1. Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, \quad x > 0$$

(6)

2. A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(5)

3

A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

4. The curve C has equation $y = 6 - 3x - \dfrac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\dfrac{d^2y}{dx^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.

(3)

5. The curve with equation

$$y = x^2 - 32\sqrt{(x)} + 20, \qquad x > 0$$

has a stationary point P.

Use calculus

(a) to find the coordinates of P,

(6)

(b) to determine the nature of the stationary point P.

(3)

6

A curve has equation $y = g(x)$.

Given that

- $g(x)$ is a cubic expression in which the coefficient of x^3 is equal to the coefficient of x
- the curve with equation $y = g(x)$ passes through the origin
- the curve with equation $y = g(x)$ has a stationary point at (2, 9)

(a) find $g(x)$,

(7)

(b) prove that the stationary point at (2, 9) is a maximum.

(2)

7. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point (2, 10) lies on C
- the gradient of the curve at (2, 10) is -3

(a) (i) show that the value of a is -2

(ii) find the value of b.

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write $\mathrm{f}(x)$ in the form $(x - 4)\mathrm{Q}(x)$ where $\mathrm{Q}(x)$ is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = \mathrm{f}(0.2x)$$

and the coordinate axes.

(2)