1

The curve C has equation $y = \frac{1}{3}x^3 - 9x + 1$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} =$

(2)

(b) Find the range of values of x for which C has a negative gradient.

(3)

(Total for Question 1 is 5 marks)

2 The curve **C** has equation $y = 5x^3 - x^2 - 6x + 4$

(a) Find $\dfrac{dy}{dx}$

$\dfrac{dy}{dx} =$ ....

(2)

There are two points on the curve **C** at which the gradient of the curve is 2

(b) Find the x coordinate of each of these two points.
Show clear algebraic working.

....

(4)

(Total for Question 2 is 6 marks)

3 A curve **C** has equation $y = x^3 - x^2 - 8x + 12$

(a) Find $\dfrac{dy}{dx}$

$\dfrac{dy}{dx} = $

(2)

The curve **C** has two turning points.

(b) Work out the x coordinates of the two turning points.
Show your working clearly.

(3)

(c) Show that the x-axis is a tangent to the curve **C**.

(2)

(Total for Question 3 is 7 marks)

4 $$y = x^3 - 2x^2 - 15x + 5$$

(a) Find $\dfrac{dy}{dx}$

$\dfrac{dy}{dx} =$ ....

(2)

C is the curve with equation $y = x^3 - 2x^2 - 15x + 5$

(b) Work out the range of values of x for which **C** has a negative gradient.

....

(4)

(Total for Question 4 is 6 marks)

5 The curve shown in the diagram has equation

$$y = x^3 - 27x + k \text{ where } k \text{ is a positive constant with } k < 54$$

The curve has a maximum point at $A\ (a, b)$
The curve has a minimum point at $B\ (c, d)$

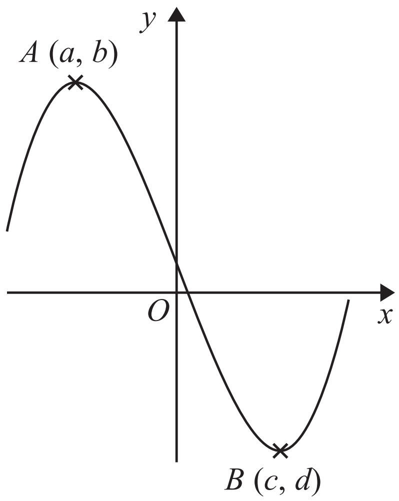


Diagram **NOT** accurately drawn

Using differentiation, find the value of $b - d$
Show your working clearly.

....

(Total for Question 5 is 6 marks)

6 The point A is the only stationary point on the curve with equation $y = kx^2 + \dfrac{16}{x}$ where k is a constant.

Given that the coordinates of A are $\left(\dfrac{2}{3}, a\right)$

find the value of a.
Show your working clearly.

$a =$ ....

(Total for Question 6 is 5 marks)

7

The diagram shows a sketch of part of the curve with equation $y = x^2 - \dfrac{p}{x}$ where p is a positive constant.

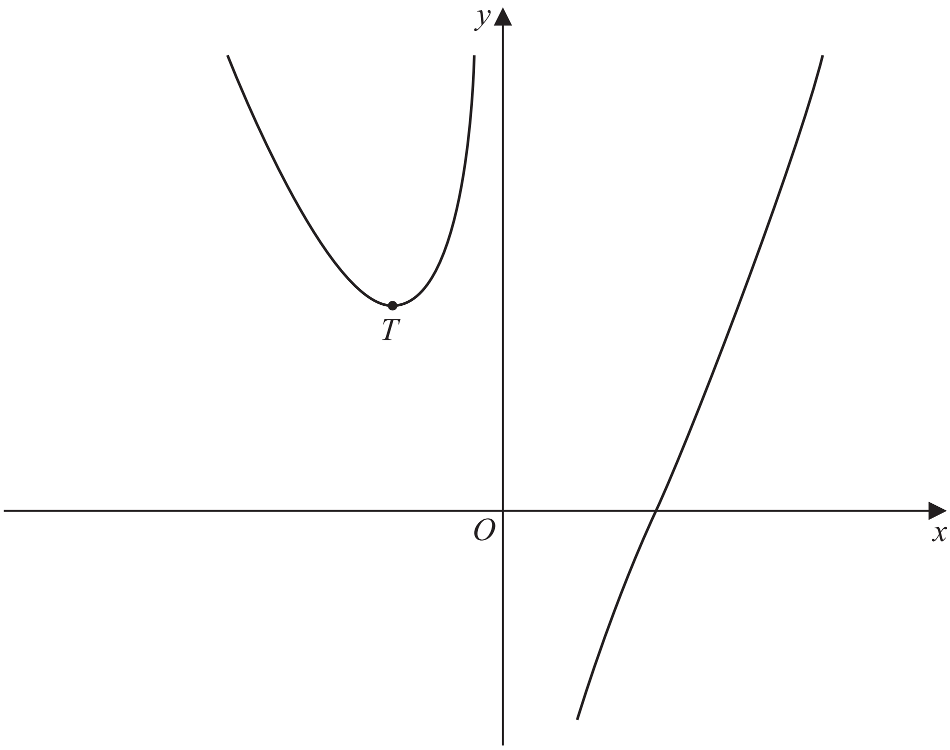


Diagram **NOT** accurately drawn

For all values of p, the curve has exactly one turning point and this turning point is a minimum shown as the point T in the sketch.

For the curve where the x coordinate of T is –3

(a) find the value of p

$p =$ ....
(4)

The line with equation $y = k$ is a tangent to the curve with equation $y = x^2 - \frac{16}{x}$

(b) Find the value of k

$k =$ ....

(3)

(Total for Question 7 is 7 marks)

8

Curve **C** has equation $y = px^3 - mx$ where p and m are positive integers.

Find the range of values of x, in terms of p and m, for which the gradient of **C** is negative.

....

(Total for Question 8 is 4 marks)

9 The curve **C** has equation $y = ax^3 + bx^2 - 12x + 6$ where a and b are constants.

The point A with coordinates (2, –6) lies on **C**
The gradient of the curve at A is 16

Find the y coordinate of the point on the curve whose x coordinate is 3
Show clear algebraic working.

$y =$ ....

(Total for Question 9 is 6 marks)

10

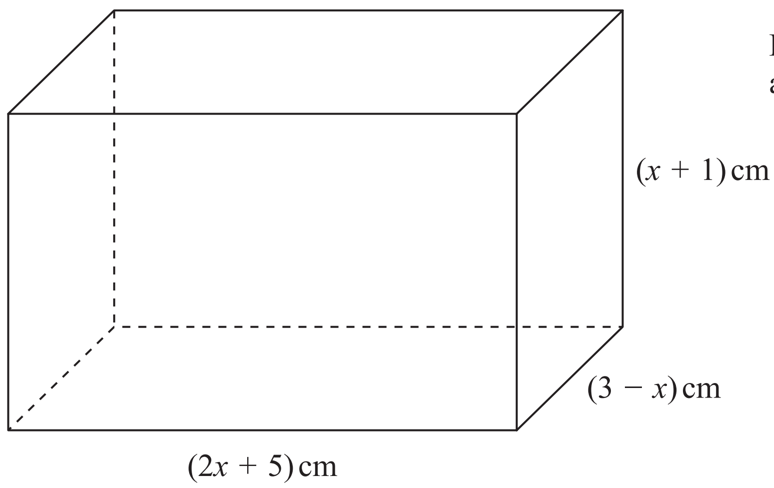


Diagram **NOT** accurately drawn

The diagram shows a cuboid of volume $V \text{ cm}^3$

(a) Show that $V = 15 + 16x - x^2 - 2x^3$

(3)

There is a value of x for which the volume of the cuboid is a maximum.

(b) Find this value of x.
Show your working clearly.
Give your answer correct to 3 significant figures.

$x =$ ....

(5)

(Total for Question 10 is 8 marks)

11 The diagram shows a solid cuboid.

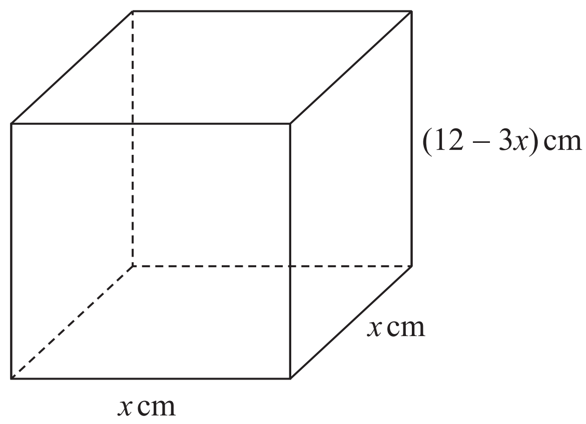


Diagram **NOT** accurately drawn

The total surface area of the cuboid is A cm^2

Find the maximum value of A.

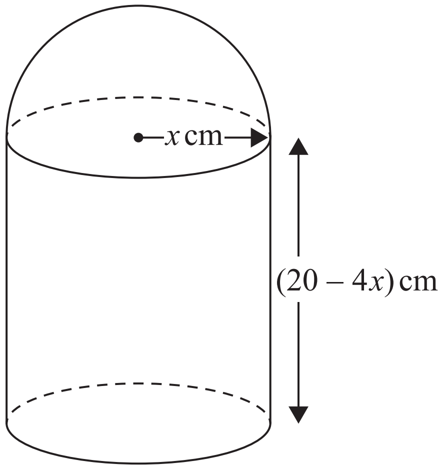
....

(Total for Question 11 is 5 marks)

12 A solid, **S**, is made from a hemisphere and a cylinder.

The centre of the circular face of the hemisphere and the centre of the top face of the cylinder are at the same point.

Diagram **NOT** accurately drawn



The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $(20 - 4x)$ cm.

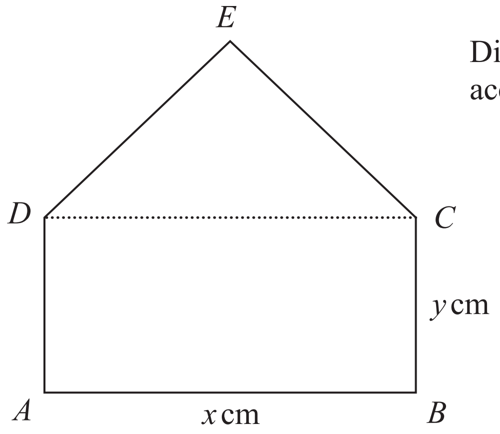
The volume of **S** is V cm^3 where $V = \frac{1}{3}\pi y$

Find the maximum value of y.
Show clear algebraic working.

....

(Total for Question 12 is 5 marks)

13 *ABCED* is a five-sided shape.



ABCD is a rectangle.
CED is an equilateral triangle.

$AB = x$ cm $\quad BC = y$ cm

The perimeter of *ABCED* is 100 cm.
The area of *ABCED* is R cm^2

(a) Show that $R = \dfrac{x}{4}\left(200 - \left[6 - \sqrt{3}\right]x\right)$

(3)

(b) (i) Find the value of x for which R has its maximum value.

Give your answer in the form $\dfrac{p}{q - \sqrt{3}}$ where p and q are integers.

x = ...

(2)

(ii) Explain why the maximum value of R is given by this value of x.

....

....

....

(1)

(Total for Question 13 is 6 marks)

14 A particle P is moving along a straight line.
The fixed point O lies on this line.

At time t seconds, the displacement, s metres, of P from O is given by

$$s = 4t^3 - 6t^2 + 5t$$

At time t seconds, the velocity of P is v m/s.

(a) Find an expression for v in terms of t.

$v =$ ....

(2)

(b) Find the time at which the acceleration of the particle is 6 m/s^2

.... seconds

(3)

(Total for Question 14 is 5 marks)

15 A particle P is moving along a straight line.
The fixed point O lies on the line.

At time t seconds ($t \geqslant 0$), the displacement of P from O is s metres where

$$s = t^3 - 9t^2 + 33t - 6$$

Find the minimum speed of P.

.... m/s

(Total for Question 15 is 5 marks)

16 A particle P is moving along a straight line that passes through the fixed point O.
The displacement, s metres, of P from O at time t seconds is given by

$$s = t^3 - 6t^2 + 5t - 4$$

Find the value of t for which the acceleration of P is $3\,\text{m/s}^2$

$t =$ ....

(Total for Question 16 is 4 marks)

17 A particle P moves along a straight line that passes through the fixed point O

The displacement, x metres, of P from O at time t seconds, where $t \geqslant 0$, is given by

$$x = 4t^3 - 27t + 8$$

The direction of motion of P reverses when P is at the point A on the line.

The acceleration of P at the instant when P is at A is a m/s^2

Find the value of a

$a =$ ....

(Total for Question 17 is 5 marks)