1. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

2. The curve C has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$$

(a) Find

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

3. The curve C has equation $y = f(x)$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{dy}{dx} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$

(2)

(b) Hence find the coordinates of the point where C crosses the y-axis.

(3)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = \mathrm{f}(x)$ where $x \in \mathbb{R}$

Given that

- $\mathrm{f}'(x) = 2x + \dfrac{1}{2}\cos x$
- the curve has a stationary point with x coordinate α
- α is small

(a) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

The point $P(0, 3)$ lies on C

(b) Find the equation of the tangent to the curve at P, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

5.

$$f(x) = x^3 + 2x^2 - 8x + 5$$

(a) Find $f''(x)$

(2)

(b) (i) Solve $f''(x) = 0$

(ii) Hence find the range of values of x for which $f(x)$ is concave.

(2)

6.

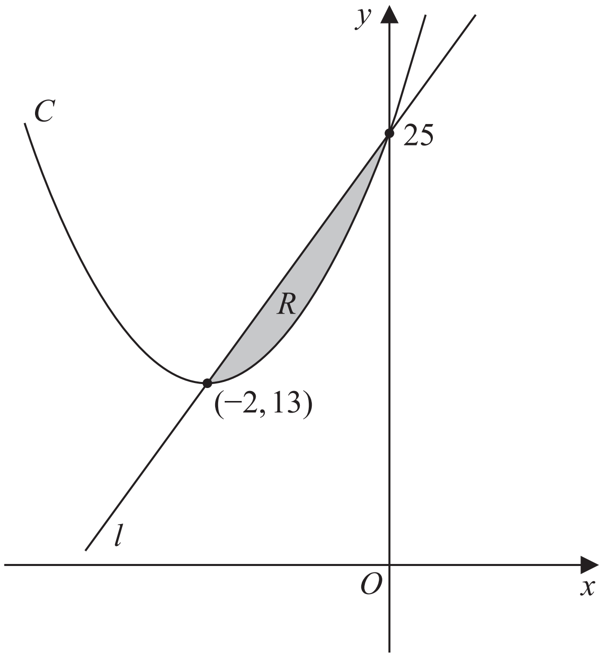


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ and a straight line l.

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $\mathrm{f}(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = \mathrm{f}(x)$

use inequalities to define R.

(5)

7.

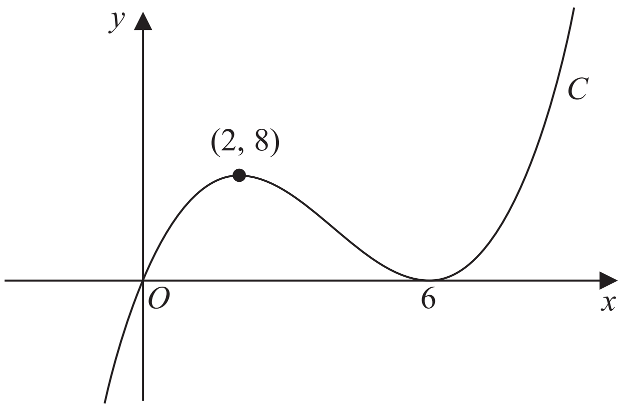


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$\mathrm{f}'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

8. $$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\dfrac{dy}{dx} < 0$

(1)

9. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x + 1}{x - 2}, \quad x > 2$$

(a) Show that

$$\mathrm{f}'(x) = \frac{-9}{(x - 2)^2}$$

(3)

Given that P is a point on C such that $\mathrm{f}'(x) = -1$,

(b) find the coordinates of P.

(3)

10. $$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$

(4)

(b) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form.

(3)

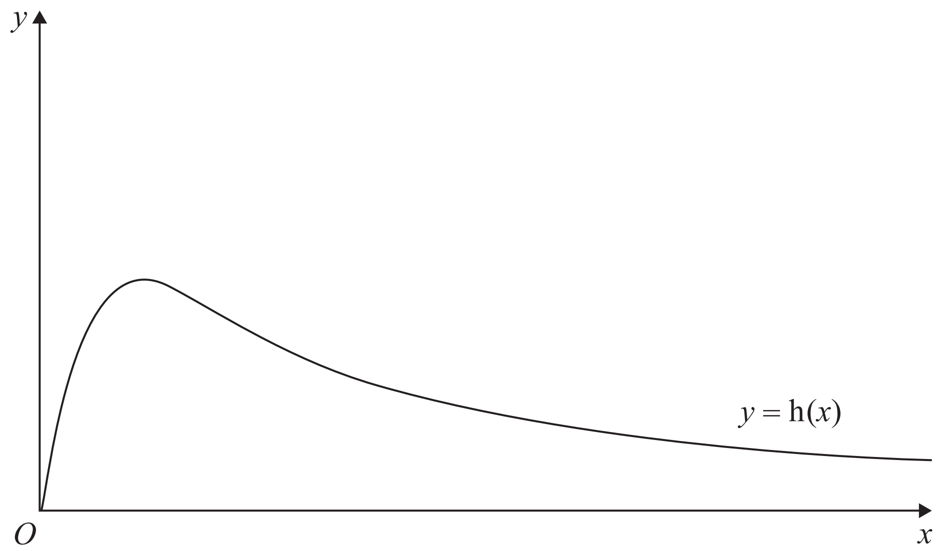


Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(c) Calculate the range of $\mathrm{h}(x)$.

(5)

11.

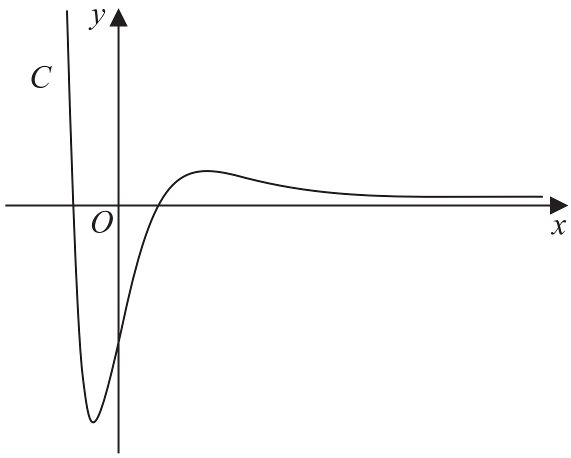


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4(x^2 - 2)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}'(x) = 8(2 + x - x^2)\mathrm{e}^{-2x}$

(3)

(b) Hence find, in simplest form, the exact coordinates of the stationary points of C.

(3)

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 3 \qquad x \geqslant 0$$

(c) Find (i) the range of g

(ii) the range of h

(3)

12.

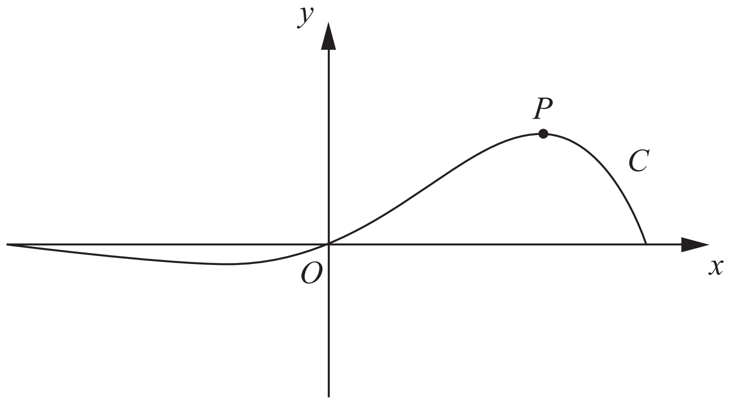


Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x \,, \quad -\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$$

(a) Find the x coordinate of the turning point P on C, for which $x > 0$
Give your answer as a multiple of π.

(6)

(b) Find an equation of the normal to C at the point where $x = 0$

(3)

13. $$f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \qquad x > 2, x \in \mathbb{R}$$

(a) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x - 2}$$

find the values of the constants A and B.

(4)

(b) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = f(x)$ at the point where $x = 3$

(5)

14. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x} \cos 4x, \quad \frac{\pi}{4} \leqslant x < \frac{\pi}{2}$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2 2y, \quad 0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \qquad 0 < y < \frac{\pi}{4}$$

where p and q are constants to be determined.

(5)

15. The curve C has equation $x = 8y \tan 2y$

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$

(a) Verify that P lies on C.

(1)

(b) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π.

(7)

16. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p.

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

17. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

18. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where k is a constant.

(a) Deduce the value of k.

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g.

(3)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)

19. The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)\,g(x)$$

where $g(x)$ is a function to be found.

(3)

Given that the curve with equation $y = f(x)$ has at least one stationary point,

(b) find the range of possible values of k.

(3)

20.

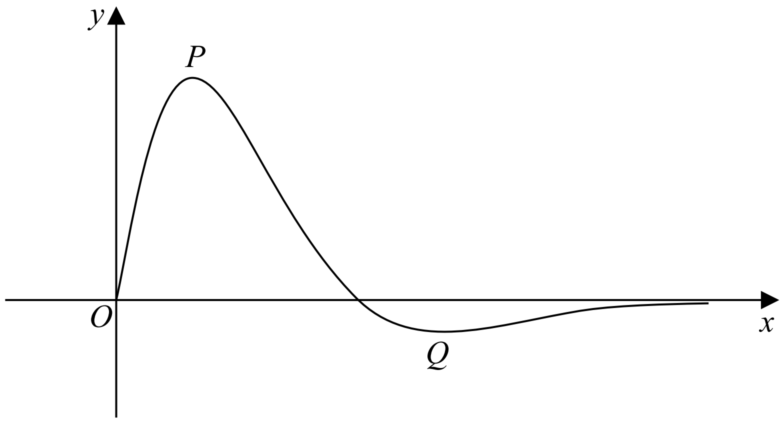


Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{\mathrm{e}^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$.

(ii) $y = 3 - 2\mathrm{f}(x)$.

(4)

21. A curve has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{7xe^{x}}{\sqrt{e^{3x} - 2}} \qquad x > \ln \sqrt[3]{2}$$

(a) Show that

$$\mathrm{f}'(x) = \frac{7e^{x}(e^{3x}(2 - x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

(5)

(b) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation $x = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

(1)

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2

(ii) the value of β

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

(2)

Question 21 continued

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

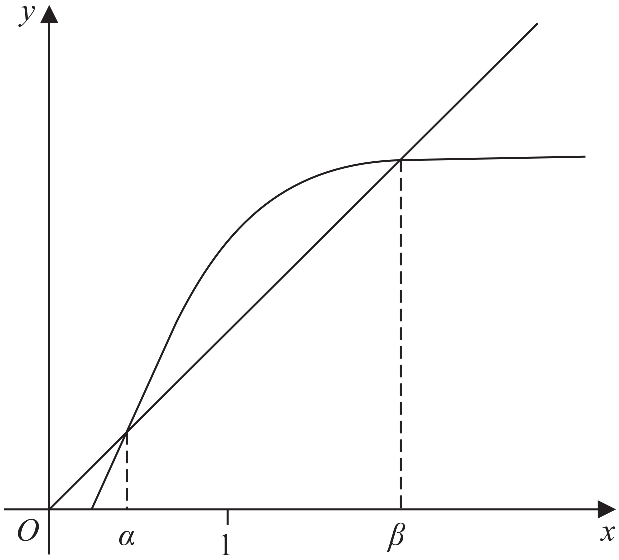
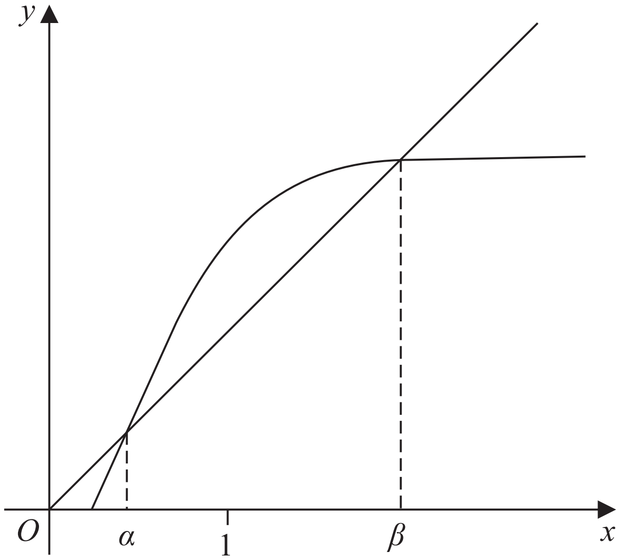


Diagram 1



copy of Diagram 1