1. The box below shows a student's attempt to prove the following identity for a > b > 0

$$\arctan a - \arctan b \equiv \arctan \frac{a-b}{1+ab}$$

Let
$$x = \arctan a$$
 and $y = \arctan b$, so that $a = \tan x$ and $b = \tan y$

So
$$tan(arctan a - arctan b) \equiv tan(x - y)$$

$$\equiv \frac{\tan x - \tan y}{1 - \tan^2(xy)}$$

$$\equiv \frac{a - b}{1 - (ab)^2}$$

$$\equiv \frac{a - ab + ab - b}{(1 - ab)(1 + ab)}$$

$$\equiv \frac{a(1 - ab) - b(1 - ab)}{(1 - ab)(1 + ab)}$$

$$\equiv \frac{a - b}{1 + ab}$$

Taking arctan of both sides gives $\arctan a - \arctan b \equiv \arctan \frac{a-b}{1+ab}$ as required.

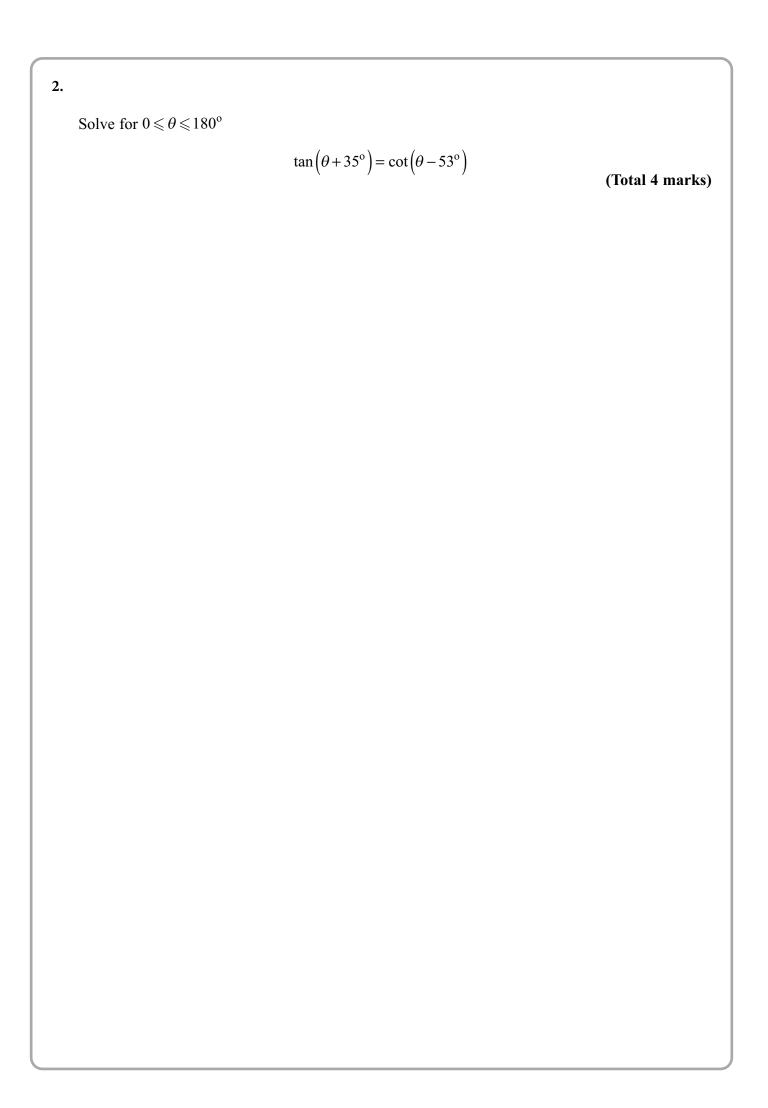
There are three errors in the proof where the working does not follow from the previous line.

(i) Describe these three errors.

(3)

(ii) Write out a correct proof of the identity.

(2)



3.		
(a) Write down the exact value of cos 405°	(1)
(b) Hence, using a double angle identity for cosvalue of cos 101.25°, giving your answer in	sine, or otherwise, determine the exact in the form	
$a\sqrt{b+a}$	$c\sqrt{2+\sqrt{2}}$	
where a , b and c are rational numbers.	(5)

4.	Given that	
	$3\sin^2 x + 2\sin x = 6\cos x + 9\sin x\cos x$	
	and that $-90^{\circ} < x < 90^{\circ}$,	
	find the possible values of $\tan x$.	
		(Total 6 marks)

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5.	Find	tha	170	110	of.
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$$\arccos\left(\frac{1}{\sqrt{2}}\right) + \arcsin\left(\frac{1}{3}\right) + 2\arctan\left(\frac{1}{\sqrt{2}}\right)$$

Give your answer as a multiple of π .

(arccos x is an alternative notion for $\cos^{-1} x$ etc.)

(Total 7 marks)

6.

- (a) Use the formula for $\sin(A B)$ to show that $\sin(90^{\circ} x) = \cos x$ (1)
- (b) Solve for $0 < \theta < 360^{\circ}$

$$2\sin(\theta + 17^{\circ}) = \frac{\cos(\theta + 8^{\circ})}{\cos(\theta + 17^{\circ})}$$

(Total 8 marks)

(7)

7. (a) Show that the equation

$$\tan x = \frac{\sqrt{3}}{1 + 4\cos x}$$

can be written in the form

$$\sin 2x = \sin(60^\circ - x) \tag{4}$$

(b) Solve, for $0 < x < 180^{\circ}$

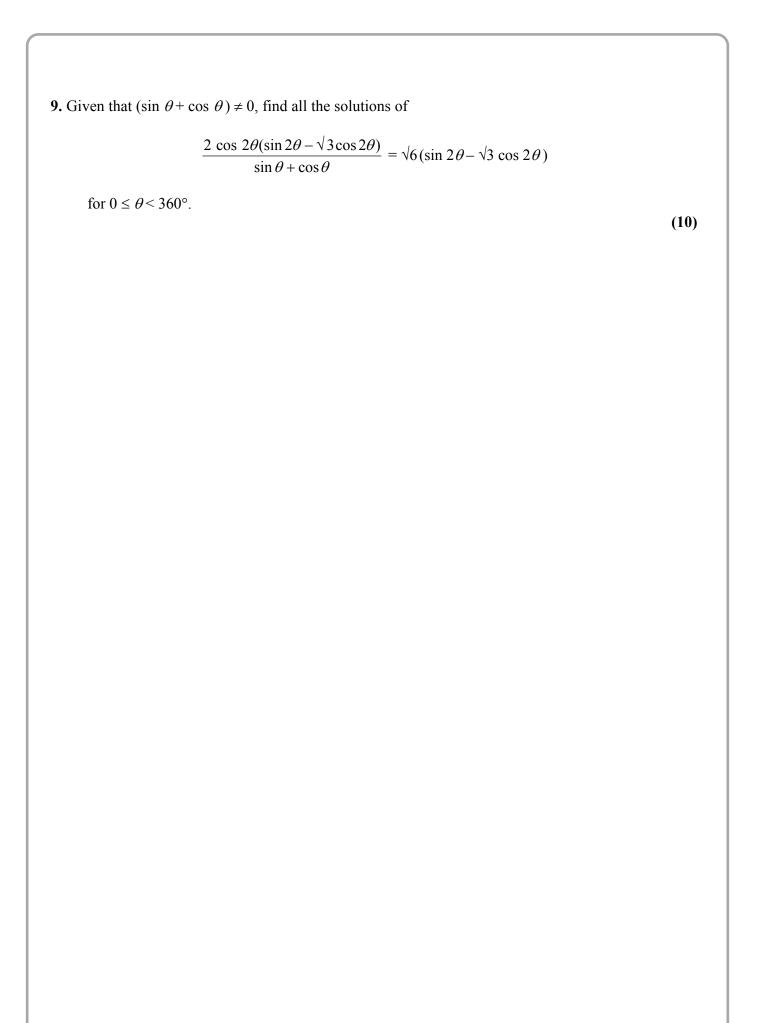
$$\tan x = \frac{\sqrt{3}}{1 + 4\cos x}$$

_ _ _ _ _ _

(5)

(Total 9 marks)

8.	Solve for $0 < x < 360^{\circ}$		
		$\cot 2x - \tan 78^\circ = \frac{(\sec x)(\sec 78^\circ)}{2}$	
	where x is not an integer m	ultiple of 90°	(Total 9 marks)



10. The angle K, $0 < K < \frac{\pi}{2}$, satisfies

$$\tan\theta\tan2\theta = \sum_{r=0}^{\infty} 2\cos^r 2\theta$$

(a) Show that $\tan \theta = 3^p$, where p is a rational number to be found.

(8)

(b) Hence show that $\frac{\pi}{6} < \theta < \frac{\pi}{4}$

(2)

(Total 10 marks)

11		
•	(a) Use the formulae for $\sin(A \pm B)$ and $\cos(A \pm B)$ to prove that $\tan(90^{\circ} - \theta) \equiv \cot\theta$	(3)
	(b) Solve for $0 < \theta < 360^{\circ}$	
	$2 - \sec^2(\theta + 11^\circ) = 2\tan(\theta + 11^\circ)\tan(\theta - 34^\circ)$	
	Give each answer as an integer in degrees.	
		(8)
		(+S1)

12. (a) Solve, for $0 \le \theta \le 2$.,

$$\sin\left(\frac{\pi}{3} - \theta\right) = \frac{1}{\sqrt{3}}\cos\theta . \tag{5}$$

(b) Find the value of x for which

$$\arcsin(1 - 2x) = \frac{\pi}{3} - \arcsin x, \qquad 0 < x < 0.5$$

 $[\arcsin x \text{ is an alternative notation for } \sin^{-1} x]$

(Total 12 marks)

(7)

13.

- (a) Prove that $\tan 15^{\circ} = 2 \sqrt{3}$ (4)
- (b) Solve, for $0 \le \theta < 360^{\circ}$,

$$\sin(\theta + 60^{\circ}) \sin(\theta - 60^{\circ}) = (1 - \sqrt{3}) \cos^{2} \theta$$
 (8)

(Total 12 marks)

14. (a) Prove the identity	
$(\sin x + \cos y)\cos(x - y) \equiv (1 + \sin(x - y))(\cos x + \sin y)$	(5)
(b) Hence, or otherwise, show that	
$\frac{\sin 5\theta + \cos 3\theta}{\cos 5\theta + \sin 3\theta} \equiv \frac{1 + \tan \theta}{1 - \tan \theta}$	
	(6)
(c) Given that $k > 1$, show that the equation $\frac{\sin 5\theta + \cos 3\theta}{\cos 5\theta + \sin 3\theta} = k$ has a unique solution	
in the interval $0 < \theta < \frac{\pi}{4}$	(4)
(-	+S2)