CELLULAR AUTOMATA

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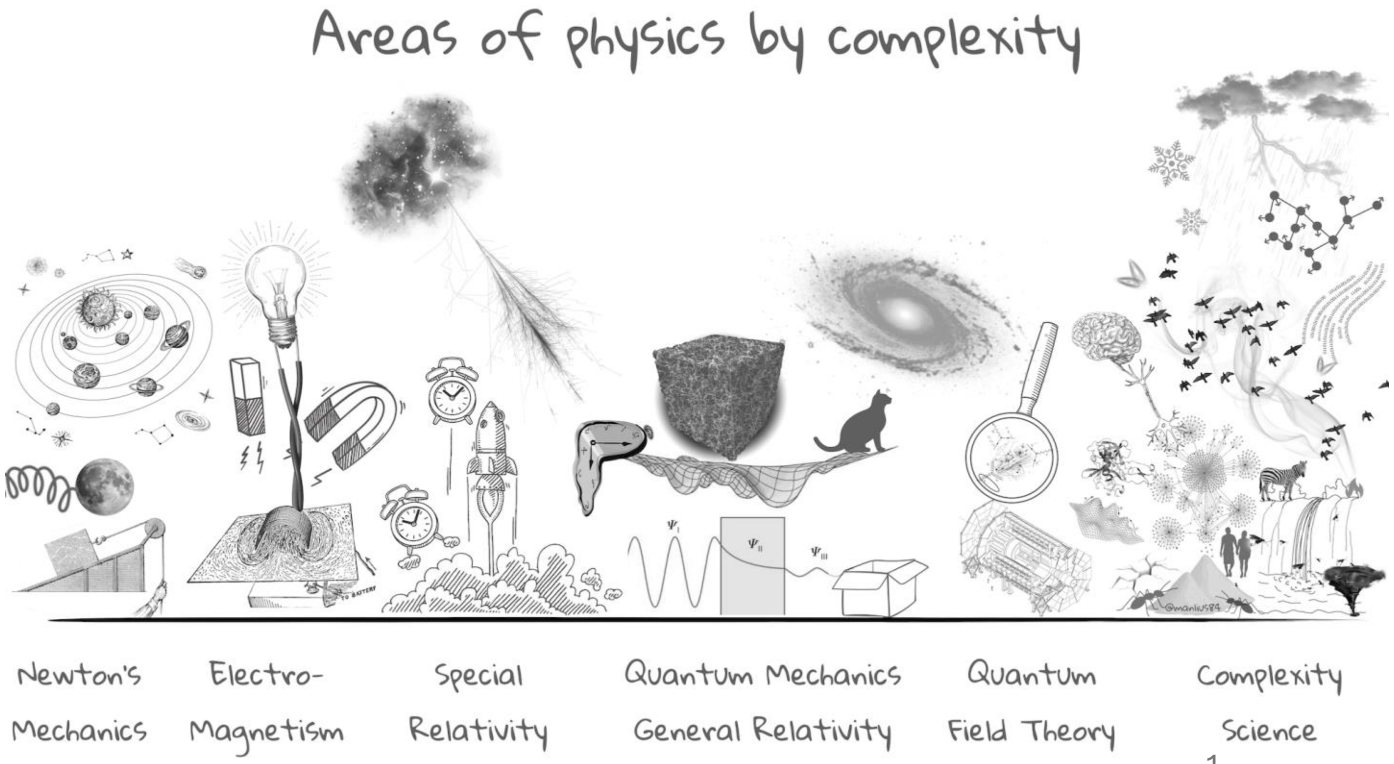

CONTENTS

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INTRODUCTION

- First example of DP model
- Binary state (0,1) in a lattice configuration
- Can be classified as deterministic (CA) or stocastic (PCA)
- Each one is characterized by **LOCAL, MARKOVIAN AND SYNCHRONOUS RULES**.

CELLULAR AUTOMATA
AND PROBABILISTIC VERSION

CA and PCA, also called Domany-Kinzel Model, are different in the description of the previous rules

In the Domany-Kinzel model that rules are described through **CONDITIONAL PROBABILITY**

$$P(s_i(t+1)|s_{i-1}(t), s_{i+1}(t))$$

CELLULAR AUTOMATA AND PROBABILISTIC VERSION

- CA and PCA, also called Domany-Kinzel Model, are different in the description of the previous rules

- In the Domany-Kinzel model that rules are described through **CONDITIONAL PROBABILITY**

- Only one absorbing state, the inactive one !

FOCUS ON DK MODEL

p, probability of open bond, is the Control Parameter

Order Parameters Density of Active States **ρ**, Probability, **P**, of not yet reached the absorbing state

FOCUS ON DK MODEL

Order Parameters
Density of Active
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$$\rho(t) = \frac{1}{L}\sum_{i=1}^{L} < s_i(t) >$$

$$P(t) =< 1 - \prod_{i=1}^{L}(1 - s_i(t)) >$$

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FOCUS ON DK MODEL

- Once establish the critical exponents, we can prove if the model belongs to the **DP universality class**

$$\xi_{\parallel} = (p - p_c)^{-\nu_{\parallel}} \qquad P(p) = (p - p_c)^{\beta'} \qquad N(t) \sim t^{\theta}$$

$$\xi_{\perp} = (p - p_c)^{-\nu_{\perp}} \qquad \rho(p) = (p - p_c)^{\beta} \qquad \theta = \frac{d\nu_{\perp} - \beta - \beta'}{\nu_{\parallel}}$$

$$z = \frac{\nu_{\parallel}}{\nu_{\perp}}$$

- Introduction of correlation, ξ, lengths for expansion in space and survival time of the cluster
- Z, dynamical exponent
- N(t) number of active sites at time t
- Hyperscaling relation, θ

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GENERALIZATION OF DK

- Mendonça and de Oliveira perform a combination of simple interactional rules
- p182-q200 with p+q=1
- New absorbing states: the fully inactive one and the fully active
- They prove that the model belongs to the DP universality class

GENERALIZATION OF DK

MODEL STRUCTURE
Finite Lattice configuration with PBC and size L

MODEL STRUCTURE
Binary model with 0 (extinction) or 1(survival) states.

Table 1. Rule table for PCA $p182$–$q200$, $p+q=1$. The first row gives the initial neighbourhood, and the other two rows give the final state reached by the central bit of the initial neighbourhood with the probability given at the leftmost column. Clearly, the configurations $00\cdots0$ and $11\cdots1$ are absorbing configurations of the PCA.

	111	110	101	100	011	010	001	000
p	1	0	1	1	0	1	1	0
q	1	1	0	0	1	0	0	0

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ANALYSIS

- First description through Mean Field approximation
- Computational one through Monte Carlo simulation
- Final goals: establish the critical exponent of the model

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Mean Field

$$P_{t+1}(\eta') = \sum_{\eta} W(\eta'|\eta) P_t(\eta)$$

- Probability evolution of the system state
- Depending on the conditional probability **W** and actual probability

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Mean Field → n=1

$$P_t(\eta_{\ell-1}, \eta_\ell, \eta_{\ell+1}) \approx P_t(\eta_{\ell-1})\, P_t(\eta_\ell)\, P_t(\eta_{\ell+1})$$

- Solution strictly depend on the approximation of consecutive sites **n**
- By that we can extract density of active states

ANALYSIS

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Mean Field

n=1

$$\rho^{(1)} = (\rho^{(1)})^3 + (2-p)(\rho^{(1)})^2(1-\rho^{(1)}) + 3p\rho^{(1)}(1-\rho^{(1)})^2$$

- Solution of density active sites at stationary state
- This combination depends on the rules described in table 1

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Mean Field

n=1

$$\rho^* = 1 \quad \rho^* = 0 \quad \rho^* = \frac{3p-1}{4p-1}$$

- The firsts indicate the absorbing states, the latter one the active region.

ANALYSIS

- The main result of Mean Field Approximation is in the furher table

Table 2. Critical parameter $p_c^{(n)}$ and value of the density of active sites $\rho^{(n)}(p)$ at $p = 1$ obtained by mean-field approximations or order n and Monte Carlo simulation (cf section 4). The numbers between parentheses (in this table and elsewhere in this paper) indicate the uncertainty in the last digit(s) of the data. For $p = 1$, PCA $p182$–$q200$ reduces to CA 182, for which the exact stationary density $\rho_{182}^{\text{exact}} = 3/4$.

n	$p_c^{(n)}$	$\rho^{(n)}(p = 1)$
1	1/3	2/3
2	1/3	2/3
3	0.4015	0.7135
4	0.4203	0.7166
MC	0.488 10(5)	0.7500(5)
Exact	NA	3/4

- Slowly convergence of the density active sites when we increase the number of consecutive sites **n**

ANALYSIS

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Monte Carlo simulation

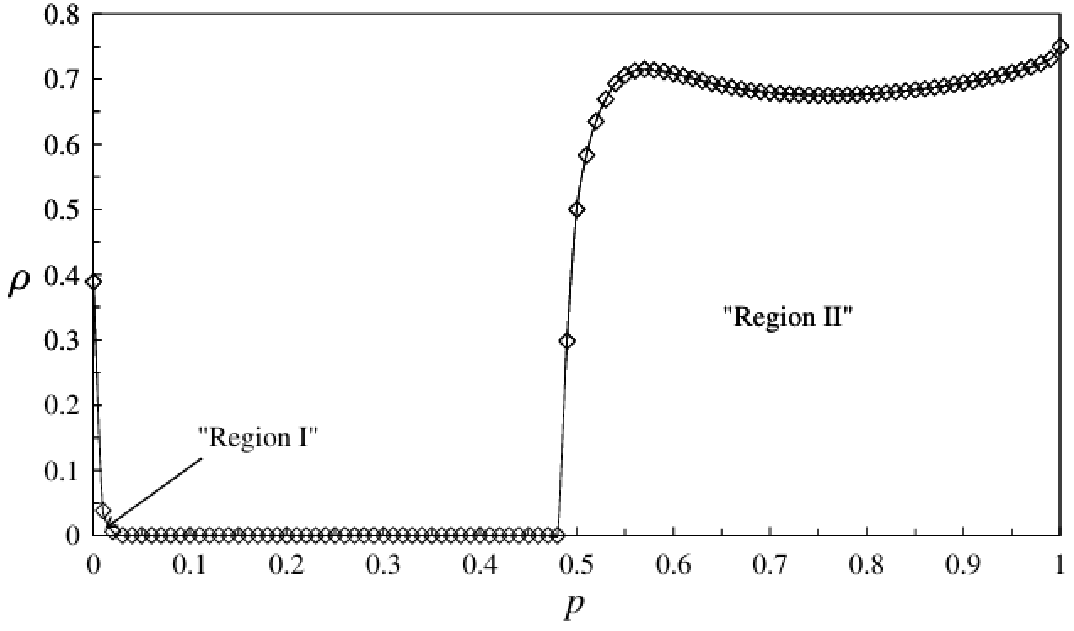

Figure 2. Density $\rho_L(p)$ of active sites for a lattice of $L = 8000$ sites initialized randomly with density 1/2. Each value of $\rho_L(p)$ is an average over 10^6 samples after relaxation through $\sim L^2$ MCS. Errors in the Monte Carlo data are similar to the ones reported in figure 1.

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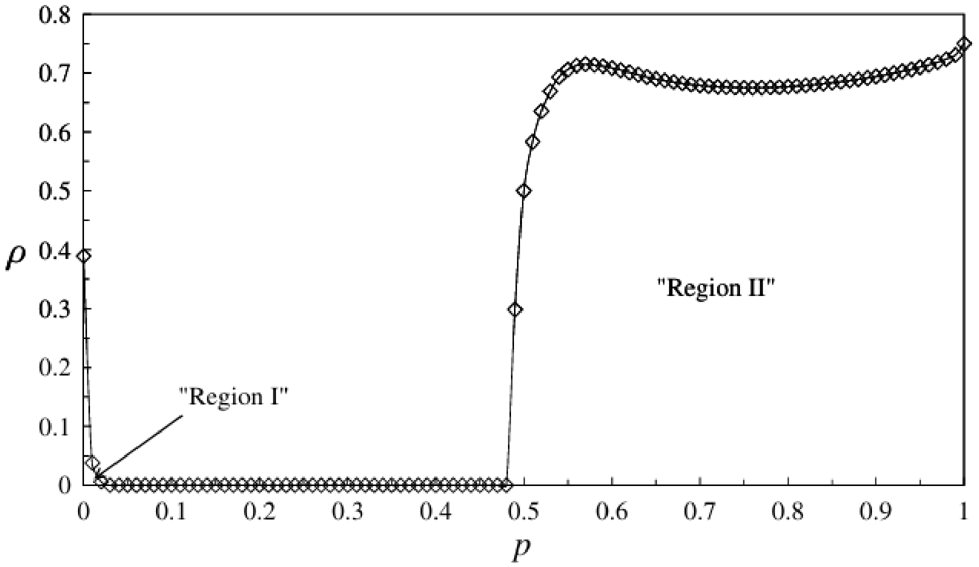
ANALYSIS

- The main result of Monte Carlo simulation is



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- 2 active regions, region I is an artifact due the initial configuration
- "bump" effect for p = 0.57 due the competion of 111 cluster and annihilation effect from boundaries in region II
- **NONMONOTONIC** behaviour of the density profile
- Extintion-survival phase transition for p = 0.48

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RESULTS

- First determination of the critical point through Linear Regression is done in further picture

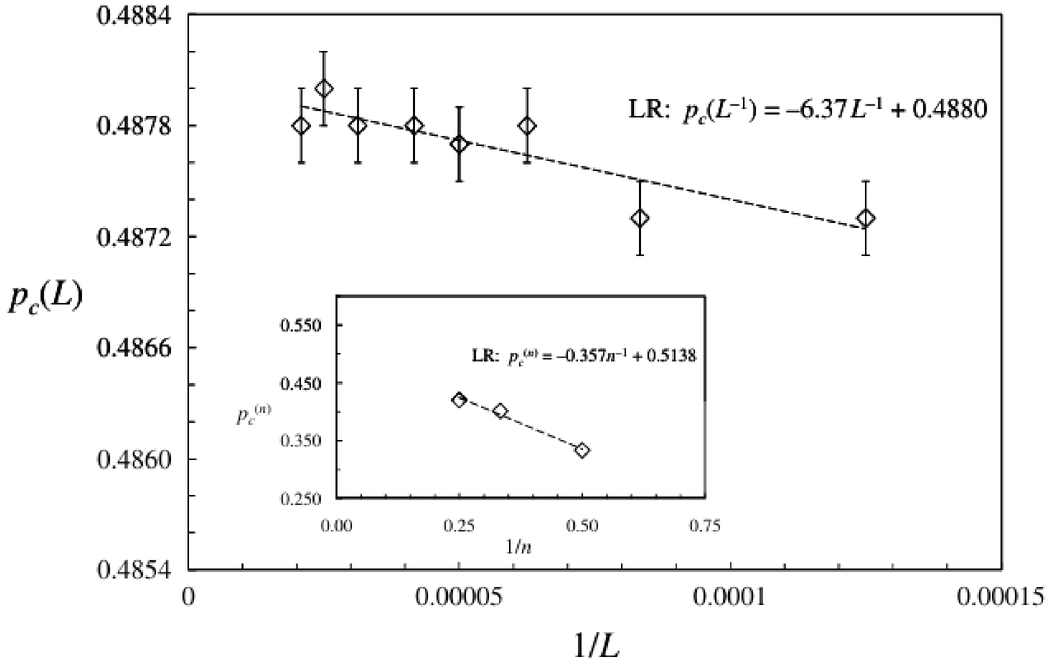

Figure 4. Critical points $p_c(L)$ for several lattice lengths $8000 \leqslant L \leqslant 48\,000$. A linear fit gives $p_c(L^{-1}) = (-6.37 \pm 1.40)L^{-1} + (0.4880 \pm 0.0001)$ with an $R^2 = 0.776$. The inset exhibits data from the mean-field approximations of orders $n = 2$, 3, and 4.

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RESULTS

- Another determination of the critical point Pc = 0.48810(5) through the plot in figure (a)
- Then they extract the best δ=0.17(1) critical exponent from figure (b) at Pc = 0.4881

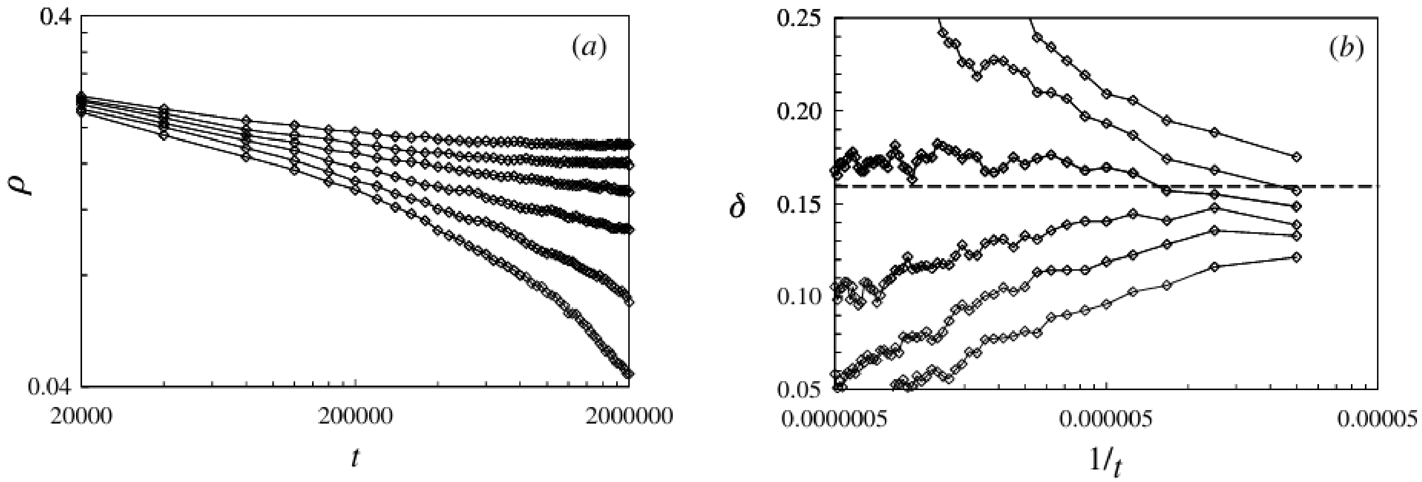

Figure 5. (a) Logarithmic plot of $\rho_L(t)$ at fixed ε with $L = 16\,000$ averaged over 1000 samples. From the lowermost curve upwards, $p = 0.4879$, 0.4880, 0.4881, 0.4882, 0.4883, and 0.4884. From this dataset we estimated $p_c = 0.488\,10(5)$. (b) Instantaneous values of δ obtained from the data depicted in figure 5(a). In panel (b) the uppermost curve corresponds to $p = 0.4879$. From the curve with $p = 0.4881$, we estimated $\delta = 0.17(1)$. The dashed line corresponds to the best value available for δ_{DP}.

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RESULTS

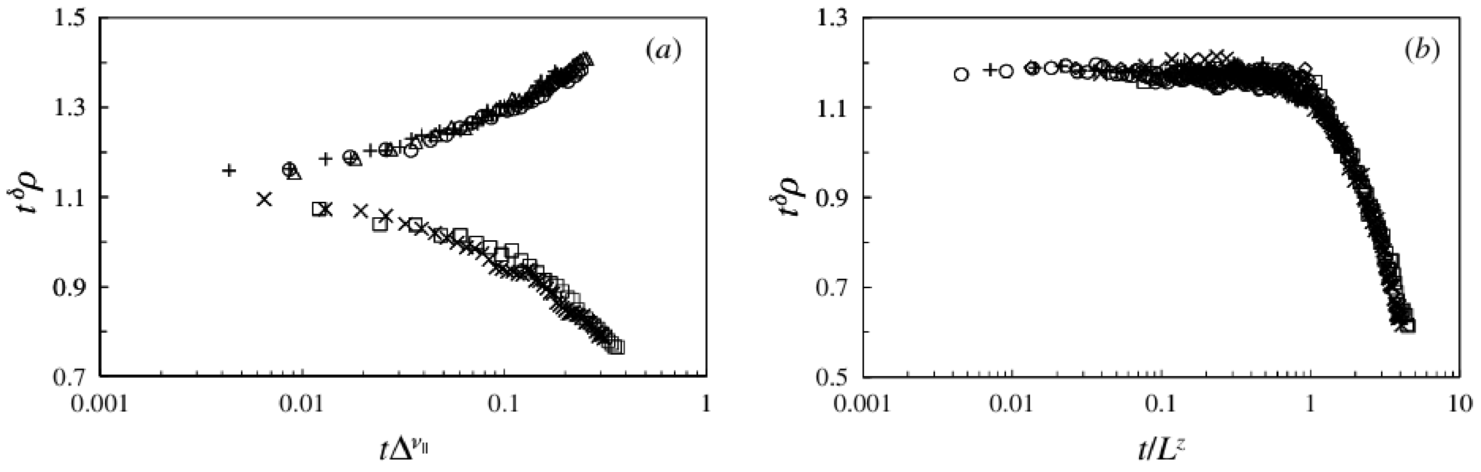

Figure 6. (*a*) Data collapse of the scaled time-dependent density profiles for $\varepsilon = \pm 0.0001$, ± 0.0002, and 0.0003. The upper (lower) branches correspond to $p > p_c$ ($p < p_c$). The best data collapse was obtained with $p_c = 0.488\,12$, $\delta = 0.16$, and $\nu_\parallel = 1.70$. (*b*) Finite-size data collapse of the scaled time-dependent density profiles on the critical point $p_c = 0.4881$ for $2000 \leqslant L \leqslant 16\,000$. Best data collapse was obtained with $\delta = 0.165$ and $z = 1.55$.

- In order to establish the other critical exponents, they study critical behaviour for ρ and extract data collapsing for different ε, they tune respectively ν|| and z.

- In figure (a) they obtain the best determination of δ = 0.16(5) and ν|| = 1.70(5) for Pc = 0.48812, by them extract β = δν|| = 0.29(2)
- In (b) the best determination is got for Pc = 0.4881 where δ = 0.165 and z = 1.55(5)

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RESULTS

- Finally we can compare their results with respect the theoretical ones for DP critical exponents

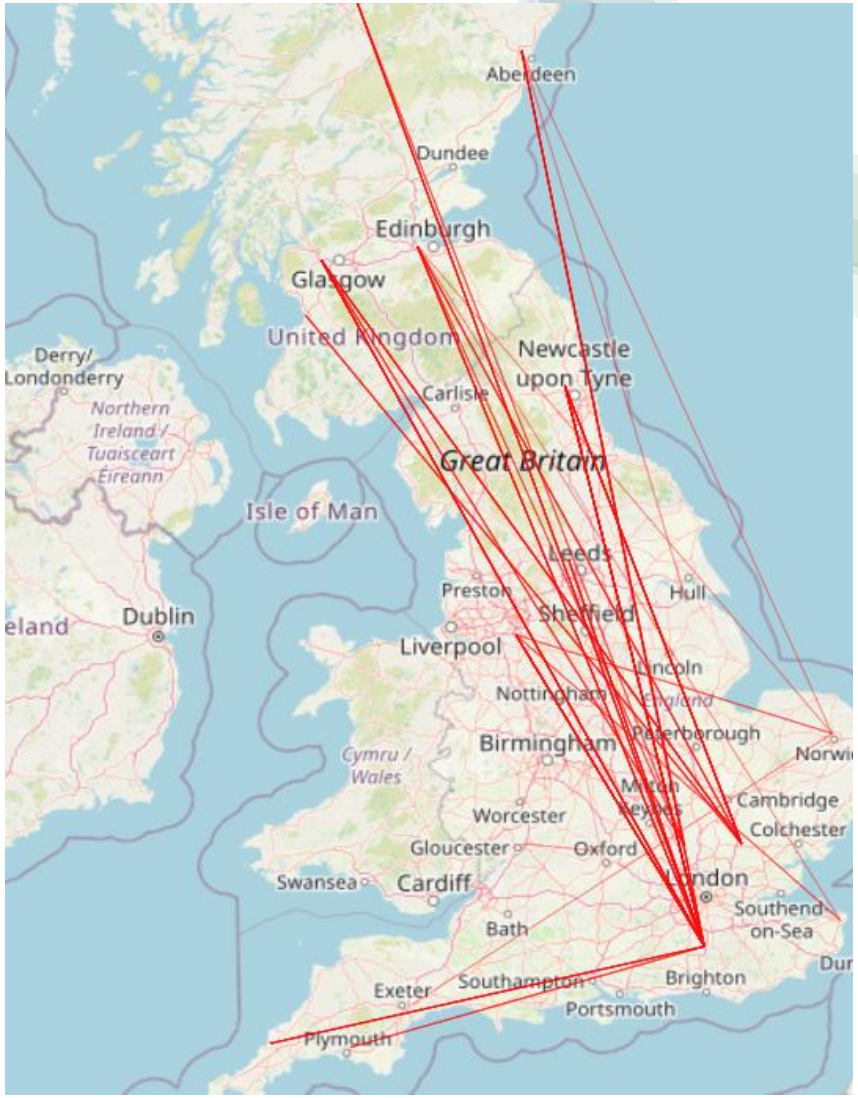
δ^{DP}	0.159464(6)
$\nu_{\parallel}^{DP}$	1.733847(6)
β^{DP}	0.276486(8)
z^{DP}	1.580745(10)

DP critical exponents expected critical exponents

- We can conclude that the model proposed belongs to the DP universality class

PERCOLATION ON COMPLEX NETWORK

- Fundamental application for the P is found in the Resilience and Robustness behaviour of Complex Network



Simulation UK air transport system before and after attacking the London one. (degree removal approach)

Result: diminishing of the Greatest Connected Component

Data from: The multilayer temporal network of public transport in Great Britain
Gallotti, Riccardo[1]; Barthelemy, Marc[1]

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THANKS FOR THE ATTENTION