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1) $A = \begin{pmatrix} 1 & 0 \\ 2 & 7 \end{pmatrix} + \begin{pmatrix} 0 & 10 \\ 6 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 10 \\ 8 & 3 \end{pmatrix}$

$B = \begin{pmatrix} 1\cdot0+0\cdot3 & 1\cdot5 \;\; 0\cdot(-2) \\ 2\cdot0+7\cdot3 & 2\cdot5+7\cdot(-2) \end{pmatrix} - \begin{pmatrix} 0\cdot1+5\cdot2 & 0\cdot0+5\cdot7 \\ 3\cdot1+2(-2) & 3\cdot0+-2\cdot7 \end{pmatrix}$

$\begin{pmatrix} 0 & 5 \\ 21 & -4 \end{pmatrix} - \begin{pmatrix} 10 & 35 \\ -1 & -14 \end{pmatrix} = \begin{pmatrix} -10 & -30 \\ 22 & 10 \end{pmatrix}$

C) Não é possivel ORDEM ≠

D) $\begin{pmatrix} -6 & 2 & -4 \\ 4 & 2 & 0 \\ 0 & 8 & 4 \end{pmatrix} - \begin{pmatrix} 6 & -3 & -18 \\ 12 & 0 & 0 \\ -9 & -12 & -3 \end{pmatrix} = \begin{pmatrix} -12 & 5 & 14 \\ -8 & 2 & 0 \\ 9 & 20 & 7 \end{pmatrix}$

E) $\begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 11 & -4 & 8 \\ -10 & 3 & 12 \\ 2 & -4 & 4 \end{pmatrix}$

$\begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -8 & -12 & 1 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{pmatrix}$

$\begin{pmatrix} 3 & -16 & 9 \\ -33 & 7 & 1 \\ -14 & -12 & 8 \end{pmatrix}$

2×3 3×1

F) $C = \begin{pmatrix} -2 & 7 \\ 3 & -3 \\ -7 & -2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} -2+14 & 0+49 \\ 3+(-6) & 0+-21 \\ -7+(-4) & 0-14 \end{pmatrix}$

$F = \begin{pmatrix} 12 & 49 \\ -3 & -21 \\ -11 & -14 \end{pmatrix}$

G) Não foi possível ordem ≠ (AC)

H) $\begin{pmatrix} 1 & -2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 5 \end{pmatrix}$

I) $\begin{pmatrix} 0 & 5 \\ 3 & -2 \end{pmatrix} \cdot \left(\begin{array}{c|c|c} -2 & 3 & -7 \\ 7 & -3 & -2 \end{array}\right) = BC = \begin{pmatrix} 35 & -15 & -10 \\ -20 & 3 & -17 \end{pmatrix}$

$F = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ $BCF = \begin{pmatrix} 35+(-15 \cdot -2)+(-10 \cdot 0) \\ (-20 \times 1)\ (15 \times -2)\ (-17 \times 0) \end{pmatrix} = \begin{pmatrix} 65 \\ -50 \end{pmatrix}$

2) A) 2×4 BA = Ñ.
B) 4×2 BA = Ñ
C) Ñ BA = Esta definido
D) 5×3 BA = Ñ
E) Ñ BA = Ñ
F) 4×4 BA = Esta definido
G) 2×3 BA = Ñ
H) 2×2 BA = Esta definido

3) $a = \begin{pmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \end{pmatrix}$

$B = \begin{pmatrix} 4 & -1 & -2 \\ 3 & 8 & 1 \\ 8 & 7 & 12 \end{pmatrix}$

$C = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$

$D = \begin{pmatrix} 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 6 & 12 & 18 & 24 \\ 8 & 16 & 24 & 32 \end{pmatrix}$

4) $A = 20$ $\quad e = -7$

$B = -32$ $\quad f = 10$

$C = 16$ $\quad G = -13$

$D = 3$

$BA_{23} = (2 \cdot 1) + (-1 \cdot 2) + (4 \cdot 5) = 20$

$AB_{23} = (-2 \cdot 3) + (-3 \cdot 4) + (2 \cdot -7) = -32$

$B^2_{31} = (-3 \cdot 1)(-1 \cdot 2)(-7 \cdot -3) = 16$

$[A]_{Tr} = 6 - 3 = 3$

$[B^3]_{Tr} = -1 + 1 - 7 = -7$

$[A-B]_{Tr} = 0 + (-2) + 12 = 10$

$(A:B) = (1 \cdot 1 + 2 \cdot 2 + -3 \cdot 1) + (-2 \cdot 0 + -3 \cdot -1 + 2 \cdot -1) + (1 \cdot 3 + 4 \cdot 4 + 5 \cdot 7) = 2 + 1 + (-16) = -13$

5) A) $2X = 3B + C - A$ $\quad 3B = \begin{pmatrix} 6 & 3 \\ 12 & 9 \end{pmatrix}$ $\quad C - A = \begin{pmatrix} 1 & -5 \\ -1 & -6 \end{pmatrix}$

$X = \begin{pmatrix} 3,5 & -1 \\ 5,5 & 1,5 \end{pmatrix}$ $\quad 2X = \begin{pmatrix} 7 & -2 \\ 11 & 3 \end{pmatrix}$

B) $B - C = \begin{pmatrix} 2 & -1 \\ 3 & 3 \end{pmatrix}$ $\quad BC^t = \begin{pmatrix} 2 & 3 \\ -1 & 3 \end{pmatrix} = BC^t/2 \begin{pmatrix} 1 & 1,5 \\ -0,5 & 1,5 \end{pmatrix}$

$Y = \begin{pmatrix} 1 & 1,5 \\ -0,5 & 1,5 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} 2 & -5,5 \\ -2,5 & -4,5 \end{pmatrix}$

C)

$X = (B - A)/4$ $\quad B - A \begin{pmatrix} 3 & -6 \\ 2 & -3 \end{pmatrix}$ $\quad X = \begin{pmatrix} 3/4 & -6/4 \\ 2/4 & -3/4 \end{pmatrix}$

d) $2X = 3A + 2B + C$ $\quad 3A = \begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix}$ $\quad 2B \begin{pmatrix} 4 & 2 \\ 8 & 6 \end{pmatrix}$

$2X = \begin{pmatrix} 1 & 25 \\ 15 & 24 \end{pmatrix}$ $\quad X = \begin{pmatrix} 1/2 & 12,5 \\ 7,5 & 12 \end{pmatrix}$

$-Y = \begin{pmatrix} 4 & 4 \\ 9 & 6 \end{pmatrix} - X = -Y =$

$-Y = \begin{pmatrix} 3,5 & -8,5 \\ 1,5 & -6 \end{pmatrix}$ $\quad Y = \begin{pmatrix} -3,5 & 8,5 \\ -1,5 & 6 \end{pmatrix}$

6) $A^2 = A \cdot A = \begin{pmatrix} 1 + 1/x \cdot x & 1/x + 1/x \\ x + x & 1 + 1/x \cdot x \end{pmatrix} = \begin{pmatrix} 2 & 2/x \\ 2x & 2 \end{pmatrix}$

$A^2 = 2A$

$A^3 = A \cdot 2A = 2 \cdot A^2$

$A^3 = 2 \cdot (2A) = 4A$

$A^4 = A \cdot A^3 \quad A \cdot 4A = 4(2A) = 8A$

$A^n = 2^{n-1}$

7) a) $A(B+C) = AB + AC = X + Y$

b) $(BA)^T = X^T$

c) $(CA)^T = Y^T$

d) $(AB)AC = X \cdot Y$

8) $A^T = \begin{pmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{pmatrix}$ $\quad 2x-3 = x+2$ $\quad A = \begin{pmatrix} 4 & 7 \\ 7 & 6 \end{pmatrix}$

$2x = x+5$

$2x - x = 5$

$\boxed{x = 5}$

b) $-B = \begin{pmatrix} 0 & 4 & -2 \\ -x & 0 & -1+z \\ -y & -2z & 0 \end{pmatrix}$ $\quad B^T \begin{pmatrix} 0 & x & y \\ -4 & 0 & 2z \\ 2 & 1-7 & 0 \end{pmatrix}$

$x = -(-4) = 4$

$y = -2$

$2z = -(1-z)$

$2z = -1 + z$

$2z - z = -1$

$z = -1$

$3z = -1 + z + 3$

$-z + 3z = 2$

$2z = 2$

$\boxed{z = 1}$

9) $\begin{pmatrix} 3x & 3y \\ 3z & 3t \end{pmatrix}$ $\quad x + 4 = 3x$ $\quad 3y = 6 + 2 + y$ $\quad 3t = 2^t + 3$

$4 = 3x - x$ $\quad -y + 3y = 8$ $\quad -2t + 3t = 3$

$2x = 4$ $\quad 2y = 8$ $\quad \boxed{t = 3}$

$\boxed{x = 2}$ $\quad \boxed{y = 4}$

10) $R = \begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix} \cdot R^T \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix}$

$\begin{pmatrix} \cos\theta^2 + \operatorname{sen}\theta^2 & -\operatorname{sen}\cdot\cos + \operatorname{sen}\cdot\cos \\ \operatorname{sen}\cdot\cos + \operatorname{sen}\cdot\cos & \cos\theta^2 + \operatorname{sen}\theta^2 \end{pmatrix}$

$\cos^2 + \operatorname{sen}^2 = 1$

$\operatorname{sen}\cos\theta - \operatorname{sen}\cos\theta$

0

$R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ = é ortogonal

b) $A = \begin{pmatrix} 1 & 0 & x \\ 0 & 1/\sqrt{2} & y \\ 0 & 1/\sqrt{2} & z \end{pmatrix} \cdot A^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ x & y & z \end{pmatrix} = \begin{pmatrix} 1+x^2 & x\cdot y & x\cdot z \\ x\cdot y & 1/\sqrt{2}^2 y^2 & 1/\sqrt{2}\cdot y\cdot z \\ z\cdot x & 1/\sqrt{2}^2 y\cdot z & 1/\sqrt{2}^2 \cdot z^2 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$x = 1 + x^2 = 1$ $\quad x = 0$

$y = 1/2 + y^2 = 1$ $\quad y = \sqrt{2}/2$

$z = 1/2 + z^2 = 1$ $\quad z = \sqrt{2}/2$

$y = z$

$\frac{1}{2} + z^2 = 1$

$\frac{1 + 2z^2}{2} = 1$

$1 + 2z^2 = 2$

$2z^2 = 2 - 1$

$z^2 = 1/2$

$z = \sqrt{1}/\sqrt{2}$

$z = 1/\sqrt{2}$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$