

1. A geometric series has first term $a = 360$ and common ratio $r = \frac{7}{8}$

Giving your answers to 3 significant figures where appropriate, find

- (a) the 20th term of the series, (2)
- (b) the sum of the first 20 terms of the series, (2)
- (c) the sum to infinity of the series. (2)

2. The first three terms of a geometric series are

18, 12 and p

respectively, where p is a constant.

Find

- (a) the value of the common ratio of the series,

(1)

- (b) the value of p ,

(1)

- (c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places.

(2)

3

A geometric series has first term a and common ratio $r = \frac{3}{4}$

The sum of the first 4 terms of this series is 175

- (a) Show that $a = 64$ (2)
- (b) Find the sum to infinity of the series. (2)
- (c) Find the difference between the 9th and 10th terms of the series.
Give your answer to 3 decimal places.

(3)

[illegible]

4

A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05

- (a) Show that the predicted profit in the year 2016 is £138 915 (1)
- (b) Find the first year in which the yearly predicted profit exceeds £200 000 (5)
- (c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound. (3)

5.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

- (a) show that the profit for Year 3 will be £23 328 (1)
- (b) find the first year when the yearly profit will exceed £65 000 (3)
- (c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000 (2)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6. (i) All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162

Find

- (a) the common ratio,

(4)

- (b) the first term.

(2)

- (ii) A different geometric series has a first term of 42 and a common ratio of $\frac{6}{7}$.

Find the smallest value of n for which the sum of the first n terms of the series exceeds 290

(4)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

The first three terms of a geometric series are $4p$, $(3p + 15)$ and $(5p + 20)$ respectively, where p is a **positive** constant.

- (a) Show that $11p^2 - 10p - 225 = 0$ (4)
- (b) Hence show that $p = 5$ (2)
- (c) Find the common ratio of this series. (2)
- (d) Find the sum of the first ten terms of the series, giving your answer to the nearest integer. (3)

8

The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

- (a) the common ratio, (2)
- (b) the first term, (2)
- (c) the sum to infinity, (2)
- (d) the smallest value of n for which the sum of the first n terms of the series exceeds 1000. (4)

9

The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$

The sum to infinity of the series is S_{∞}

(a) Find the value of S_∞

(2)

The sum to N terms of the series is S_N

(b) Find, to 1 decimal place, the value of S_{12}

(2)

(c) Find the smallest value of N , for which

$$S_\infty - S_N < 0.5$$

(4)

10. The first three terms of a geometric sequence are

$$7k - 5, 5k - 7, 2k + 10$$

where k is a constant.

(a) Show that $11k^2 - 130k + 99 = 0$

(4)

Given that k is not an integer,

(b) show that $k = \frac{9}{11}$

(2)

For this value of k ,

(c) (i) evaluate the fourth term of the sequence, giving your answer as an exact fraction,

(ii) evaluate the sum of the first ten terms of the sequence.

(6)

11

A geometric series is $a + ar + ar^2 + \dots$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r} \quad (4)$$

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(b) the common ratio,

(2)

(c) the first term, (2)

(d) the sum to infinity. (3)

12. The first three terms of a geometric sequence are

$$3k + 4 \qquad 12 - 3k \qquad k + 16$$

where k is a constant.

(a) Show that k satisfies the equation

$$3k^2 - 62k + 40 = 0 \quad (2)$$

Given that the sequence converges,

(b) (i) find the value of k , giving a reason for your answer,

(ii) find the value of S_∞

(5)

13. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_{\infty} = \frac{8}{7} \times S_6$$

show that $r = \pm \frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

14.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a .

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r .

(4)

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16. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^{\circ} = \frac{9}{28}$$

(3)