

This image shows a full page of blank, lined paper. It features approximately 28 horizontal blue or grey lines spaced evenly apart, typical of notebook paper. The lines extend across the entire width of the page, leaving small margins at the top and bottom. There are no vertical lines, text, or other markings on the page.

**(Total 4 marks)**

2. (i) Given that  $y = 5x^3 + 7x + 3$ , find

$$(a) \quad \frac{dy}{dx}, \tag{3}$$

$$(b) \quad \frac{d^2 y}{dx^2}.$$

(ii) Find  $\int \left(1 + 3\sqrt{x} - \frac{1}{x^2}\right) dx$ . (4)

[illegible]

3. Given that  $y = 2x^2 - \frac{6}{x^3}$ ,  $x \neq 0$ ,

(a) find  $\frac{dy}{dx}$ ,

**(2)**

(b) find  $\int y \, dx$ .

(3)

4.

- (a) Show that  $(4 + 3\sqrt{x})^2$  can be written as  $16 + k\sqrt{x} + 9x$ , where  $k$  is a constant to be found.

(2)

- (b) Find  $\int (4 + 3\sqrt{x})^2 dx$ .

(3)

5. A curve has equation  $y = f(x)$  and passes through the point  $(4, 22)$ .

Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find  $f(x)$ , giving each term in its simplest form.

(5)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6. The equation  $2x^2 - 3x - (k + 1) = 0$ , where  $k$  is a constant, has no real roots.

Find the set of possible values of  $k$ .

(4)

7.

- (a) Write  $\frac{2\sqrt{x+3}}{x}$  in the form  $2x^p+3x^q$  where  $p$  and  $q$  are constants.

(2)

Given that  $y = 5x - 7 + \frac{2\sqrt{x+3}}{x}$ ,  $x > 0$ ,

- (b) find  $\frac{dy}{dx}$ , simplifying the coefficient of each term.

(4)

**8.** The curve  $C$  has equation  $y = f(x)$ ,  $x \neq 0$ , and the point  $P(2, 1)$  lies on  $C$ . Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(a) find  $f(x)$ .

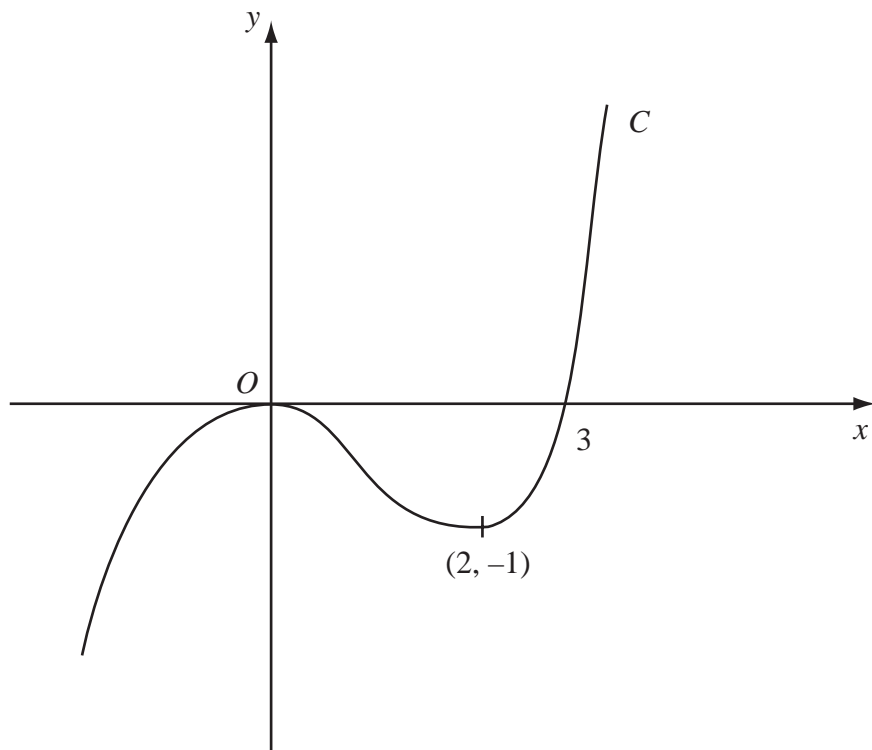
(5)

(b) Find an equation for the tangent to  $C$  at the point  $P$ , giving your answer in the form  $y = mx + c$ , where  $m$  and  $c$  are integers.

(4)



9.



**Figure 1**

Figure 1 shows a sketch of the curve  $C$  with equation  $y = f(x)$ . There is a maximum at  $(0, 0)$ , a minimum at  $(2, -1)$  and  $C$  passes through  $(3, 0)$ .

On separate diagrams sketch the curve with equation

(a)  $y = f(x + 3)$ , (3)

(b)  $y = f(-x)$ . (3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the  $x$ -axis.

10. Given that  $\frac{2x^2 - x^2}{\sqrt{x}}$  can be written in the form  $2x^p - x^q$ ,

(a) write down the value of  $p$  and the value of  $q$ .

(2)

Given that  $y = 5x^4 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ ,

(b) find  $\frac{dy}{dx}$ , simplifying the coefficient of each term.

(4)

11.

The equation  $kx^2 + 4x + (5 - k) = 0$ , where  $k$  is a constant, has 2 different real solutions for  $x$ .

(a) Show that  $k$  satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of  $k$ .

(4)

This image shows a full page of blank, lined paper. It features approximately 20 evenly spaced horizontal grey lines across the entire width of the page, typical of notebook or legal stationery. The background is a solid off-white color. There are no margins, text, or other markings present.

12. The curve with equation  $y = f(x)$  passes through the point  $(1, 6)$ . Given that

$$f'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}}, \quad x > 0,$$

find  $f(x)$  and simplify your answer.

**(7)**

**13.**

The curve  $C$  has equation  $y = 4x + 3x^{\frac{3}{2}} - 2x^2$ ,  $x > 0$ .

- (a) Find an expression for  $\frac{dy}{dx}$ . **(3)**

- (b) Show that the point  $P(4, 8)$  lies on  $C$ . (1)

- (c) Show that an equation of the normal to  $C$  at the point  $P$  is

$$3y = x + 20. \quad (4)$$

The normal to  $C$  at  $P$  cuts the  $x$ -axis at the point  $Q$ .

- (d) Find the length  $PQ$ , giving your answer in a simplified surd form. (3)

14.

$$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

- (a) Find the values of the constants  $a$  and  $b$ .

(2)

- (b) In the space provided below, sketch the graph of  $y = x^2 + 2x + 3$ , indicating clearly the coordinates of any intersections with the coordinate axes.

(3)

- (c) Find the value of the discriminant of  $x^2 + 2x + 3$ . Explain how the sign of the discriminant relates to your sketch in part (b).

(2)

The equation  $x^2 + kx + 3 = 0$ , where  $k$  is a constant, has no real roots.

- (d) Find the set of possible values of  $k$ , giving your answer in surd form.

(4)

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