1. The box below shows a student's attempt to prove the following identity for $a > b > 0$

$$\arctan a - \arctan b \equiv \arctan \frac{a-b}{1+ab}$$

Let $x = \arctan a$ and $y = \arctan b$, so that $a = \tan x$ and $b = \tan y$

$$\text{So } \tan(\arctan a - \arctan b) \equiv \tan(x-y)$$

$$\equiv \frac{\tan x - \tan y}{1 - \tan^2(xy)}$$

$$\equiv \frac{a-b}{1-(ab)^2}$$

$$\equiv \frac{a - ab + ab - b}{(1-ab)(1+ab)}$$

$$\equiv \frac{a\,\cancel{(1-ab)} - b\,\cancel{(1-ab)}}{\cancel{(1-ab)}\,(1+ab)}$$

$$\equiv \frac{a-b}{1+ab}$$

Taking arctan of both sides gives $\arctan a - \arctan b \equiv \arctan \dfrac{a-b}{1+ab}$ as required.

There are three errors in the proof where the working does not follow from the previous line.

(i) Describe these three errors.

(3)

(ii) Write out a correct proof of the identity.

(2)

2.

Solve for $0 \leqslant \theta \leqslant 180^{\circ}$

$$\tan\left(\theta + 35^{\circ}\right) = \cot\left(\theta - 53^{\circ}\right)$$

(Total 4 marks)

3.

(a) Write down the exact value of $\cos 405°$

(1)

(b) Hence, using a double angle identity for cosine, or otherwise, determine the exact value of $\cos 101.25°$, giving your answer in the form

$$a\sqrt{b + c\sqrt{2 + \sqrt{2}}}$$

where a, b and c are rational numbers.

(5)

4. Given that

$$3 \sin^2 x + 2 \sin x = 6 \cos x + 9 \sin x \cos x$$

and that $-90° < x < 90°$,

find the possible values of $\tan x$.

(Total 6 marks)

5. Find the value of

$$\arccos\left(\frac{1}{\sqrt{2}}\right) + \arcsin\left(\frac{1}{3}\right) + 2\arctan\left(\frac{1}{\sqrt{2}}\right)$$

Give your answer as a multiple of π.

(arccos x is an alternative notion for $\cos^{-1} x$ etc.)

(Total 7 marks)

6.

(a) Use the formula for $\sin(A-B)$ to show that $\sin(90°-x)=\cos x$

(1)

(b) Solve for $0<\theta<360°$

$$2\sin(\theta+17°)=\frac{\cos(\theta+8°)}{\cos(\theta+17°)}$$

(7)

(Total 8 marks)

7. (a) Show that the equation

$$\tan x = \frac{\sqrt{3}}{1 + 4\cos x}$$

can be written in the form

$$\sin 2x = \sin(60° - x)$$

(4)

(b) Solve, for $0 < x < 180°$

$$\tan x = \frac{\sqrt{3}}{1 + 4\cos x}$$

(5)

(Total 9 marks)

8. Solve for $0 < x < 360°$

$$\cot 2x - \tan 78° = \frac{(\sec x)(\sec 78°)}{2}$$

where x is not an integer multiple of $90°$

(Total 9 marks)

9. Given that $(\sin\theta + \cos\theta) \neq 0$, find all the solutions of

$$\frac{2\cos 2\theta(\sin 2\theta - \sqrt{3}\cos 2\theta)}{\sin\theta + \cos\theta} = \sqrt{6}(\sin 2\theta - \sqrt{3}\cos 2\theta)$$

for $0 \leq \theta < 360°$.

(10)

10. The angle K, $0 < K < \dfrac{\pi}{2}$, satisfies

$$\tan\theta \tan 2\theta = \sum_{r=0}^{\infty} 2\cos^r 2\theta$$

(a) Show that $\tan\theta = 3^p$, where p is a rational number to be found.

(8)

(b) Hence show that $\dfrac{\pi}{6} < \theta < \dfrac{\pi}{4}$

(2)

(Total 10 marks)

11.

(a) Use the formulae for $\sin(A \pm B)$ and $\cos(A \pm B)$ to prove that $\tan(90° - \theta) \equiv \cot\theta$

(3)

(b) Solve for $0 < \theta < 360°$

$$2 - \sec^2(\theta + 11°) = 2\tan(\theta + 11°)\tan(\theta - 34°)$$

Give each answer as an integer in degrees.

(8)

(+S1)

12. (a) Solve, for $0 \leqslant \theta < 2.$,

$$\sin\left(\frac{\pi}{3} - \theta\right) = \frac{1}{\sqrt{3}} \cos\theta \ .$$

(5)

(b) Find the value of x for which

$$\arcsin(1 - 2x) = \frac{\pi}{3} - \arcsin x, \qquad 0 < x < 0.5$$

[$\arcsin x$ is an alternative notation for $\sin^{-1} x$]

(7)

(Total 12 marks)

13.

(a) Prove that $\tan 15° = 2 - \sqrt{3}$

(4)

(b) Solve, for $0 \leqslant \theta < 360°$,

$$\sin(\theta + 60°)\sin(\theta - 60°) = (1 - \sqrt{3})\cos^2\theta$$

(8)

(Total 12 marks)

14. (a) Prove the identity

$$(\sin x + \cos y)\cos(x - y) \equiv (1 + \sin(x - y))(\cos x + \sin y)$$

(5)

(b) Hence, or otherwise, show that

$$\frac{\sin 5\theta + \cos 3\theta}{\cos 5\theta + \sin 3\theta} \equiv \frac{1 + \tan\theta}{1 - \tan\theta}$$

(6)

(c) Given that $k > 1$, show that the equation $\dfrac{\sin 5\theta + \cos 3\theta}{\cos 5\theta + \sin 3\theta} = k$ has a unique solution in the interval $0 < \theta < \dfrac{\pi}{4}$

(4)

(+S2)