

1. A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey $\pounds C$ when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

- (a) Find, according to this model,

- (i) the value of v that minimises the cost of the journey,

- (ii) the minimum cost of the journey.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

- (b) Prove by using $\frac{d^2C}{dv^2}$ that the cost is minimised at the speed found in (a)(i).

(2)

- (c) State one limitation of this model.

(1)

2.

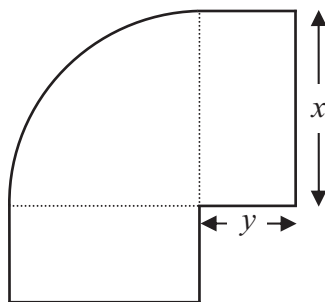


Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2 ,

(a) show that

$$y = \frac{16 - \pi x^2}{8x} \quad (3)$$

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x \quad (3)$$

(c) Use calculus to find the minimum value of P .

(d) Find the width of each rectangle when the perimeter is a minimum. Give your answer to the nearest centimetre.

3.

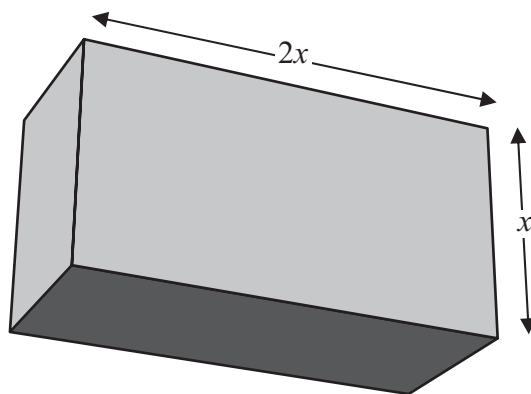


Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.

The volume of the cuboid is 81 cubic centimetres.

- (a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2} \quad (3)$$

- (b) Use calculus to find the minimum value of L .

(6)

- (c) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)

4.

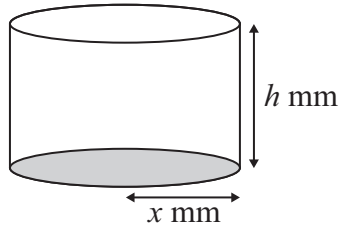


Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3 ,

- (a) express h in terms of x ,
- (1)**

- (b) show that the surface area, $A \text{ mm}^2$, of a tablet is given by $A = 2\pi x^2 + \frac{120}{x}$ (3)

The manufacturer needs to minimise the surface area $A \text{ mm}^2$, of a tablet.

- (c) Use calculus to find the value of x for which A is a minimum.

- (d) Calculate the minimum value of A , giving your answer to the nearest integer. (2)

- (e) Show that this value of A is a minimum. (2)

5.

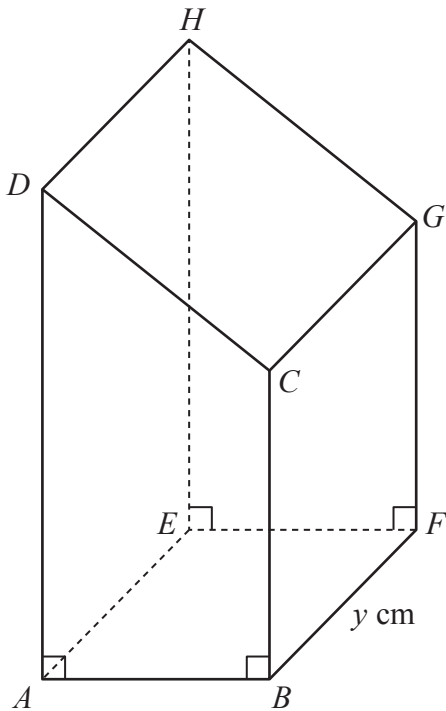


Figure 4

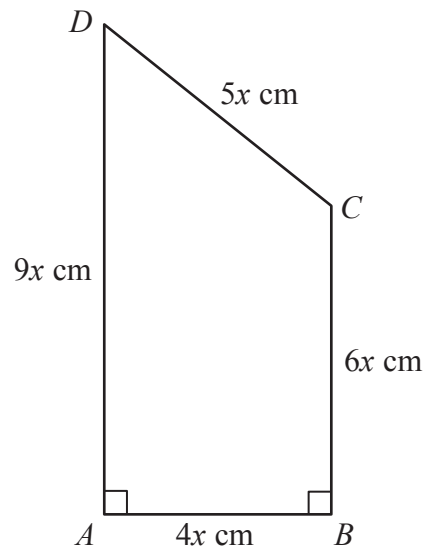


Figure 5

Figure 4 shows a closed letter box $ABFEH GCD$, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base $ABFE$ of the prism is a rectangle. The total surface area of the six faces of the prism is S cm².

The cross section $ABCD$ of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.

The angle $DAB = 90^\circ$ and the angle $ABC = 90^\circ$.

The volume of the letter box is 9600 cm³.

(a) Show that

$$y = \frac{320}{x^2} \quad (2)$$

(b) Hence show that the surface area of the letter box, S cm², is given by

$$S = 60x^2 + \frac{7680}{x} \quad (4)$$

(c) Use calculus to find the minimum value of S .

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

Given that the radius of the cylinder is r cm,

- $$C = 6\pi r^2 + \frac{300\pi}{r} \quad (4)$$

- (5)

- (1)

A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius r cm and height h cm and the capacity of each container is 355 cm^3

The metal used

- for the circular base and the curved side costs 0.04 pence/cm²
- for the circular top costs 0.09 pence/cm²

Both metals used are of negligible thickness.

- (a) Show that the total cost, C pence, of the metal for one container is given by

$$C = 0.13\pi r^2 + \frac{28.4}{r} \quad (4)$$

- (b) Use calculus to find the value of r for which C is a minimum, giving your answer to 3 significant figures.

- (c) Using $\frac{d^2C}{dr^2}$ prove that the cost is minimised for the value of r found in part (b).

- (d) Hence find the minimum value of C , giving your answer to the nearest integer. (2)

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8.

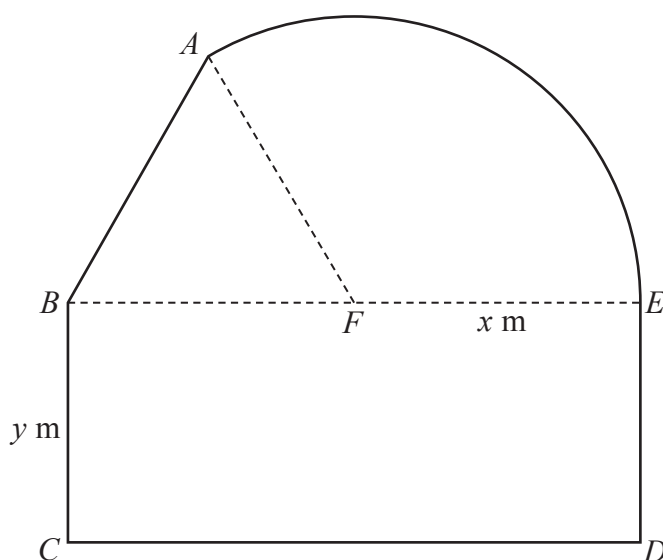


Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure $ABCDEA$, as shown in Figure 4, consists of a rectangle $BCDE$ joined to an equilateral triangle BFA and a sector FEA of a circle with radius x metres and centre F .

The points B , F and E lie on a straight line with $FE = x$ metres and $10 \leq x \leq 25$

- (a) Find, in m^2 , the exact area of the sector FEA , giving your answer in terms of x , in its simplest form.

(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is 1000 m^2 ,

- (b) show that

$$y = \frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3})$$

(3)

- (c) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}(4\pi + 36 - 3\sqrt{3})$$

(3)

- (d) Use calculus to find the minimum value of P , giving your answer to the nearest metre.

(5)

- (e) Justify, by further differentiation, that the value of P you have found is a minimum.

(2)
