1 (a) Expand and simplify $(y-2)(y-5)$

....

(2)

(b) Prove algebraically that

$$(2n+1)^2 - (2n+1) \quad \text{is an even number}$$

for all positive integer values of n.

(3)

(Total for Question 1 is 5 marks)

2 n is an integer.

Prove algebraically that the sum of $\frac{1}{2}n(n + 1)$ and $\frac{1}{2}(n + 1)(n + 2)$ is always a square number.

(Total for Question 2 is 2 marks)

3 Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n.

(Total for Question 3 is 3 marks)

4 Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

(Total for Question 4 is 2 marks)

5 n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always an even number.

(Total for Question 5 is 4 marks)

6 Prove algebraically that

$$(2n + 1)^2 - (2n + 1) \text{ is an even number}$$

for all positive integer values of n.

(Total for Question 6 is 3 marks)

7 (a) Prove that

$$(2m + 1)^2 - (2n - 1)^2 = 4(m + n)(m - n + 1)$$

(3)

Sophia says that the result in part (a) shows that the difference of the squares of any two odd numbers must be a multiple of 4

(b) Is Sophia correct?
You must give reasons for your answer.

....

....

....

(1)

(Total for Question 7 is 4 marks)

8 (a) Show that $x(x-1)(x+1) = x^3 - x$

(1)

(b) Prove that the difference between a whole number and the cube of this number is always a multiple of 6

(3)

(Total for Question 8 is 4 marks)

9 N is a multiple of 5

$A = N + 1$
$B = N - 1$

Prove, using algebra, that $A^2 - B^2$ is always a multiple of 20

(Total for Question 9 is 3 marks)

10 Prove algebraically that the product of any two odd numbers is always an odd number.

(Total for Question 10 is 4 marks)

11 Prove that the square of an odd number is always 1 more than a multiple of 4

(Total for Question 11 is 4 marks)

12 Prove that the difference between two consecutive square numbers is always an odd number.
Show clear algebraic working.

(Total for Question 12 is 3 marks)

13 Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4

(Total for Question 13 is 3 marks)

14 Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8

(Total for Question 14 is 3 marks)

15 Prove that when the sum of the squares of any two consecutive odd numbers is divided by 8, the remainder is always 2
Show clear algebraic working.

(Total for Question 15 is 3 marks)

16 The product of two consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number.

(Total for Question 16 is 3 marks)

17 Using algebra, prove that, given any 3 consecutive even numbers, the difference between the square of the largest number and the square of the smallest number is always 8 times the middle number.

(Total for Question 17 is 3 marks)

18 An expression for the nth term of the sequence of triangular numbers is $\dfrac{n(n+1)}{2}$

Prove that the sum of any two consecutive triangular numbers is a square number.

(Total for Question 18 is 3 marks)

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19 Using algebra, prove that, given any 3 consecutive whole numbers, the sum of the square of the smallest number and the square of the largest number is always 2 more than twice the square of the middle number.

(Total for Question 19 is 3 marks)

20 Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

(Total for Question 20 is 4 marks)

21 Here are the first four terms of a sequence of fractions.

$$\frac{1}{1} \qquad \frac{2}{3} \qquad \frac{3}{5} \qquad \frac{4}{7}$$

The numerators of the fractions form the sequence of whole numbers 1 2 3 4 …
The denominators of the fractions form the sequence of odd numbers 1 3 5 7 …

(a) Write down an expression, in terms of n, for the nth term of this sequence of fractions.

(2)

(b) Using algebra, prove that when the square of any odd number is divided by 4 the remainder is 1

(3)

(Total for Question 21 is 5 marks)

22 The table gives information about the first six terms of a sequence of numbers.

Term number	1	2	3	4	5	6
Term of sequence	$\frac{1 \times 2}{2}$	$\frac{2 \times 3}{2}$	$\frac{3 \times 4}{2}$	$\frac{4 \times 5}{2}$	$\frac{5 \times 6}{2}$	$\frac{6 \times 7}{2}$

Prove algebraically that the sum of any two consecutive terms of this sequence is always a square number.

(Total for Question 22 is 4 marks)

23 $(2x + 23)$, $(8x + 2)$ and $(20x - 52)$ are three consecutive terms of an arithmetic sequence.

Prove that the common difference of the sequence is 12

(Total for Question 23 is 4 marks)