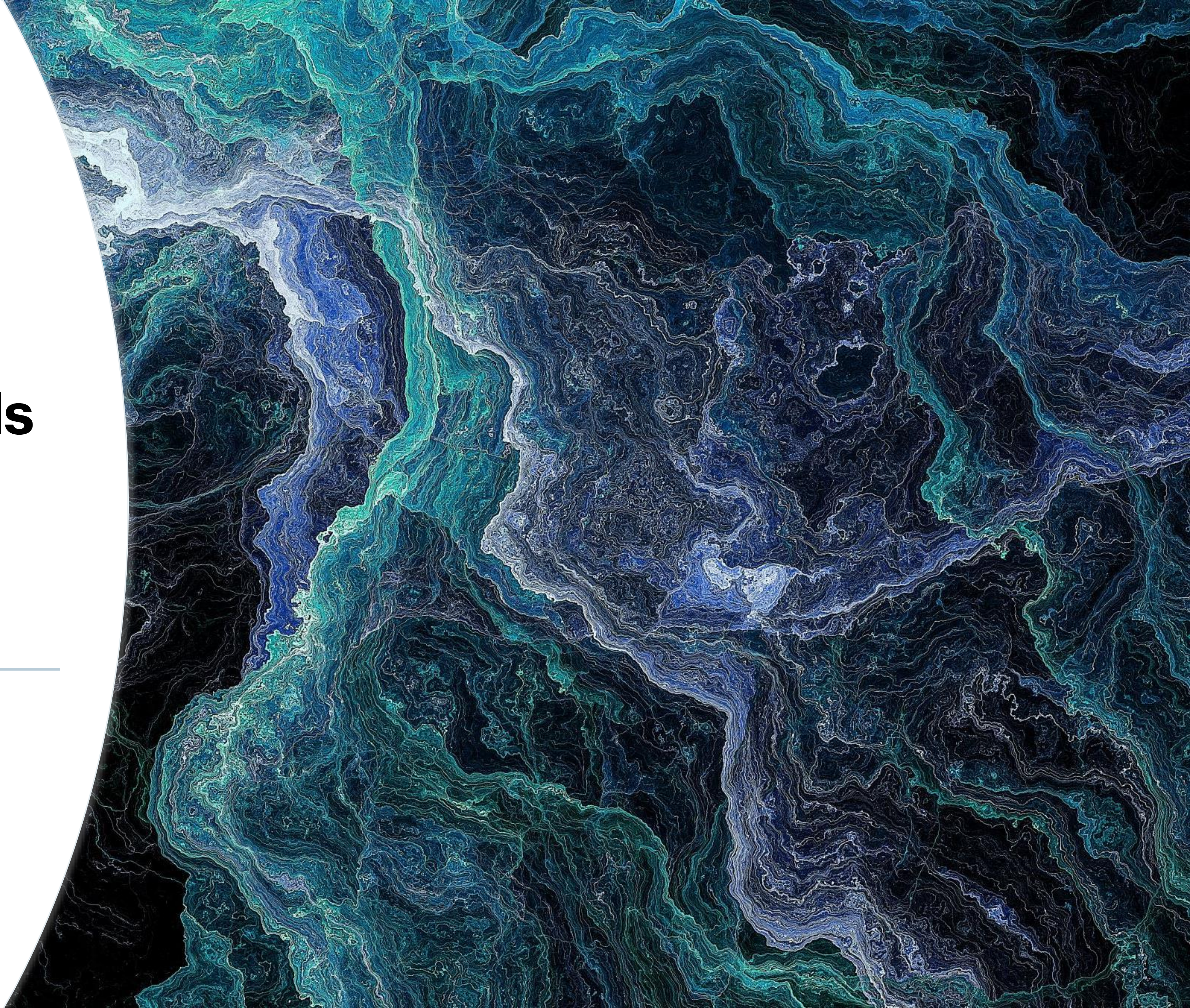
A correspondance between the models of Hodgkin-Huxley and FitzHugh-Nagumo revisited



INTRODUCTION

- The first point is a description of H-H model and the modify of 4 EDOs through the introduction of a new model, Rinzel (V, W variables)

- From this reduction we can compare Rinzel with to FHN model where the first one is a two-variables model and the last one is a cubic formulation

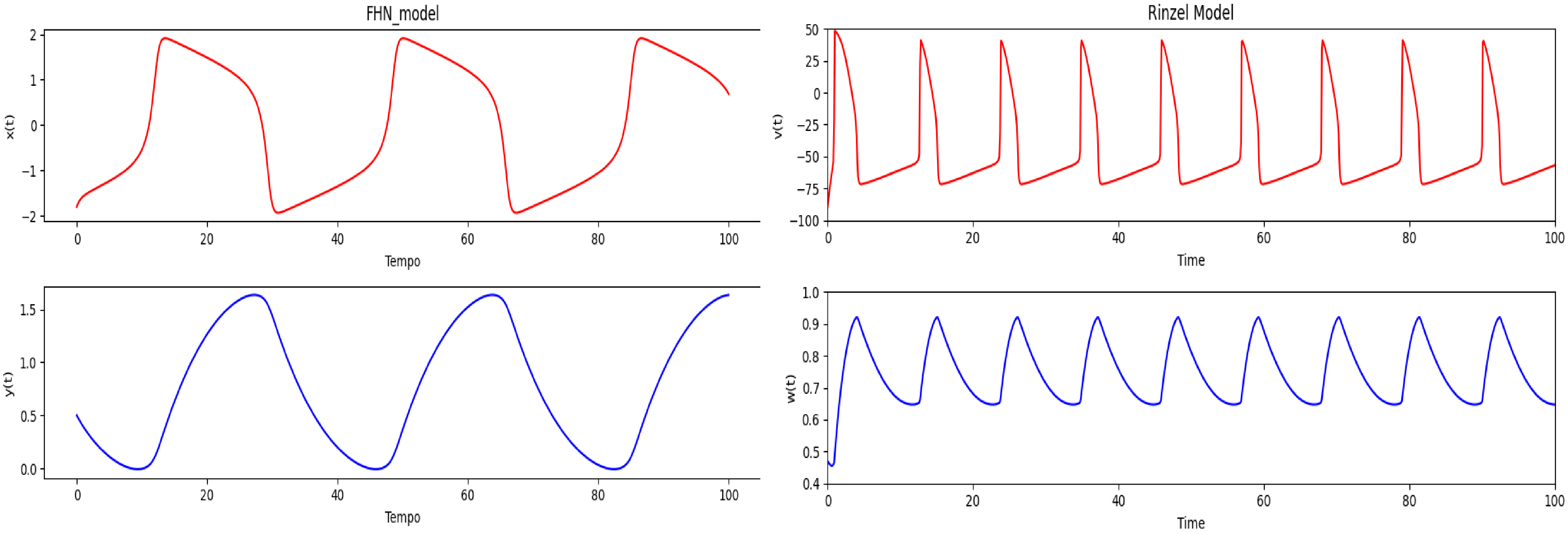
$$\frac{dv}{dt} = I - g_{Na}(V - V_{Na})(1-w)m_{\infty}^{3} - g_k(\frac{w}{S})^4(V - V_k) - g_l(v - vk)$$

$$\frac{dw}{dt} = \epsilon \frac{W_{\infty} - W}{\tau}$$

$$\frac{dx}{dt} = x - \frac{x^3}{3} - y + z$$

$$\frac{dy}{dt} = \epsilon(a + x - by)$$

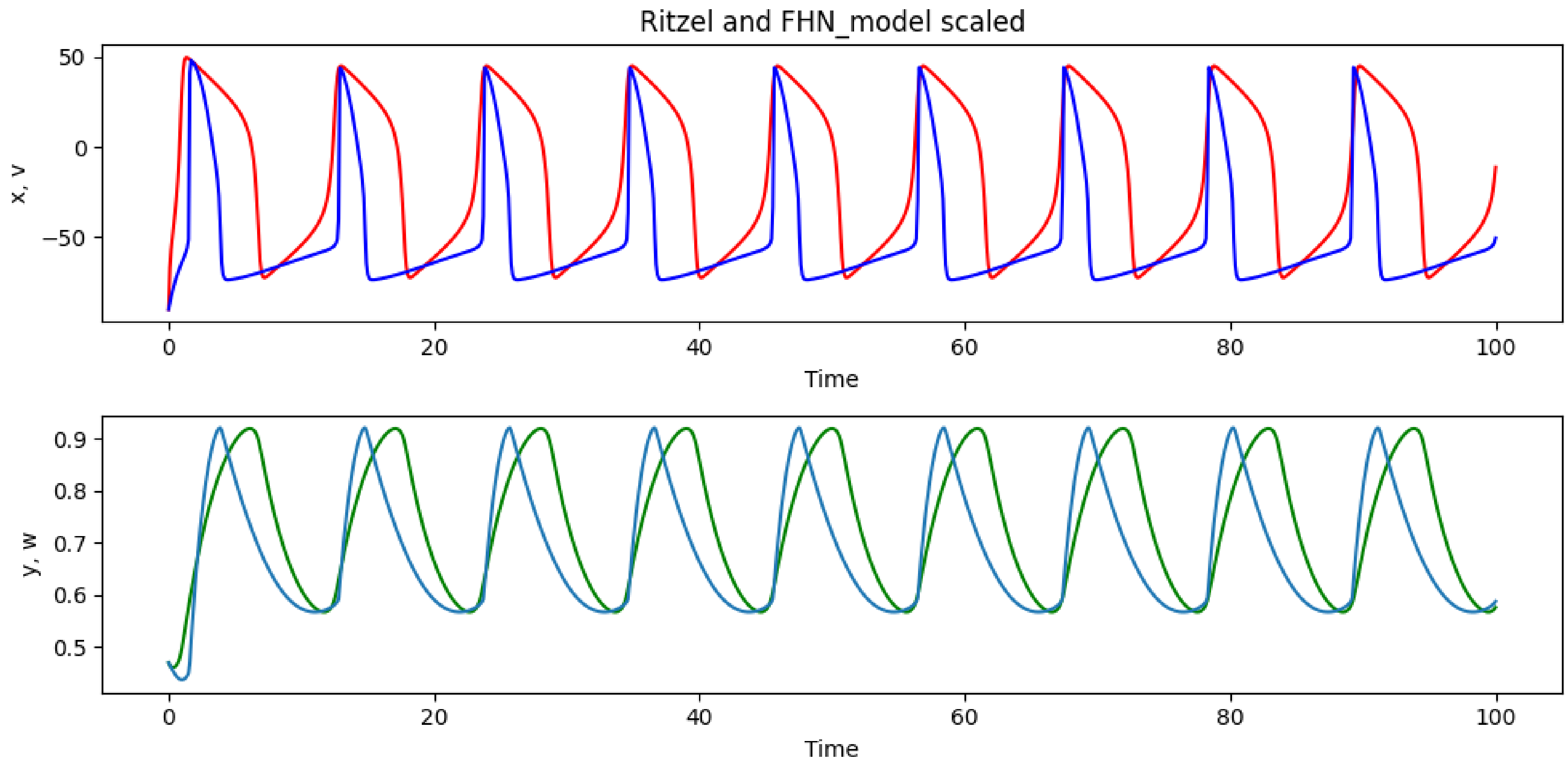
RINZEL AND FHN MODELS



Between two models we can see different scale and we can insert some parameters to equalize them.
Here we can see a difference in width and magnitude for both variables w and y (recovery variables) and x, v (description of spikes dynamics).

COMPARISION RINZEL AND FHN

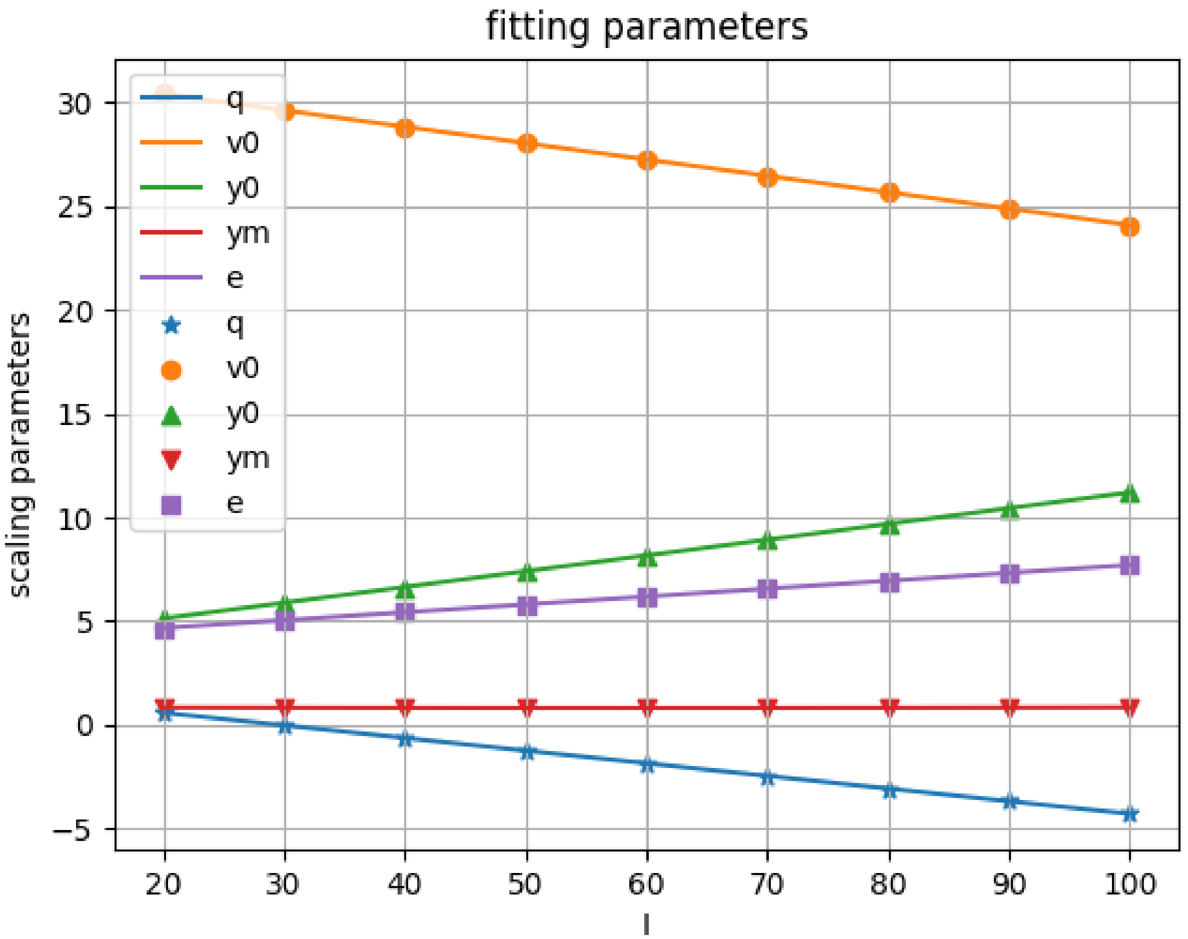
- To equalize the models we need introduction of some parameters: x0, y0, ym, E, v0
- From that we get same scale between two models



Here we see a shift effect between two models but we can correct it

SCALING PARAMETERS

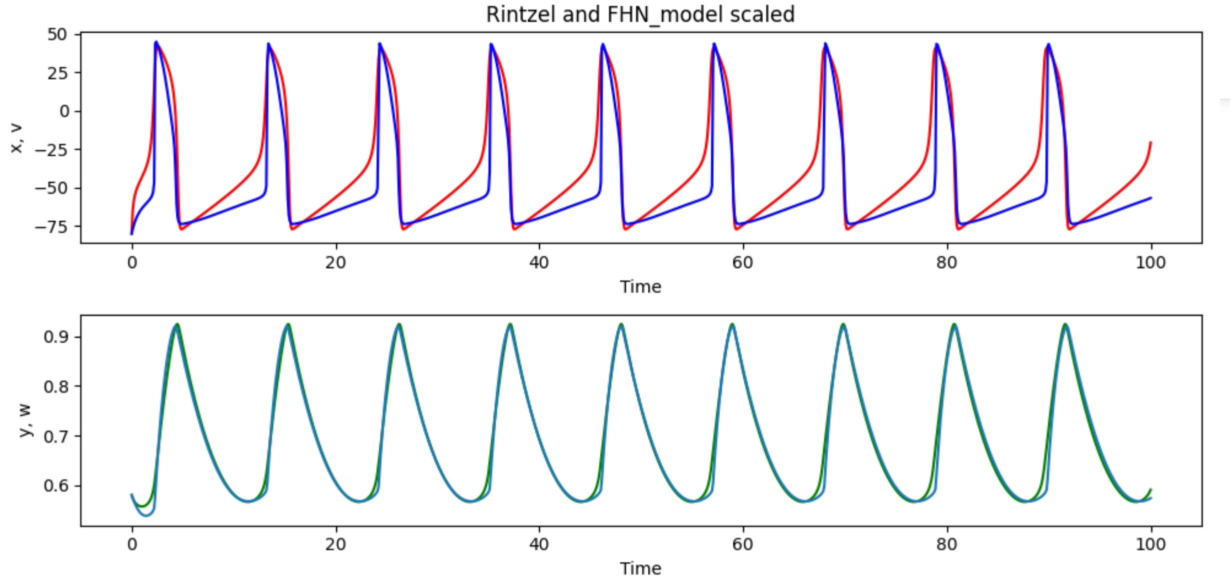
- To reduce the shift we can study the parameters and from fitting procedure we get the right formulation varying the input current:

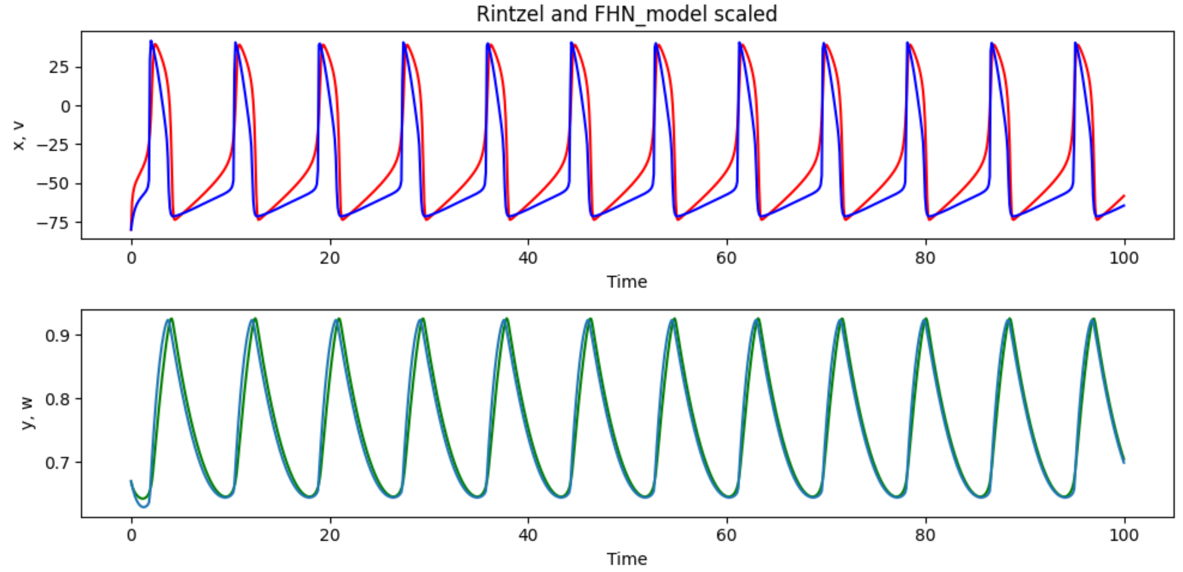


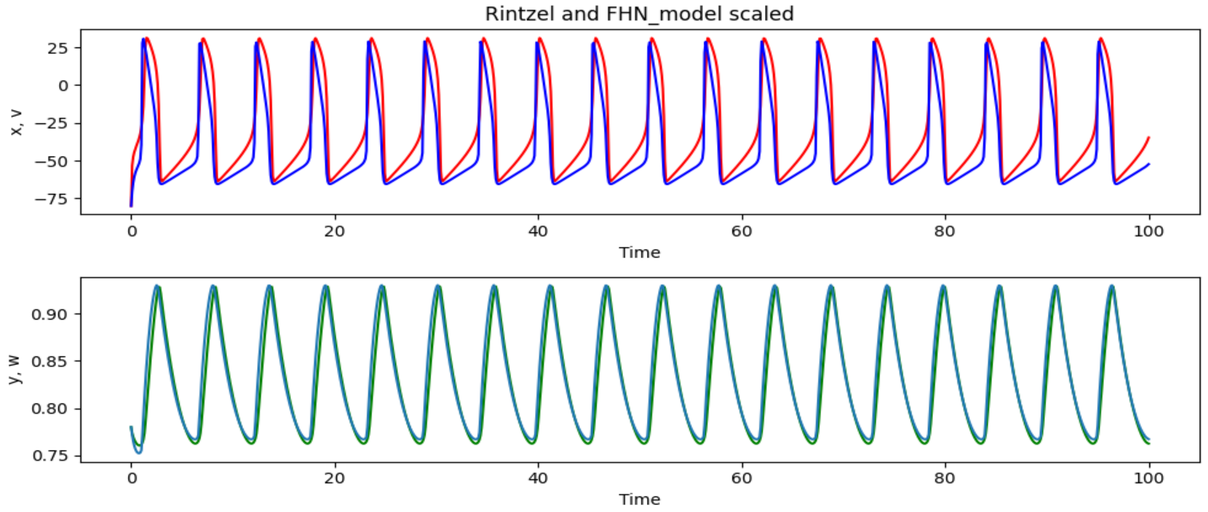
$$z(I) = 1/\left[\exp(-0.061I + 1.8) + 1\right] - 1,$$
$$v_0(I) = -0.079I + 32,$$
$$y_0(I) = (0.076I + 3.6)^{-1},$$
$$y_m(I) = 1.3 \cdot 10^{-5} I^2 - 0.0015I + 0.85,$$
$$\epsilon(I) = 0.038I + 3.9.$$

APPLICATION OF PARAMETERS

- Thanks the previous procedure we can see an accurancy correspondence in recovery variables (w, y)







Input current and variation of variables with I= 20, 40, 100 nA

We can explain the difference and similarities between these configuration through the phase plane

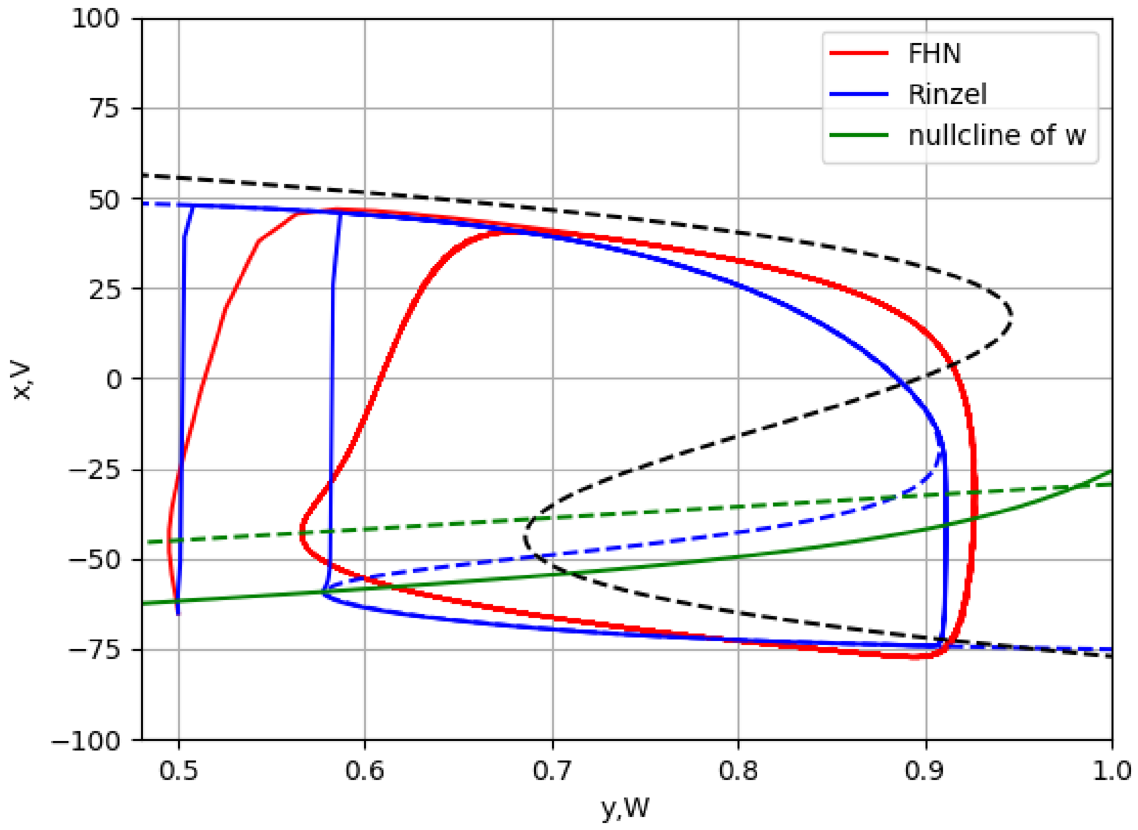
ANALYSIS OF PHASE PLANE

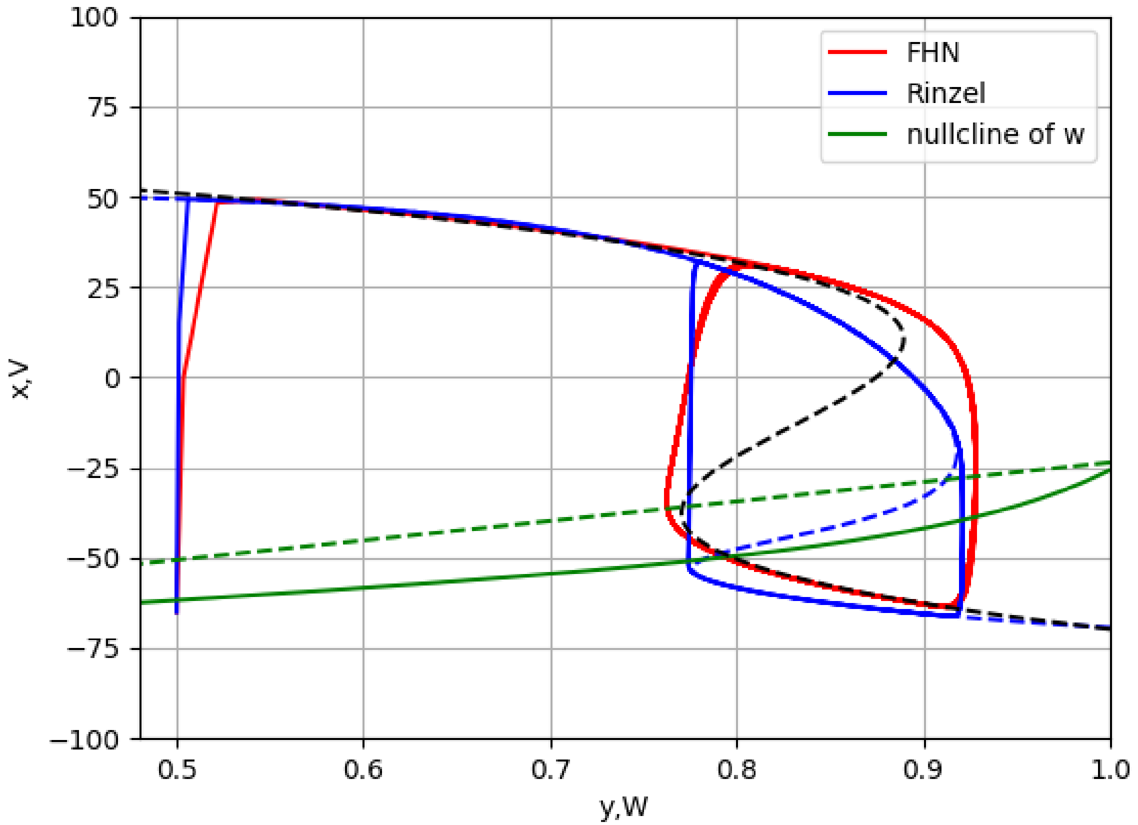
On the phase plane we can study nullclines of the two models and get:

- Asymmetry of y-nullclines
- Antisymmetry of FHN cubic parabola
- Parallelism between W and y nullclines at low voltages
- Curvature of W not so strong at high voltage values

Correspondence between W and Y
Difference between V and X

The phase plane is current-dependent, where the dashed black curve is nullcline of X , the blue one is nullcline of V, and the green one is nullcline of Y





CONCLUSION

- From this workframe we can see a substitution of H.H model with FHN that it is simplier than the first one here we don't have 4 EDOs but only two
- We can describe the spikes dynamics as the HH model has done
- From equalization between Y-nullclines and two point we get the unknown parameters in Y-nullclines that tell us why the scaling parameters are current-dependent
- We can use the FHN model for the NEUROVASCULAR DYNAMICS mediated by glia

$$y = \frac{y_0}{v_0}\left[(x - x_0)\left(1 - \frac{(x - x_0)^2}{3v_0^2}\right) - y_m\frac{v_0}{y_0} + zv_0\right]$$

$$w = S\sqrt[4]{\frac{I - g_l(v_{Na} - v_l)}{g_K(v_{Na} - v_K)}},$$

$$w = \frac{I - g_{Na}(v_K - v_{Na}) - g_l(v_K - v_l)}{g_{Na}(v_K - v_{Na})}$$

NEUROVASCULAR DYNAMICS

FHN model can apply in the analysis of neurovascular dynamics with some difference parameters with respect to the previous studies.