1. The point $P(-2, -5)$ lies on the curve with equation $y = f(x), \quad x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = f(x) + 2$

(1)

(b) $y = |f(x)|$

(1)

(c) $y = 3f(x - 2) + 2$

(2)

2.

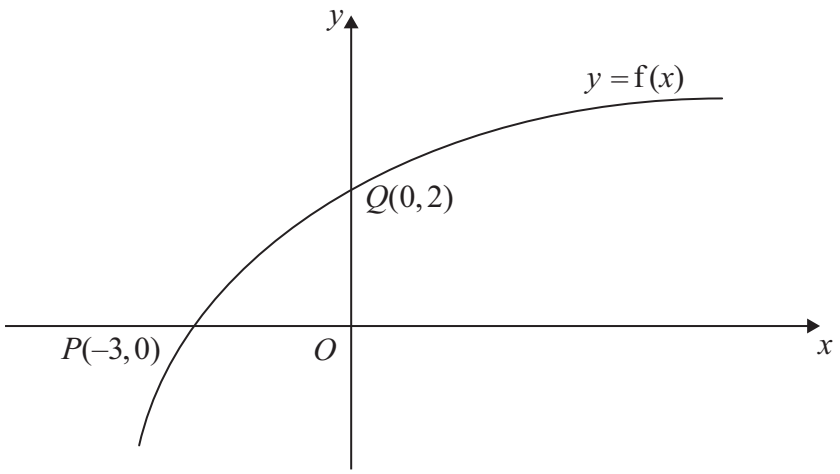

Figure 1

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

(a) Find the value of $\mathrm{ff}(-3)$.

(2)

On separate diagrams, sketch the curve with equation

(b) $y = \mathrm{f}^{-1}(x)$,

(2)

(c) $y = \mathrm{f}(|x|) - 2$,

(2)

(d) $y = 2\mathrm{f}\left(\frac{1}{2}x\right)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

3.

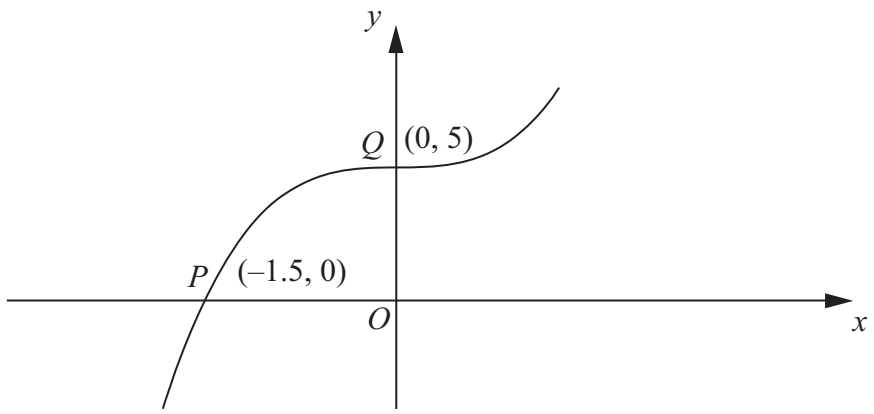

Figure 2

Figure 2 shows part of the curve with equation $y = \mathrm{f}(x)$
The curve passes through the points $P(-1.5,\ 0)$ and $Q(0,\ 5)$ as shown.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$

(2)

(b) $y = \mathrm{f}(|x|)$

(2)

(c) $y = 2\mathrm{f}(3x)$

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

4. Given that

$$f(x) = \ln x, \quad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x-axis.
In each case, state the equation of the asymptote.

(7)

5. Given that

$$f(x) = 2e^x - 5, \quad x \in \mathbb{R}$$

(a) sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$

(ii) $y = |f(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(6)

(b) Deduce the set of values of x for which $f(x) = |f(x)|$

(1)

(c) Find the exact solutions of the equation $|f(x)| = 2$

(3)

6.

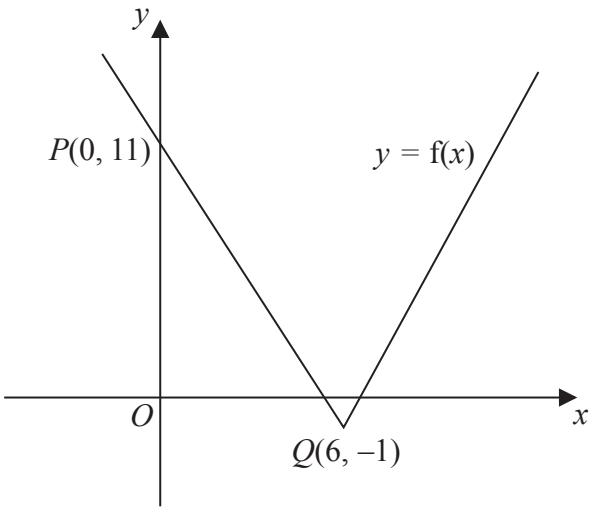

Figure 1

Figure 1 shows part of the graph with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y-axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$

(2)

(b) $y = 2\mathrm{f}(-x) + 3$

(3)

On each diagram, show the coordinates of the points corresponding to P and Q.

Given that $\mathrm{f}(x) = a|x - b| - 1$, where a and b are constants,

(c) state the value of a and the value of b.

(2)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

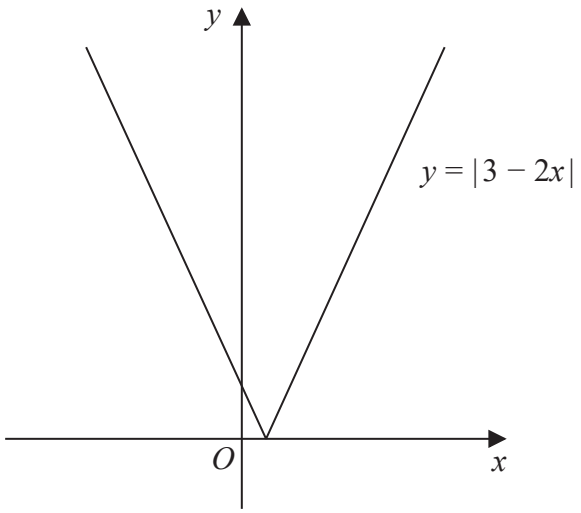

Figure 1

Figure 1 shows a sketch of the graph with equation $y = |3 - 2x|$

Solve

$$|3 - 2x| = 7 + x$$

(4)

8. The function f has domain $-2 \leqslant x \leqslant 6$ and is linear from (–2, 10) to (2, 0) and from (2, 0) to (6, 4). A sketch of the graph of $y = f(x)$ is shown in Figure 1.

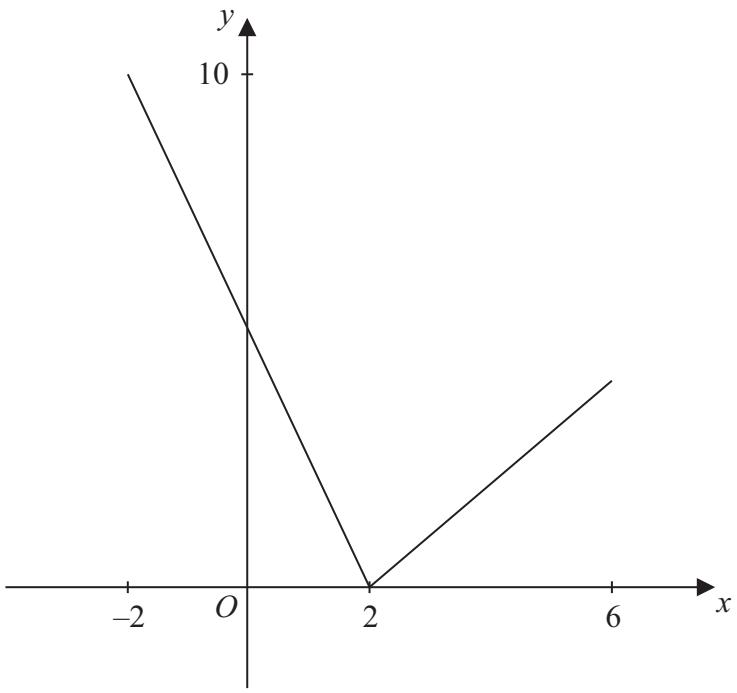

Figure 1

(a) Write down the range of f.

(1)

(b) Find ff(0).

(2)

The function g is defined by

$$g : x \to \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $g^{-1}(x)$

(3)

(d) Solve the equation $gf(x) = 16$

(5)

Question 8 continued

9.

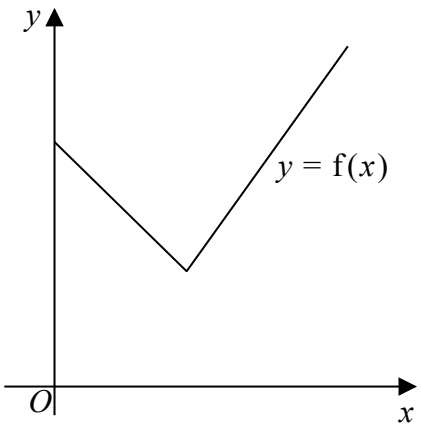

Figure 2

Figure 2 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$\mathrm{f}(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

10.

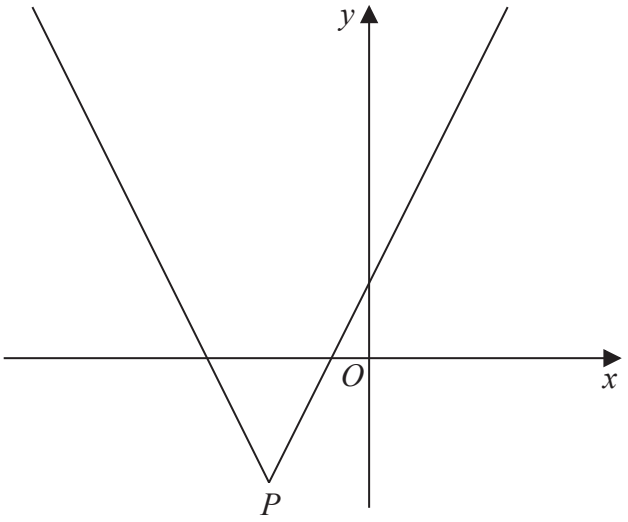

Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

Question 10 continued

11.

Given that a and b are positive constants,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x - a|$

(ii) $y = |2x - a| + b$

Show, on each sketch, the coordinates of each point at which the graph crosses or meets the axes.

(4)

Given that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$,

(b) find c in terms of a.

(4)

12.

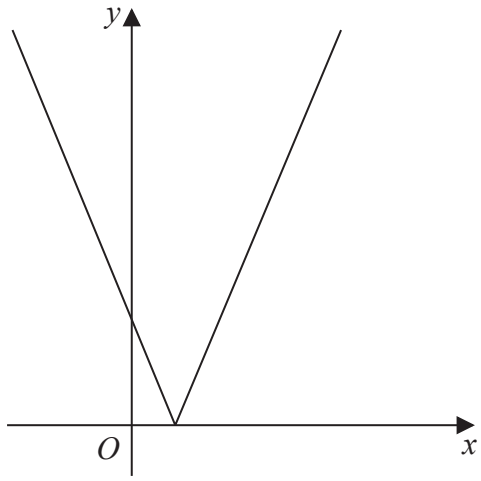

Figure 4

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where k is a positive constant.

(a) Sketch the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k, the set of values of x for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5\mathrm{f}\left(\frac{1}{2}x\right)$$

(2)

Question 12 continued

13.

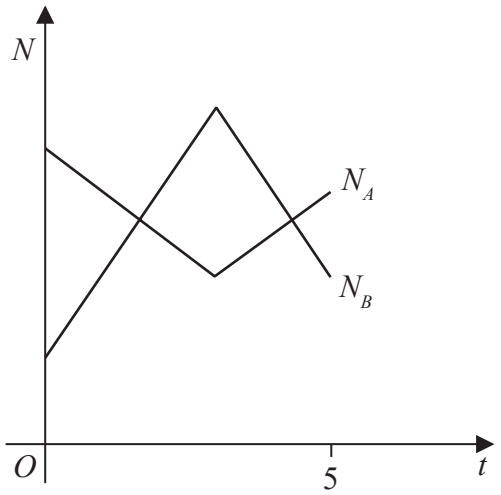

Figure 2

The number of subscribers to two different music streaming companies is being monitored.

The number of subscribers, N_A, in thousands, to **company A** is modelled by the equation

$$N_A = |t - 3| + 4 \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

The number of subscribers, N_B, in thousands, to **company B** is modelled by the equation

$$N_B = 8 - |2t - 6| \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

Figure 2 shows a sketch of the graph of N_A and the graph of N_B over a 5-year period.

Use the equations of the models to answer parts (a), (b), (c) and (d).

(a) Find the initial difference between the number of subscribers to **company A** and the number of subscribers to **company B**.

(2)

When $t = T$ **company A** reduced its subscription prices and the number of subscribers increased.

(b) Suggest a value for T, giving a reason for your answer.

(2)

(c) Find the range of values of t for which $N_A > N_B$ giving your answer in set notation.

(5)

(d) State a limitation of the model used for **company B**.

(1)

Question 13 continued