1. Find

$$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right) dx$$

giving your answer in simplest form.

(4)

2. Find

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$$

giving your answer in its simplest form.

(4)

3. Find

$$\int \frac{3x^4 - 4}{2x^3}\,\mathrm{d}x$$

writing your answer in simplest form.

(4)

4. Use integration to find

$$\int_{1}^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) \mathrm{d}x$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(5)

5. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) \mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) \mathrm{d}x = 8$$

(3)

6. Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right) dx = 2A^2$$

show that there are exactly two possible values for A.

(5)

7.

The curve C has equation $y = \mathrm{f}(x)$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$

(2)

(b) Hence find the coordinates of the point where C crosses the y-axis.

(3)

8.

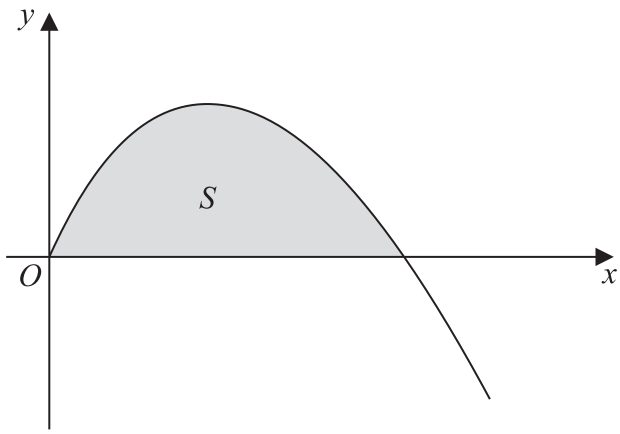


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}}, \qquad x \geqslant 0$$

The finite region S, bounded by the x-axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left(3x - x^{\frac{3}{2}}\right) \mathrm{d}x$$

(3)

(b) Hence find the area of S.

(3)

9.

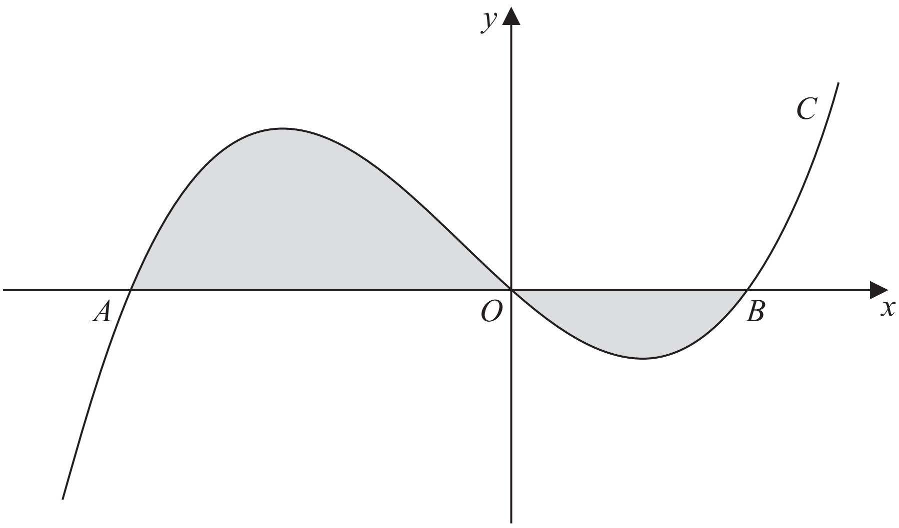


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

10. (a) Find

$$\int 10x(x^{\frac{1}{2}} - 2)\,dx$$

giving each term in its simplest form.

(4)

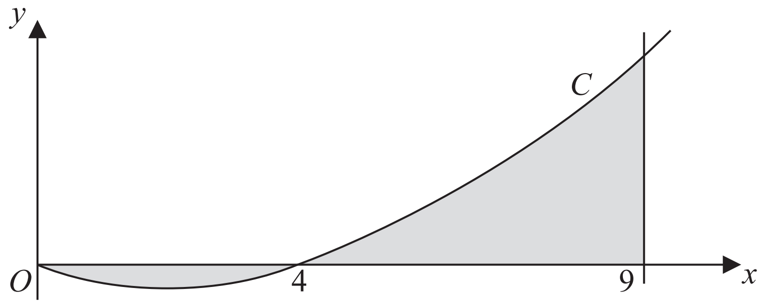


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \qquad x \geqslant 0$$

The curve C starts at the origin and crosses the x-axis at the point (4, 0).

The area, shown shaded in Figure 2, consists of two finite regions and is bounded by the curve C, the x-axis and the line $x = 9$

(b) Use your answer from part (a) to find the total area of the shaded regions.

(5)

11. A curve C has equation $y = f(x)$ where

$$f(x) = -3x^2 + 12x + 8$$

(a) Write $f(x)$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The curve C has a maximum turning point at M.

(b) Find the coordinates of M.

(2)

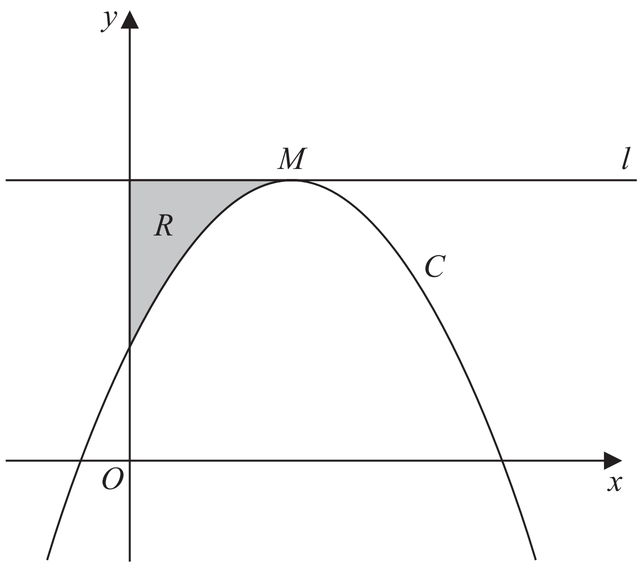


Figure 3

Figure 3 shows a sketch of the curve C.

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Using algebraic integration, find the area of R.

(5)

12.

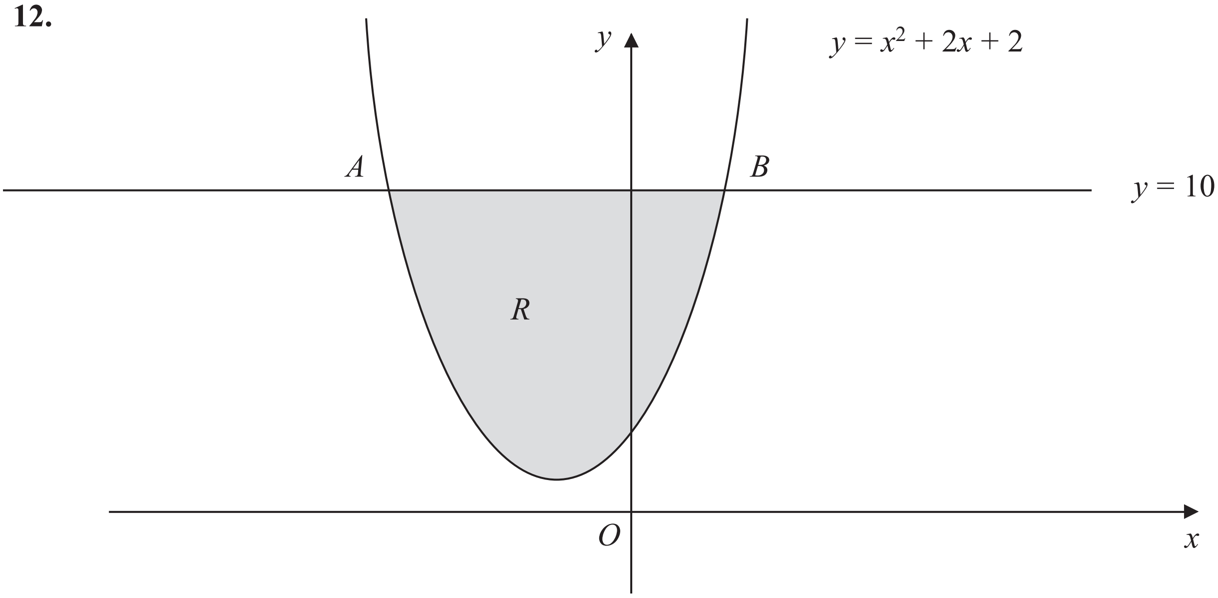


Figure 1

The line with equation $y = 10$ cuts the curve with equation $y = x^2 + 2x + 2$ at the points A and B as shown in Figure 1. The figure is not drawn to scale.

(a) Find by calculation the x-coordinate of A and the x-coordinate of B.

(2)

The shaded region R is bounded by the line with equation $y = 10$ and the curve as shown in Figure 1.

(b) Use calculus to find the exact area of R.

(7)

13.

$$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that $g(x)$ is divisible by $(x - 5)$.

(2)

(b) Hence, showing all your working, write $g(x)$ as a product of three linear factors.

(4)

The finite region R is bounded by the curve with equation $y = g(x)$ and the x-axis, and lies below the x-axis.

(c) Find, using algebraic integration, the exact value of the area of R.

(4)

14. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

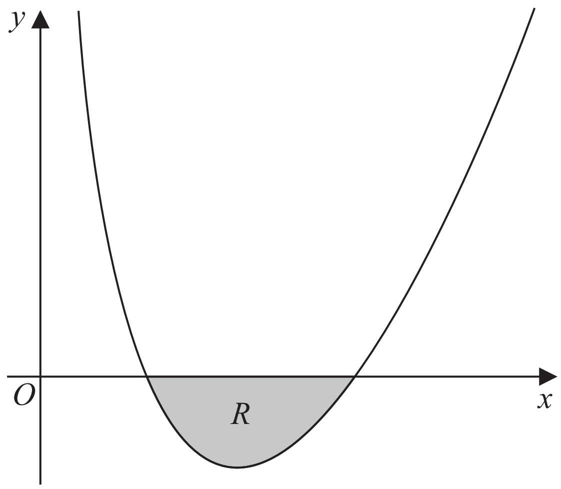


Figure 3

Figure 3 shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \qquad x > 0$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

Find the exact area of R, writing your answer in the form $a\sqrt{2} + b$, where a and b are constants to be found.

(6)

15.

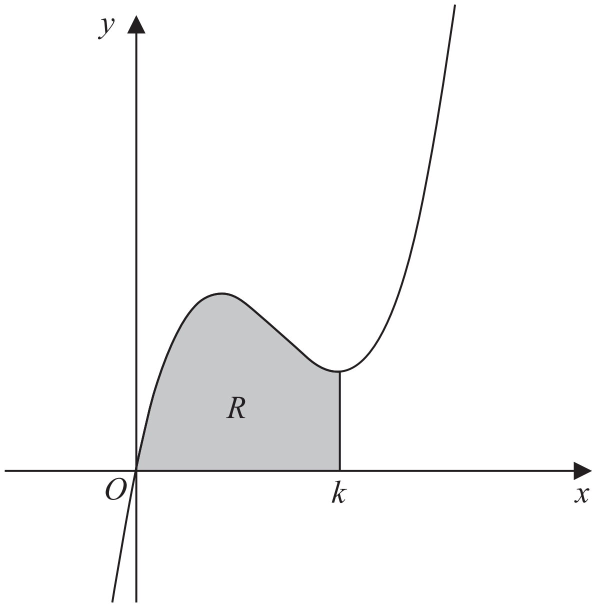


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

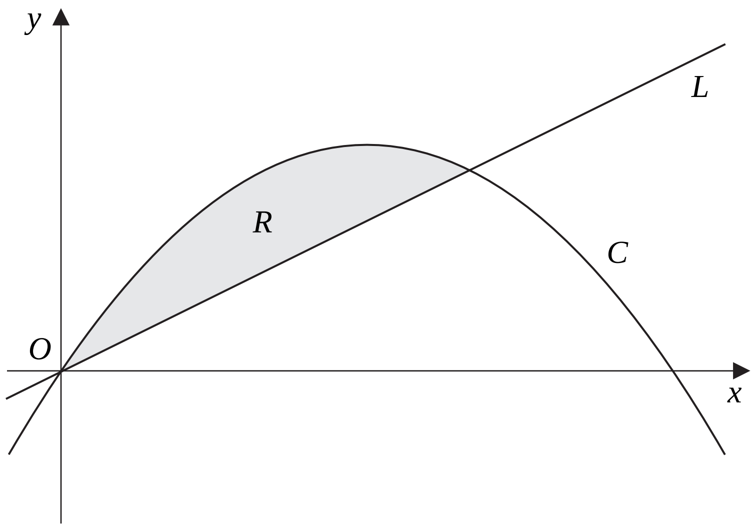
Show that the area of R is $\dfrac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

16.

Figure 2



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).

(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.

(6)

17.

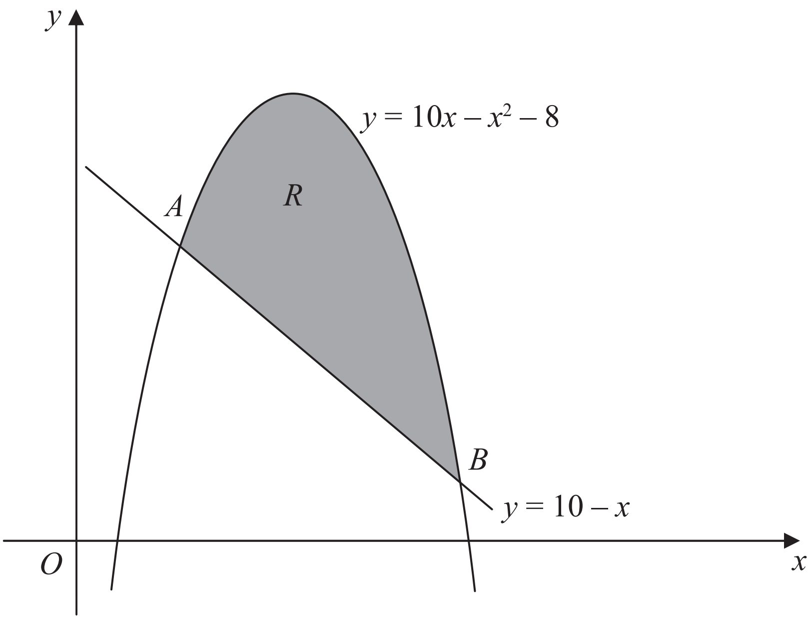


Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$

The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

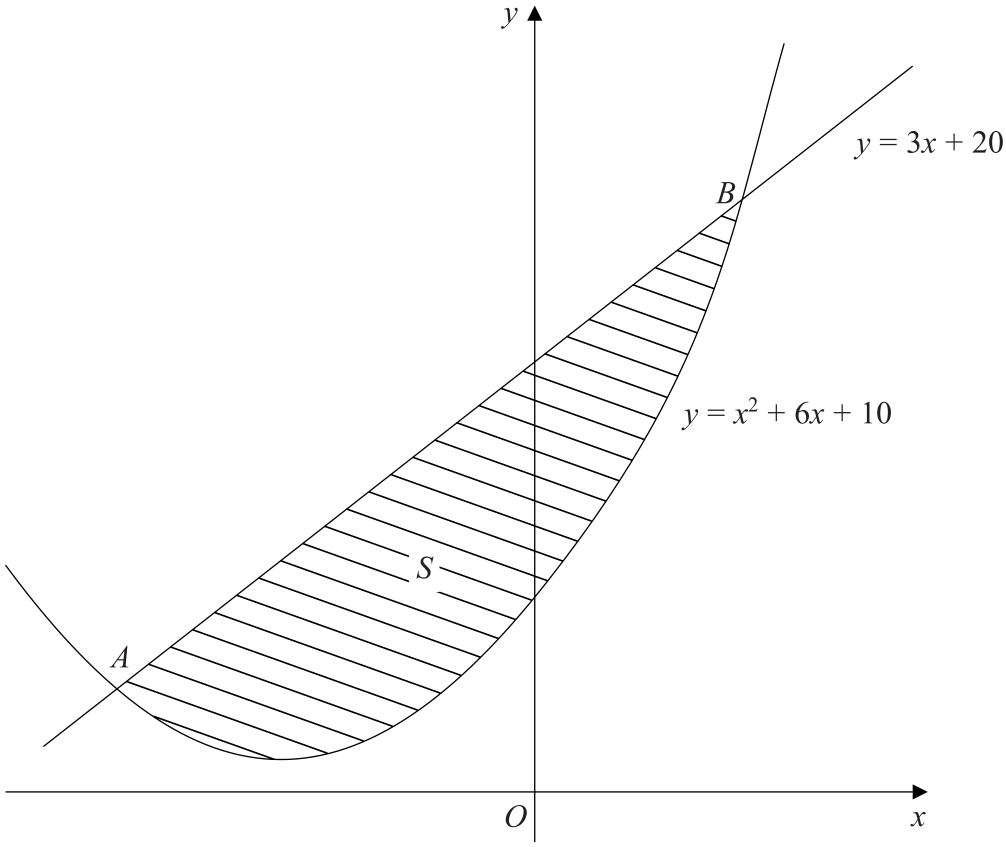
The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

18.

Figure 2



The line with equation $y = 3x + 20$ cuts the curve with equation $y = x^2 + 6x + 10$ at the points A and B, as shown in Figure 2.

(a) Use algebra to find the coordinates of A and the coordinates of B.

(5)

The shaded region S is bounded by the line and the curve, as shown in Figure 2.

(b) Use calculus to find the exact area of S.

(7)

19.

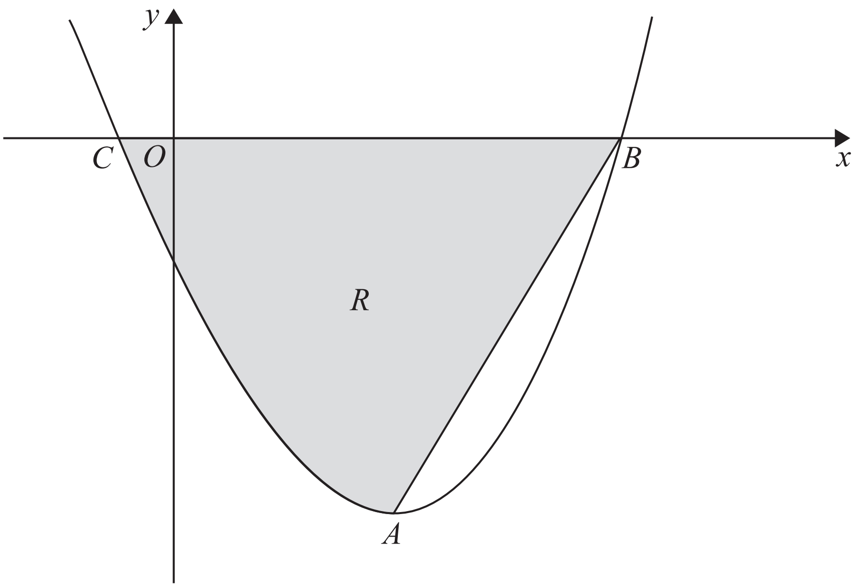


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leqslant x \leqslant 2.2$$

The curve has a turning point at the point A.

(a) Using calculus, show that the x coordinate of A is 1

(3)

The curve crosses the x-axis at the points $B(2, 0)$ and $C\left(-\frac{1}{4}, 0\right)$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line AB, and the x-axis.

(b) Use integration to find the area of the finite region R, giving your answer to 2 decimal places.

(7)

20.

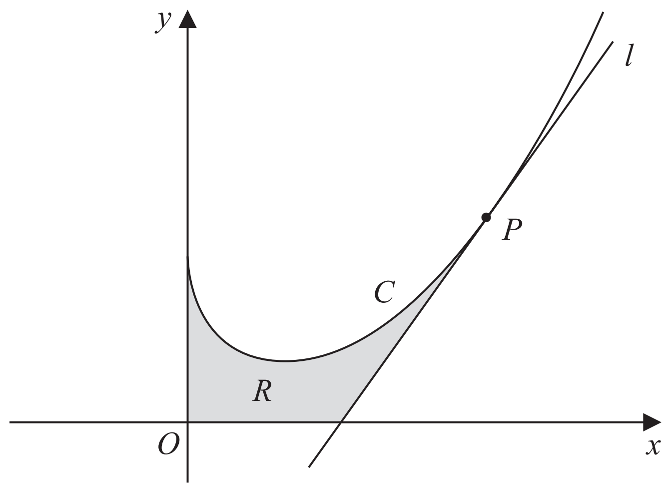


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \qquad x \geqslant 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P.

(a) Show that l has equation

$$13x - 6y - 26 = 0$$

(5)

The region R, shown shaded in Figure 2, is bounded by the y-axis, the curve C, the line l and the x-axis.

(b) Find the exact area of R.

(5)

21.

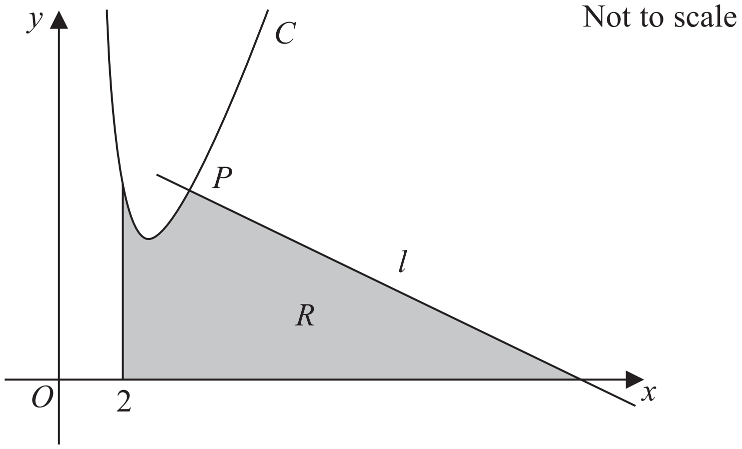


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \qquad x > 0$$

The point P (4, 6) lies on C.
The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Show that the area of R is 46
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

22. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

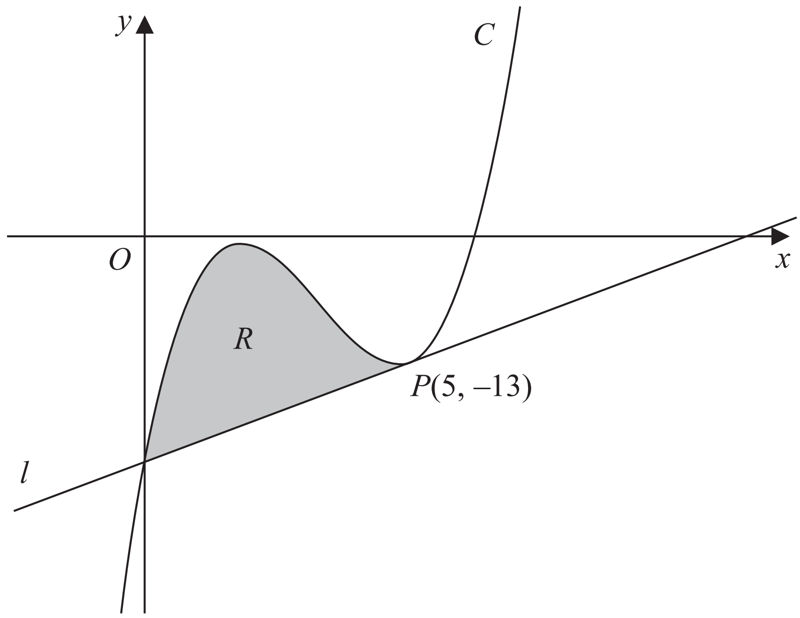


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

23.

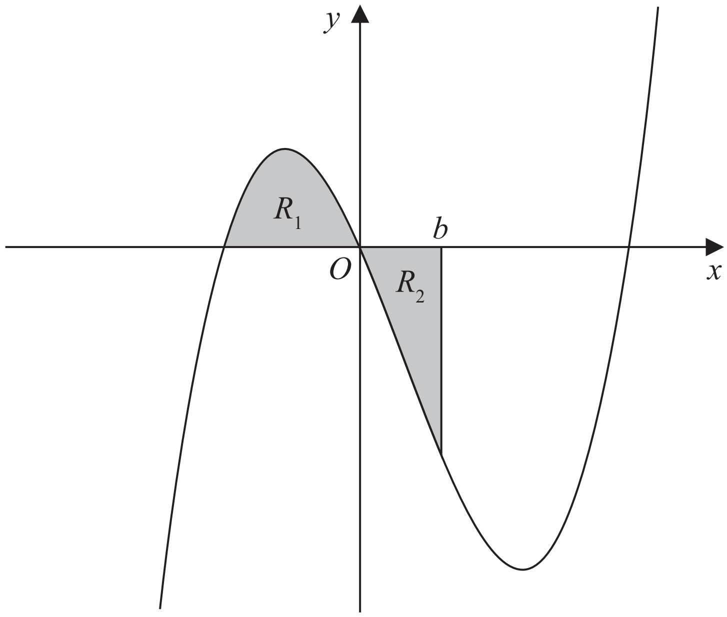


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)