1. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

2

A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3rd gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

(3)

3 Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(a) Find the number of points that Lewis scored for capturing his 20th spaceship.

(2)

(b) Find the total number of points Lewis scored for capturing his first 20 spaceships.

(3)

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her nth dragon,

(c) find the value of n.

(3)

4 Jess started work 20 years ago. In year 1 her annual salary was £17 000. Her annual salary increased by £1 500 each year, so that her annual salary in year 2 was £18 500, in year 3 it was £20 000 and so on, forming an arithmetic sequence. This continued until she reached her maximum annual salary of £32 000 in year k. Her annual salary then remained at £32 000.

(a) Find the value of the constant k.

(2)

(b) Calculate the total amount that Jess has earned in the 20 years.

(5)

5

A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(a) Find how much he saves in week 15

(2)

(b) Calculate the total amount he saves over the 60 week period.

(3)

The boy's sister also saves some money each week over a period of m weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence. She saves a total of £63 in the m weeks.

(c) Show that

$$m(m + 1) = 35 \times 36$$

(4)

(d) Hence write down the value of m.

(1)

6

In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than the year before, so that the shop sold 160 computers in 2001, 170 computers in 2002, and so on forming an arithmetic sequence.

(a) Show that the shop sold 220 computers in 2007.

(2)

(b) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive.

(3)

In the year 2000, the selling price of each computer was £900. The selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

(c) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year. By forming and solving an equation, find the year in which this occurred.

(4)

7

On John's 10th birthday he received the first of an annual birthday gift of money from his uncle. This first gift was £60 and on each subsequent birthday the gift was £15 more than the year before. The amounts of these gifts form an arithmetic sequence.

(a) Show that, immediately after his 12th birthday, the total of these gifts was £225

(1)

(b) Find the amount that John received from his uncle as a birthday gift on his 18th birthday.

(2)

(c) Find the total of these birthday gifts that John had received from his uncle up to and including his 21st birthday.

(3)

When John had received n of these birthday gifts, the total money that he had received from these gifts was £3375

(d) Show that $n^2 + 7n = 25 \times 18$

(3)

(e) Find the value of n, when he had received £3375 in total, and so determine John's age at this time.

(2)

8. (a) Express $2\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x \qquad \cos x + \sin x \qquad \cos x + 2\sin x \qquad x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(b) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

9. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}\,[2a + (n-1)d]$$

(3)

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(2)

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(1)

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

(1)