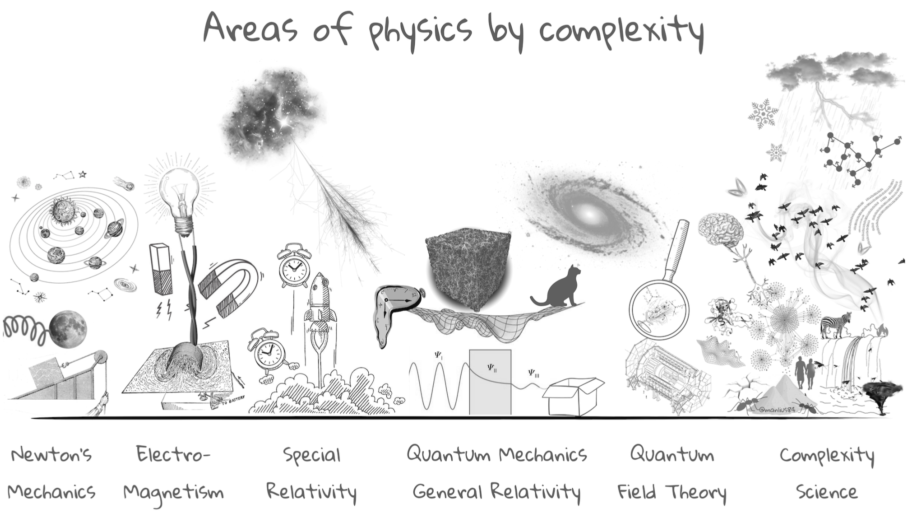
UNIVERSITÀ DEGLI STUDI DI PADOVA



Project # 9

Gabriele Fiaschi

Team name: Sociophysics

Last update: September 14, 2024

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1 | Voter model

1.1 | Introduction

Sociophysics is one of the topics where we can see the application of the complex network model like Voter model.
In this report I treat about the features of this model, how they change due the presence of dimensionality of the system and the presence of the disorder to establish the localization of critical point.
After that it's been explored the dynamical change for q-voter models, extending the previous model to different dynamics.

1.2 | The "Classical" Voter Model

Voter model is strongly applied in sociodynamical behaviour, particularly in the decision behaviour.
In fact we have an "ensemble" of voters that they can decide between two options, physically we can describe this condition as a spin's system where which one is associated to values: $\sigma = \pm 1$
The nodes of the network, the voters, are associated to the spin state, so the first step for this model is the elementary choice of an opinion for one selected node, after that we select a neighbor of the first one and finally the interaction consists on the copy of the same "opinion" or state for one of them. (here the selection's process, for node and its neighbor, happens randomly)
This kind of system is defined as binary-system, where the state or votants in this case can choose between two options.
Here physics explains how people can copy to reduce the uncertainty bewteen two valid options.
The system is characterized by the presence of transition probability for changing the state of single node's state:

$$P(\sigma_i \to -\sigma_i) = \frac{1}{2}(1 - \frac{\sigma_i}{k_i} \sum_{j \in N_i} \sigma_j) \tag{1.1}$$

The system has two absorbing states $\sigma = 1$ or $\sigma = -1$, for analysing the whole system we can use specific order parameter, that in this case is **average interface density**, ρ.
It's defined as the density of links that are connecting different spin 's state in our network, it's described by:

$$\rho = \frac{\sum_{i=1}^{N} \sum_{j \in V_i} \frac{1-\sigma_i \sigma_j}{2}}{\sum_{i=1}^{N} k_i} \tag{1.2}$$

where V_i is the set of neighbours of node i, and k_i is the degree of the selected node. Now we can distinghuish disordened and ordered system, through the value of ρ. Values around:

1. $\rho = \frac{1}{2}$ random disorder
2. $\rho << 1$ large spatial domain
3. $\rho = 0$ ordended system

Starting from initial random condition can be detected the evolution of ρ, particularly, the temporal evolution of the density highlights the kinetics of the ordering process; this kind of process is strictly related to the dimension of the system, in fact for $d \leq 2$ the main result is the ordering phase of the system, instead for $d > 2$ we lose this effect. Studying ρ's dynamics, we can see that it initially decreases,indicating the presence of partially ordened system, but after initial transcient ρ starts fluctuating around an average plateu, ξ. This is a measure of partial order, a metastable state situating between the absorbing states, and it links to the average linear size.
The metastable state is characterized by finite lifetime and we can extracted it from the ρ's average behaviour.

$$\langle \rho \rangle \propto \xi e^{\frac{-t}{\tau}}$$

where τ is the lifetime of the metastable state.
Here we see that thanks the presence of fluctuations ρ changes from its initial metastable plateau value to $\rho = 0$, featuring the absorbing state.

1.3 | Disorder and dimensionality

In previous section it's been mentioned the role of dimensional structure of the system. Now we can focus on the dimensionality, the role of **disorder** and the **power law degree distribution** , fundamental future for scale free network.
So all of this is been studyied on "synthetic" network as **Barabasi-Albert** network thanks its degree distribution. This kind of network highlights the similar behaviour for level of ordering and presence of plateau ξ already described in the previous chapter,but at the same time is been evidenced that the total ordering is archieved for lower dimensionality.

1.3.1 Role of disorder

The key role of the disorder is completely described by the quantity p, probability of connection between nodes, that tell us the randomness of the system and we can see its behaviour when it is increased, in fact the system approaches toward the behaviour of Barabasi-Albert network in tha case. This limiting behaviour is fundamental for the description of the scale free network, particularly scale-free degree distribution feature, that resembles the real world network.

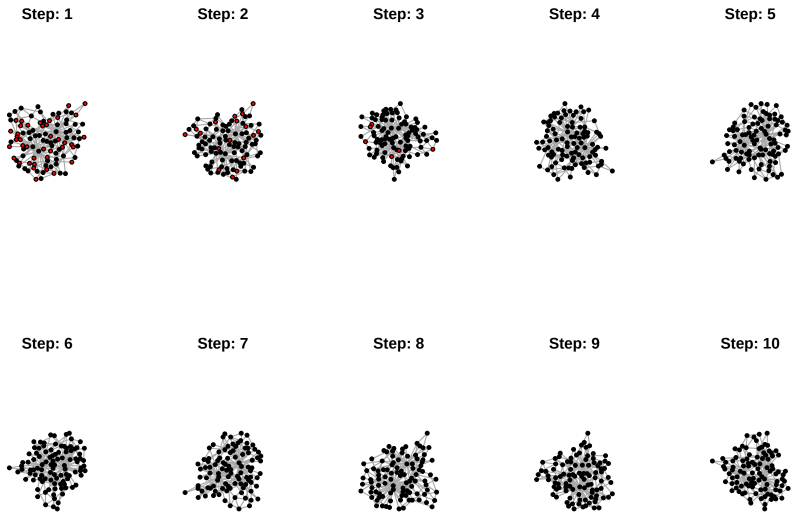


Figure 1.1: Evolution on 2D barabasi with fr layout
black nodes indicate state 1 the red one state -1

1.3.2 Role of degree distribution

We can compare multiple evolutions in networks with scale free distribution and evolutions in equivalent networks but involving single state's degree distribution. Here our interest is focused on the mean interface density where we can see that for finite size fluctuations, ρ escapes from meatastable state to reach the absorbing one, getting the order phase of the system that is more efficient when hubs are present in the networks.

1.4 | Results and applications

In this section I've analysed the previous results numerically.
Important results for the voter model dynamics are that model is affected by the topology of the networks, in fact this feature highlights the different interctions between nodes and as conseguence different dynamical approach for the voter model.
We' ve seen that dimension influence how the system's ordened or disordened, tuned by parameter p, that influences too.
Already in figure 1.1 we can see how evolves ρ quantity, as we can see in this table 1.1

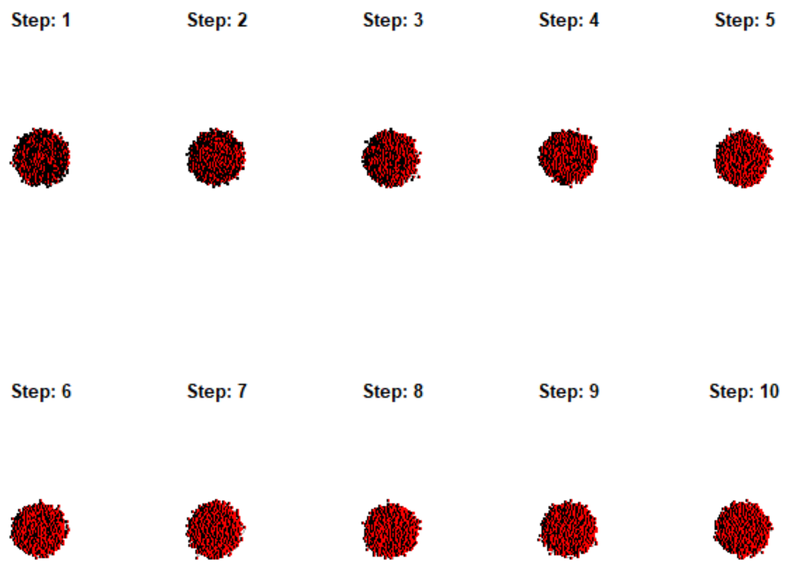


Figure 1.2: Evolution on 2D Barabasi with fr layout and 10000 nodes

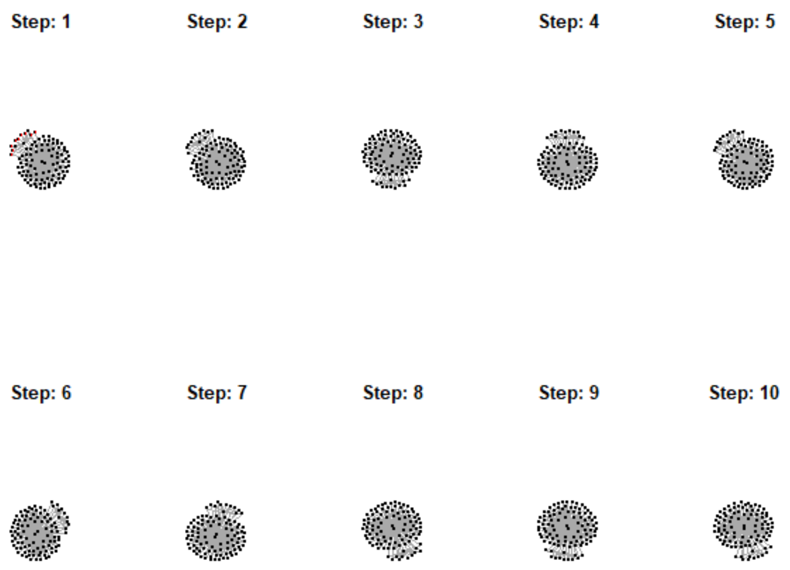


Figure 1.3: evolution in the degree distribution of Barabasi-Albert network with 10 time steps, power= 3

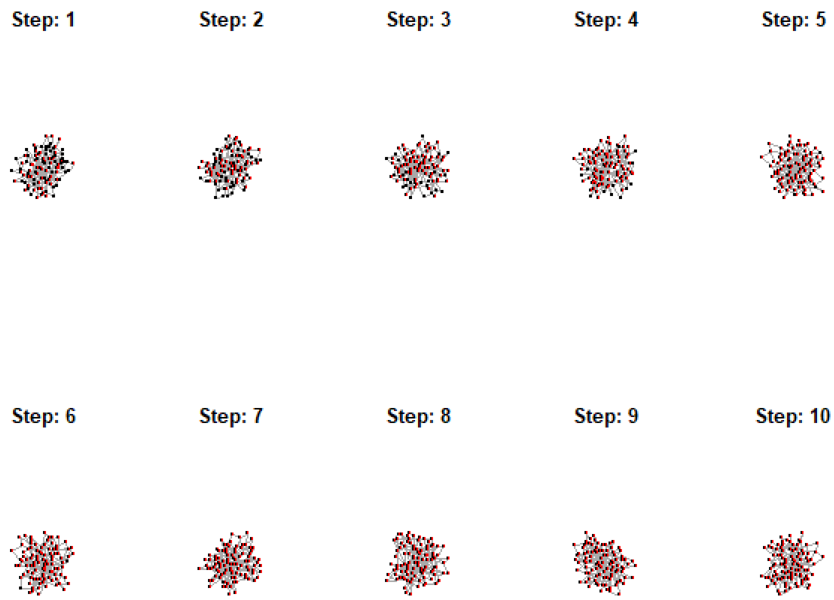


Figure 1.4: evolution in the degree distribution of Barabasi-Albert network with 10 time steps, power= $\frac{1}{4}$

steps	ρ
1	0.258883248730964
2	0.0989847715736041
3	0.0355329949238579
4	0
5	0
6	0
7	0
8	0
9	0
10	0

Table 1.1: evolution of ρ for 2D Barabasi-Albert network and nodes $n = 100$

steps	ρ
1	0.218007701155173
2	0.154248137220583
3	0.0857128569285393
4	0.0363304495674351
5	0.00892633895084263
6	0.00300045006751013
7	0.000675101265189778
8	7.50112516877532e-05
9	5.00075011251688e-05
10	0

Table 1.2: Evolution of ρ in 2D Barabasi-Albert network and nodes $n = 10000$

Here we get the ordering process for 2D Barabasi Albert structure.
Then if we increase the number of nodes in the system we get different values for ρ but finally it reaches absorbing state, as we can see in 1.2.
Finally we see that in 1.3 and 1.4 the changing in the degree distributions that are affecting the evolution of ρ quantity as we can see in further tables 1.3 and 1.4:

steps	ρ
1	0.0456852791878173
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0

Table 1.3: evolution of ρ for 2D Barabasi-Albert network with $n = 100$, power$= 3$

steps	ρ
1	0.271573604060914
2	0.233502538071066
3	0.134517766497462
4	0.0634517766497462
5	0.0177664974619289
6	0.0126903553299492
7	0
8	0
9	0
10	0

Table 1.4: Evolution of ρ in 2D Barabasi-Albert network with nodes $n = 100$, power$= \frac{1}{4}$

steps	ρ
1	0.5876963
2	0.5688108
3	0.5172027
4	0.5411369
5	0.4839192
6	0.4134256
7	0.2178384
8	0.2120419
9	0.4824233
10	0.5964847

Table 1.5: evolution of ρ for real network, UK air transportation

1.4.1 application for real world networks

what we can see here is that we have sustained fluctuations for the selection of time step, $t = 10$. Due the high quantity of nodes in the system we can't improve the time step to see where and if there is a collapse toward an absorbing state.
Finally I report data in 1.5 for evolution of ρ for UK air transportation data, as specific application of Voter model on real world network.
So at the end the data I used for the analysis of real network are extracted from:

1. TransportDirect.data.gov.ukhttp://data.gov.uk/dataset/nptdr(2010)
2. Gallotti,R.&Barthelemy,M.figshare.http://dx.doi.org/10.6084/m9.figshare.1249862(2014)
3. Gallotti,R.&Barthelemy,M.Dryad.http://dx.doi.org/10.5061/dryad.pc8m3(2014)

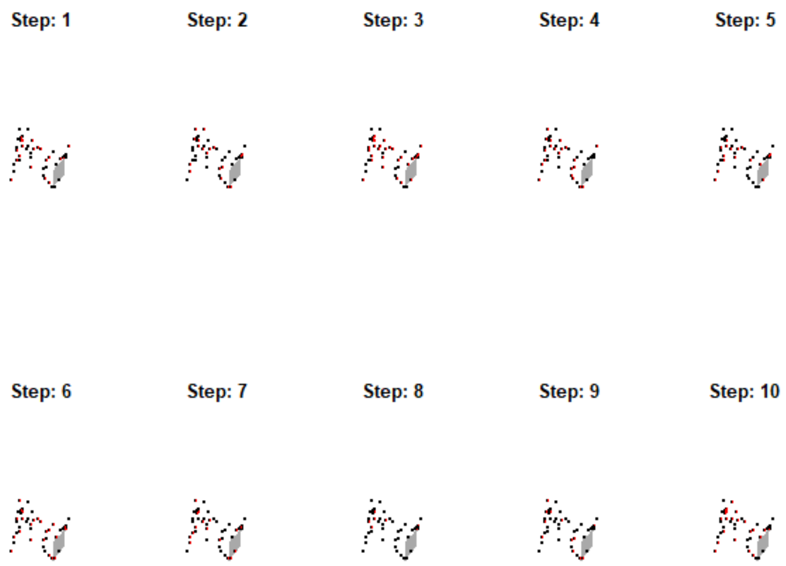


Figure 1.5: Application of voter model on data ectracted from movements in the UK airports

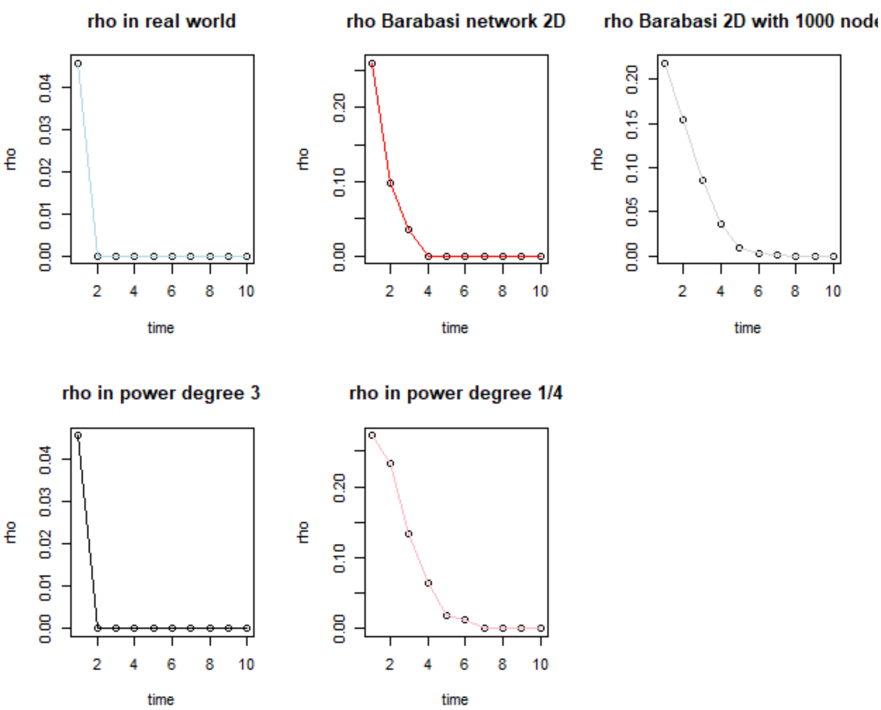


Figure 1.6: Graphics describing evolution of rho

2 | Axelrod model for dissemination of culture

2.1 | Introduction

One of the most discussed phenomena is the formation and visualization of pattern in complex network, particularly, in a social system we can evidence the formation of groups, based on cultural diversity, through the whole system.
The formation of cultural diversity as an emergence phenomena is explicated by the Axelrod's Model for Dissemination of Culture [1].
I'll describe the model and apply to synthetic network.

2.2 | Axelrod Model

The whole model is described through the first definition of culture.
Culture is defined through **attributes** that describe the dimension of the culture, then, for each attribute, we can define a set of traits as the possible values that each feature can assume.
Thanks this abstract definition we can define a **degree of cultural similarity** between individuals as the percentage of their equal feature's trait.
Now in network's point of view we can "geolocalized" the model, in fact model includes, geographical patches and inside each of them there are specific distribution of agents that can't migrate towards other patches. Agents can interact with other ones that are located in the neighbor patches, the interaction is tuned by the **principle of similar culture**, they interact each other only if are culturally similar, and each one, thanks this interaction, improves its cultural similarity as explain in further page of the report.
This kind of result is typical for **assortative networks**, where agents or nodes interaction is tuned by the presence of specific similarity and the quantity that we have to take into account for the analysis of similarity is defined through culture.
This kind of model, based on

1. Agent-modeling
2. No central authority
3. Adaptive model

takes part of **social influence model**.
Computationally the model is described through two steps:

- random pick of site and corresponding neighbors
- interaction between sites mediated by the probability

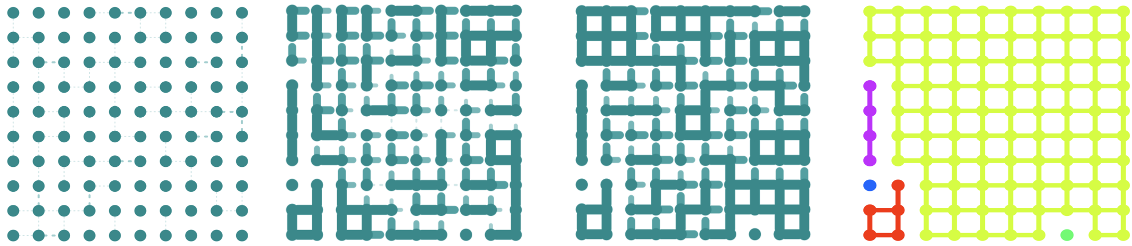
More specifically the interaction consists on: selection of not common feature between two agents (or neighbors) and finally one of them get the same feature of the other one. All of that is mediated thanks the probability, that is proportional to the common number of cultural feature, cultural similarity, that agents share.
Axelrod model can highlight the presence of transition due the rule of local convergence, in fact this rule leads to global polarization or a fragmentation of cultural site; here the transition happens from homogeneous culture (order phase) to heterogeneous or fragmentation culture (disorder phase) inside a social system or nation. The order parameter that we can use to determine the transition is the average largest size of homogeneous region in the final state, $< S_{max} >$, this approach is very similar to what we do in the percolation analysis of a system, in the last case we analyze the size of the largest connected component to determine if the system is annihilated or not evaluating how far the size of largest connected component is with respect to 0, when it happens we get that attacks or cascading failures have success. Here for the analysis of Axelrod model we can use similar approach to stress the fragmentation of cultural region.

2.3 | Model application on synthetic network

Now I apply the previous model to different synthetic graphs.
Firstly I use a regular Lattice with N, nodes. Here each node is considered an agent which is characterized by the presence of initial different cultural features, F.
As explain in previous section each feature is characterized by different value of trait, q, and only in the case of equality, cultural similarity, agents start interacting each other.
The procedure of interaction is repeted until there are some similar features, on the contrary the process stops, particularly it stops when there are equal cultural feature or also in the case of completely different cultural feature; in the last case there's a clusterization phenomena of cultural region. In this analysis is fundamental the cluster formation that happens for different values of F and q .
In fact for

1. $q \sim 1$ and $F \sim 2$, (generally for small value of F) we get homogenization of cultural region
2. $q >> 1$ we get fragmentation of cultural region and small cluster formation.

This procedure is applied to an Erdős–Rényi (ER) model (which is characterized by the number of nodes N).



(a) Epoch 0 (b) Epoch ~ 20000 (c) Epoch ~ 50000 (d) Epoch ~ 90000

Figure 2.1: Example of evolution on Lattice model. The process is repeated until convergence (complete similarity or complete difference between nodes).

2.4 Analysis on ERG

Now I study and apply the previous methods getting results for different change of parameters on both the Lattice and ERG models.

The three parameters as succinct descriptors of our model, mainly:

- n_c, the number of stable cultures in the frozen state,
- $\frac{S_{max}}{N}$, the relative size of the largest stable community with respect to the number of nodes,
- T_{co}, the number of epochs necessary for convergence.

2.4.1 Variation of F and q

Firstly I start analyzing and varying the number of Future F and possible traits q, fixing the number of nodes of the system. By increasing the number of traits q, as seen in figure 2.3, we get highly fragmentation in number of cultural region, manifestation of clusters on network. This kind of result is expected thanks the increase of F and q, as proposed in fig 2.3.
Same results is obtained for ERG but in this case without specific layout, as done in figure 2.2

2.5 Phase transition and active bonds

Generally is observed the emergence of big cultural region, this evaluation can be done through the analysis of $\frac{S_{max}}{N} \sim 1$, for small values of q.
Here we can see correlation between number of cultural region as q values increase, that means that q is the order parameter for a multicultural region or traditionalist cultural region.
By measuring the density of active bonds n_a (i.e. the fraction of edges that **allow** an interaction, $p \neq 0$ and $p \neq 1$) I observe an initial rise for values of q related to mono-cultural system, followed by a steady decrease [2]. For multi-cultural system this seems to fade (as q increases), allowing quick convergence to a fixed configuration.

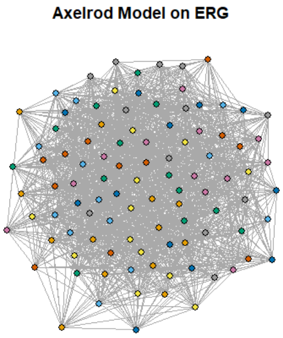


Figure 2.2: ERG graph with $F = 20$, $q = 50$

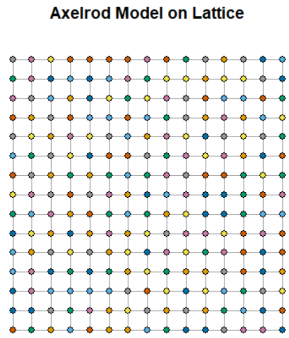


Figure 2.3: Lattice graph with $F = 20$, $q = 50$

Number of cultural features (F)	Number of traits per feature (q)		
	10	15	20
5	2.19	3.06	6.56
10	2.00	2.26	2.75
15	2.38	2.31	2.81

Table 2.1: The table displays the average number of stable regions n_c for different values of F and q and maintaining $N = 200$ and $\langle k \rangle = 5$ (ER Model). The results are averaged across 32 runs.

3 | Bibliography

[1] Robert Axelrod. The dissemination of culture. *The Journal of Conflict Resolution*, 41(2):203–226, 1997.

[2] Claudio Castellano, Matteo Marsili, and Alessandro Vespignani. Nonequilibrium phase transition in a model for social influence. *Phys. Rev. Lett.*, 85:3536–3539, 2000.