \cdot 3 + 2 \cdot (-1)^3 \cdot -4$
$det(a) = 3 + 8 = 11$

b) $\begin{pmatrix} \sqrt{2} & 3\sqrt{6} \\ 2 & \sqrt{3} \end{pmatrix}$ $det(b) = \sqrt{2} \cdot (-1)^2 \cdot \sqrt{3} + 2 \cdot (-1)^3 \cdot 3\sqrt{6}$
$det(b) = \sqrt{6} + (-6\sqrt{6}) = -5\sqrt{6}$

c) $\begin{pmatrix} 1 & 0 & 2 \\ 5 & -1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ $det(c) = 1 \cdot (-1)^4 \cdot (2) + 0 \cdot cof + 0 \cdot cof$
$det(c) = 2$

d) $\begin{pmatrix} -2 & 1 & -1 \\ 1 & 5 & 4 \\ -3 & 4 & 2 \end{pmatrix}$ $det(d) = -2 \cdot (-1)^2 \cdot -6 + 1 \cdot (-1)^3 \cdot 10 + -1(-1)^4 \cdot 19$
$det(d) = 12 + (-14) + (-19)$
$det(d) = -21$

e) $\begin{pmatrix} 0 & 2 & 0 \\ 1 & 3 & 5 \\ 2 & -1 & 2 \end{pmatrix}$ $det(e) = 0\,cof + 2 \cdot (-1)^3 \cdot -8 + 0 \cdot cof$
$det(e) = -2 \cdot -8$
$det(e) = 16$

f) $\begin{pmatrix} 3 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$ $det(f) = 3 \cdot (-1)^2 \cdot Det(\tilde{a})$
$det(f) = 3 \cdot -3$
$det(f) = -6$

$\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{matrix} 1 & 0 \\ -1 & 1 \\ 0 & 1 \end{matrix}$

$0 + 0 - 0 \mid -1 + 0 + -1$

$-2 - (0)$

G) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 2 & 5 & 3 \\ 7 & 2 & \sqrt{5} & 0 & 0 \\ 10 & -3 & 6 & 1 & 0 \\ -2 & -3 & 0 & 0 & 0 \end{pmatrix}$

$\det(G) = 1 \cdot (-1)^2 \cdot \det(\tilde{G})$

$\det(G) = \det(\tilde{G})$

$\det(\tilde{G}) = 3 \cdot (-1)^5 \cdot \det(g)$

$\det(\tilde{G}) = 3 \cdot -1 \cdot -3\sqrt{5}$

$\det(G) = 9\sqrt{5}$

$\left|\begin{matrix} 2 & \sqrt{5} & 0 \\ -3 & 6 & 1 \\ -3 & 0 & 0 \end{matrix}\right| \begin{matrix} 2 & \sqrt{5} \\ -3 & 6 \\ -3 & 0 \end{matrix}$

$0 \quad 0 \quad 0 \quad 0 + -3\sqrt{5} + 0$

h) $\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{pmatrix}$

$\det(H) - 2 \cdot (-1)^7 \cdot \det(\tilde{H})$

$\det(H) - 2 \cdot -1 \cdot 12$

$\det(H) = -24$

$\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 \end{pmatrix}$

$\det(h) = 3 \cdot (-1)^2 \cdot -4$

$\det(h) = 12$

$\begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ -2 & 0 & 0 \end{pmatrix} \begin{matrix} 0 & 2 \\ 0 & 0 \\ 2 & 0 \end{matrix}$

$-0 \quad 0 \quad 0 \; 0 - 4 \quad 0$

2) a) = $A+B$ $\begin{pmatrix} 7 & -2 & 14 \\ 3 & 2 & 10 \\ 4 & -8 & 2 \end{pmatrix} \begin{matrix} 7 & -2 \\ 3 & 2 \\ 4 & -8 \end{matrix}$ $Det(A+B) = 72$

$(112 - 560 - 12 \mid 28 + (-80) + (-336))$

$-388 - (-460) = 72$

b) $det(A \cdot B) = det(A) \cdot det(B) = 66 \cdot -9 = -594$

$\begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{pmatrix} \begin{matrix} 3 & -5 \\ 4 & 2 \\ 1 & -9 \end{matrix}$ $\begin{pmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{pmatrix} \begin{matrix} 4 & 3 \\ -1 & 0 \\ 3 & 1 \end{matrix}$

$(14 + (-216) + (-120) \mid 36 + (140) + (252)$ $0 \; 8 + 12 \mid 0 \; 18 - 7$

$(+322 - 256)$ $(7 + 11 - 20)$

$det(A) = 66$ (-9)

c) $det(B^T A^T) = det(A \cdot B) = det(A) \cdot det(B)$

$det(B^T A^T) = -594$

d) $det = \begin{pmatrix} 9 & -15 & 21 \\ 12 & 6 & 24 \\ 3 & -27 & 18 \end{pmatrix} - \begin{pmatrix} 4 & 6 & -2 \\ 12 & 18 & -4 \\ 16 & 24 & -6 \end{pmatrix} = \begin{pmatrix} 5 & -21 & 23 \\ 0 & -12 & 28 \\ -13 & -51 & 24 \end{pmatrix}$

$(3A - 2C) + B = \begin{pmatrix} 9 & -18 & 30 \\ -1 & -12 & 30 \\ -10 & -50 & 20 \end{pmatrix} \begin{matrix} 9 & -18 \\ -1 & -12 \\ -10 & -50 \end{matrix}$

$= (360 + 3600 - 13.500) + (-2160 + 5400 + 1500)$

$9540 + 4740$

$det(3A - 2C + B) = 14280$

e) $det(A \cdot C^T) = det(A) \cdot det(C)$ $detC = \begin{pmatrix} 2 & 3 & 1 \\ 6 & 9 & -2 \\ 8 & 12 & -3 \end{pmatrix} \begin{matrix} 2 & 3 \\ 6 & 9 \\ 8 & 12 \end{matrix}$

$66 \cdot 0$

$det(A \cdot C^T) = 0$

$-(-72 - 48 - 541 - 54 - 48 - 72)$

$det(C) = 0$

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3) a) $det(A^T) = det(A) = -2$

b) $det(6A) = K^n \cdot det(A) = 6^4 \cdot (-2) = 1296 \cdot (-2) = -2592$

c) $det(A^7) = det(A)^7 = -2^7 = -128$

d) $det(A^{-1}) = \frac{1}{det(A)} = \frac{1}{-2} = -\frac{1}{2}$

4) a) $\begin{pmatrix} a & b & c \\ d & e & f \\ 4G & 4h & 4i \end{pmatrix} = 4 \cdot \begin{pmatrix} a & b & c \\ d & e & f \\ G & h & i \end{pmatrix} = 4 \cdot -3 = -12$

b) $\begin{pmatrix} a & b & 2c \\ 3d & 3e & 6F \\ g & h & 2i \end{pmatrix} = -2 \cdot 3 \cdot -6 = 18$

÷3 ÷2

c) $\begin{pmatrix} -a & -b & -c \\ d & e & f \\ -g & -h & -i \end{pmatrix} = (-1) \cdot (-1) \cdot -3 = -3$

-1 -1

d) $det(d) = det(A) = -3$

e) $\begin{pmatrix} a & b & c \\ 2d+a & 2e+B & 2F+C \\ G & h & i \end{pmatrix}$ *A soma de uma linha a outra não altera o Det.

·/.2

$2 \cdot det = 2 \cdot -3 = -6$

f) $\begin{pmatrix} Ka+a & Kb+b & Kb+c \\ d & e & f \\ G & h & i \end{pmatrix} = det \cdot (K+1) = -3(K+1)$

·/.(K+1)

5) $\begin{pmatrix} 10 & 8 & 40 & \\ 4 & 6 & 20 & \\ -5 & -7 & -30 & \\ 3 & -6 & -30 & 12 \end{pmatrix}$ $det(A) = 3 \cdot det \begin{pmatrix} 10 & 8 & 40 & -2 \\ 4 & 6 & 20 & -4 \\ -5 & -7 & -30 & 1 \\ 1 & -2 & -10 & 4 \end{pmatrix}$

$\begin{pmatrix} 0 & 28 & 140 & -42 \\ 0 & 14 & 60 & -20 \\ 0 & -17 & -80 & 21 \\ 1 & -2 & -10 & 4 \end{pmatrix}$ $det(A) = 3 \cdot 1 \cdot (-1)^5 \cdot det \begin{pmatrix} 28 & 140 & -42 \\ 14 & 60 & -20 \\ -17 & -80 & 21 \end{pmatrix}$

$\begin{pmatrix} 0 & 20 & -2 \\ 14 & 60 & -20 \\ -17 & -80 & 21 \end{pmatrix} = L_1 - 2 \cdot L_1$

$det = 0 \cdot cof - 20 \cdot det \begin{pmatrix} 14 & -20 \\ -17 & 21 \end{pmatrix} - 2 \cdot det \begin{pmatrix} 14 & 60 \\ -17 & 80 \end{pmatrix}$

$294 - 340 = -46$

$det \begin{pmatrix} 14 & 60 \\ -17 & 80 \end{pmatrix} =$

$det = -20 \cdot (-46) + -2 \cdot 10$

$-1120 + 1020 = -100$ $det = 920 + 200 = 1120$

$Det(A) = 1120 \cdot 3 = 3360$

6) $\begin{vmatrix} 4 & 6 & x \\ 7 & 4 & 2x \\ 5 & 2 & -x \end{vmatrix} = -128$

$4 \cdot (-1)^2 \cdot (-4x - 4x) + 6(-1)^3 \cdot (-7x - 10x) + x \cdot (-1)^4 \cdot -6$

$4 \cdot 8x - 102x + 6x$

$-32 + 102 = 6x$ $64x = -128$ $x = -128/64$ $x = -2$

b) $\begin{pmatrix} 3 & 5 & 7 \\ 2x & x & 3x \\ 4 & 6 & 7 \end{pmatrix}$ $3 \cdot (-1)^2 \cdot (7x - 18x) + 2x \cdot (-1)^3 \cdot (35 - 42) + 4 \cdot (-1)^4 \cdot (8x)$

$3 \cdot 1 \cdot (-11x) + 2x \cdot -1 \cdot (-7x) + 32x$

$-33x + 14x + 32 = 13x = 39$ $x = \frac{39}{13} = 3$

$det = 3$

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C) $\begin{pmatrix} x+3 & x+1 & x+4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{pmatrix} = -7$ $(x+3)\cdot(-1)^2\cdot 5 + x+1\cdot(-1)^3\cdot(28-27)$

$(5x+15) + (-x+1) + x+4\,(-1)^4\cdot -5$

$4x+14 + -5x - 20$

$-x-6 = -7$ $\boxed{x=-1}$

D) $\begin{pmatrix} x & x+2 \\ 1 & x \end{pmatrix}$ $x^2 - x - 2 = 0$ $x_1 = 2$

$x_2 = -1$

e) $\begin{pmatrix} x-4 & 10 & 3 \\ 2 & 0 & x-9 \\ 0 & 3 & 0 \end{pmatrix}$ O det não pode ser 10 e 3

$(x-4)\cdot(x-9) = x^2 - 9x - 4x + 36 = 6$

$-x^2 - 13 + 30 = 0$

$x_1 = -10$

$x_2 = -3$

7-A) $A^{-1} = 1/\det(A)\cdot\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ $\det(A) = ad - (-b\cdot -c)$

$\det(A) \neq 0$

ou seja $ad - (-b\cdot -c) \neq 0$

b) $A^{-1} = \frac{1}{1}\cdot\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

$B^{-1} = \frac{1}{1}\begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$

$(A\cdot B)^{-1} = B^{-1}\cdot A^{-1}$

$A\cdot B = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}\cdot\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 4+35 & -2-21 \\ -2-20 & 1+12 \end{bmatrix}$

$(A\cdot B)^{-1}$ $\begin{bmatrix} 39 & -23 \\ -22 & 13 \end{bmatrix}$

8) $a = \begin{pmatrix} 2 & -2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} (-1)^2 \cdot 1 & (-1)^3 \cdot 3 \\ (-1)^3 \cdot -2 & (-1)^4 \cdot 2 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 2 & 2 \end{pmatrix}$

$b = \begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} (-1)^2 \cdot -3 & (-1)^3 \cdot -1 & (-1)^4 \cdot 1 \\ (-1)^3 \cdot -2 & (-1)^4 \cdot -2 & (-1)^5 \cdot 2 \\ (-1)^4 \cdot -2 & (-1)^5 \cdot 2 & (-1)^6 \cdot 6 \end{pmatrix}$

$b = \begin{pmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{pmatrix}$

9) $a = \frac{1}{8} \cdot ADJ$

$ADJ = \begin{bmatrix} (-1)^2 \cdot 1 & (-1)^3 \cdot 3 \\ (-1)^3 \cdot -2 & (-1)^4 \cdot 2 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 1/8 & -1/4 \\ -3/8 & 1/4 \end{bmatrix}$

b) $\begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{matrix} 2 & -2 \\ 1 & 2 \\ 0 & 1 \end{matrix}$ $\quad \frac{1}{8} \cdot ADJ \quad \begin{pmatrix} (-1)^2 \cdot -3 & (-1)^3 \cdot -1 & (-1)^4 \, 1 \\ (-1)^3 \cdot 2 & (-1)^4 \cdot -2 & (-1)^5 \, 2 \\ (-1)^4 \cdot -2 & (-1)^5 \cdot 2 & (-1)^6 \, 6 \end{pmatrix}$

$0 + 2 + 2 \quad -4 \; 0 \; 0$

$\begin{pmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{pmatrix} \quad B^{-1} \begin{pmatrix} 3/8 & 1/4 & -1/4 \\ -1/8 & 1/4 & 1/4 \\ -1/8 & 1/4 & -3/4 \end{pmatrix}$

c) $\begin{pmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix} \begin{matrix} 0 & -1 \\ 2 & 0 \\ 1 & 1 \end{matrix}$ $\quad C^{T} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{pmatrix} \quad C^{-1} = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ -1/3 & -1/3 & 2/3 \\ 2/3 & -1/3 & 2/3 \end{pmatrix}$

det = 3

D) $\begin{pmatrix} 1 & 0 & 0 & 1 \\ \cancel{0} & \cancel{1} & \cancel{0} & \cancel{0} \\ \cancel{2} & \cancel{0} & \cancel{2} & \cancel{3} \\ 0 & 0 & -1 & 0 \end{pmatrix}$

$Det(D) = 1 \cdot (-1)^3 \cdot -1$

$Det(D) = 1$

$\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ 2 & 2 & 3 & 2 & 2 \\ 0 & -1 & 0 & 0 & -1 \end{array}$

$\underline{D^{-1}} = 1 \cdot \begin{pmatrix} -3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & 2 \end{pmatrix}$

$-2 - (-3) = -1$

$1 \cdot (-1)^2 = -3$ $\quad 0 \cdot (-1)^3 = 0$ $\quad 0 \cdot (-1)^4 = 0$ $\quad 1 (-1)^5 = -2$

$0 \cdot (-1)^3 = 0$ $\quad 1 \cdot (-1)^4 = -1$ $\quad 0 (-1)^5 = 0$ $\quad 0 (-1)^6 = 0$

$2 \cdot (-1)^4 = 0$ $\quad 0 \cdot (-1)^5 = 0$ $\quad 2 (-1)^6 = 0$ $\quad 3 (-1)^7$

$0 \cdot (-1)^5 = 0$ $\quad 0 \cdot (-1)^6 = 0$ $\quad -1 (-1)^7 = 1$ $\quad 0 (-1)^8 = 0$

$\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 3 & 0 & 2 \\ 0 & -1 & 0 & 0 & -1 \end{array}$ $\quad \begin{array}{ccc} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{array}$

-3

$\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ 2 & 2 & 3 & 2 & 2 \\ 0 & -1 & 0 & 0 & -1 \end{array}$ $\quad -1 \quad$ $\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & & \end{array}$

-3

$\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 2 & 0 & 3 & 2 & 0 \end{array}$ $\quad \begin{array}{ccc|cc} 0 & 1 & 0 & 0 & 1 \\ 2 & 0 & 2 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{array}$

$0\ 0\ 0$

$3 - 2$

10) $2A^T - C = XB$ $\qquad B^{-1}$

$B^{-1}(2A^T - C) = X$

$X = B^{-1}(2A^T - C)$ É necessário que "B" seja invertível.

B) $2A^T \begin{pmatrix} 2 & 4 & 0 \\ 0 & 6 & -2 \\ -2 & 0 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 4 & 0 \\ 0 & 8 & -2 \\ -2 & 0 & 6 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & -4 \\ -4 & 0 & -2 \end{pmatrix}$

$B^{-1} = \frac{1}{det} \cdot ADJ^T$ $\qquad ADJ = \begin{pmatrix} -9 & 2 & -6 \\ 12 & 3 & -10 \\ -4 & 0 & 3 \end{pmatrix}$

$B = \begin{pmatrix} 3 & -2 & 6 \\ 2 & -1 & 5 \\ 1 & 0 & 3 \end{pmatrix} \begin{matrix} 3 & -2 \\ 2 & -1 \\ 1 & 0 \end{matrix}$ $\qquad -10-(-6)$

$det = -1$ $\qquad -9-10-(-6-12)$

$3 \cdot (-1)^2 \cdot -3 = -9$

$2 \cdot (-1)^3 \cdot -6 = 12$

$1 \cdot (-1)^4 \cdot -4 = -4$

$-2 \cdot (-1)^3 \cdot 1 = 2$

$-1 \cdot (-1)^4 \cdot 3 = 3$

$6 \cdot (-1)^4 \cdot -1 = -6$

$5 \cdot (-1)^5 \cdot 2 = -10$

$3 \cdot (-1)^6 \cdot 1 = 3$

$B^{-1} \begin{pmatrix} -9 & -12 & 4 \\ 2 & -3 & 0 \\ 6 & 0 & 3 \end{pmatrix}$

$X = \begin{bmatrix} 18 & -24 & 8 \\ -4 & 6 & 0 \\ -12 & 20 & -6 \end{bmatrix}$