1. $$f(x) = 2x^3 + x^2 - 5x + c, \text{ where } c \text{ is a constant.}$$

Given that $f(1) = 0$,

(a) find the value of c,

(2)

(b) factorise $f(x)$ completely,

(4)

(c) find the remainder when $f(x)$ is divided by $(2x - 3)$.

(2)

2. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(1 + px)^9,$$

where p is a constant.

(2)

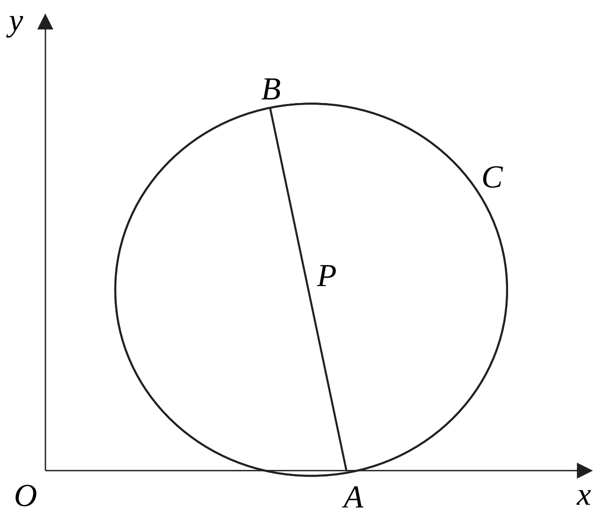
These first 3 terms are 1, $36x$ and qx^2, where q is a constant.

(b) Find the value of p and the value of q.

(4)

3.

Figure 1



In Figure 1, $A(4, 0)$ and $B(3, 5)$ are the end points of a diameter of the circle C.

Find

(a) the exact length of AB,

(2)

(b) the coordinates of the midpoint P of AB,

(2)

(c) an equation for the circle C.

(3)

4. Solve the equation

$$5^x = 17,$$

giving your answer to 3 significant figures.

(3)

5.

$$f(x) = x^3 + 3x^2 + 5.$$

Find

(a) $f''(x)$,

(3)

(b) $\int_1^2 f(x)\,dx$.

(4)

6. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1-2x)^5$. Give each term in its simplest form.

(4)

(b) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1+x)(1-2x)^5 \approx 1-9x\,.$$

(2)

7. The line joining the points $(-1, 4)$ and $(3, 6)$ is a diameter of the circle C.

Find an equation for C.

(6)

8.

(a) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

9. (a) Show that the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1$$

can be written as

$$5 \sin^2 \theta = 3.$$

(2)

(b) Hence solve, for $0° \leqslant \theta < 360°$, the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1,$$

giving your answers to 1 decimal place.

(7)

10. Given that a and b are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

(6)

11. Find all the solutions, in the interval $0 \leqslant x < 2\pi$, of the equation

$$2\cos^2 x + 1 = 5\sin x,$$

giving each solution in terms of π.

(6)

12.

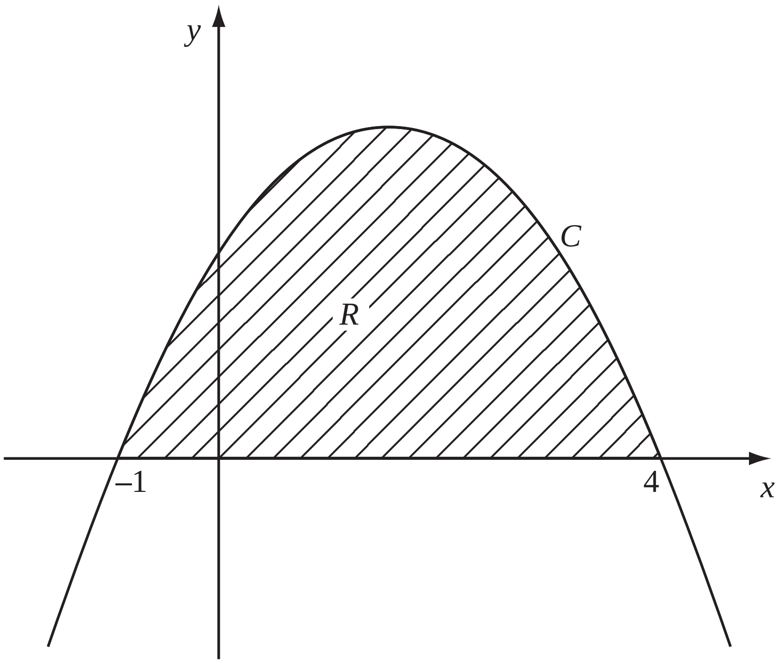


Figure 1

Figure 1 shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(5)

13. $$f(x) = x^4 + 5x^3 + ax + b,$$

where a and b are constants.

The remainder when $f(x)$ is divided by $(x - 2)$ is equal to the remainder when $f(x)$ is divided by $(x + 1)$.

(a) Find the value of a.

(5)

Given that $(x + 3)$ is a factor of $f(x)$,

(b) find the value of b.

(3)

14. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum.

(5)

(b) Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v.

(2)

(c) Calculate the minimum total cost of the journey.

(2)

15. **Figure 3**

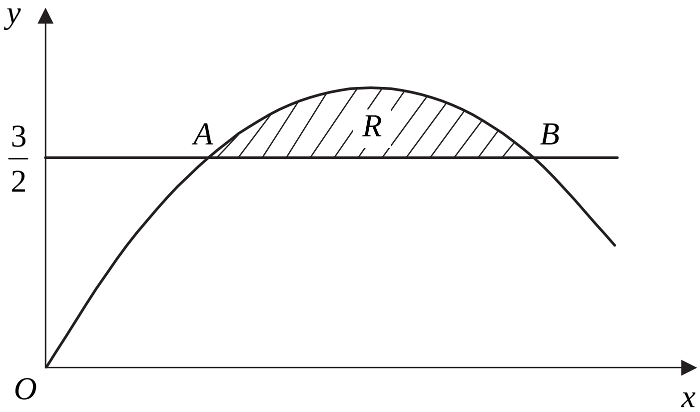


Figure 3 shows the shaded region R which is bounded by the curve $y = -2x^2 + 4x$ and the line $y = \frac{3}{2}$. The points A and B are the points of intersection of the line and the curve.

Find

(a) the x-coordinates of the points A and B,

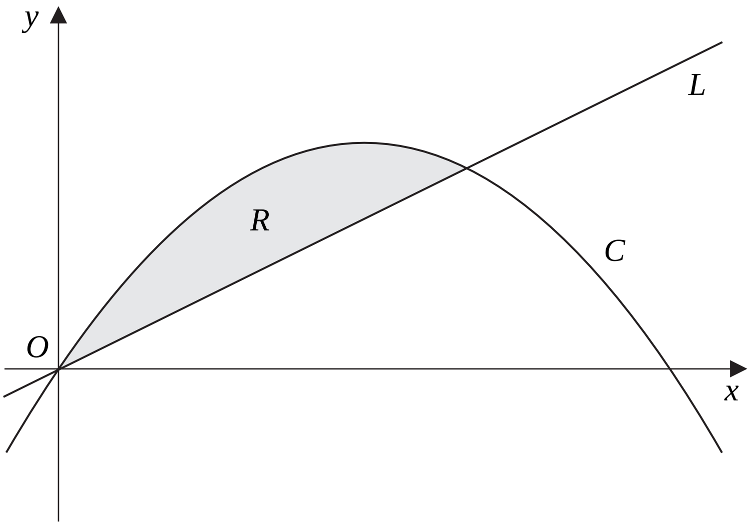
(4)

(b) the exact area of R.

(6)

16.

Figure 2



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points $(0, 0)$ and $(4, 8)$.

(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.

(6)

17.

A circle C has centre M (6, 4) and radius 3.

(a) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$

(2)

Figure 3

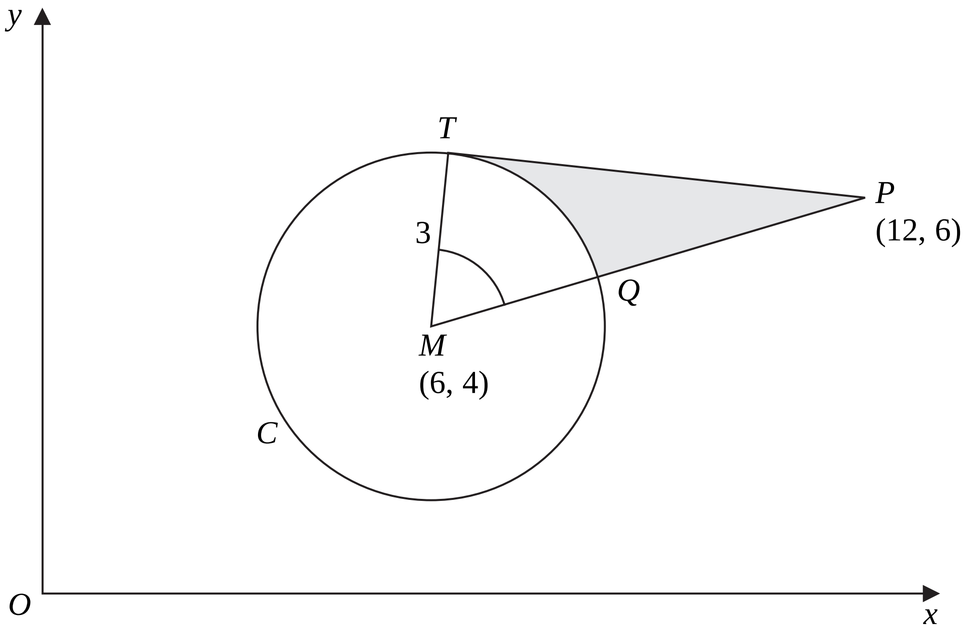


Figure 3 shows the circle C. The point T lies on the circle and the tangent at T passes through the point P (12, 6). The line MP cuts the circle at Q.

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ, as shown in Figure 3.

(c) Find the area of the shaded region TPQ. Give your answer to 3 decimal places.

(5)

18.

Figure 4

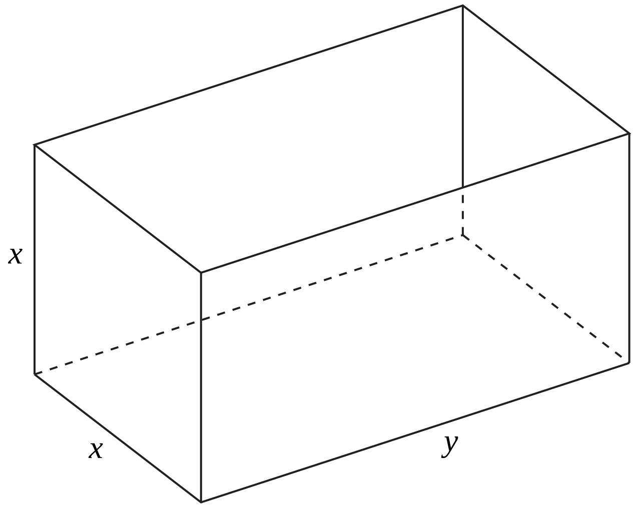


Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is $100\,\text{m}^3$.

(a) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2 .$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)