1. $$f(x) = 2x^3 - 7x^2 + 4x + 4$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

2.

$$f(x) = 2x^3 - 7x^2 - 10x + 24$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

3. $$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(a) Find the remainder when $f(x)$ is divided by $(x-1)$.

(2)

(b) Use the factor theorem to show that $(x+1)$ is a factor of $f(x)$.

(2)

(c) Factorise $f(x)$ completely.

(4)

4. $$f(x) = 6x^3 + 13x^2 - 4$$

(a) Use the remainder theorem to find the remainder when $f(x)$ is divided by $(2x + 3)$.
(2)

(b) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.
(2)

(c) Factorise $f(x)$ completely.
(4)

5. $$f(x) = 2x^3 + 5x^2 + 2x + 15$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(b) Find the constants a, b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root.

(2)

(d) Write down the real root of the equation $f(x - 5) = 0$

(1)

6

$f(x) = ax^3 + bx^2 - 4x - 3$, where a and b are constants.

Given that $(x - 1)$ is a factor of $f(x)$,

(a) show that

$$a + b = 7$$

(2)

Given also that, when $f(x)$ is divided by $(x + 2)$, the remainder is 9,

(b) find the value of a and the value of b, showing each step in your working.

(4)

7. $$\mathrm{f}(x) = 6x^3 + 3x^2 + Ax + B, \text{ where } A \text{ and } B \text{ are constants.}$$

Given that when $\mathrm{f}(x)$ is divided by $(x + 1)$ the remainder is 45,

(a) show that $B - A = 48$

(2)

Given also that $(2x + 1)$ is a factor of $\mathrm{f}(x)$,

(b) find the value of A and the value of B.

(4)

(c) Factorise $\mathrm{f}(x)$ fully.

(3)

8. $$f(x) = ax^3 - 11x^2 + bx + 4, \quad \text{where } a \text{ and } b \text{ are constants.}$$

When $f(x)$ is divided by $(x - 3)$ the remainder is 55

When $f(x)$ is divided by $(x + 1)$ the remainder is -9

(a) Find the value of a and the value of b.

(5)

Given that $(3x + 2)$ is a factor of $f(x)$,

(b) factorise $f(x)$ completely.

(4)

9.

$$f(x) = 2x^3 - 5x^2 + ax + 18$$

where a is a constant.

Given that $(x - 3)$ is a factor of $f(x)$,

(a) show that $a = -9$

(2)

(b) factorise $f(x)$ completely.

(4)

Given that

$$g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^{y}) + 18$$

(c) find the values of y that satisfy $g(y) = 0$, giving your answers to 2 decimal places where appropriate.

(3)

10. $$f(x) = -6x^3 - 7x^2 + 40x + 21$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely.

(4)

(c) Hence solve the equation

$$6(2^{3y}) + 7(2^{2y}) = 40(2^y) + 21$$

giving your answer to 2 decimal places.

(3)

11. (a) Factorise completely $9x - x^3$

(2)

The curve C has equation

$$y = 9x - x^3$$

(b) Sketch C showing the coordinates of the points at which the curve cuts the x-axis.

(2)

The line l has equation $y = k$ where k is a constant.

Given that C and l intersect at 3 distinct points,

(c) find the range of values for k, writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

12.

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

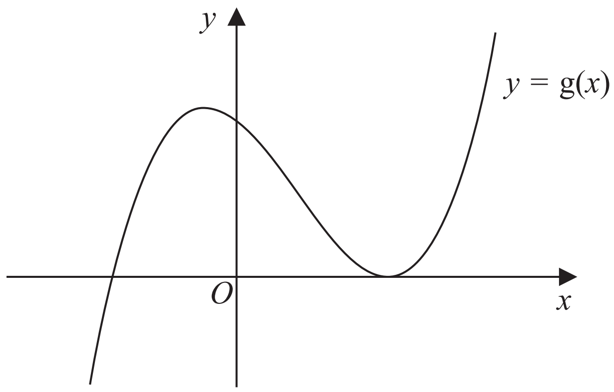

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

(c) Use your answer to part (b), and the sketch, to deduce the values of x for which

(i) $g(x) \leqslant 0$

(ii) $g(2x) = 0$

(3)

13. $$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

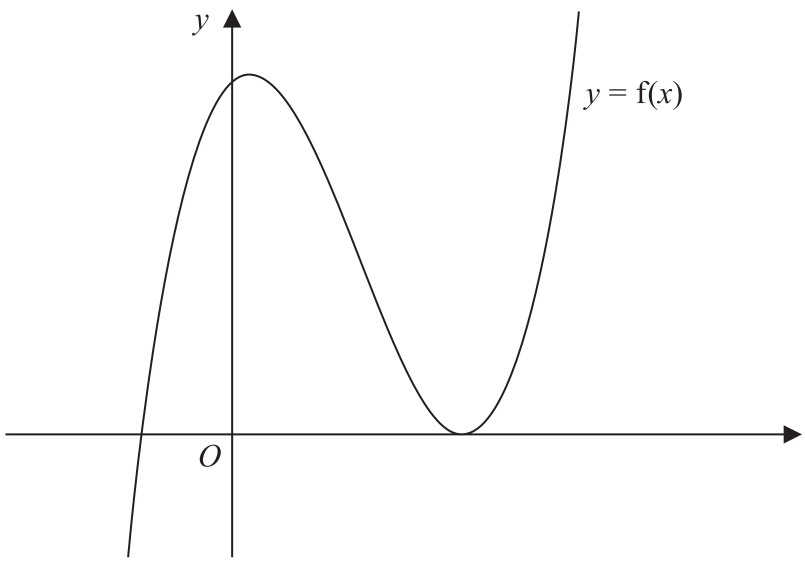

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

14. $$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that $g(x)$ is divisible by $(x - 5)$.

(2)

(b) Hence, showing all your working, write $g(x)$ as a product of three linear factors.

(4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x-axis, and lies below the x-axis.

(c) Find, using algebraic integration, the exact value of the area of R.

(4)

15. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \qquad x \in \mathbb{R}$$

where a is a positive constant.

Given $(x - a)$ is a factor of $f(x)$,

(a) show that

$$a(4a^2 + 5a - 6) = 0$$

(2)

(b) Hence

(i) find the value of a

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3$$

(4)

16

A curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $\mathrm{f}(x)$

find, in simplest form, $\mathrm{f}(x)$

(6)

17.

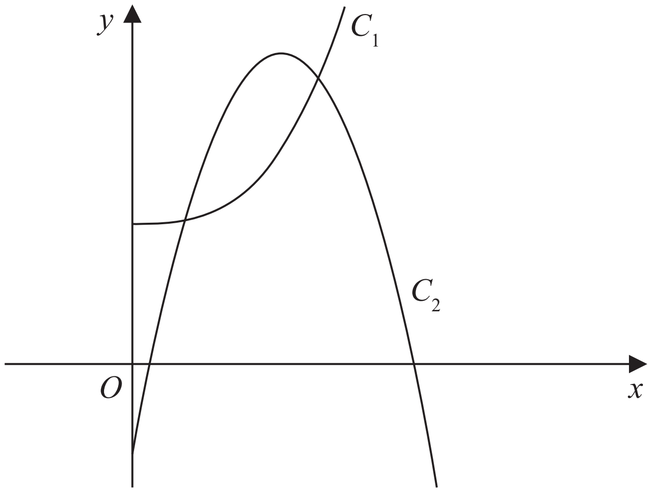

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(a) Verify that the curves intersect at $x = \dfrac{1}{2}$

(2)

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

Question 17 continued