1

Write $(3x+2) \div \left(\dfrac{3x^2-7x-6}{5}\right) - \dfrac{5}{x+3}$ as a single fraction in its simplest form.

Show clear algebraic working.

....

(Total for Question 1 is 4 marks)

2 Find the two values of x such that

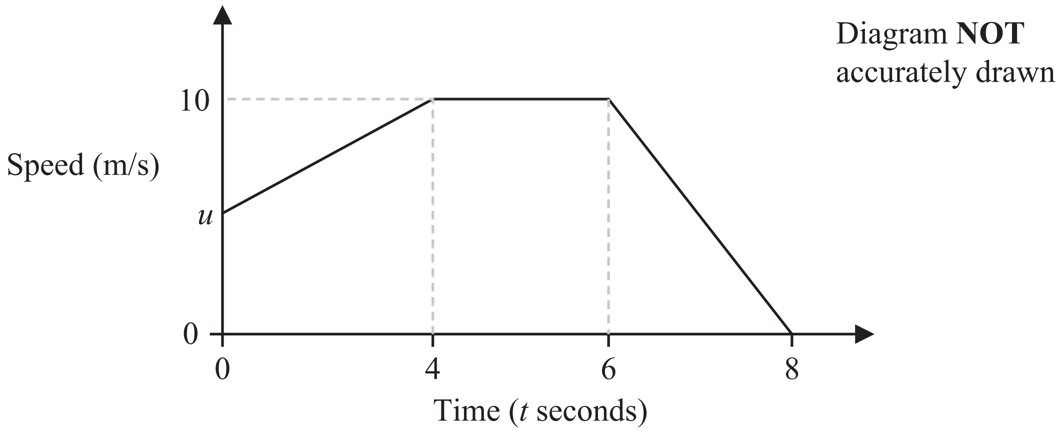
$$\frac{12^{3x} \times 3^{4x^2-3x} \times 3}{24^{2x}} = 27$$

Show your working clearly.

....

(Total for Question 2 is 4 marks)

3



The diagram shows a sketch of the speed-time graph of part of a cyclist's journey along a straight horizontal road.

(a) Calculate the deceleration, in m/s^2, for the last 2 seconds of this part of the cyclist's journey.

.... m/s^2

(2)

At time $t = 0$ seconds, the speed of the cyclist is u m/s
The cyclist travelled a total distance of 65 m in the 8 seconds.

(b) Calculate the value of u

$u =$ ....

(3)

(Total for Question 3 is 5 marks)

4 The line $\mathbf{L_1}$ has equation $5x + 4y = 16$

The line $\mathbf{L_2}$ is parallel to $\mathbf{L_1}$ and passes through the point with coordinates (8, 15)
$\mathbf{L_2}$ crosses the x-axis at the point A and the y-axis at the point B.

Calculate the length, to the nearest whole number, of AB.

....

(Total for Question 4 is 5 marks)

5 y is directly proportional to x^3
x is inversely proportional to the square root of w.

$y = 729$ when $x = 4.5$
$x = 25$ when $w = 0.16$

Find a formula for y in terms of w.

....

(Total for Question 5 is 5 marks)

....

(Total for Question 5 is 5 marks)

6

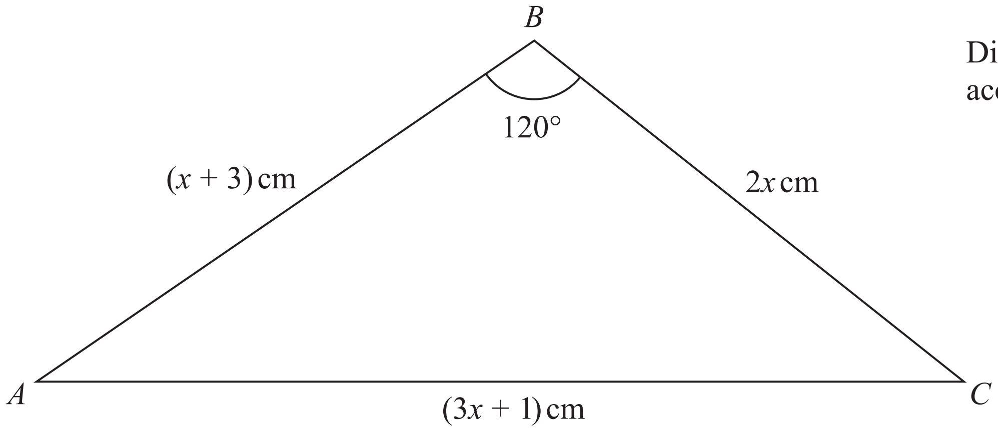


Diagram **NOT** accurately drawn

The diagram shows triangle ABC in which

$AB = (x + 3)$ cm
$BC = 2x$ cm
$AC = (3x + 1)$ cm
$\angle ABC = 120°$

Find the size, in degrees to 3 significant figures, of $\angle ACB$.

(Total for Question 6 is 5 marks)

7

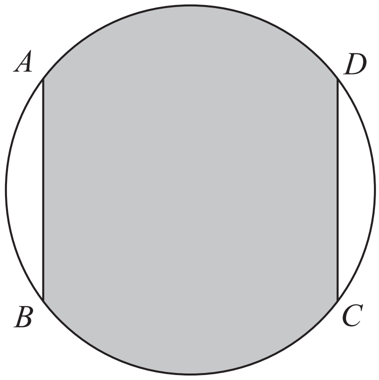


Diagram **NOT** accurately drawn

The diagram shows a circle of radius $2x$ cm.

The lines AB and DC are parallel and $AB = DC = 2x$ cm.
The area of the region shown shaded in the diagram is kx^2 cm^2

Find the exact value of k.

$k =$ ....

(Total for Question 7 is 5 marks)

8 A bag contains n beads.
There are 4 orange beads in the bag.
The rest of the beads are purple.

Donald is going to take at random 2 beads from the bag.

The probability that both beads will be the same colour is $\frac{51}{91}$

Find the value of n.

Show clear algebraic working.

$n =$ ....

(Total for Question 8 is 6 marks)

9

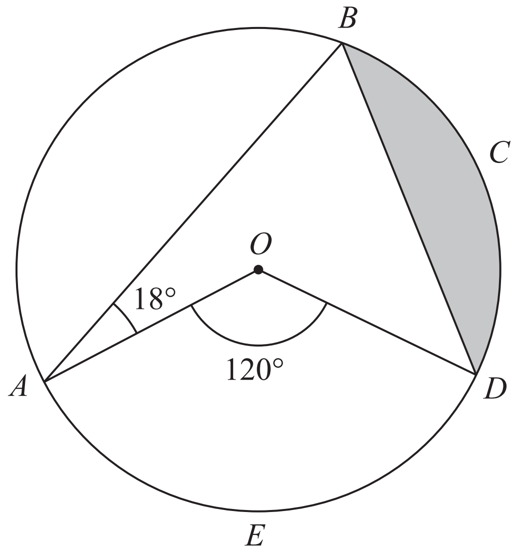


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a circle *ABCDE* with centre *O*.

$$\angle BAO = 18° \qquad \angle AOD = 120°$$

The area of segment *BCD*, shown shaded in Figure 2, is T cm^2

Given that the perimeter of the sector *AODE* is $5(3 + \pi)$ cm,

calculate the value, to one decimal place, of T.

(Total for Question 9 is 6 marks)

10

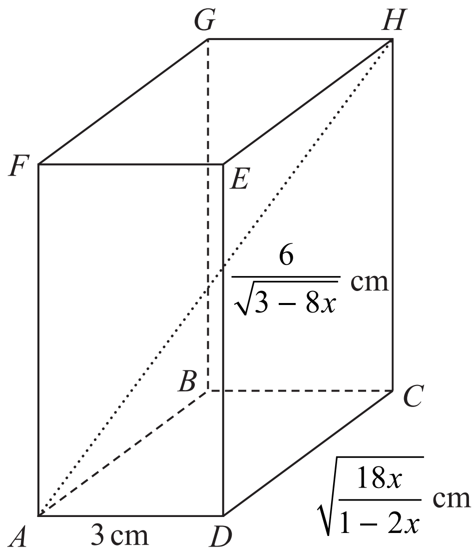


Diagram **NOT** accurately drawn

The diagram shows cuboid *ABCDEFGH* in which

$AD = 3\,\text{cm}$ $\qquad DC = \sqrt{\dfrac{18x}{1-2x}}\,\text{cm}$ $\qquad AH = \dfrac{6}{\sqrt{3-8x}}\,\text{cm}$

where $0 < x < \dfrac{3}{8}$

Given that the length of CH is L cm, where $L = \dfrac{k}{\sqrt{(3-8x)(1-2x)}}$ and k is a positive integer,

(a) find the value of k
Show your working clearly.

$k =$ ....

(5)

Given that $x = 0.3$

(b) calculate the volume, in cm^3, of the cuboid.

.... cm^3

(2)

(Total for Question 10 is 7 marks)

11 x is directly proportional to w^3

y is inversely proportional to $\sqrt{w}$

$y = 2$ when $x = \frac{1}{4}$

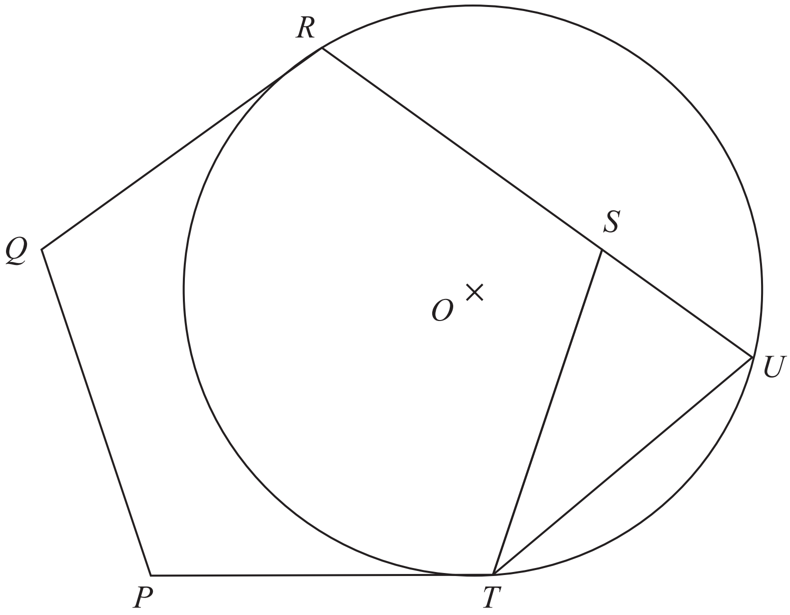
Find the value of p and the value of q such that $xy^p = q$

$p =$ ....

$q =$ ....

(Total for Question 11 is 4 marks)

12



PQRST is a regular pentagon.
R, *U* and *T* are points on a circle, centre *O*.
QR and *PT* are tangents to the circle.
RSU is a straight line.

Prove that $ST = UT$.

(Total for Question 20 is 5 marks)

13

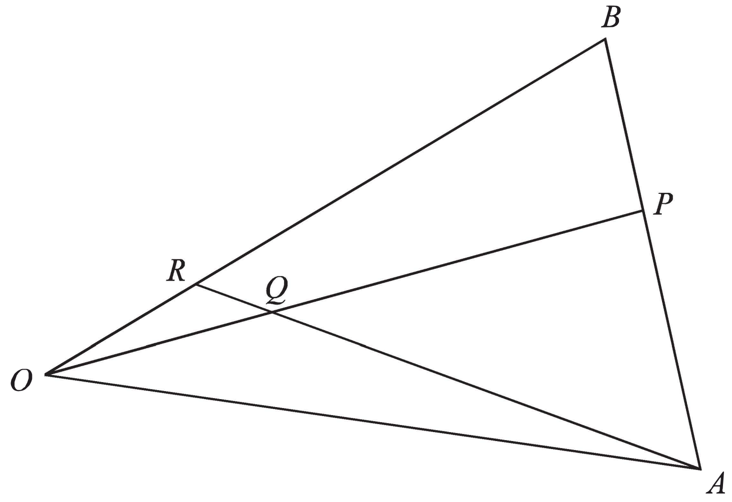


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows triangle OAB in which $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

P is the point on AB such that $AP : PB = 2 : 1$
Q is the point on OP such that $OQ : QP = 1 : 3$

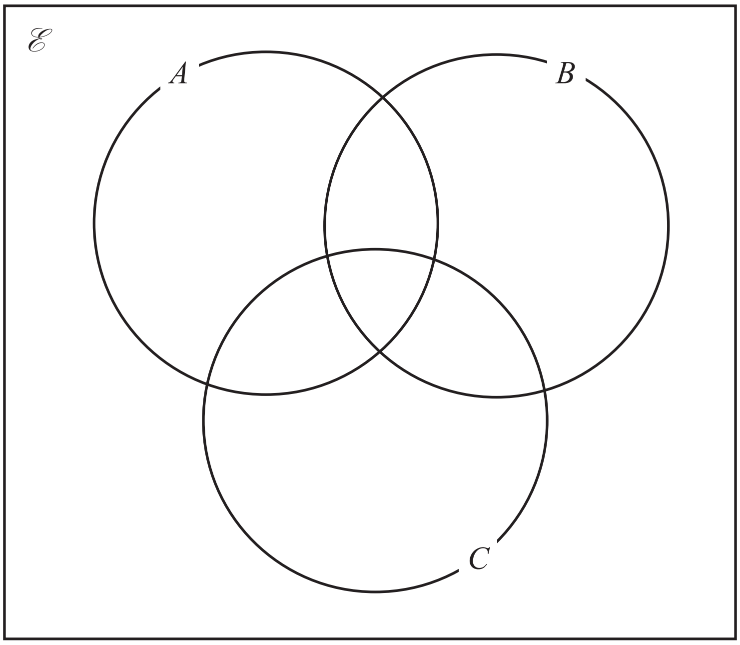
R is the point on OB such that RQA is a straight line.

Calculate, in its simplest form, the ratio $OR : RB$

(Total for Question 13 is 6 marks)

14 $\mathscr{E}$ = {odd numbers between 0 and 30}
A = {multiples of 3}
B = {prime numbers}
C = {factors of 30}

(a) Complete the Venn diagram for this information showing the position of each of the numbers in the universal set.



(3)

(b) Find (i) $n([A \cup C] \cap B)$

(ii) $n([B \cap C'] \cup A')$

(2)

A number is chosen at random from the universal set, $\mathscr{E}$

(c) Write down the probability that the number is in the set $C \cap A'$

(2)

Given that the number chosen from $\mathscr{E}$ is a multiple of 3

(d) find the probability that the number is also a factor of 30

(2)

(Total for Question 14 is 9 marks)

15

Diagram **NOT** accurately drawn

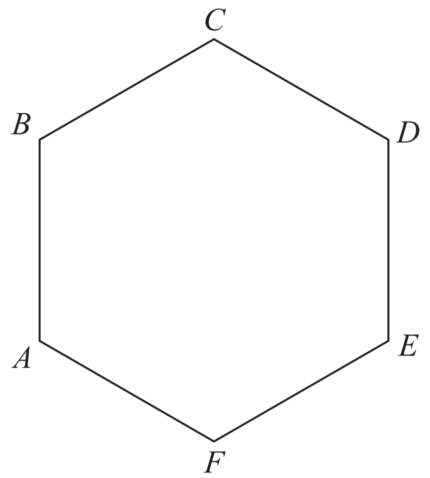


Figure 4

Figure 4 shows a regular hexagon *ABCDEF*

Given that the area of hexagon $ABCDEF = 150\sqrt{3}$ cm^2

(a) find the perimeter, in cm, of the hexagon.

(4)

Diagram **NOT** accurately drawn

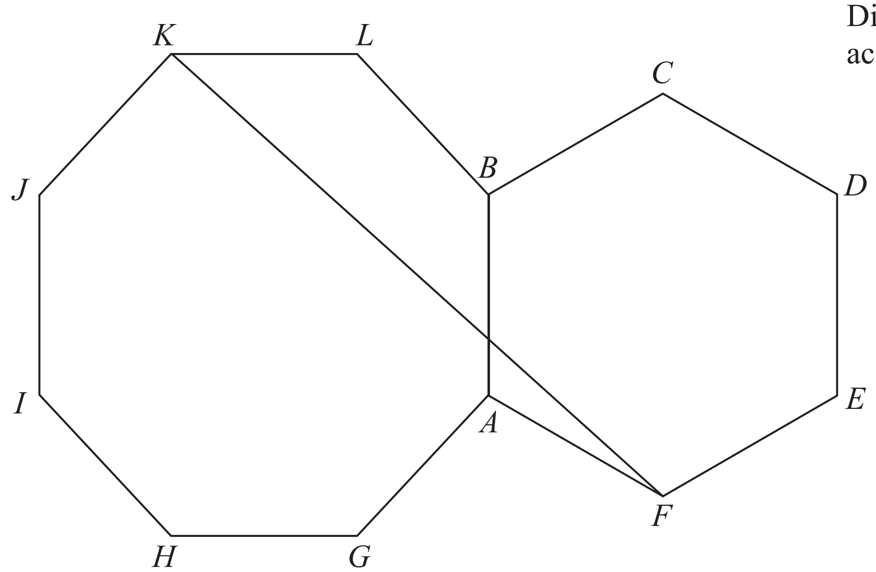


Figure 5

Figure 5 shows a shape *AGHIJKLBCDEF* made from a regular octagon *GHIJKLBA* and the regular hexagon *ABCDEF* from part (a).

(b) Work out the length, in cm to one decimal place, of the straight line *KF*

(6)

(Total for Question 15 is 10 marks)

16 (a) Solve the inequality $5(x + 1) < x$

Show clear algebraic working.

(2)

(b) Solve the simultaneous equations

$$3x^2 + y^2 - 7 = 0$$
$$y - 3x - 5 = 0$$

Show clear algebraic working.

(5)

(c) Hence find the value of x for which

$$5(x + 1) < x \quad \textbf{and} \quad 3x^2 + y^2 - 7 = 0 \quad \textbf{and} \quad y - 3x - 5 = 0$$

(1)

(Total for Question 16 is 8 marks)

17 A box contains 8 green counters and 2 white counters only.

Peter takes at random 2 counters from the box.

(a) Calculate the probability that Peter will take 1 green counter and 1 white counter.

(3)

A bag contains 28 blue beads and n red beads only.

Naasir selects a bead from the bag at random.

(b) Explain why the probability of the bead being red cannot be $\frac{6}{11}$

(3)

Naasir keeps the first bead and selects a second bead at random from the bag.

The probability of both beads being different colours is $\frac{1}{2}$

Given that there are fewer blue beads than red beads,

(c) calculate the probability that both beads are blue.
Show clear algebraic working.

(5)

(Total for Question 17 is 11 marks)

18 A curve **C** and a straight line **L** are drawn on a grid.

C has equation $y = 5x^2 - 16x - 5$

L has equation $y + 5x = 7$

(a) Find the coordinates of the points of intersection of **C** and **L**
Show clear algebraic working.

(5)

P is the point on the curve with equation $y = 5x^2 - 16x - 5$ with x coordinate 2

The line **Q** is the tangent to the curve at the point P

The line **Q** crosses the x-axis at the point X and the y-axis at the point Y
The point M lies on **Q** and is such that $XM = MY$

(b) Calculate the coordinates of the point M
Give your coordinates as exact values.

(5)

(Total for Question 18 is 10 marks)

19

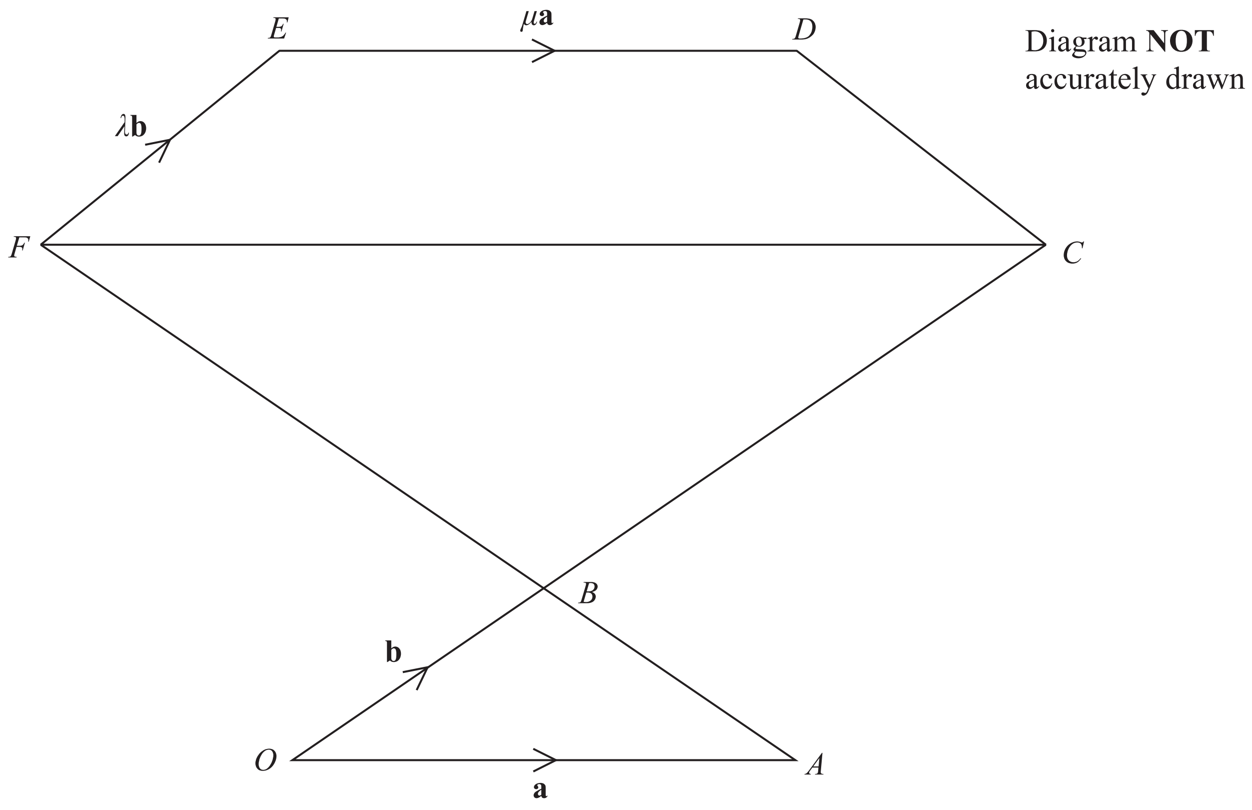


Figure 2

In Figure 2, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{FE} = \lambda\mathbf{b}$ and $\overrightarrow{ED} = \mu\mathbf{a}$, where λ and μ are positive constants.

B is the point of intersection of OC and AF such that $OB:OC = AB:AF = 1:3$

(a) Find, in terms of **a** or **b** or **a** and **b**, simplifying your answers where possible,

(i) $\overrightarrow{AB}$ (ii) $\overrightarrow{CF}$

(2)

(b) Find, in terms of **a**, **b**, λ and where necessary μ, simplifying your answers where possible,

(i) $\overrightarrow{CD}$ (ii) $\overrightarrow{AE}$

(3)

Given that $\overrightarrow{AE} = 4\overrightarrow{CD}$

(c) find the value of μ and the value of λ.

(4)

Given also that $|\mathbf{a}| = 4\,\text{cm}$, $|\mathbf{b}| = 1\,\text{cm}$ and that the area of the trapezium $CDEF$ is $5\,\text{cm}^2$

(d) calculate the size, in degrees to 3 significant figures, of $\angle CFE$.

(4)

(Total for Question 19 is 13 marks)

20

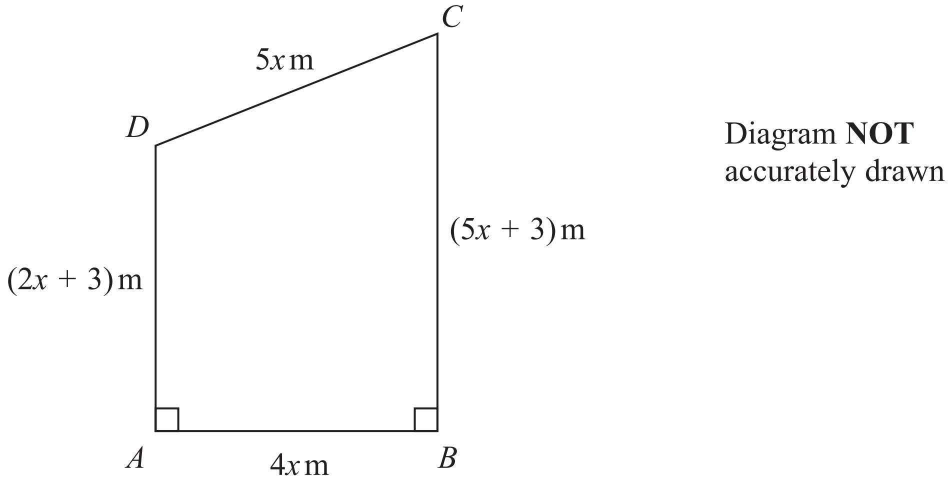


Figure 3

Figure 3 shows the plan for a lawn that is in the shape of a trapezium *ABCD* in which

$AB = 4x$ metres $\quad BC = (5x + 3)$ metres $\quad CD = 5x$ metres $\quad DA = (2x + 3)$ metres

The perimeter of the lawn is P metres.

(a) Find and simplify an expression for P in terms of x.

(2)

The area of the lawn is $A\,\text{m}^2$

(b) Show that $A = 14x^2 + 12x$

(2)

The owner of the lawn wants the perimeter of the lawn to be greater than 52 m.
He also wants the area of the lawn to be at most $162\,\text{m}^2$

(c) Find the range of possible values of x.
Show clear algebraic working.

(6)

(Total for Question 20 is 10 marks)

21

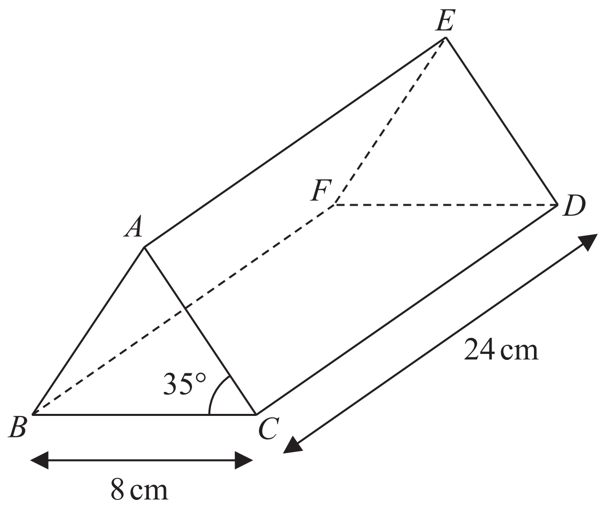


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a solid right triangular prism *ABCDEF*.

A cross section *ABC* of the prism is an isosceles triangle in which $AB = AC$.

$$\angle ACB = 35° \qquad CB = 8\text{ cm} \qquad CD = 24\text{ cm}$$

(a) Calculate the total surface area, in cm^2 to 3 significant figures, of the prism.

(5)

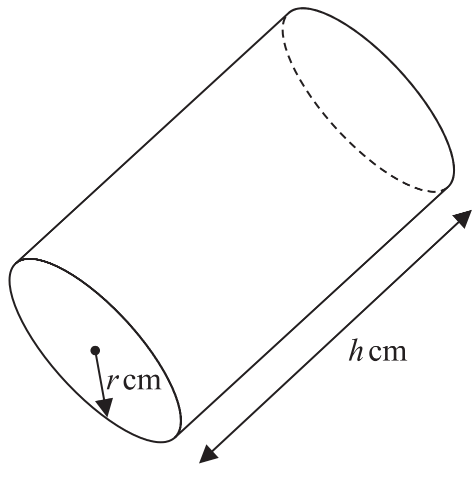


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows a solid right circular cylinder with radius r cm and length h cm.

The total surface area of the cylinder is $(224 + 60\sqrt{3})\pi$ cm^2

Given that $r = 3\sqrt{3} + 2$

(b) find the exact value of h.
Show your working clearly and give your answer in the form $a\sqrt{27}$ where a is an integer.

(6)

(Total for Question 21 is 11 marks)

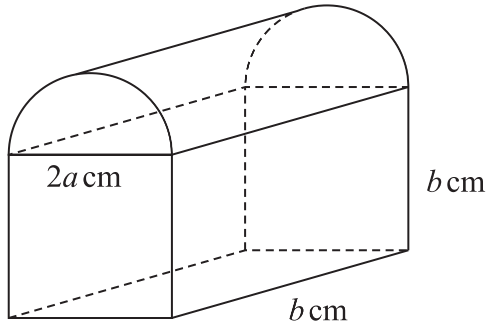


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a solid silver paperweight made from a cuboid and a half cylinder.
The cuboid is $2a$ cm wide, b cm long and b cm high.
The plane face of the half cylinder coincides with the top face of the cuboid.
The total surface area of the paper weight is A cm^2

(a) Find an expression for A in terms of π, a and b.

(2)

Given that $a = 6\sqrt{5}$ and that the surface area of the paperweight can be written as

$$(2b^2 + 6ab + 60\pi\sqrt{15})\text{ cm}^2$$

(b) show that the exact value of b is $10\sqrt{3} - 6\sqrt{5}$

(5)

The paperweight is melted down to form a different cuboid.
This second cuboid is $2a$ cm wide, b cm long and h cm high, as shown in Figure 4.

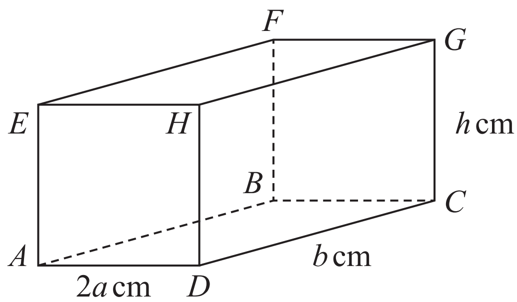


Diagram **NOT** accurately drawn

Figure 4

(c) Calculate the size, to the nearest degree, of angle GAC.

(5)

(Total for Question 22 is 12 marks)