

Mathematical Formulation of the N-bodies Problem

$$\boxed{y'(t)=g(t,y)}$$

With

$$y(t)=[r_1 \dots r_N \, v_1 \dots v_N]^T \quad \text{and} \quad g(t,y)=[v_1 \dots v_N \, f_1 \dots f_N]^T$$

Where we have

$$f_k = \sum_{j=1, j \neq k}^{j=N} G m_j \frac{r_j - r_k}{|r_j - r_k|^3}$$

Numerical Method

Explicit Euler's method

$$y_{t_{k+1}} = y_{t_k} + \Delta t \, g(t_k, y_{t_k})$$

4-th order Runge-Kutta method

$$y_{t_{k+1}} = y_{t_k} + \frac{\Delta t}{6} (Y_1 + 2Y_2 + 2Y_3 + Y_4)$$

$$Y_1 = g(t_k, y_{t_k})$$

$$Y_2 = g\left(t_k + \frac{\Delta t}{2}, y_{t_k} + \frac{\Delta t}{2} Y_1\right)$$

$$Y_3 = g\left(t_k + \frac{\Delta t}{2}, y_{t_k} + \frac{\Delta t}{2} Y_2\right)$$

$$Y_4 = g(t_k + \Delta t, y_{t_k} + \Delta t Y_3)$$