1. Find $\int \left(12x^5 - 8x^3 + 3\right) \mathrm{d}x$, giving each term in its simplest form.

(4)

(Total 4 marks)

2. (i) Given that $y = 5x^3 + 7x + 3$, find

(a) $\frac{dy}{dx}$,

(3)

(b) $\frac{d^2y}{dx^2}$.

(1)

(ii) Find $\int \left(1 + 3\sqrt{x} - \frac{1}{x^2}\right) dx$.

(4)

3. Given that $y = 2x^2 - \frac{6}{x^3}, \quad x \neq 0,$

(a) find $\frac{dy}{dx}$,

(2)

(b) find $\int y \, dx$.

(3)

4.

(a) Show that $(4 + 3\sqrt{x})^2$ can be written as $16 + k\sqrt{x} + 9x$, where k is a constant to be found.

(2)

(b) Find $\int (4+3\sqrt{x})^2 \, \mathrm{d}x$.

(3)

5. A curve has equation $y = f(x)$ and passes through the point (4, 22).

Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $f(x)$, giving each term in its simplest form.

(5)

6. The equation $2x^2 - 3x - (k + 1) = 0$, where k is a constant, has no real roots.

Find the set of possible values of k.

(4)

7.

(a) Write $\frac{2\sqrt{x}+3}{x}$ in the form $2x^{p}+3x^{q}$ where p and q are constants.

(2)

Given that $y = 5x - 7 + \frac{2\sqrt{x}+3}{x}$, $x > 0$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term.

(4)

8. The curve C has equation $y = f(x)$, $x \neq 0$, and the point P (2, 1) lies on C. Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(a) find $f(x)$.

(5)

(b) Find an equation for the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

9.

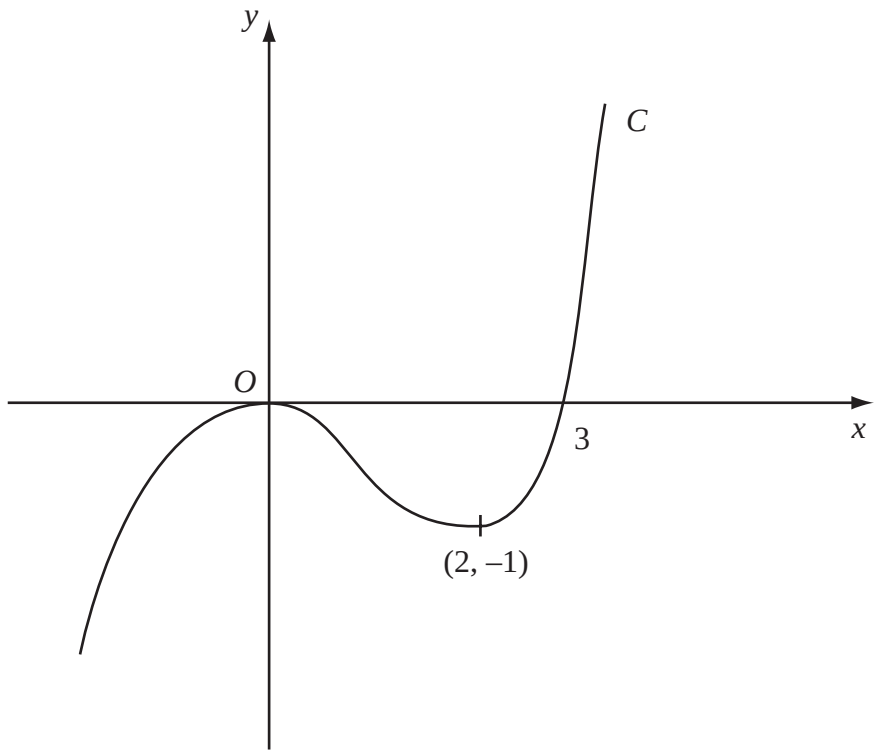

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$. There is a maximum at (0, 0), a minimum at (2, –1) and C passes through (3, 0).

On separate diagrams sketch the curve with equation

(a) $y = f(x + 3)$,

(3)

(b) $y = f(-x)$.

(3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

10. Given that $\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q.

(2)

Given that $y = 5x^4 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term.

(4)

11.

The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of k.

(4)

12. The curve with equation $y = f(x)$ passes through the point (1, 6). Given that

$$f'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}}, \quad x > 0,$$

find $f(x)$ and simplify your answer.

(7)

13.

The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2, \quad x > 0.$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

(b) Show that the point P (4, 8) lies on C.

(1)

(c) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

(4)

The normal to C at P cuts the x-axis at the point Q.

(d) Find the length PQ, giving your answer in a simplified surd form.

(3)

14. $x^2 + 2x + 3 \equiv (x + a)^2 + b$.

(a) Find the values of the constants a and b.

(2)

(b) In the space provided below, sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(3)

(c) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (b).

(2)

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(d) Find the set of possible values of k, giving your answer in surd form.

(4)