1. (a) Sketch the graph of the curve with equation

$$y = \left|\ln\left(2x + 5\right)\right| \qquad x > -\frac{5}{2}$$

On your sketch you should clearly state the equations of any asymptotes and mark the coordinates of points where the curve meets the coordinate axes.

(Total 3 marks)

2.

The function f is given by

$$f(x) = \ln(2x - 5), \qquad x > 2.5$$

(a) Find $f^{-1}(x)$.

(2)

The function g has domain $x > 2$ and

$$fg(x) = \ln\left(\frac{x + 10}{x - 2}\right), \qquad x > 2$$

(b) Find $g(x)$ and simplify your answer.

(3)

(Total 5 marks)

3.

The function f is given by

$$f(x) = x^2 - 4x + 9 \qquad x \in \mathbb{R},\ x \geqslant 3$$

(a) Find the range of f.

(2)

The function g is given by

$$g(x) = \frac{10}{x+1} \qquad x \in \mathbb{R},\ x \geqslant 4$$

(b) Find an expression for $gf(x)$.

(1)

(c) Find the domain and range of gf.

(4)

(Total 7 marks)

4. The function f is given by

$$f(x) = \sqrt{x+2} \quad \text{for} \quad x \in \mathbb{R}, x \geqslant 0$$

(a) Find $f^{-1}(x)$ and state the domain of f^{-1}

(2)

The function g is given by

$$g(x) = x^2 - 4x + 5 \quad \text{for} \quad x \in \mathbb{R}, x \geqslant 0$$

(b) Find the range of g.

(2)

(c) Solve the equation $fg(x) = x$.

(3)

(Total 7 marks)

5.

The function f is given by

$$f(x) = x^2 - 2x + 6, \qquad x \geqslant 0$$

(a) Find the range of f.

(3)

The function g is given by

$$g(x) = 3 + \sqrt{(x + 4)}, \qquad x \geqslant 2$$

(b) Find gf(x).

(2)

(c) Find the domain and range of gf.

(3)

(Total 8 marks)

6. (a) On the same diagram, sketch

$$y = (x + 1)(2 - x) \quad \text{and} \quad y = x^2 - 2|x|.$$

Mark clearly the coordinates of the points where these curves cross the coordinate axes.

(3)

(b) Find the x-coordinates of the points of intersection of these two curves.

(5)

(Total 8 marks)

7. (a) On separate diagrams sketch the curves with the following equations. On each sketch you should mark the coordinates of the points where the curve crosses the coordinate axes.

(i) $y = x^2 - 2x - 3$

(ii) $y = x^2 - 2|x| - 3$

(iii) $y = x^2 - x - |x| - 3$

(7)

(b) Solve the equation

$$x^2 - x - |x| - 3 = x + |x|$$

(4)

(Total 11 marks)

8.

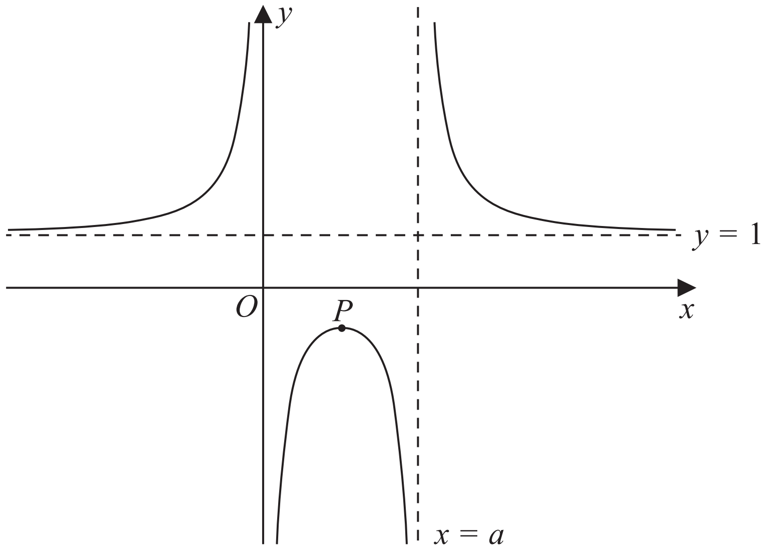

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 1 + \frac{4}{x(x-3)}$$

The curve has a turning point at the point P, and the lines with equations $y = 1$, $x = 0$ and $x = a$ are asymptotes to the curve.

(a) Write down the value of a.

(1)

(b) Find the coordinates of P, justifying your answer.

(4)

(c) Sketch the curve with equation $y = \left|\mathrm{f}\left(x + \frac{3}{2}\right)\right| - 1$

On your sketch, you should show the coordinates of any points of intersection with the coordinate axes, the coordinates of any turning points and the equations of any asymptotes.

(7)

9. The functions f and g are defined by

$$\mathrm{f}(x) = 2\sqrt{1 - \mathrm{e}^{-x}} \qquad x \in \mathbb{R}, x \geqslant 0$$
$$\mathrm{g}(x) = \ln\left(4 - x^2\right) \qquad x \in \mathbb{R}, -2 < x < 2$$

(a) (i) Explain why fg cannot be formed as a composite function.

(ii) Explain why gf can be formed as a composite function.

(2)

(b) (i) Find $\mathrm{gf}(x)$, giving the answer in the form $\mathrm{gf}(x) = a + bx$, where a and b are constants.

(ii) State the domain and range of gf.

(5)

(c) Sketch the graph of the function gf.

On your sketch, you should show the coordinates of any points where the graph meets or crosses the coordinate axes.

(2)

The circle C with centre $(0, -\ln 9)$ touches the line with equation $y = \mathrm{gf}(x)$ at precisely one point.

(d) Find an equation of the circle C.

(3)

(+S1)

10.

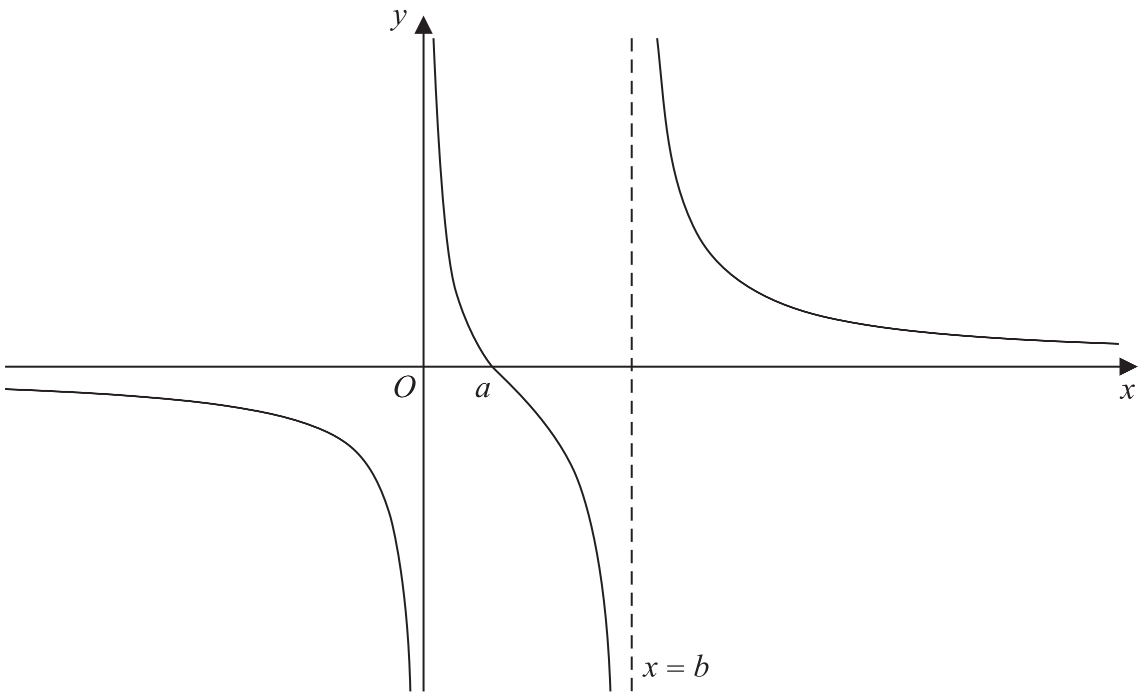

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4(x-1)}{x(x-3)}$$

The curve cuts the x-axis at $(a, 0)$. The lines $y = 0$, $x = 0$ and $x = b$ are asymptotes to the curve.

(a) Write down the value of a and the value of b.

(2)

(b) On separate axes, sketch the curves with the following equations. On your sketches, you should mark the coordinates of any intersections with the coordinate axes and state the equations of any asymptotes.

(i) $y = \mathrm{f}(x+2) - 4$

(6)

(ii) $y = \mathrm{f}(|x|) - 3$

(6)

(Total 14 marks)

11.

$$\mathrm{f}(x) = \frac{ax + b}{x + 2}\,; \quad x \in \mathbb{R}, x \neq -2,$$

where a and b are constants and $b > 0$.

(a) Find $\mathrm{f}^{-1}(x)$.

(2)

(b) Hence, or otherwise, find the value of a so that $\mathrm{ff}(x) = x$.

(2)

The curve C has equation $y = \mathrm{f}(x)$ and $\mathrm{f}(x)$ satisfies $\mathrm{ff}(x) = x$.

(c) On separate axes sketch

(i) $y = \mathrm{f}(x)$,

(3)

(ii) $y = \mathrm{f}(x - 2) + 2$.

(3)

On each sketch you should indicate the equations of any asymptotes and the coordinates, in terms of b, of any intersections with the axes.

The normal to C at the point P has equation $y = 4x - 39$. The normal to C at the point Q has equation $y = 4x + k$, where k is a constant.

(d) By considering the images of the normals to C on the curve with equation $y = \mathrm{f}(x - 2) + 2$, or otherwise, find the value of k.

(5)

(Total 15 marks)

12.

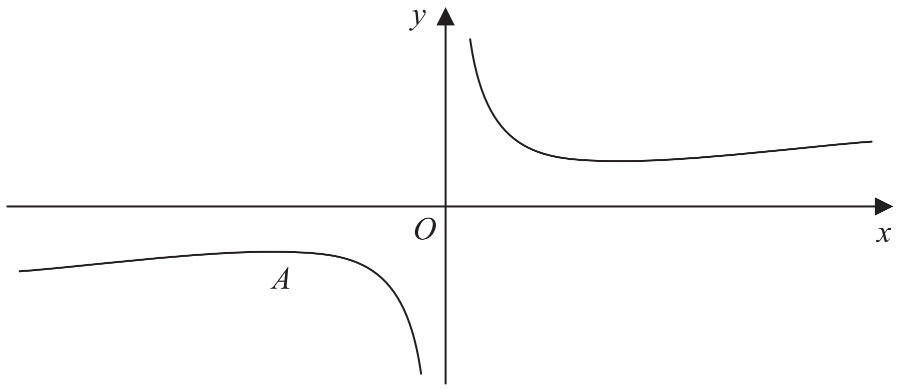

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x^2 + 16}{3x} \qquad x \neq 0$$

The curve has a maximum at the point A with coordinates (a, b).

(a) Find the value of a and the value of b.

(4)

The function g is defined as

$$\mathrm{g} : x \mapsto \frac{x^2 + 16}{3x} \qquad a \leqslant x < 0$$

where a is the value found in part (a).

(b) Write down the range of g.

(1)

(c) On the same axes sketch $y = \mathrm{g}(x)$ and $y = \mathrm{g}^{-1}(x)$.

(3)

(d) Find an expression for $\mathrm{g}^{-1}(x)$ and state the domain of g^{-1}

(5)

(e) Solve the equation $\mathrm{g}(x) = \mathrm{g}^{-1}(x)$.

(3)

(Total 16 marks)

13. (a) Find the set of values of k for which the equation

$$\frac{x^2 + 3x + 8}{x^2 + x - 2} = k$$

has no real roots.

(6)

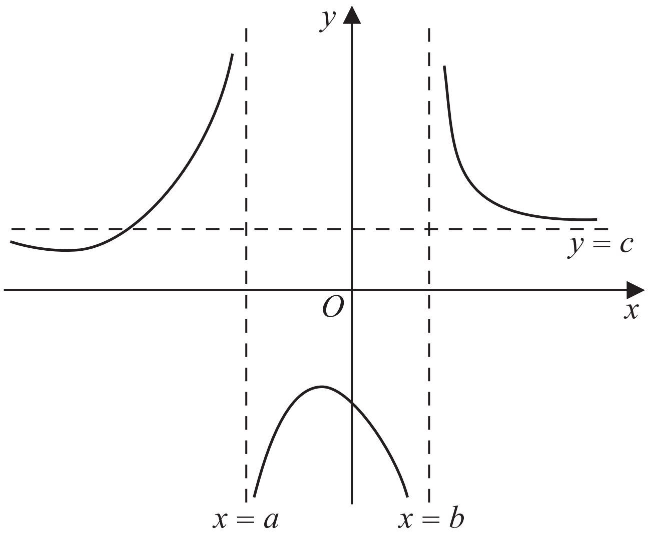

Figure 3

Figure 3 shows a sketch of the curve C_1 with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = \dfrac{x^2 + 3x + 8}{x^2 + x - 2}$

The curve has asymptotes $x = a$, $x = b$ and $y = c$, where a, b and c are integers.

(b) Find the value of a, the value of b and the value of c.

(4)

(c) Find the coordinates of the points of intersection of C_1 with the line $y = 2$

(3)

(d) Find all the integer pairs (r, s) that satisfy $s = \dfrac{r^2 + 3r + 8}{r^2 + r - 2}$

(4)

The curve C_2 has equation $y = \mathrm{g}(x)$ where $\mathrm{g}(x) = \dfrac{2x^2 - 4x + 6}{x^2 - 3x}$

(e) Show that, for suitable integers m and n, $\mathrm{g}(x)$ can be written in the form $\mathrm{f}(x + m) + n$.

(4)

(f) Sketch C_2 showing any asymptotes and stating their equations.

(3)

(Total 24 marks)