1.

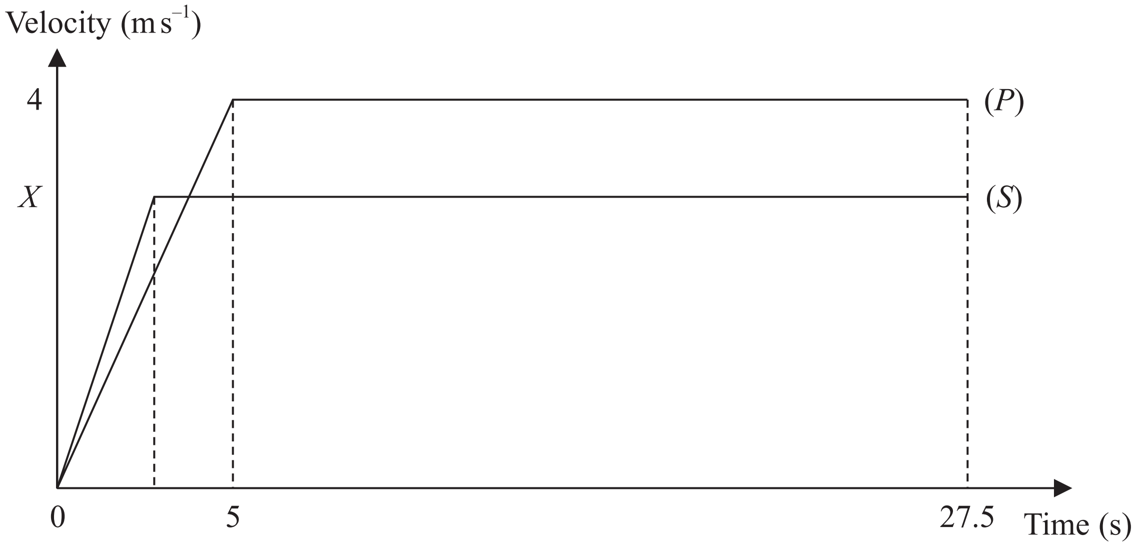

Figure 1

Two children, Pat (P) and Sam (S), run a race along a straight horizontal track.

Both children start from rest at the same time and cross the finish line at the same time.

In a model of the motion:

Pat accelerates at a constant rate from rest for 5 s until reaching a speed of $4\,m\,s^{-1}$ and then maintains a constant speed of $4\,m\,s^{-1}$ until crossing the finish line.

Sam accelerates at a constant rate of $1\,m\,s^{-2}$ from rest until reaching a speed of $X\,m\,s^{-1}$ and then maintains a constant speed of $X\,m\,s^{-1}$ until crossing the finish line.

Both children take 27.5 s to complete the race.

The velocity-time graphs shown in Figure 1 describe the model of the motion of each child from the instant they start to the instant they cross the finish line together.

Using the model,

(a) explain why the areas under the two graphs are equal,

(1)

(b) find the acceleration of Pat during the first 5 seconds,

(1)

(c) find, in metres, the length of the race,

(2)

(d) find the value of X, giving your answer to 3 significant figures.

(4)

2. An athlete runs along a straight road. She starts from rest and moves with constant acceleration for 5 seconds, reaching a speed of $8\,\text{m}\,\text{s}^{-1}$. This speed is then maintained for T seconds. She then decelerates at a constant rate until she stops. She has run a total of 500 m in 75 s.

(a) In the space below, sketch a speed-time graph to illustrate the motion of the athlete.

(3)

(b) Calculate the value of T.

(5)

3.

A train travels along a straight horizontal track between two stations, A and B.

In a model of the motion, the train starts from rest at A and moves with constant acceleration $0.3\,\text{m s}^{-2}$ for 80 s.
The train then moves at constant velocity before it moves with a constant deceleration of $0.5\,\text{m s}^{-2}$, coming to rest at B.

(a) For this model of the motion of the train between A and B,

(i) state the value of the constant velocity of the train,

(ii) state the time for which the train is decelerating,

(iii) sketch a velocity-time graph.

(3)

The total distance between the two stations is 4800 m.

(b) Using the model, find the total time taken by the train to travel from A to B.

(3)

(c) Suggest one improvement that could be made to the model of the motion of the train from A to B in order to make the model more realistic.

(1)

4.

At time $t = 0$, a parachutist falls vertically from rest from a helicopter which is hovering at a height of 550 m above horizontal ground.

The parachutist, who is modelled as a particle, falls for 3 seconds before her parachute opens.

While she is falling, and before her parachute opens, she is modelled as falling freely under gravity.

The acceleration due to gravity is modelled as being $10\,\mathrm{m\,s^{-2}}$.

(a) Using this model, find the speed of the parachutist at the instant her parachute opens.

(1)

When her parachute is open, the parachutist continues to fall vertically.

Immediately after her parachute opens, she decelerates at $12\,\mathrm{m\,s^{-2}}$ for 2 seconds before reaching a constant speed and she reaches the ground with this speed.

The total time taken by the parachutist to fall the 550 m from the helicopter to the ground is T seconds.

(b) Sketch a speed-time graph for the motion of the parachutist for $0 \leqslant t \leqslant T$.

(2)

(c) Find, to the nearest whole number, the value of T.

(5)

In a refinement of the model of the motion of the parachutist, the effect of air resistance is included before her parachute opens and this refined model is now used to find a new value of T.

(d) How would this new value of T compare with the value found, using the initial model, in part (c)?

(1)

(e) Suggest one further refinement to the model, apart from air resistance, to make the model more realistic.

(1)

5.

At time $t = 0$, a small ball is projected vertically upwards with speed $U\,\text{m}\,\text{s}^{-1}$ from a point A that is 16.8 m above horizontal ground.

The speed of the ball at the instant immediately before it hits the ground for the first time is $19\,\text{m}\,\text{s}^{-1}$

The ball hits the ground for the first time at time $t = T$ seconds.

The motion of the ball, from the instant it is projected until the instant just before it hits the ground for the first time, is modelled as that of a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m}\,\text{s}^{-2}$

Using the model,

(a) show that $U = 5$

(2)

(b) find the value of T,

(2)

(c) find the time from the instant the ball is projected until the instant when the ball is 1.2 m below A.

(4)

(d) Sketch a velocity-time graph for the motion of the ball for $0 \leqslant t \leqslant T$, stating the coordinates of the start point and the end point of your graph.

(2)

In a refinement of the model of the motion of the ball, the effect of air resistance on the ball is included and this refined model is now used to find the value of U.

(e) State, with a reason, how this new value of U would compare with the value found in part (a), using the initial unrefined model.

(1)

(f) Suggest one further refinement that could be made to the model, apart from including air resistance, that would make the model more realistic.

(1)

6.

A train travels along a straight horizontal track from station P to station Q.

In a model of the motion of the train, at time $t = 0$ the train starts from rest at P, and moves with constant acceleration until it reaches its maximum speed of $25\,\mathrm{m\,s^{-1}}$

The train then travels at this constant speed of $25\,\mathrm{m\,s^{-1}}$ before finally moving with constant deceleration until it comes to rest at Q.

The time spent decelerating is four times the time spent accelerating.

The journey from P to Q takes 700 s.

Using the model,

(a) sketch a speed-time graph for the motion of the train between the two stations P and Q.
(1)

The distance between the two stations is 15 km.

Using the model,

(b) show that the time spent accelerating by the train is 40 s,
(3)

(c) find the acceleration, in $\mathrm{m\,s^{-2}}$, of the train,
(1)

(d) find the speed of the train 572 s after leaving P.
(2)

(e) State one limitation of the model which could affect your answers to parts (b) and (c).
(1)

7.

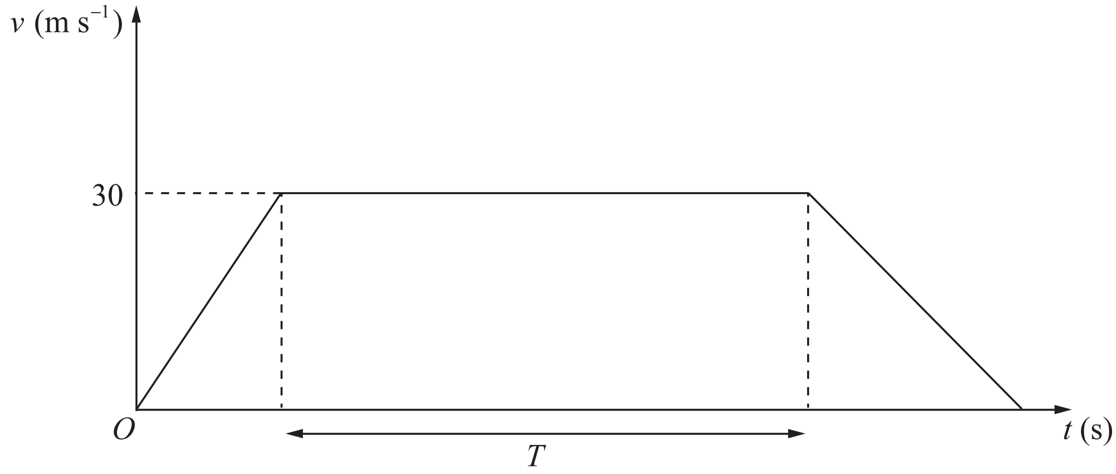

Figure 4

The velocity-time graph in Figure 4 represents the journey of a train P travelling along a straight horizontal track between two stations which are 1.5 km apart. The train P leaves the first station, accelerating uniformly from rest for 300 m until it reaches a speed of 30 m s^{-1}. The train then maintains this speed for T seconds before decelerating uniformly at 1.25 m s^{-2}, coming to rest at the next station.

(a) Find the acceleration of P during the first 300 m of its journey.

(2)

(b) Find the value of T.

(5)

A second train Q completes the same journey in the same total time. The train leaves the first station, accelerating uniformly from rest until it reaches a speed of V m s^{-1} and then immediately decelerates uniformly until it comes to rest at the next station.

(c) Sketch on the diagram above, a velocity-time graph which represents the journey of train Q.

(2)

(d) Find the value of V.

(6)

8.

A car accelerates uniformly from rest for 20 seconds. It moves at constant speed v m s^{-1} for the next 40 seconds and then decelerates uniformly for 10 seconds until it comes to rest.

(a) For the motion of the car, sketch

(i) a speed-time graph,

(ii) an acceleration-time graph.

(6)

Given that the total distance moved by the car is 880 m,

(b) find the value of v.

(4)

9.

A girl runs a 400 m race in a time of 84 s. In a model of this race, it is assumed that, starting from rest, she moves with constant acceleration for 4 s, reaching a speed of 5 m s^{-1}. She maintains this speed for 60 s and then moves with constant deceleration for 20 s, crossing the finishing line with a speed of V m s^{-1}.

(a) Sketch, in the space below, a speed-time graph for the motion of the girl during the whole race.

(2)

(b) Find the distance run by the girl in the first 64 s of the race.

(3)

(c) Find the value of V.

(5)

(d) Find the deceleration of the girl in the final 20 s of her race.

(2)

10.

Two trains M and N are moving in the same direction along parallel straight horizontal tracks. At time $t = 0$, M overtakes N whilst they are travelling with speeds 40 m s^{-1} and 30 m s^{-1} respectively. Train M overtakes train N as they pass a point X at the side of the tracks.

After overtaking N, train M maintains its speed of 40 m s^{-1} for T seconds and then decelerates uniformly, coming to rest next to a point Y at the side of the tracks.

After being overtaken, train N maintains its speed of 30 m s^{-1} for 25 s and then decelerates uniformly, also coming to rest next to the point Y.

The times taken by the trains to travel between X and Y are the same.

(a) Sketch, on the same diagram, the speed-time graphs for the motions of the two trains between X and Y.

(4)

Given that $XY = 975$ m,

(b) find the value of T.

(8)

11.

A car moves along a straight horizontal road from a point A to a point B, where $AB = 885\text{ m}$. The car accelerates from rest at A to a speed of 15 m s^{-1} at a constant rate $a\text{ m s}^{-2}$. The time for which the car accelerates is $\frac{1}{3}T$ seconds. The car maintains the speed of 15 m s^{-1} for T seconds. The car then decelerates at a constant rate of 2.5 m s^{-2} stopping at B.

(a) Find the time for which the car decelerates.

(2)

(b) Sketch a speed-time graph for the motion of the car.

(2)

(c) Find the value of T.

(4)

(d) Find the value of a.

(2)

(e) Sketch an acceleration-time graph for the motion of the car.

(3)

12.

A car is travelling along a straight horizontal road. The car takes 120 s to travel between two sets of traffic lights which are 2145 m apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for 30 s until its speed is 22 m s^{-1}. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

(a) Sketch, in the space below, a speed-time graph for the motion of the car between the two sets of traffic lights.

(2)

(b) Find the value of T.

(3)

A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, a m s^{-2}, and passes the car at the point A which is 990 m from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed 22 m s^{-1}.

(c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A.

(4)

(d) Find the value of a.

(2)

13.

A train travels along a straight horizontal track between two stations, A and B. The train starts from rest at A and moves with constant acceleration 0.5 m s^{-2} until it reaches a speed of V m s^{-1}, ($V < 50$). The train then travels at this constant speed before it moves with constant deceleration 0.25 m s^{-2} until it comes to rest at B.

(a) Sketch in the space below a speed-time graph for the motion of the train between the two stations A and B.

(2)

The total time for the journey from A to B is 5 minutes.

(b) Find, in terms of V, the length of time, in seconds, for which the train is

(i) accelerating,

(ii) decelerating,

(iii) moving with constant speed.

(5)

Given that the distance between the two stations A and B is 6.3 km,

(c) find the value of V.

(6)

14.

A car is moving on a straight horizontal road. At time $t = 0$, the car is moving with speed 20 m s^{-1} and is at the point A. The car maintains the speed of 20 m s^{-1} for 25 s. The car then moves with constant deceleration 0.4 m s^{-2}, reducing its speed from 20 m s^{-1} to 8 m s^{-1}. The car then moves with constant speed 8 m s^{-1} for 60 s. The car then moves with constant acceleration until it is moving with speed 20 m s^{-1} at the point B.

(a) Sketch a speed-time graph to represent the motion of the car from A to B.

(3)

(b) Find the time for which the car is decelerating.

(2)

Given that the distance from A to B is 1960 m,

(c) find the time taken for the car to move from A to B.

(8)

15.

Two cars P and Q are moving in the same direction along the same straight horizontal road. Car P is moving with constant speed 25 m s^{-1}. At time $t = 0$, P overtakes Q which is moving with constant speed 20 m s^{-1}. From $t = T$ seconds, P decelerates uniformly, coming to rest at a point X which is 800 m from the point where P overtook Q. From $t = 25 \text{ s}$, Q decelerates uniformly, coming to rest at the same point X at the same instant as P.

(a) Sketch, on the same axes, the speed-time graphs of the two cars for the period from $t = 0$ to the time when they both come to rest at the point X.

(4)

(b) Find the value of T.

(8)