1. A sequence of terms $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

(a) (i) Show that this sequence is periodic.

(ii) State the order of this periodic sequence.

(2)

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

(2)

2. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 1$$
$$a_{n+1} = \frac{k(a_n + 1)}{a_n}, \qquad n \geqslant 1$$

where k is a positive constant.

(a) Write down expressions for a_2 and a_3 in terms of k, giving your answers in their simplest form.

(3)

Given that $\sum_{r=1}^{3} a_r = 10$

(b) find an exact value for k.

(3)

3. A sequence $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+1} = 2u_n - 1, \quad n \geqslant 1$$

Given that $u_2 = 9$,

(a) find the value of u_3 and the value of u_4,

(2)

(b) evaluate $\sum_{r=1}^{4} u_r$.

(3)

4. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4$$
$$a_{n+1} = k(a_n + 2), \qquad \text{for } n \geqslant 1$$

where k is a constant.

(a) Find an expression for a_2 in terms of k.

(1)

Given that $\sum_{i=1}^{3} a_i = 2$,

(b) find the two possible values of k.

(6)

5. (i) A sequence $U_1, U_2, U_3, \ldots$ is defined by

$$U_{n+2} = 2U_{n+1} - U_n, \quad n \geqslant 1$$

$$U_1 = 4 \text{ and } U_2 = 4$$

Find the value of

(a) U_3

(1)

(b) $\sum_{n=1}^{20} U_n$

(2)

(ii) Another sequence $V_1, V_2, V_3, \ldots$ is defined by

$$V_{n+2} = 2V_{n+1} - V_n, \quad n \geqslant 1$$

$$V_1 = k \text{ and } V_2 = 2k, \text{ where } k \text{ is a constant}$$

(a) Find V_3 and V_4 in terms of k.

(2)

Given that $\sum_{n=1}^{5} V_n = 165$,

(b) find the value of k.

(3)

6. A sequence of numbers $a_1, a_2, a_3 \ldots$ is defined by

$$a_1 = 3$$

$$a_{n+1} = 2a_n - c \qquad (n \geqslant 1)$$

where c is a constant.

(a) Write down an expression, in terms of c, for a_2

(1)

(b) Show that $a_3 = 12 - 3c$

(2)

Given that $\sum_{i=1}^{4} a_i \geqslant 23$

(c) find the range of values of c.

(4)

7. A sequence of numbers $a_1, a_2, a_3 \ldots$ is defined by

$$a_{n+1} = 5a_n - 3, \qquad n \geqslant 1$$

Given that $a_2 = 7$,

(a) find the value of a_1

(2)

(b) Find the value of $\sum_{r=1}^{4} a_r$

(3)

8. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4,$$

$$a_{n+1} = 5 - ka_n, \quad n \geqslant 1$$

where k is a constant.

(a) Write down expressions for a_2 and a_3 in terms of k.

(2)

Find

(b) $\sum_{r=1}^{3} (1 + a_r)$ in terms of k, giving your answer in its simplest form,

(3)

(c) $\sum_{r=1}^{100} (a_{r+1} + ka_r)$

(1)

9. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \qquad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3)

(b) Find the value of k, giving a reason for your answer.

(2)

(c) Find the value of u_3

(1)

10. A sequence $u_1, u_2, u_3 ...$ is defined by

$$u_1 = 35$$

$$u_{n+1} = u_n + 7\cos\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

(a) (i) Show that $u_2 = 40$

(ii) Find the value of u_3 and the value of u_4

(3)

Given that the sequence is periodic with order 4

(b) (i) write down the value of u_5

(ii) find the value of $\sum_{r=1}^{25} u_r$

(3)

11. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)