$\begin{cases} a+c=3 \\ b-c=7 \end{cases}$ $c=-8$ $\vec{N}=(11,-1,8)$

$a=11$

——— // ——— // ———

Lista 6 GA Guilherme Falcocci 2025.1.08.027

1) $\vec{J}=(1,1,1)$ $\|\vec{J}\| = \sqrt{1+1+1} = \sqrt{3}$

b) $\vec{v} = 3i + 4k$ $\vec{v} = (3,0,4)$

$\|v\| = \sqrt{3^2+4^2}$ $\|v\| = \sqrt{25} = 5$

c) $\vec{v} = -i + j$ $\vec{v} = (-1\ 1\ 0)$

$\|v\| = \sqrt{(-1)^2+1^2} = \sqrt{2}$

d) $\vec{v} = 4i + 3j - k$ $\vec{v} = (4,3,-1)$

$\|v\| = \sqrt{4^2+3^2+(-1)^2} = \sqrt{26}$

2) a) $E = (e_1, e_2, e_3)_E$ é uma base ortogonal pois cada vetor representa uma aresta do cubo, e como a aresta do cubo é unitária, a norma de cada vetor é 1, logo, E é uma base ortogonal.

b) $\vec{V_0} = (0,-1,-1)$ $V_1 = (0,1,-1)$ $V_2 = (1,1,-1)$.

$V_0 = \vec{CD} + \vec{CB} = -e_2 - e_3 = (0,-1,-1)$

$V_1 = \vec{DC} + \vec{CB} = e_2 - e_3 = (0,1,-1)$

$V_2 = \vec{GC} = e_1 + e_2 - e_3 = (1,1,-1)$

c) Todas as normas são 1 após a normalização, portanto F é uma base ortogonal.

$F1 = \vec{V}/\|V\| = \frac{(0,-1,-1)}{-\sqrt{2}} = \left(0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$F2 = \vec{V}/\|V\| = \frac{(0,1,-1)}{\sqrt{2}} = \left(0, \frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$

$F3 = \vec{W} = (1,1,-1)$

$F1 . F2 = 0$ $\quad F1 . F3 = 0$ $\quad F2 . F3 = 0$

d) para ser ortogonal $M^T M = I$. Como F é ortogonal M é ortogonal.

$$M = \begin{bmatrix} 0 & 0 & -1 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \end{bmatrix}$$

$\vec{HB} = B - H = (-1,1,1) = (0, \sqrt{2}, 1)$

em F:

$$HB_F = M^T . HB_e = \begin{bmatrix} 0 & -\sqrt{2}/2 & \sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \\ 1 \end{bmatrix}$$

3) $\vec{AB} = B - A = (3,-3,-6)$ $\quad$ c) $(7/2, 11/2, -1)$

$\vec{BC} = C - B = (-5,-4,4)$ $\quad$ d) 10/19

$\vec{CA} = A - C = (2,7,2)$ $\quad$ e) $\vec{AB} + \vec{BC} + \vec{CA} = (000)$

b) $\|\vec{AB}\| = \sqrt{3^2 + 9 + 36} = \sqrt{54} = 3\sqrt{6}$ $\quad$ ABC é isósceles //

$\|BC\| = \sqrt{(-5)^2 + (-4)^2 + 4^2} = \sqrt{57}$

$\|\vec{CA}\| = \sqrt{2^2 + 7^2 + 2^2} = \sqrt{57}$

4) a) $\vec{U} \cdot \vec{V} = \|U\| . \|V\| . \cos\theta = |U.V| \leq \|\vec{U}\| . \|V\|$

Igualdade ocorre quando o cos = 0/360 ou 180.

b) $\|\vec{U} + \vec{V}\|^2 = \|U\|^2 + 2U.V + \|V\|^2 \Rightarrow$

$2\vec{U} . \vec{V} \leq 2\|\vec{U}\| . \|\vec{V}\|$ $\quad$ $\|\vec{U} + \vec{V}\| \leq \|U\| + \|V\|$

c) $\|\vec{U} + \vec{V}\|^2 = \|U\|^2 + 2.U.V + \|V\|^2$

$\|\vec{U} - \vec{V}\|^2 = \|U\|^2 - 2U.V + \|V\|^2$

$\|U + V\|^2 - \|U - V\|^2 = 4.\vec{U}.\vec{V}$

5) $\theta = \pi/2$ rad

$$\cos\theta = \frac{1.(-2)+0.10+1.2}{\sqrt{2}.\sqrt{108}} = 0 \rightarrow \theta = \pi/2 \text{ ou } 90^\circ$$

b) $\theta = \cos^{-1}(1/3) - 1,23$ rad

$$\cos\theta = \frac{-1+1+1}{\sqrt{3}.\sqrt{3}} = 1/3 \cong 70 \text{ GRAUS}$$

c) $\theta = \pi/4$ rad

$$\cos = \frac{6+3+0}{\sqrt{18}.\sqrt{9}} = 9/3\sqrt{18} = \frac{\sqrt{2}}{2} = \cos 45^\circ$$

d) $\theta = \pi/3$ rad

$$\cos\theta = \frac{3+1+0}{\sqrt{4}.\sqrt{16}} = 4/8 = 1/2 \rightarrow \theta = \frac{\pi}{3} \text{ ou } 60^\circ$$

6) $x = \pm\sqrt{6}$

$(x-1).(x-1)+1.(-1)+2.(-2)=0$

$x^2-1-1-4=0$

$x^2=6 \qquad x = \mp\sqrt{6}$

b) $x=-2$

$x.4+x.x+4.1=0$

$x^2+4x+4=0$

$(x+2)^2=0$

$x=-2$

7) $\vec{v} = (1,-1,-1)$

$$\vec{v}\times\vec{w} \begin{vmatrix} i & \vec{j} & \vec{k} \\ 4 & -1 & 5 \\ 1 & -2 & 3 \end{vmatrix} = i(-3+10)-j(12-5)+k(-8+1) = (7,-7,-7)$$

$\vec{v} = k(7-7-7)$

$\vec{v}.(1,1,1) = 7k-7k-7k = -7k = -1 \quad k = 1/7$

$\vec{v} = (1,-1,-1)$

b) $\vec{v} = (3,-3,-3)$

$$v\times w = \begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 2 & -4 & 6 \end{vmatrix} = i(18-4)-j(12+2)+k(-8-6) = (14-14-14) \quad \vec{v} = k(14-14-14)$$

$\|\vec{v}\| = |k|\sqrt{14^2+(-14)^2+(-14)^2}$

$|k|\sqrt{588} = |k|.14\sqrt{3} = 3\sqrt{3} \quad |k| = 3/14 \rightarrow (3,-3,-3)$

c) $\theta = \arccos(4/\sqrt{26})$

$$\cos\theta = \frac{4}{\sqrt{6\sqrt{10}}.\sqrt{6\sqrt{10}}} = \frac{4}{\sqrt{36-10}} = 4/\sqrt{26}$$

8) $proj\,\vec{v} = (18/11, -6/11, 6/11)$

$\vec{u}.\vec{v} = 3+1+2 = 6 \,;\, \|\vec{u}\|^2 = 9+1+1 = 11$

$proj\,\vec{v} = 6/11(3,-1,1) = (18/11, -6/11, 6/11)$

b) $proj\,\vec{v} = (0,0,0)$

$u.v = -3+3+0=0 \,;\, \|\vec{u}\|^2 = 9+1 = 10$

$proj\,\vec{v} = 0/10,\, m=0$

c) $proj\ \vec{v} = (-10/9, 5/9, 10/9)$

$u \cdot \vec{v} = 2+1+2 = 5 \quad \|\vec{v}\|^2 = 4+1+4 = 9$

$proj\ v = 5/9\ (-2, 1, 2) = (-10/9, 5/9, 10/9)$

d) $proj\ \vec{v} = (1, 2, 4)$ //

$\vec{v} = -2(1,2,4)$ $\vec{u}$ e v SÃO PARALELOS.

9) $proj\ \vec{v}\ (4 - 4\ 2)$

$u \cdot v = 6 + 12 + 0 = 18$; $\|v\| = 4+4+1 = 9$

$proj\ \vec{v} = 18/9\ (2, -2, 1) = (4, -4, 2)$

• $proj\ \vec{v} = (6/5, -12/5, 0)$

$\vec{u} \cdot \vec{v} = 18$; $\|\vec{v}\| = 9 + 36 = 45$

b) $\vec{p} = (4, -4, 2)$

$\vec{p} = proj\ \vec{v} = (4, -4, 2)$

• $\vec{q} = (-1, -2, -2)$

$q = \vec{v} - \vec{p} = (3, -6, 0) - (4, -4, 2)$

$q = (-1, -2, -2)$

c) $\begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 3 & -6 & 0 \end{vmatrix} = i(0+6) - j(0-3) + k(-12+6)$

$(6, 3, -6)$

$\|u \times \vec{v}\| = \sqrt{36+9+36} = \sqrt{81} = 9$ //

10) a) $\begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 5 & 4 & 0 \end{vmatrix} = k(12-15) = -3k = 0, 0, 3$

b) $\begin{vmatrix} i & j & k \\ 7 & 0 & -5 \\ 1 & 2 & -1 \end{vmatrix} = \vec{i}(0+10) - \vec{j}(-7+5) + \vec{k}(14-0)$

$= (10, 2, 14)$

$\|\vec{v} + \vec{u}\| = \sqrt{100+4+196} = \sqrt{300} = 10\sqrt{3}$

c) $\vec{v} \times \vec{v}$ $\begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ 1 & 1 & 4 \end{vmatrix} = i(-12-1) - j(4-1) + k(1+3)$

$(-13, -3, +4)$

$\|v \times u\| = \sqrt{169+9+16} = \sqrt{194}$

d) $\vec{v} \times \vec{v} = (0, 0, 0)$ NORMA = 0

v e $2\vec{v}$ → SÃO PARALELOS

11) $u \cdot v = \|u\| \cdot \|v\| \cdot \cos\theta$

$\|u \times \|v\| = \|u\| \cdot \|v\| \cdot sen\theta$

$sen^2 + \cos^2 = 1 \therefore \|\vec{u} \times \vec{v}\|^2 = \|v\|^2 \|u\|^2 (1 - \cos^2\theta)$

$\|u\|^2 \|v\|^2 - (\vec{u} \cdot \vec{v})^2$ //

b) $\|u \times v\| = \sqrt{16} = 4$ //

$\|u \times v\|^2 = 1^2 \cdot (5)^2 - (3)^2 = 25 - 9 = \sqrt{16}$

c) $\|AB \times AC\| = l \cdot l \cdot sen\ 60° = l^2 \cdot \frac{\sqrt{3}}{2}$ //

12) $x \times (i + j - k) = \begin{vmatrix} i & j & k \\ a & b & c \\ -1 & 1 & -1 \end{vmatrix} = (-b-c)i - (a+c)j + (a+b)k$

$$\begin{cases} -b-c=-2 \to b=1 \\ a-c=0 \to a=c=1 \\ a+b=2 \to c+b=2 \end{cases} \qquad x=\vec{i}+\vec{j}+\vec{k}$$

b) $\begin{Bmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ 1 & 0 & 1 \end{Bmatrix} \begin{matrix} (y)i-(x-z)j+(-y)k. \\ (y, z-x, -y) \\ (1,1,-1)=(2,2,-2) \end{matrix}$

$$\begin{cases} y=2 \\ z-x=2 \\ -y=-2 \end{cases} \qquad \vec{x}=(-1,2,1)_{//}$$

c) $x=(-1,-1,-1)$

$x \cdot u=-3x+0y+3z=0 \to -x+z=0 \to x=z$

$x \cdot v=2x-2y+0z=0 \to x-y=0 \to x=y$

$x=(x,x,x)$

13) $AD=D-A=(5-3,\ 3-2,\ 3-(-1))=(2\ 1\ 4)$

$AB \times AD \begin{Bmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & 1 & 4 \end{Bmatrix} \begin{matrix} =(4+1)i-(4+2)\cdot j+(1-2)k \\ =(5-6-1) \\ =\sqrt{5^2(-6)^2(-1)^2}=\sqrt{62} \end{matrix}$

b) $AB \times AC=\begin{Bmatrix} i & j & k \\ -1 & 1 & 0 \\ 0 & 1 & 3 \end{Bmatrix} \begin{matrix} =(3-0)i-(-3-0)j+(-1-0)k \\ =(3,3,-1) \\ =\sqrt{3^2+3^2+1^2}=\sqrt{19} \end{matrix}$

$At=\sqrt{19}/2$

$h=\sqrt{19}/\sqrt{10}=\frac{\sqrt{19}\cdot\sqrt{10}}{10}=\frac{\sqrt{19}}{10}_{//}$

$\|bc\|=\sqrt{1+0+3^2}=\sqrt{10}_{//}$

14) $v\cdot(v \times w)=\begin{Bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{Bmatrix}=\vec{u}\cdot(\overrightarrow{v\times w})=(\overrightarrow{u\times v})\cdot w$

b) $[u,w,v]=-[u,v,w]=0$

$[v,2w,\vec{u}]=2[v,w,u]=0$

$[u,3v-2u,w+3j]=0$

$\begin{Bmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ -1 & 2 & 0 \end{Bmatrix} \begin{matrix} =(-1)i+2(0\cdot 2(-1,-1)) \\ =1\cdot 4-3\cdot 2+2\cdot 1=4-6+2=0_{//} \end{matrix}$

15) $\begin{Bmatrix} i & j & k \\ 1 & 0 & 1 \\ 1 & 3 & 4 \end{Bmatrix} \begin{matrix} =i(0\cdot 4-1\cdot 3)-j(1\cdot 4-1\cdot 1) \\ =k(1\cdot 3-0\cdot 1)=(-3,-3,3) \\ \|AB\times AD\|=\sqrt{-3^2+3^2+3^2}=\sqrt{27}=3\sqrt{3} \end{matrix}$

b) $\begin{Bmatrix} i & j & k \\ 2 & 2 & 2 \\ 3 & 5 & 6 \end{Bmatrix} \begin{matrix} =i(2\cdot 6-2\cdot 5)-j(2\cdot 6-2\cdot 3)+k(2\cdot 5-2\cdot 3) \\ =(12-10,(-12-6),(10-6)=(2,-6,4) \\ =1\cdot 2+0\cdot -6+1\cdot 4=6_{//} \end{matrix}$

c) $6=3\sqrt{3}\cdot h \to h=\frac{6}{6\sqrt{3}}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}_{//}$

D) $V=1/6\cdot 6=1_{//}$ e) $\frac{1/3}{2/\sqrt{5}}=\frac{1}{3}\cdot\frac{2}{\sqrt{5}}=\frac{2}{3\sqrt{5}}=\frac{2\sqrt{5}}{15}_{//}$