

1

The curve C has equation $y = \frac{1}{3}x^3 - 9x + 1$

(a) Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \quad (2)$$

(b) Find the range of values of x for which C has a negative gradient.

(3)

(Total for Question 1 is 5 marks)

2 The curve **C** has equation $y = 5x^3 - x^2 - 6x + 4$

(a) Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \dots\dots\dots$$

(2)

There are two points on the curve **C** at which the gradient of the curve is 2

(b) Find the x coordinate of each of these two points.
Show clear algebraic working.

\dots\dots\dots
(4)

(Total for Question 2 is 6 marks)

3 A curve **C** has equation $y = x^3 - x^2 - 8x + 12$

(a) Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \quad (2)$$

The curve **C** has two turning points.

(b) Work out the x coordinates of the two turning points.
Show your working clearly.

(3)

(c) Show that the x -axis is a tangent to the curve **C**.

(2)

(Total for Question 3 is 7 marks)

4

$$y = x^3 - 2x^2 - 15x + 5$$

(a) Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \dots\dots\dots (2)$$

C is the curve with equation $y = x^3 - 2x^2 - 15x + 5$

(b) Work out the range of values of x for which **C** has a negative gradient.

.....
(4)

(Total for Question 4 is 6 marks)

5 The curve shown in the diagram has equation

$$y = x^3 - 27x + k \text{ where } k \text{ is a positive constant with } k < 54$$

The curve has a maximum point at $A(a, b)$

The curve has a minimum point at $B(c, d)$

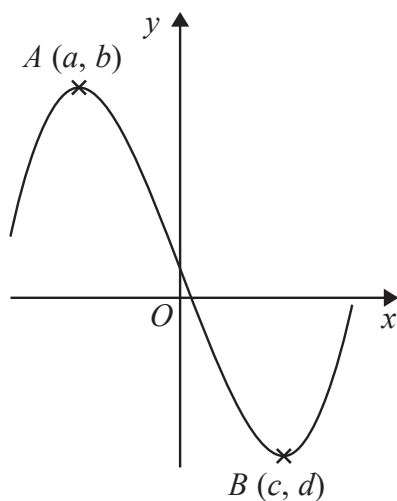


Diagram **NOT**
accurately drawn

Using differentiation, find the value of $b - d$
Show your working clearly.

.....
(Total for Question 5 is 6 marks)

- 6 The point A is the only stationary point on the curve with equation $y = kx^2 + \frac{16}{x}$ where k is a constant.

Given that the coordinates of A are $\left(\frac{2}{3}, a\right)$

find the value of a .

Show your working clearly.

$a = \dots\dots\dots$

(Total for Question 6 is 5 marks)

7

The diagram shows a sketch of part of the curve with equation $y = x^2 - \frac{p}{x}$ where p is a positive constant.

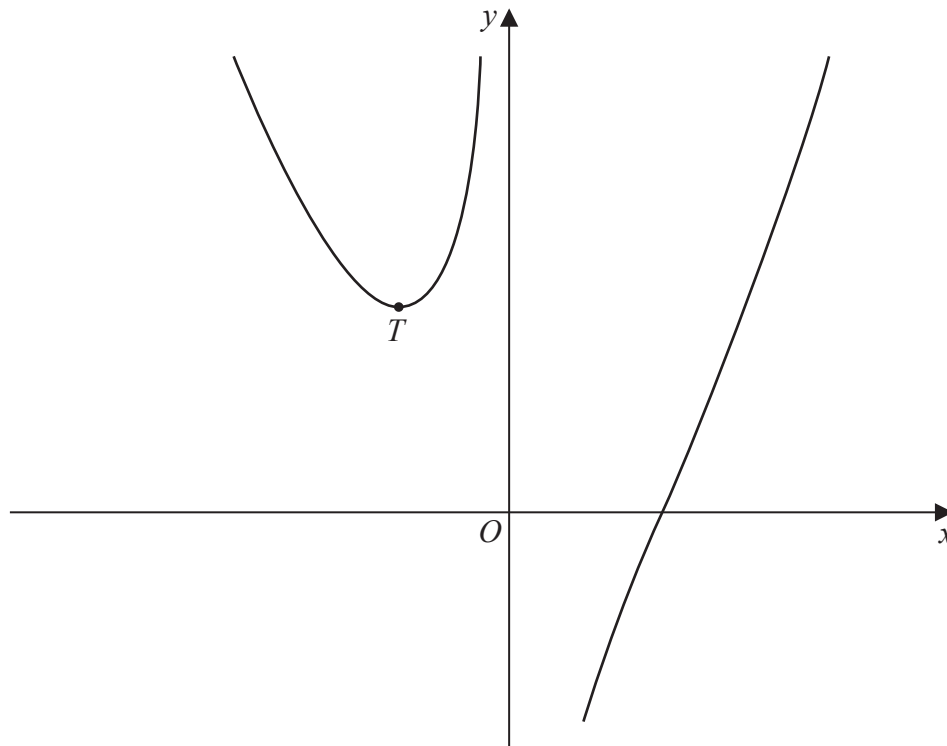


Diagram **NOT**
accurately drawn

For all values of p , the curve has exactly one turning point and this turning point is a minimum shown as the point T in the sketch.

For the curve where the x coordinate of T is -3

(a) find the value of p

$p = \dots\dots\dots$
(4)

The line with equation $y = k$ is a tangent to the curve with equation $y = x^2 - \frac{16}{x}$

(b) Find the value of k

$$k = \dots\dots\dots$$

(3)

(Total for Question 7 is 7 marks)

8

Curve **C** has equation $y = px^3 - mx$ where p and m are positive integers.

Find the range of values of x , in terms of p and m , for which the gradient of **C** is negative.

.....
(Total for Question 8 is 4 marks)

9 The curve **C** has equation $y = ax^3 + bx^2 - 12x + 6$ where a and b are constants.

The point A with coordinates $(2, -6)$ lies on **C**

The gradient of the curve at A is 16

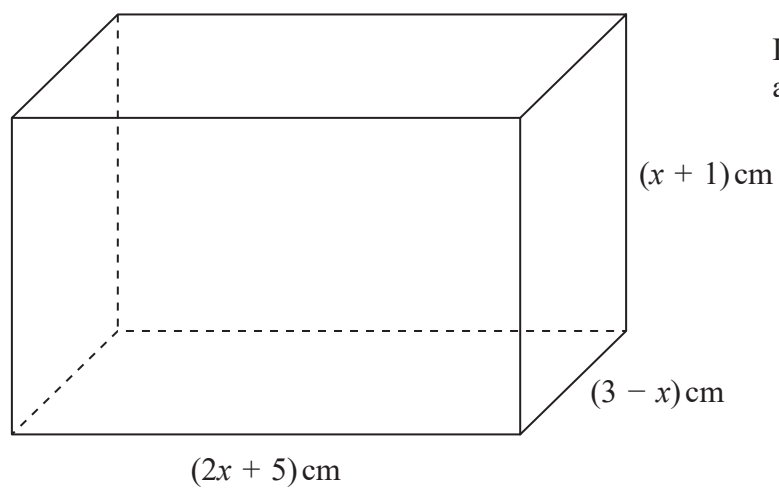
Find the y coordinate of the point on the curve whose x coordinate is 3

Show clear algebraic working.

$y = \dots\dots\dots$

(Total for Question 9 is 6 marks)

10



The diagram shows a cuboid of volume $V \text{ cm}^3$

(a) Show that $V = 15 + 16x - x^2 - 2x^3$

(3)

There is a value of x for which the volume of the cuboid is a maximum.

(b) Find this value of x .

Show your working clearly.

Give your answer correct to 3 significant figures.

$x = \dots\dots\dots$

(5)

(Total for Question 10 is 8 marks)

11 The diagram shows a solid cuboid.

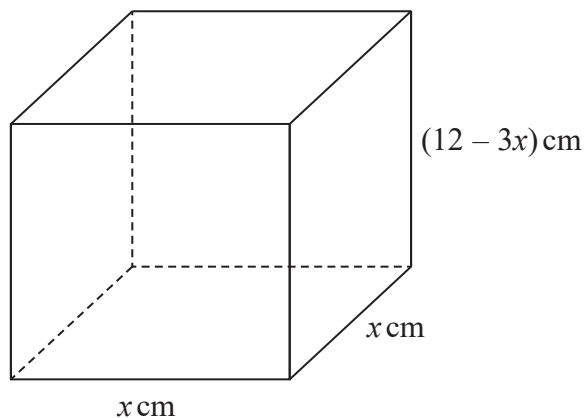


Diagram **NOT**
accurately drawn

The total surface area of the cuboid is $A \text{ cm}^2$

Find the maximum value of A .

(Total for Question 11 is 5 marks)

12 A solid, **S**, is made from a hemisphere and a cylinder.

The centre of the circular face of the hemisphere and the centre of the top face of the cylinder are at the same point.

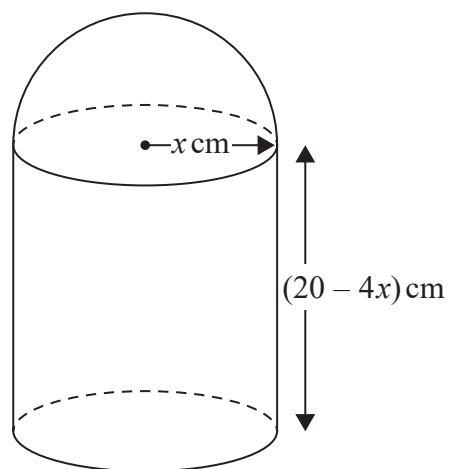


Diagram **NOT**
accurately drawn

The radius of the cylinder and the radius of the hemisphere are both x cm.
The height of the cylinder is $(20 - 4x)$ cm.

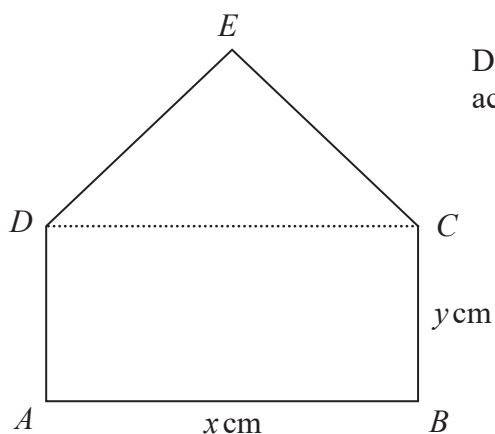
The volume of **S** is V cm³ where $V = \frac{1}{3} \pi y$

Find the maximum value of y .
Show clear algebraic working.

.....

(Total for Question 12 is 5 marks)

13 $ABCED$ is a five-sided shape.



$ABCD$ is a rectangle.

CED is an equilateral triangle.

$$AB = x \text{ cm} \quad BC = y \text{ cm}$$

The perimeter of $ABCED$ is 100 cm.

The area of $ABCED$ is $R \text{ cm}^2$

(a) Show that $R = \frac{x}{4} \left(200 - [6 - \sqrt{3}]x \right)$

(b) (i) Find the value of x for which R has its maximum value.

Give your answer in the form $\frac{p}{q - \sqrt{3}}$ where p and q are integers.

$x = \dots\dots\dots$
(2)

(ii) Explain why the maximum value of R is given by this value of x .

.....
.....
.....
(1)

(Total for Question 13 is 6 marks)

- 14** A particle P is moving along a straight line.
The fixed point O lies on this line.

At time t seconds, the displacement, s metres, of P from O is given by

$$s = 4t^3 - 6t^2 + 5t$$

At time t seconds, the velocity of P is v m/s.

- (a) Find an expression for v in terms of t .

$$v = \dots\dots\dots$$

(2)

- (b) Find the time at which the acceleration of the particle is 6 m/s^2

$$\dots\dots\dots \text{ seconds}$$

(3)

(Total for Question 14 is 5 marks)

- 15** A particle P is moving along a straight line.
The fixed point O lies on the line.

At time t seconds ($t \geq 0$), the displacement of P from O is s metres where

$$s = t^3 - 9t^2 + 33t - 6$$

Find the minimum speed of P .

..... m/s

(Total for Question 15 is 5 marks)

- 16** A particle P is moving along a straight line that passes through the fixed point O .
The displacement, s metres, of P from O at time t seconds is given by

$$s = t^3 - 6t^2 + 5t - 4$$

Find the value of t for which the acceleration of P is 3 m/s^2

$t = \dots\dots\dots$

(Total for Question 16 is 4 marks)

- 17** A particle P moves along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds, $t \geq 1$, is s metres where

$$s = 4t^2 + \frac{125}{t}$$

The velocity of P at time t seconds, $t \geq 1$, is v m/s

Work out the distance of P from O at the instant when $v = 0$

..... m

(Total for Question 17 is 5 marks)

18 A particle P moves along a straight line that passes through the fixed point O

The displacement, x metres, of P from O at time t seconds, where $t \geq 0$, is given by

$$x = 4t^3 - 27t + 8$$

The direction of motion of P reverses when P is at the point A on the line.

The acceleration of P at the instant when P is at A is $a \text{ m/s}^2$

Find the value of a

$a = \dots\dots\dots$

(Total for Question 18 is 5 marks)

- 19** A particle P is moving along a straight line.
The fixed point O lies on this line.

At time t seconds where $t \geq 0$, the displacement, s metres, of P from O is given by

$$s = t^3 + 5t^2 - 8t + 10$$

Find the displacement of P from O when P is instantaneously at rest.

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

..... metres

(Total for Question 19 is 5 marks)

- 20** A particle P is moving along a straight line that passes through the fixed point O .
The displacement, s metres, of P from O at time t seconds is given by

$$s = t^3 - 6t^2 + 5t - 4$$

Find the value of t for which the acceleration of P is 3 m/s^2

$t = \dots\dots\dots$

(Total for Question 20 is 4 marks)

21 A particle moves along a straight line.

The fixed point O lies on this line.

The displacement of the particle from O at time t seconds, $t \geq 0$, is s metres where

$$s = t^3 + 4t^2 - 5t + 7$$

At time T seconds the velocity of P is V m/s where $V \geq -5$

Find an expression for T in terms of V .

Give your expression in the form $\frac{-4 + \sqrt{k + mV}}{3}$ where k and m are integers to be found.

$T = \dots\dots\dots$

(Total for Question 21 is 6 marks)

- 22** Two particles, P and Q , move along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds is s metres, where

$$s = t^3 - 4t^2 + 5t \quad \text{for } t > 1$$

The displacement of Q from O at time t seconds is x metres, where

$$x = t^2 - 4t + 4 \quad \text{for } t > 1$$

Find the range of values of t where $t > 1$ for which both particles are moving in the same direction along the straight line.

.....

(Total for Question 22 is 6 marks)
