1. [*In this question position vectors are given relative to a fixed origin O*]

At time t seconds, where $t \geqslant 0$, a particle, P, moves so that its velocity $\mathbf{v}\,\text{m s}^{-1}$ is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^2\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})\,\text{m}$.

(a) Find the acceleration of P when $t = 4$

(3)

(b) Find the position vector of P when $t = 4$

(3)

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\ \text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

2. At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}\ \text{m s}^{-2}$ is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}\ \text{m s}^{-1}$

Find the speed of P when $t = 4$

(6)

3. [*In this question, position vectors are given relative to a fixed origin.*]

At time t seconds, where $t > 0$, a particle P has velocity $\mathbf{v}\,\text{m s}^{-1}$ where

$$\mathbf{v} = 3t^2\mathbf{i} - 6t^{\frac{1}{2}}\mathbf{j}$$

(a) Find the speed of P at time $t = 2$ seconds.

(2)

(b) Find an expression, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, for the acceleration of P at time t seconds, where $t > 0$

(2)

At time $t = 4$ seconds, the position vector of P is $(\mathbf{i} - 4\mathbf{j})\,\text{m}$.

(c) Find the position vector of P at time $t = 1$ second.

(4)

4. At time t seconds, where $t \geqslant 0$, a particle P has velocity $\mathbf{v}\,\text{m s}^{-1}$ where

$$\mathbf{v} = (t^2 - 3t + 7)\mathbf{i} + (2t^2 - 3)\mathbf{j}$$

Find

(a) the speed of P at time $t = 0$

(3)

(b) the value of t when P is moving parallel to $(\mathbf{i} + \mathbf{j})$

(2)

(c) the acceleration of P at time t seconds

(2)

(d) the value of t when the direction of the acceleration of P is perpendicular to $\mathbf{i}$

(2)

5. At time t seconds, a particle P has velocity $\mathbf{v}\,\text{m s}^{-1}$, where

$$\mathbf{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\,\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$

(2)

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} - \mathbf{j}$

(3)

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\mathbf{r}$ metres.

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t.

(3)

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\text{m s}^{-1}$

(6)

6. (i) At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = (1 - 4t)\mathbf{i} + (3 - t^2)\mathbf{j}$$

At the instant when $t = 0$, the velocity of P is $36\mathbf{i}$ m s^{-1}

(a) Find the velocity of P when $t = 4$

(3)

(b) Find the value of t at the instant when P is moving in a direction perpendicular to $\mathbf{i}$

(3)

(ii) At time t seconds, where $t \geqslant 0$, a particle Q moves so that its position vector $\mathbf{r}$ metres, relative to a fixed origin O, is given by

$$\mathbf{r} = (t^2 - t)\mathbf{i} + 3t\mathbf{j}$$

Find the value of t at the instant when the speed of Q is 5 m s^{-1}

(6)

7. A particle P of mass 1.5 kg moves under the action of a single force **F** newtons.

At time t seconds, $t \geqslant 0$, P has velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$, where

$$\mathbf{v} = (5t^2 - t^3)\mathbf{i} + (2t^3 - 8t)\mathbf{j}$$

(a) Find **F** when $t = 2$

(4)

At time $t = 0$, P is at the origin O.

(b) Find the position vector of P relative to O at the instant when P is moving in the direction of the vector **j**

(4)

8. At time t seconds, $t \geqslant 0$, a particle P has position vector $\mathbf{r}$ metres with respect to a fixed origin O, where

$$\mathbf{r} = (t^3 - 8t)\mathbf{i} + \left(\frac{1}{3}t^3 - t^2 + 2t\right)\mathbf{j}$$

(a) Find the acceleration of P when $t = 4$

(5)

At time T seconds, $T \geqslant 0$, P is moving in the direction of $(2\mathbf{i} + \mathbf{j})$

(b) Find the value of T

(3)

9. At time t seconds, $t \geqslant 0$, a particle P has velocity $\mathbf{v}$ m s^{-1}, where

$$\mathbf{v} = (27 - 3t^2)\mathbf{i} + (8 - t^3)\mathbf{j}$$

When $t = 1$, the particle P is at the point with position vector $\mathbf{r}$ m relative to a fixed origin O, where $\mathbf{r} = -5\mathbf{i} + 2\mathbf{j}$

Find

(a) the magnitude of the acceleration of P at the instant when it is moving in the direction of the vector $\mathbf{i}$,

(5)

(b) the position vector of P at the instant when $t = 3$

(5)

10. A particle P of mass 0.3 kg moves under the action of a single force **F** newtons. At time t seconds ($t \geqslant 0$), P has velocity $\mathbf{v}$ m s^{-1}, where

$$\mathbf{v} = (3t^2 - 4t)\mathbf{i} + (3t^2 - 8t + 4)\mathbf{j}$$

(a) Find **F** when $t = 4$

(3)

At the instants when P is at the points A and B, particle P is moving parallel to the vector **i**.

(b) Find the distance AB.

(9)

11. At time t seconds ($t \geqslant 0$), a particle P has velocity $\mathbf{v}\,\text{m s}^{-1}$, where

$$\mathbf{v} = (3t^2 - 9t + 6)\mathbf{i} + (t^2 + t - 6)\,\mathbf{j}$$

(a) Find the acceleration of P when $t = 3$

(3)

When $t = 0$, P is at the fixed point O.

The particle comes to instantaneous rest at the point A.

(b) Find the distance OA.

(7)