1

Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(a) $5^x = 10$,

(2)

(b) $\log_3(x-2) = -1$.

(2)

2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

3. (a) Sketch the graph of

$$y = 3^x, \quad x \in \mathbb{R}$$

showing the coordinates of any points at which the graph crosses the axes.

(2)

(b) Use algebra to solve the equation

$$3^{2x} - 9(3^x) + 18 = 0$$

giving your answers to 2 decimal places where appropriate.

(5)

4. Find the values of x such that

$$2\log_3 x - \log_3(x - 2) = 2$$

(5)

5. A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is shown below.

$2\log_2 x - \log_2 \sqrt{x} = 3$	
$2\log_2 \left(\frac{x}{\sqrt{x}}\right) = 3$	using the subtraction law for logs
$2\log_2 \left(\sqrt{x}\right) = 3$	simplifying
$\log_2 x = 3$	using the power law for logs
$x = 3^2 = 9$	using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each.

(2)

(b) Write out the correct solution.

(3)

6. Given that $y = 3x^2$,

(a) show that $\log_3 y = 1 + 2\log_3 x$

(3)

(b) Hence, or otherwise, solve the equation

$$1 + 2\log_3 x = \log_3 (28x - 9)$$

(3)

7

Given that

$$2\log_2(x+15) - \log_2 x = 6$$

(a) Show that

$$x^2 - 34x + 225 = 0$$

(5)

(b) Hence, or otherwise, solve the equation

$$2\log_2(x+15) - \log_2 x = 6$$

(2)

8

Given that $\log_3 x = a$, find in terms of a,

(a) $\log_3 (9x)$

(2)

(b) $\log_3\left(\frac{x^5}{81}\right)$

(3)

giving each answer in its simplest form.

(c) Solve, for x,

$$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$$

giving your answer to 4 significant figures.

(4)

9. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p,

(i) $\log_3\left(\frac{x}{9}\right)$

(ii) $\log_3\left(\sqrt{x}\right)$

(2)

(b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

10. (i) Find the exact value of x for which

$$\log_2(2x) = \log_2(5x + 4) - 3$$

(4)

(ii) Given that

$$\log_a y + 3\log_a 2 = 5$$

express y in terms of a.
Give your answer in its simplest form.

(3)

11. (i) Use logarithms to solve the equation $8^{2x+1} = 24$, giving your answer to 3 decimal places.

(3)

(ii) Find the values of y such that

$$\log_2(11y - 3) - \log_2 3 - 2\log_2 y = 1, \qquad y > \frac{3}{11}$$

(6)

12. (i) $2\log(x + a) = \log(16a^6)$, where a is a positive constant

Find x in terms of a, giving your answer in its simplest form.

(3)

(ii) $\log_3(9y + b) - \log_3(2y - b) = 2$, where b is a positive constant

Find y in terms of b, giving your answer in its simplest form.

(4)

13. (i) Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, \qquad a > 2$$

express b in terms of a.

(3)

(ii) Solve the equation

$$2^{2x+5} - 7(2^x) = 0$$

giving your answer to 2 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)