1.

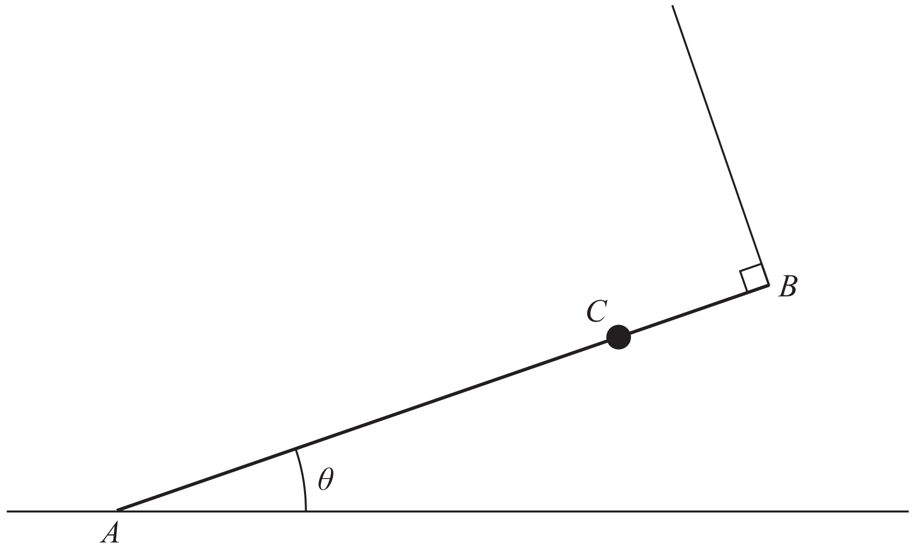

Figure 2

A uniform rod AB has mass M and length $2a$

A particle of mass $2M$ is attached to the rod at the point C, where $AC = 1.5a$

The rod rests with its end A on rough horizontal ground.

The rod is held in equilibrium at an angle θ to the ground by a light string that is attached to the end B of the rod.

The string is perpendicular to the rod, as shown in Figure 2.

(a) Explain why the frictional force acting on the rod at A acts horizontally to the right on the diagram.

(1)

The tension in the string is T

(b) Show that $T = 2Mg\cos\theta$

(3)

Given that $\cos\theta = \frac{3}{5}$

(c) show that the magnitude of the vertical force exerted by the ground on the rod at A is $\frac{57Mg}{25}$

(3)

The coefficient of friction between the rod and the ground is μ

Given that the rod is in limiting equilibrium,

(d) show that $\mu = \frac{8}{19}$

(4)

2.

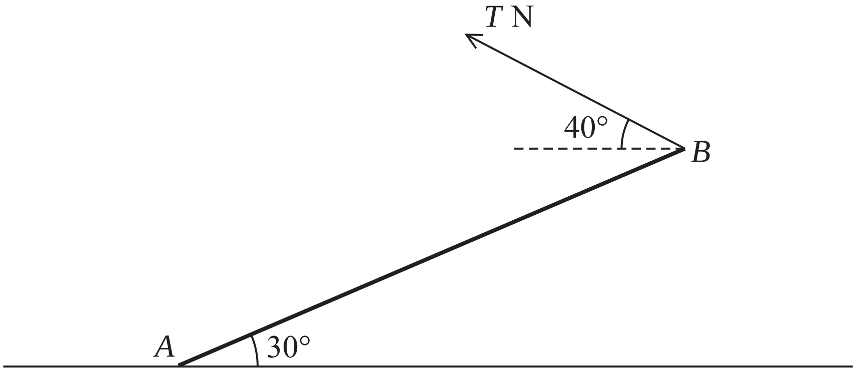

Figure 1

A uniform rod AB, of mass 6 kg and length 1.6 m, rests with its end A on rough horizontal ground. The rod is held in equilibrium at 30° to the horizontal by a light string attached to the rod at B. The string is at 40° to the horizontal and lies in the same vertical plane as the rod, as shown in Figure 1. The tension in the string is T newtons. The coefficient of friction between the ground and the rod is μ.

(a) Show that, to 3 significant figures, $T = 27.1$

(4)

(b) Find the set of values of μ for which equilibrium is possible.

(5)

3.

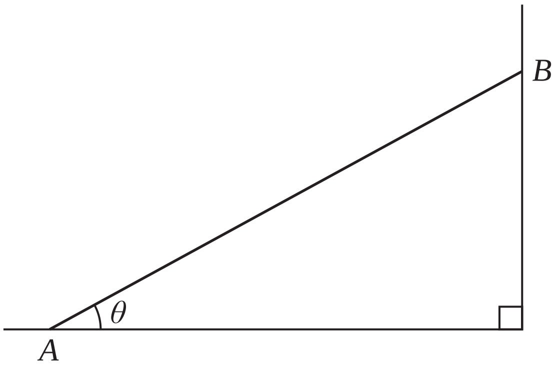

Figure 1

A uniform rod AB, of mass 25 kg and length 3 m, has end A resting on rough horizontal ground. The end B rests against a rough vertical wall.

The rod is in a vertical plane perpendicular to the wall.

The coefficient of friction between the rod and the ground is $\frac{4}{5}$

The coefficient of friction between the rod and the wall is $\frac{3}{5}$

The rod rests in limiting equilibrium.

The rod is at an angle of θ to the ground, as shown in Figure 1.

Find the exact value of $\tan\theta$.

(9)

4.

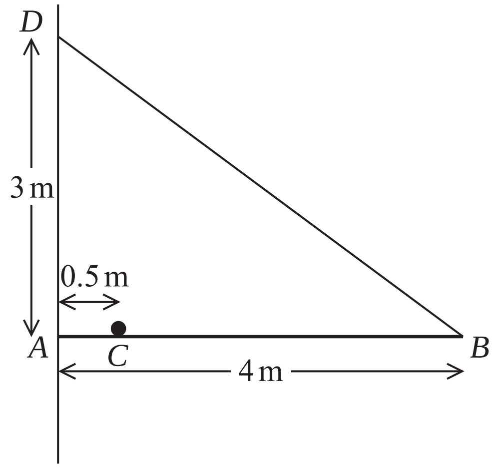

Figure 1

A uniform rod AB has mass 5 kg and length 4 m. The rod is held in a horizontal position by a light inextensible string. The end A of the rod rests against a rough vertical wall. One end of the string is attached to the rod at B and the other end is attached to the wall at a point D. The point D is vertically above A, with $AD = 3$ m. A particle of mass 2 kg is attached to the rod at C, where $AC = 0.5$ m, as shown in Figure 1. The rod is in equilibrium in a vertical plane perpendicular to the wall. The coefficient of friction between the rod and the wall is μ.

Find

(a) the tension in the string,

(4)

(b) the magnitude of the force exerted by the wall on the rod at A,

(5)

(c) the range of possible values of μ.

(2)

5.

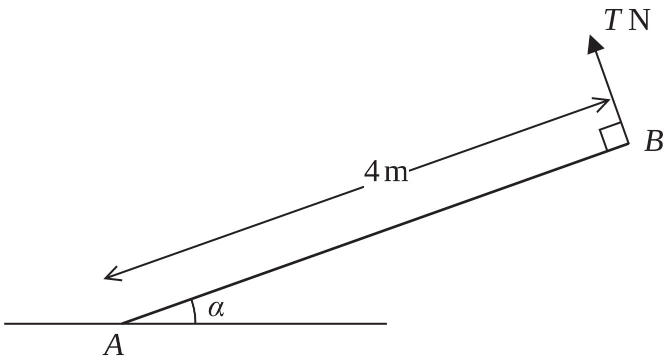

Figure 2

A uniform rod AB has length 4 m and weight 50 N.
The rod has its end A on rough horizontal ground. The rod is held in equilibrium at an angle α to the ground by a light inextensible cable attached to the rod at B, as shown in Figure 2. The cable and the rod lie in the same vertical plane and the cable is perpendicular to the rod. The tension in the cable is T newtons.

Given that $\sin\alpha = \dfrac{3}{5}$

(a) show that $T = 20$

(3)

Given also that the rod is in limiting equilibrium,

(b) find the value of the coefficient of friction between the rod and the ground.

(6)

6.

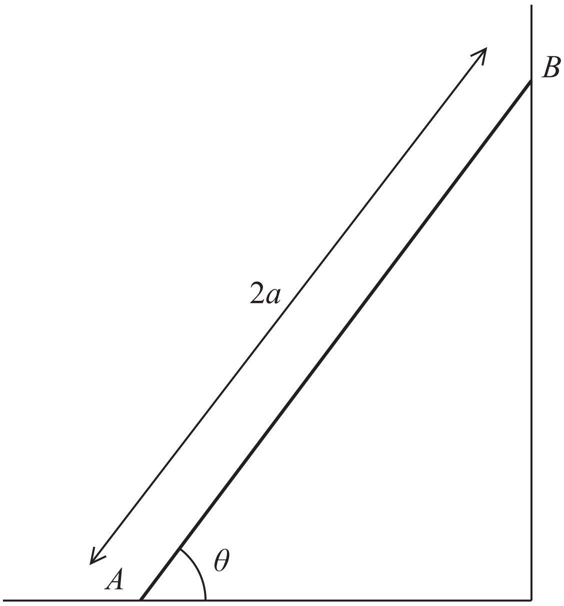

Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

(5)

A horizontal force of magnitude kmg, where k is a constant, is now applied to the beam at A.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of k.

(5)

7.

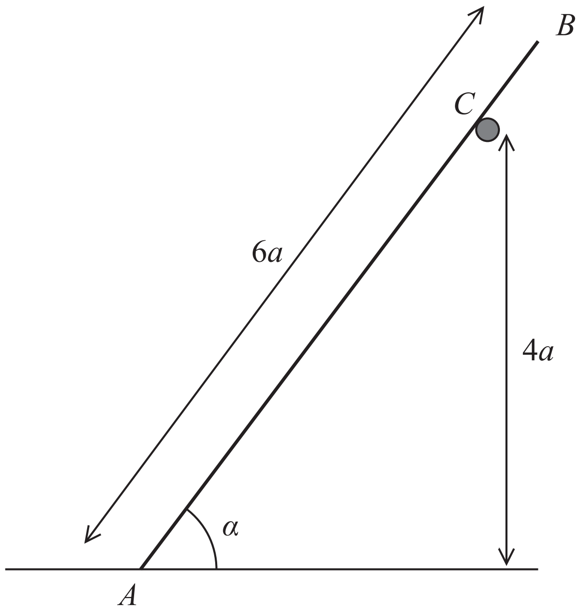

Figure 1

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C.

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ.

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$

(3)

(b) Hence, or otherwise, find the value of μ.

(7)

8.

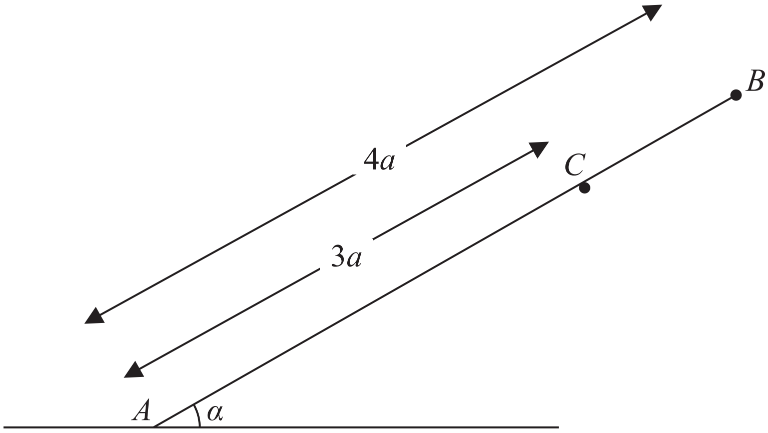

Figure 2

A uniform rod AB has length $4a$ and weight W. A particle of weight kW, $k < 1$, is attached to the rod at B. The rod rests in equilibrium against a fixed smooth horizontal peg. The end A of the rod is on rough horizontal ground, as shown in Figure 2. The rod rests on the peg at C, where $AC = 3a$, and makes an angle α with the ground, where $\tan\alpha = \frac{1}{3}$. The peg is perpendicular to the vertical plane containing AB.

(a) Give a reason why the force acting on the rod at C is perpendicular to the rod.

(1)

(b) Show that the magnitude of the force acting on the rod at C is

$$\frac{\sqrt{10}}{5}W\left(1+2k\right)$$

(4)

The coefficient of friction between the rod and the ground is $\frac{3}{4}$.

(c) Show that for the rod to remain in equilibrium $k \leqslant \frac{2}{11}$.

(7)

9.

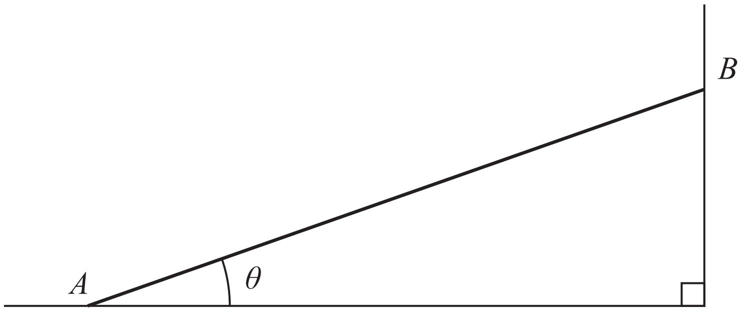

Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2}Mg\cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

A new value for S is calculated using this new model, with $\tan\theta = \frac{3}{4}$

(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

(1)

10.

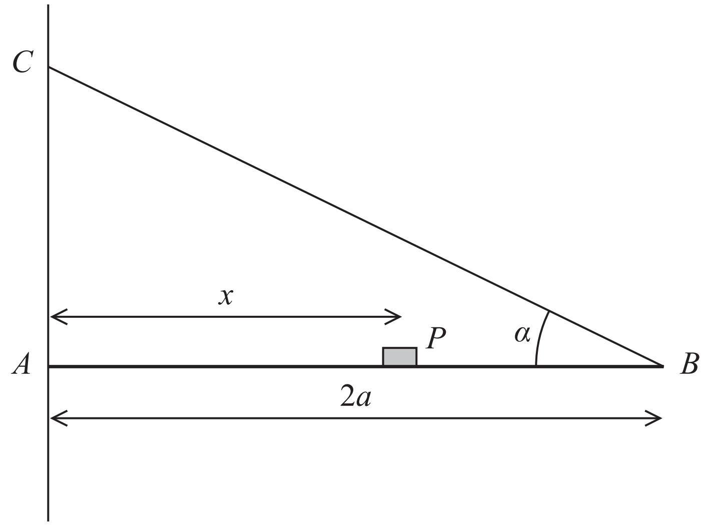

Figure 3

A plank, AB, of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C, which is vertically above A.

A small block of mass $3M$ is placed on the plank at the point P, where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\dfrac{5Mg(3x + a)}{6a}$

(3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a.

(2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan\beta$

(5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P. You must justify your answer carefully.

(3)

11.

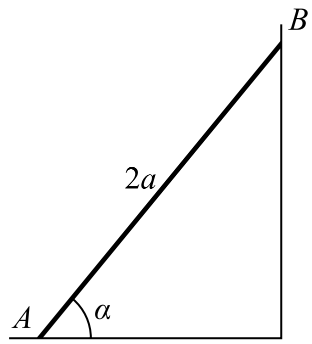

Figure 1

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)