1. $$f(x) = x^3 - \frac{10\sqrt{x} - 4x}{x^2} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[1.4, 1.5]$

(2)

(b) Determine $f'(x)$.

(3)

(c) Using $x_0 = 1.4$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to calculate a second approximation to α, giving your answer to 3 decimal places.

(2)

2.

$$f(x) = x^2 - \frac{3}{\sqrt{x}} - \frac{4}{3x^2}, \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1.6, 1.7]

(2)

(b) Taking 1.6 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to find a second approximation to α. Give your answer to 3 decimal places.

(5)

3.

$$f(x) = x^2 + \frac{3}{x} - 1, \qquad x < 0$$

The only real root, α, of the equation $f(x) = 0$ lies in the interval $[-2, -1]$.

(a) Taking -1.5 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 2 decimal places.

(5)

(b) Show that your answer to part (a) gives α correct to 2 decimal places.

(2)

4.

$$f(x) = x^3 - \frac{1}{2x} + x^{\frac{3}{2}}, \qquad x > 0$$

The root α of the equation $f(x) = 0$ lies in the interval [0.6, 0.7].

(a) Taking 0.6 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(5)

(b) Show that your answer to part (a) is correct to 3 decimal places.

(2)

5. $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval $[3.5, 4]$

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

6.

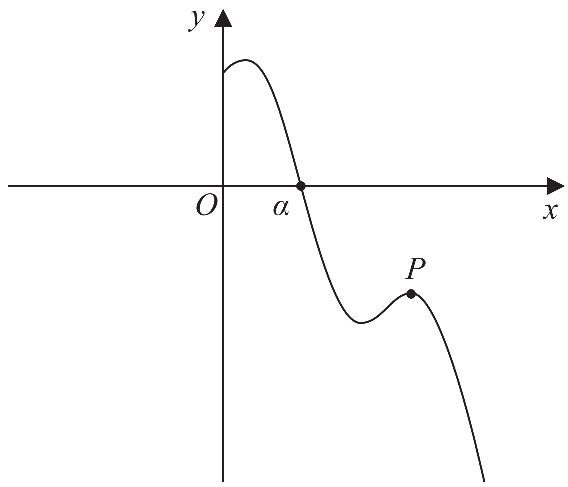

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and x is measured in radians.

The point P, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P, giving your answer to 3 significant figures.

(4)

The curve crosses the x-axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $\mathrm{f}(4) = 4.274$ and $\mathrm{f}(5) = -1.212$

(b) explain why α must lie in the interval $[4, 5]$

(1)

(c) Taking $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson method once to $\mathrm{f}(x)$ to obtain a second approximation to α.

Show your method and give your answer to 3 significant figures.

(2)