**1.** A geometric series has first term $a = 360$ and common ratio $r = \frac{7}{8}$

Giving your answers to 3 significant figures where appropriate, find

(a) the 20th term of the series,

**(2)**

(b) the sum of the first 20 terms of the series,

**(2)**

(c) the sum to infinity of the series.

**(2)**

**2.** The first three terms of a geometric series are

$$18, 12 \text{ and } p$$

respectively, where $p$ is a constant.

Find

(a) the value of the common ratio of the series,

**(1)**

(b) the value of $p$,

**(1)**

(c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places.

**(2)**

**3**

A geometric series has first term $a$ and common ratio $r = \frac{3}{4}$

The sum of the first 4 terms of this series is 175

(a) Show that $a = 64$

**(2)**

(b) Find the sum to infinity of the series.

**(2)**

(c) Find the difference between the 9th and 10th terms of the series.
Give your answer to 3 decimal places.

**(3)**

**4**

A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05

(a) Show that the predicted profit in the year 2016 is £138 915

**(1)**

(b) Find the first year in which the yearly predicted profit exceeds £200 000

**(5)**

(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

**(3)**

**5.** **In this question you should show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

**(1)**

(b) find the first year when the yearly profit will exceed £65 000

**(3)**

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

**(2)**

**6.** (i) All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162

Find

(a) the common ratio,

**(4)**

(b) the first term.

**(2)**

(ii) A different geometric series has a first term of 42 and a common ratio of $\frac{6}{7}$.

Find the smallest value of $n$ for which the sum of the first $n$ terms of the series exceeds 290

**(4)**

**7**

The first three terms of a geometric series are $4p$, $(3p + 15)$ and $(5p + 20)$ respectively, where $p$ is a **positive** constant.

(a) Show that $11p^2 - 10p - 225 = 0$

**(4)**

(b) Hence show that $p = 5$

**(2)**

(c) Find the common ratio of this series.

**(2)**

(d) Find the sum of the first ten terms of the series, giving your answer to the nearest integer.

**(3)**

**8**

The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(a) the common ratio,

**(2)**

(b) the first term,

**(2)**

(c) the sum to infinity,

**(2)**

(d) the smallest value of $n$ for which the sum of the first $n$ terms of the series exceeds 1000.

**(4)**

**9**

The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$

The sum to infinity of the series is $S_\infty$

(a) Find the value of $S_\infty$

**(2)**

The sum to $N$ terms of the series is $S_N$

(b) Find, to 1 decimal place, the value of $S_{12}$

**(2)**

(c) Find the smallest value of $N$, for which

$$S_\infty - S_N < 0.5$$

**(4)**

**10.** The first three terms of a geometric sequence are

$$7k - 5, \;\; 5k - 7, \;\; 2k + 10$$

where $k$ is a constant.

(a) Show that $11k^2 - 130k + 99 = 0$

**(4)**

Given that $k$ is not an integer,

(b) show that $k = \dfrac{9}{11}$

**(2)**

For this value of $k$,

(c) (i) evaluate the fourth term of the sequence, giving your answer as an exact fraction,

(ii) evaluate the sum of the first ten terms of the sequence.

**(6)**

**11**

A geometric series is $a + ar + ar^2 + ...$

(a) Prove that the sum of the first $n$ terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

**(4)**

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(b) the common ratio,

**(2)**

(c) the first term,

**(2)**

(d) the sum to infinity.

**(3)**

**12.** The first three terms of a geometric sequence are

$$3k + 4 \qquad 12 - 3k \qquad k + 16$$

where $k$ is a constant.

(a) Show that $k$ satisfies the equation

$$3k^2 - 62k + 40 = 0$$

**(2)**

Given that the sequence converges,

(b) (i) find the value of $k$, giving a reason for your answer,

(ii) find the value of $S_\infty$

**(5)**

**13.** In a geometric series the common ratio is $r$ and sum to $n$ terms is $S_n$

Given

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where $k$ is an integer to be found.

**(4)**

**14.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A geometric series has common ratio $r$ and first term $a$.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

**(4)**

Given also that $S_{10}$ is four times $S_5$

(b) find the exact value of $r$.

**(4)**

**15.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Given that the first three terms of a geometric series are

$$12\cos\theta \qquad 5+2\sin\theta \qquad \text{and} \qquad 6\tan\theta$$

(a) show that

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

**(3)**

Given that $\theta$ is an obtuse angle measured in radians,

(b) solve the equation in part (a) to find the exact value of $\theta$

**(2)**

(c) show that the sum to infinity of the series can be expressed in the form

$$k\left(1-\sqrt{3}\right)$$

where $k$ is a constant to be found.

**(5)**