**1**

On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, $H$ m, of a carriage above the ground, $t$ seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where $a$ and $b$ are positive constants.

(a) Find a complete equation for the model.

**(3)**

(b) Use the model to determine the height of the carriage above the ground when $t = 40$

**(1)**

In an alternative model, the vertical height, $H$ m, of a carriage above the ground, $t$ seconds after the carriage starts the first circuit, is given by

$$H = 29\cos(9t + \alpha)° + \beta \qquad 0 \leqslant \alpha < 360°$$

where $\alpha$ and $\beta$ are constants.

(c) Find a complete equation for the alternative model.

**(2)**

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

**(1)**

**Question 1 continued**

**2.** (a) Express $\sin x + 2\cos x$ in the form $R\sin(x+\alpha)$ where $R$ and $\alpha$ are constants, $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of $R$ and give the value of $\alpha$ in radians to 3 decimal places.

**(3)**

The temperature, $\theta$°C, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \qquad 0 \leqslant t < 24$$

where $t$ is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

**(1)**

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

**(3)**

**3.**

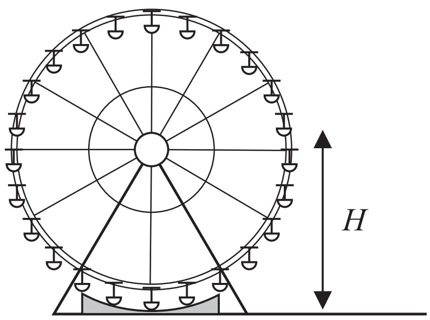


**Figure 4**

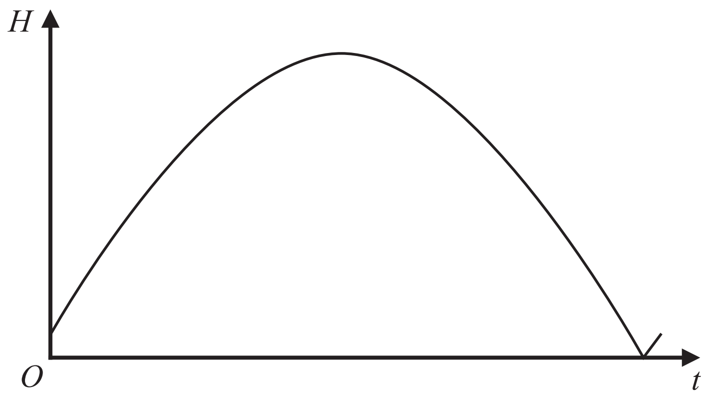


**Figure 5**

Figure 4 shows a sketch of a Ferris wheel.

The height above the ground, $H$ m, of a passenger on the Ferris wheel, $t$ seconds after the wheel starts turning, is modelled by the equation

$$H = |A \sin(bt + \alpha)°|$$

where $A$, $b$ and $\alpha$ are constants.

Figure 5 shows a sketch of the graph of $H$ against $t$, for one revolution of the wheel.

Given that

- the maximum height of the passenger above the ground is 50 m
- the passenger is 1 m above the ground when the wheel starts turning
- the wheel takes 720 seconds to complete one revolution

(a) find a complete equation for the model, giving the exact value of $A$, the exact value of $b$ and the value of $\alpha$ to 3 significant figures.

**(4)**

(b) Explain why an equation of the form

$$H = |A \sin(bt + \alpha)°| + d$$

where $d$ is a positive constant, would be a more appropriate model.

**(1)**

Question 3 continued

**4.** (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of $R$ and the value of $\alpha$ in radians to 3 decimal places.

**(3)**

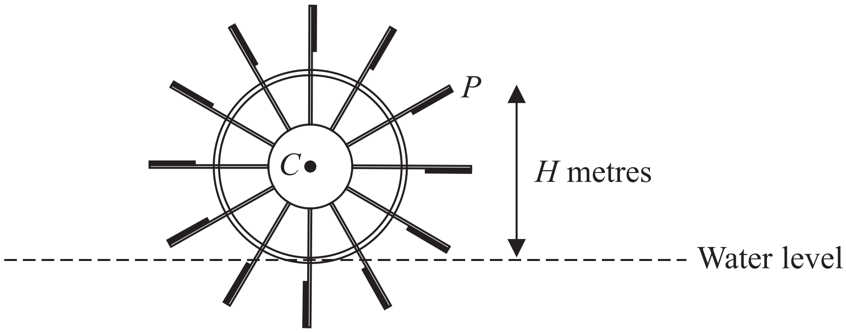


**Figure 6**

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point $C$.

The point $P$ is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, $H$ metres, of $P$ above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where $t$ is the time in seconds after the wheel starts rotating.

Using the model, find

(b) (i) the maximum height of $P$ above the water level,

(ii) the value of $t$ when this maximum height first occurs, giving your answer to one decimal place.

**(3)**

In a single revolution of the wheel, $P$ is below the water level for a total of $T$ seconds.

According to the model,

(c) find the value of $T$ giving your answer to 3 significant figures.

(*Solutions based entirely on calculator technology are not acceptable*.)

**(4)**

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

**(1)**

**Question 4 continued**

**5.** (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of $R$ and give the value of $\alpha$, in degrees, to 2 decimal places.

**(3)**

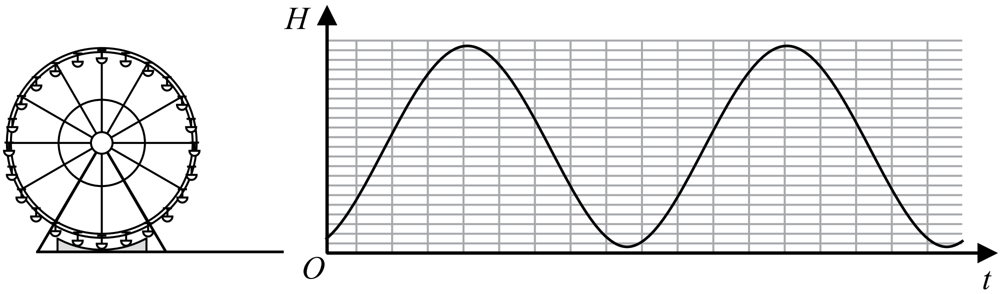


**Figure 3**

The height above the ground, $H$ metres, of a passenger on a Ferris wheel $t$ minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where $a$ is a constant.

Figure 3 shows the graph of $H$ against $t$ for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

**(2)**

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

**(3)**

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

**(1)**

**Question 5 continued**