

Problems handle

- * classification { supervised learning } → test
 - * prediction (Regression) → Deploy the
 - * clustering { unsupervised model }
 - * Dimensionality reduction

Steps

① Train

© test

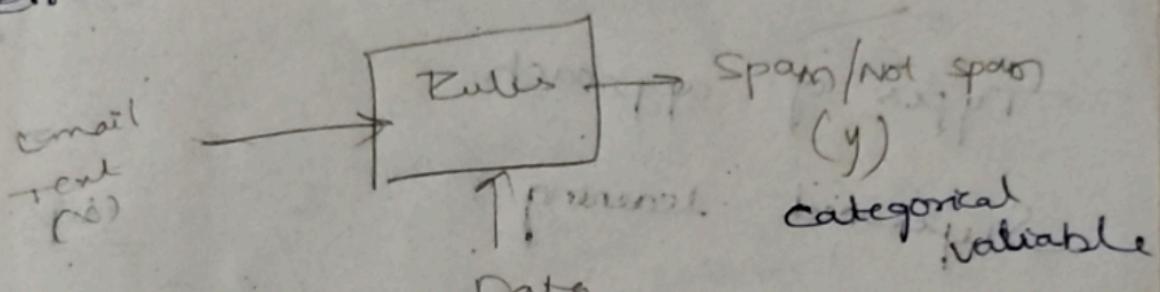
① Deploy the

unsupervised model

reduction

Classification \rightarrow Y is Categorical

→ Email classification



Data

data x
variable error

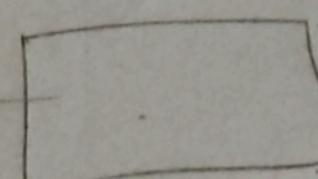
Meeting at 10:00am

from $\frac{1}{2}$ 1000

→ Regression

① Weather prediction

Previous
day temp -
humidity
wind speed



\rightarrow Term?

M (continues)

Regression is continuous

→ Clustering

1) Classification (x, y) \rightarrow discrete

→ Regression (x, y) \rightarrow continuous

3) Clustering (x)

4) Dimensionality Reduction (Reducing x)

Types of approach

Supervised learning :

steps

① Learning from solved examples — Training

② Exercises — validation

③ Exam — Testing

Input	Label	Human knowledge (supervision)
Email Meeting at 10 won lottery	not spam	
	spam	

Example:

1) Face Recognition



Ex Card fraud detection

Last 10 transaction, location

3) Disease Prediction

Symptoms \rightarrow Disease (y)

4) Stock price prediction

Previous 10 days price \rightarrow Tomorrow price (y)

Dataset:

Split the data into train set, validation set, test set

	X	y
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Train (60%) - 80%

validation (20%) 10%

test (20%) 10%

Select by Randomization

Train - 5, 10, 1, 6, 2, 3, 4, 3

Validation - 9

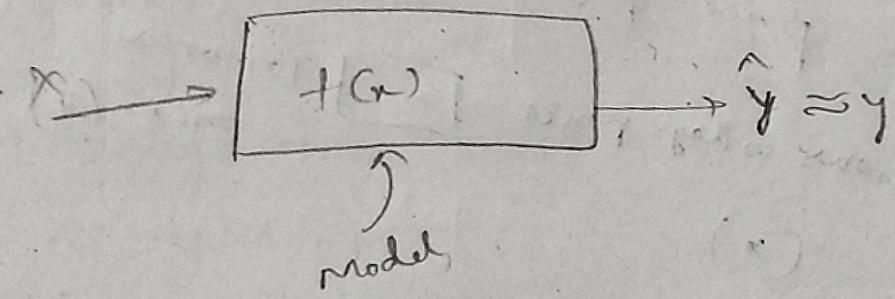
Test - 8

Regression

Simplest

No. of hours study	marks
Train	50
	55
	65
	70
	80

test 3.5 | ?



Linear function:

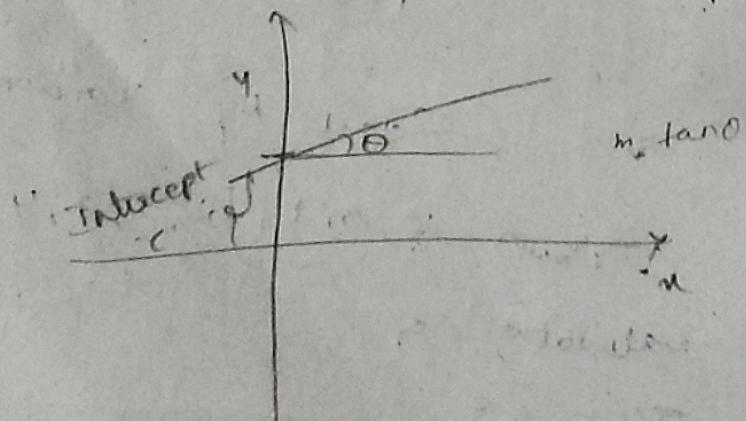
Simplest linear function

(line)

$$y = mx + c$$

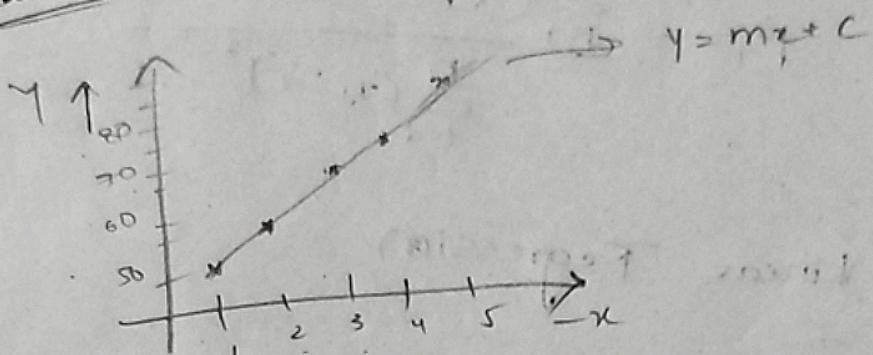
↑
Slope
↓
Input

Output ↙ Intercept



Intercept : tells where the line is located
in M and C we can control to define the line

Scatter plot



$m \& c \rightarrow$ parameters

Parameters

→ Controls the model

$$y = mx + c$$

m and c are the parameters (2^P)

Training = Finding right parameters.



Deep learning

Neural Networks inspired by
Brain neurons

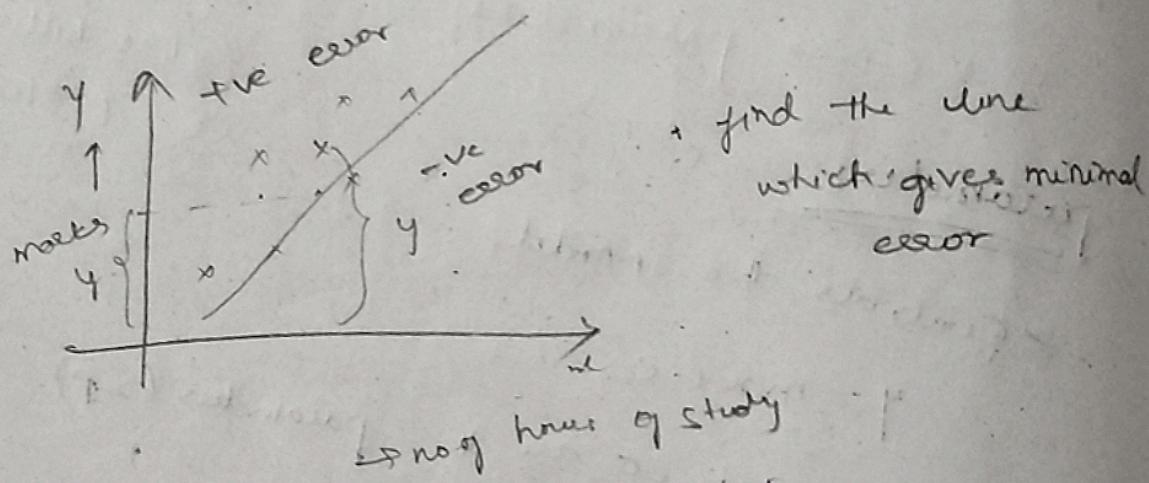
$$y = \beta_0 + \beta_1 x$$

$$\beta_0 = \bar{y} - \beta_1 \bar{x}$$

$$\beta_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

Simple Linear Regression

$$\hat{y} = \theta_0 + \theta_1 x$$



$$\hat{y} = 4 + 10x$$

$$\begin{aligned} x = 5, \hat{y} &= 54 \\ y &= 60 \end{aligned} \quad \left. \begin{array}{l} \text{error} \\ \text{error} \end{array} \right\}$$

$$\text{Error} = y - \hat{y}_{\text{Model-1}}$$

x	y	\hat{y}	$\sum p$	\hat{y}	$\sum e_1$
1	50	14	36	42	8
2	55	24	31	44	11
3	65	34	31	46	19
4	70	44	26	48	22
5	80	54	26	50	30

Model - 1 : $4x + 10$

Model - 2 : $40t + 24$

Select the model which produce
Minimal error

Error :

$$e_{ra} = y - \hat{y}$$

$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

Sum of squared error

- ① All errors will become positive
- ② It magnifies the large errors.

Finding the best parameters:
(Minimum error)

- ① Using Formula

$$\theta_0 = \bar{y} - \theta_1 \bar{x}$$

$$\theta_1 = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

① model → prediction

② Error

③ Train

↳ Find the parameters for which error is minimum

Regression !

Linear

x	y
1	50
2	55
3	65
4	70
5	80

$$\hat{y} = \theta_0 + \theta_1 x$$

Find θ_0 and θ_1 such that

→ the error is minimum

SSE → sum of square errors

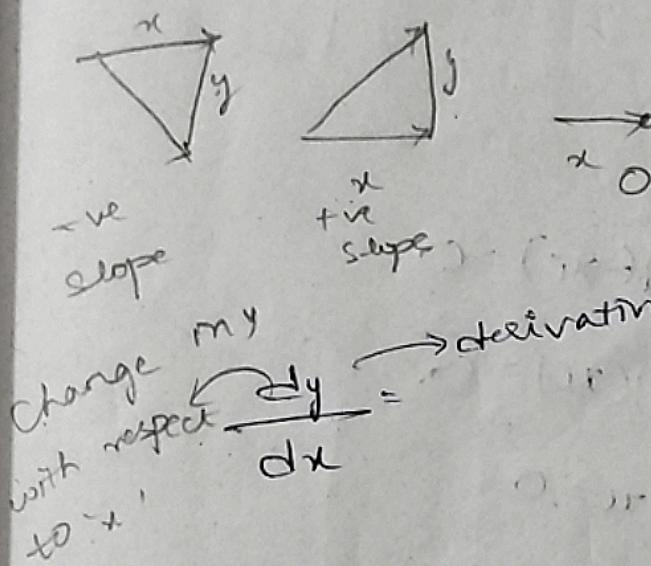
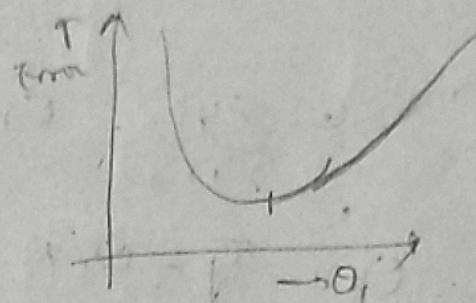
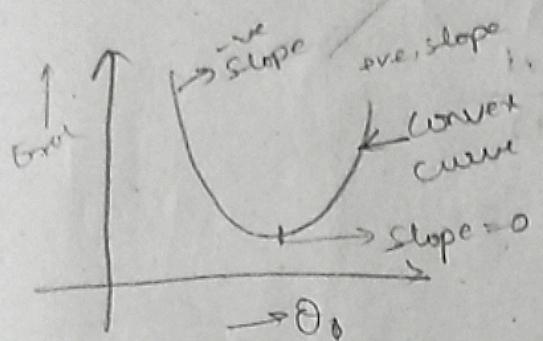
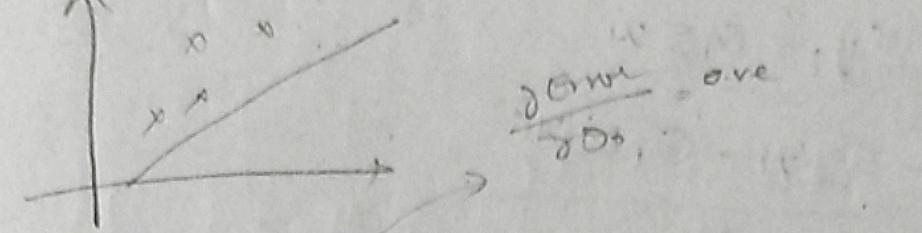
$$y_i - \hat{y}_i$$

$$|y_i - \hat{y}_i|$$

$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2 = \sum_{i=1}^n (y_i - \theta_0 - \theta_1 x_i)^2$$

Formula :

$$\theta_0 = \bar{y} - \theta_1 \bar{x}$$



Find θ_0 and θ_1 such that

$$\frac{\partial \text{Error}}{\partial \theta_1} = 0, \quad \frac{\partial \text{Error}}{\partial \theta_0} = 0$$

Find θ_0

$$\Rightarrow \frac{\partial \text{SSE}}{\partial \theta_0} = 0$$

$$\rightarrow 2 \sum (y_i - \theta_0 - \theta_1 x_i)(-1) = 0$$

$$- 2 \sum (y_i - \theta_0 - \theta_1 x_i) = 0$$

$$\sum (y_i - \theta_0 - \theta_1 x_i) = 0$$

$$\sum y_i - \sum \theta_0 - \sum \theta_1 x_i = 0$$

$$\theta_0 = \frac{\sum y_i - \theta_1 \sum x_i}{n}$$

$$= \frac{\sum y_i}{n} - \theta_1 \frac{\sum x_i}{n}$$

$$\theta_0 = \bar{y} - \theta_1 \bar{x}$$

Find θ_1

$$\frac{\partial SSE}{\partial \theta_1} = 0$$

$$\Rightarrow 2 \sum (y_i - \theta_0 - \theta_1 x_i) (x_i) = 0$$

$$\Rightarrow -2 \sum (y_i - \theta_0 - \theta_1 x_i) (x_i) = 0$$

$$\Rightarrow \sum (y_i - \theta_0 - \theta_1 x_i) x_i = 0$$

$$\Rightarrow \sum (y_i - \theta_0 x_i - \theta_1 x_i^2) = 0$$

$$\Rightarrow \sum x_i y_i - \theta_0 \sum x_i - \theta_1 \sum x_i^2 = 0$$

$$\Rightarrow \sum x_i y_i - \left(\frac{\sum y_i - \theta_0 \sum x_i}{n} \right) \sum x_i - \theta_1 \sum x_i^2 = 0$$

$$\Rightarrow n \sum x_i y_i - (\sum y_i - \theta_0 \sum x_i) \sum x_i - n \theta_1 \sum x_i^2 = 0$$

$$\Rightarrow n \sum x_i y_i - \sum y_i \sum x_i + \theta_1 (\sum x_i)^2 - n \theta_1 \sum x_i^2 = 0$$

$$\Rightarrow n \sum x_i y_i - \sum y_i \sum x_i = \theta_1 (n \sum x_i^2 - (\sum x_i)^2)$$

$$\theta_1 = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

Regression : $m \times n = n \times 1$
 Estimating θ_0 and θ_1 using formula :

$$\theta_0 = \bar{y} - \theta_1 \bar{x}$$

$$\theta_1 = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = 7.5$$

- Estimating $\theta = (\theta_0, \theta_1)$ using normal form

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}$$

$$\text{Mean of } x = \bar{x} = \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$$

$$\bar{y} = \frac{50+55+65+70+75}{5} = 64$$

	x	y	\hat{y}
1, 1	1	50	49
1, 2	2	55	56.5
1, 3	3	65	64
1, 4	4	70	71.5
1, 5	5	75	77

$$\hat{y} = \theta_0 + \theta_1 x = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}$$

$$\theta = (x^T x)^{-1} x^T y$$

\Rightarrow

$$\hat{y} = x \theta$$

$$\Rightarrow x = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}, \quad y = \begin{bmatrix} 50 \\ 55 \\ 65 \\ 70 \end{bmatrix}$$

$$(x^T x)^T, \quad 5 \times 1$$

$$(x^T x)^{-1} \cdot x^T y$$

$$2 \times 2, \quad 2 \times 1$$

Code

Python : import numpy as np

$x = np.array([1, 1, 1, 1, 1])$

$y = np.array([50, 55, 65, 70, 80])$

$\theta = np.linalg.inv(x.T @ x) @ x.T @ y$

Print (θ)

$\theta = [41.5, 7.5]$

$$\Rightarrow SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$y = \begin{bmatrix} 50 \\ 55 \\ 65 \\ 70 \\ 80 \end{bmatrix} \quad \hat{y} = \begin{bmatrix} 49 \\ 56.5 \\ 64 \\ 71.5 \\ 79 \end{bmatrix}$$

$$= (y - \hat{y})^T (y - \hat{y}) \quad \Rightarrow y - \hat{y} = \begin{bmatrix} 1 \\ 1.5 \\ 1 \\ 1.5 \\ 1 \end{bmatrix}^T \begin{bmatrix} 1 \\ -1.5 \\ 1 \\ -1.5 \\ 1 \end{bmatrix}$$

$$(y - x\theta)^T (y - x\theta)$$

$$= y^T y - \theta^T x^T y + \theta^T x^T x \theta$$

$$\begin{array}{c} \cancel{\text{cancel}} \\ = \begin{array}{c} |x| \\ |x| \\ |x| \end{array} \end{array} \quad \begin{array}{c} |x^T y| \\ |x^T y| \\ |x^T y| \end{array} \quad \begin{array}{c} |x^T x| \\ |x^T x| \\ |x^T x| \end{array}$$

$$= y^T y - \theta^T x^T y + \theta^T x^T x \theta$$

$$= y^T y - \theta^T x^T y - \theta^T x^T y + \theta^T x^T x \theta$$

MT - 1

- ML is a subset of AI.
- enables computers to learn from data and make decisions or predictions
- Learn from historical data
- Identify patterns and relationships within the data
- predict outcomes or actions on new data

Key features:

- Data-Driven
- Iterative
- Adaptive

Types of Machine Learning:

- * Supervised learning
learning from labeled data
- * Unsupervised learning
learning from unlabeled data
- * Reinforcement learning
learning through trial and error

x_i	y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	x_i^2	y_i^2	$x_i y_i$
1	50	-2	-14	4	2500	70
2	55	-1	-9	1	3025	97.5
3	65	0	1	0	4225	65
4	70	1	6	1	4900	140
5	80	2	16	4	6400	320

1) sum of $(x_i - \bar{x})(y_i - \bar{y})$:

$$\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = 28 + 9 + 0 + 6 + 32 = 75$$

2) sum of $(x_i - \bar{x})^2$:

$$\sum_{i=1}^n (x_i - \bar{x})^2 = 4 + 40 + 1 + 4 = 49$$

Slope θ_1 is

$$\theta_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\theta_1 = \frac{75}{49} = 1.5$$

3) calculate the Intercept

$$\theta_0 = \bar{y} - \theta_1 \bar{x}$$

$$\theta_0 = 64 - (1.5)(3)$$

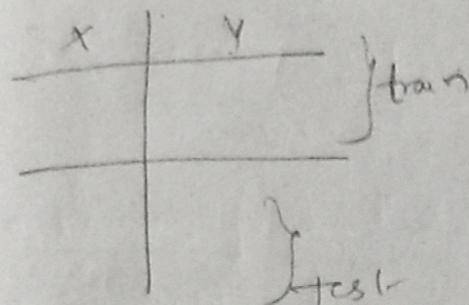
$$\theta_0 = 64 - 22.5 = 41.5$$

4) Final equation:

$$y = 41.5 + 1.5x$$

Evaluation metric

What is the performance on test data?



$$\hat{y} = f(x)$$

$$\text{error} \rightarrow SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$MSE = \frac{SSE}{n}$$

$$RMSE = \sqrt{MSE}$$

Evaluation metric

$$R^2$$

$$\text{Adjusted } R^2$$

$$\Rightarrow R^2 = \frac{\sum (y_i - \hat{y})^2}{\sum (y_i - \bar{y})^2}$$

↓

variance in data

If variance is less data is located near to mean.

→ If variance is high data is located away from mean

ex:	x	y	\hat{y}	$y - \hat{y}$	$(y - \hat{y})^2$	$(y - \bar{y})(\hat{y} - \bar{y})$
	50	49.5	1	1	-14	196
	55	56.5	-1.5	2.25	-9	81
	65	69	1	1	1	1
	70	71.5	-1.5	2.25	6	81
	80	79	1	1	16	216
					7.5	570

$$R^2 =$$

$$\bar{y} = 64$$

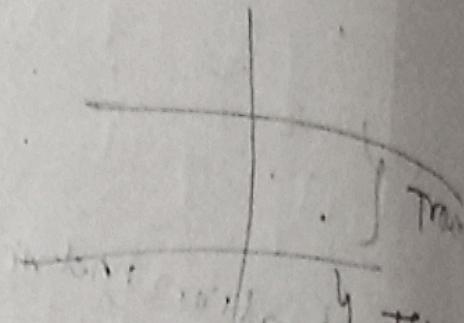
$$R^2 = \frac{7.5}{570} =$$

0.98

(Classification)

Evaluation, Metric:

x	y	\hat{y}
1	1	1
1	1	0
0	1	0
0	0	0



2)

→ Binary classification

→ only 2 labels (spam, not spam)

→ Confusion matrix → predicted, label

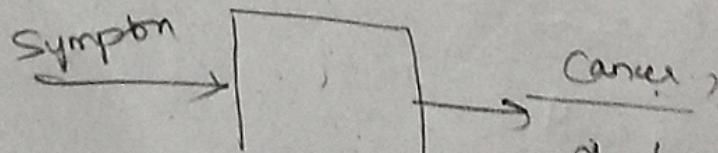
		1	0
Actual label	1	TP	FN
	0	FP	TN

Spam and marked as spam

More dangerous

not spam and marked as not spam

cancer prediction



Classification:

- Accuracy
- Precision
- Recall
- F1 score

y	\hat{y}
1	✓
0	✗
1	✓
0	✗
1	✓
0	✗

$$\text{Accuracy} = \frac{\text{Total correct predictions}}{\text{Total predictions}}$$

$$= \frac{TP + TN}{TP + TN + FP + FN}$$

$$= \frac{4}{8} = 0.5 = 50\%$$

To measure uncertainty $\{P_1, P_2\}$

$\{50\%, 50\%\} \rightarrow$

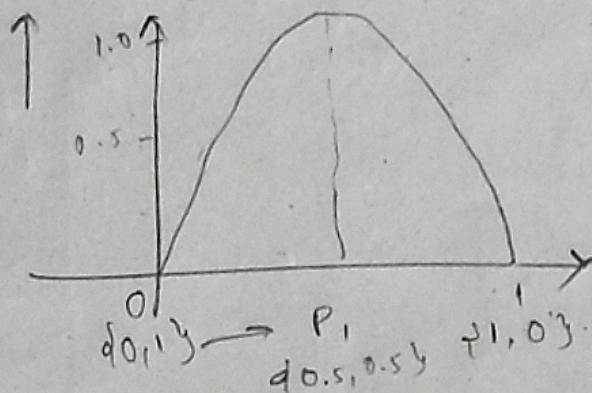
more
uncertain

$\{10, 10\} \rightarrow$ more
impurity

$\{90\%, 10\% \} \rightarrow$ less
uncertainty

$\{90, 10\}$ less impurity

$$\text{Entropy} = -P_1 \log_2 P_1$$
$$= -P_2 \log_2 P_2$$



$\{P_1, P_2\}$

x_1	x_2	x_3	x_4	label

$E(x_1)$

$E(x_2)$

$E(x_3)$

$E(x_4)$

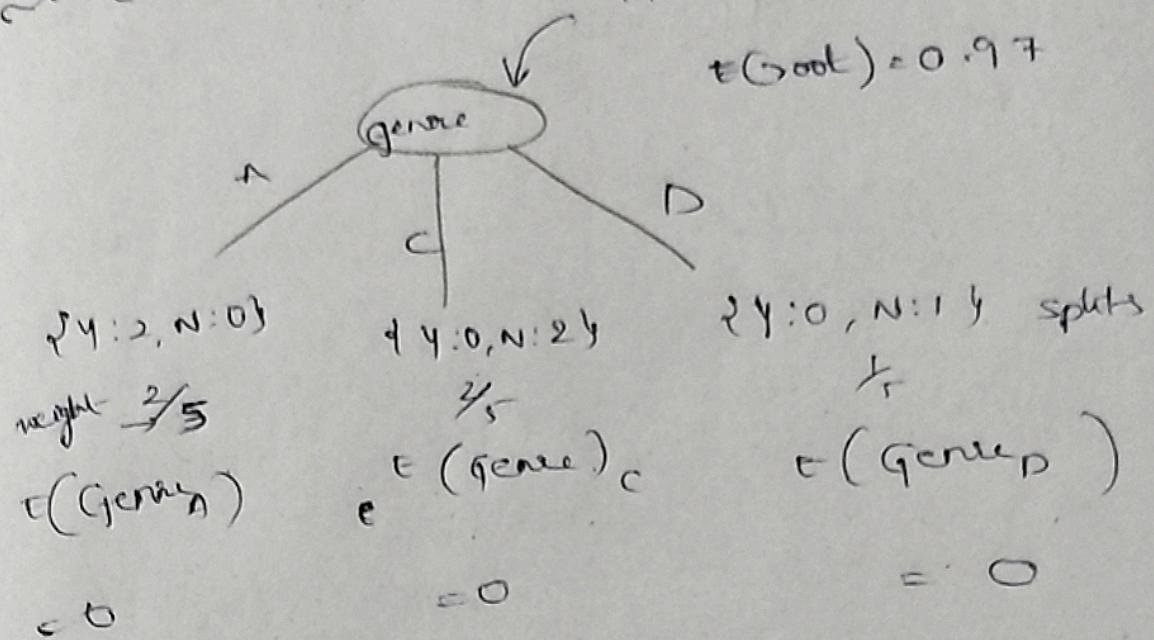
Gentile
A

Time
 E

watch more
 y

Information Gain

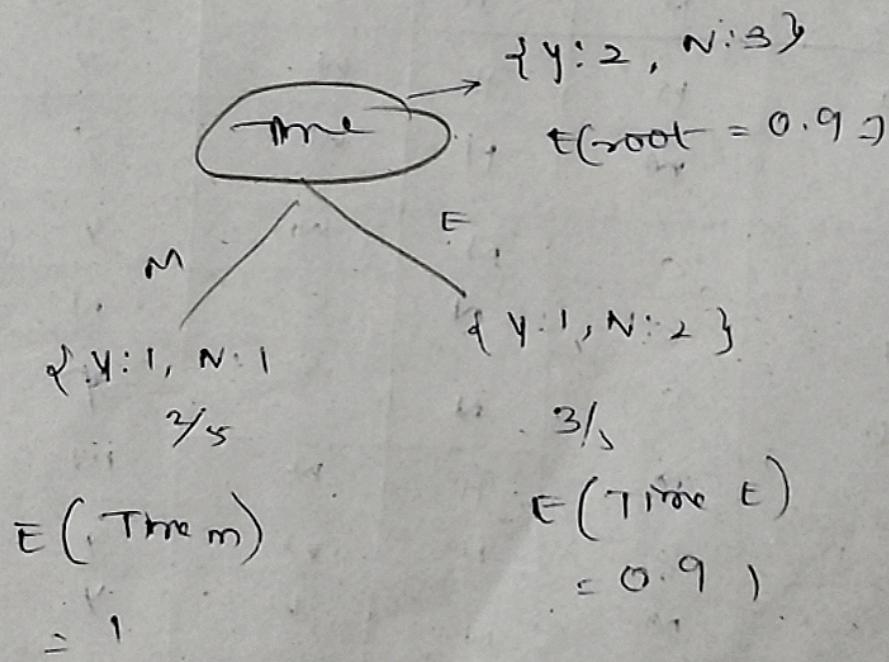
$\{Y:2, N:3\}$



$$\text{Average Entropy} = \frac{2}{5} \times 0 + \frac{2}{5} \times 0 + \frac{1}{5} \times 0 \\ = 0$$

$\Delta G(\text{Entropy})$ = Reduction in uncertainty / impurity

$$= 0.97 - 0 = 0.97$$



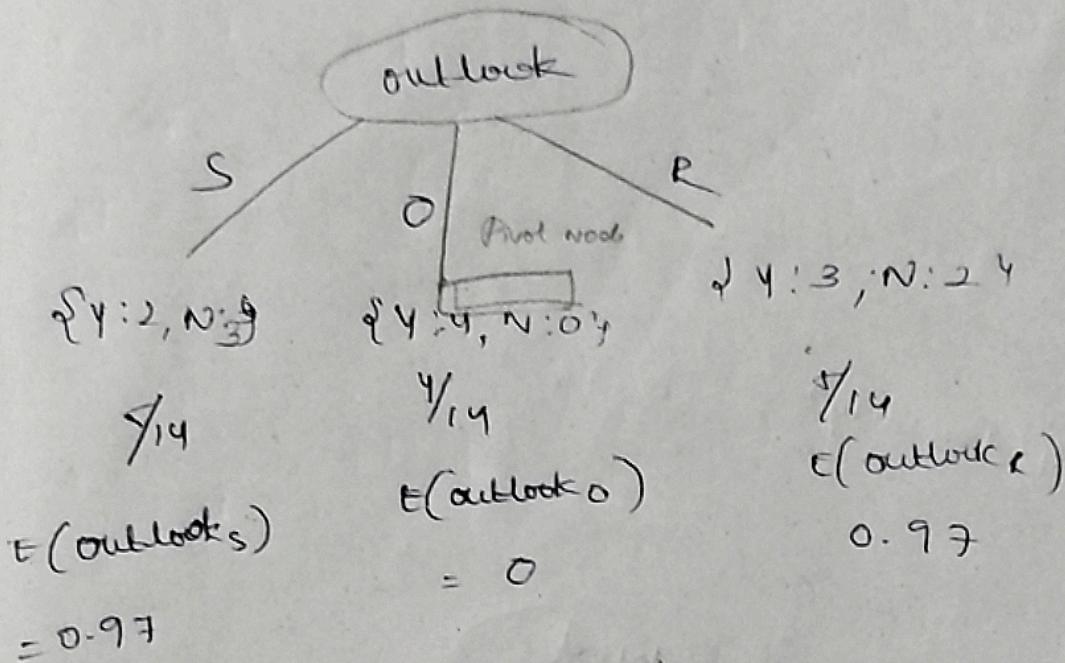
$$\text{Average Entropy} = \frac{2}{5} \times 1 + \frac{3}{5} \times 0.9 \\ = 0.946$$

$$\Delta G(\text{Time}) = 0.97 - 0.946 = 0.024$$

day	outlook	temp	humidity	wind	play tennis?
1	S	H	H	N	N
2	S	H	H	S	N
3	O	H	H	W	Y
4	R	M	H	W	Y
5	R	C	Z	W	N
6	R	C	Z	S	N
7	O	M	H	N	N
8	S	C	Z	W	Y
9	S	M	Z	W	Y
10	R	Z	Z	S	N
11	S	M	Z	S	Y
12	O	H	H	W	N
13	O	H	Z	W	Y

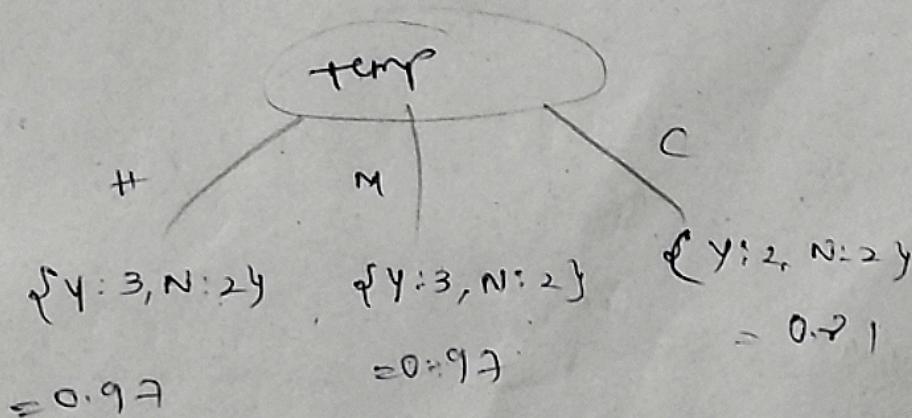
S → Sunny
 O → overcast
 R → Rainy
 N → Mild
 C → Cool
 H → Normal
 W → Weak
 D → No

Hence, $Y:9, N:5$



Average Entropy = $\frac{5}{14} \times 0.97 + \frac{4}{14} \times 0 + \frac{5}{14} \times 0.97$
 $= 0.69$

$I_G(\text{outlook}) = 0.94 - 0.69 = 0.24$



Average Entropy = $\frac{5}{14} \times 0.97 + \frac{5}{14} \times 0.97 + \frac{4}{14} \times 0.21$
 $= 0.924$

$I_G(\text{Temp}) = 0.94 - 0.924 = 0.016$

Humidity

N
S
 $\{ y: 3, n: 4 \}$ $\{ y: 6, n: 1 \}$

$$T.G(\text{Humidity}) = 0.16$$

Wind

N
S
 $\{ y: , n: 3 \}$ $\{ y: , n: 3 \}$

$$T.G(\text{Wind}) = 0.06$$

