

1

- (a) Use the iteration formula $x_{n+1} = \sqrt[3]{10 - 2x_n}$ to find the values of x_1 , x_2 and x_3
Start with $x_0 = 2$

$$x_1 = \dots\dots\dots$$

$$x_2 = \dots\dots\dots$$

$$x_3 = \dots\dots\dots$$

(3)

The values of x_1 , x_2 and x_3 found in part (a) are estimates of the solution of an equation of the form $x^3 + ax + b = 0$ where a and b are integers.

- (b) Find the value of a and the value of b .

$$a = \dots\dots\dots$$

$$b = \dots\dots\dots$$

(1)

(Total for Question 1 is 4 marks)

2 Using $x_{n+1} = -2 - \frac{4}{x_n^2}$

with $x_0 = -2.5$

(a) find the values of x_1, x_2 and x_3

$$x_1 = \dots\dots\dots$$

$$x_2 = \dots\dots\dots$$

$$x_3 = \dots\dots\dots$$

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 + 2x^2 + 4 = 0$

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(2)

(Total for Question 2 is 5 marks)

3 (a) Show that the equation $x^4 - x^2 - 5 = 0$ can be written in the form $x = \sqrt[4]{x^2 + 5}$

(1)

(b) Starting with $x_0 = 1.5$

use the iteration formula $x_{n+1} = \sqrt[4]{x_n^2 + 5}$ three times to find an estimate for a solution of $x^4 - x^2 - 5 = 0$

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(3)

(Total for Question 3 is 4 marks)

4 (a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

(2)

(b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

(1)

(c) Starting with $x_0 = 2$,
use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a
solution of $x^3 + x = 7$

.....
(3)

(Total for Question 4 is 6 marks)

5

(a) Show that the equation $x^3 + 2x - 6 = 0$ has a solution between $x = 1$ and $x = 2$

(2)

(b) Show that the equation $x^3 + 2x - 6 = 0$ can be rearranged to give $x = \frac{6}{x^2 + 2}$

(1)

(c) Starting with $x_0 = 1.45$

use the iteration formula $x_{n+1} = \frac{6}{x_n^2 + 2}$ twice to find an estimate
for the solution of $x^3 + 2x - 6 = 0$

Give your answer correct to 4 decimal places.

(3)

(Total for Question 5 is 6 marks)

6 (a) Show that the equation $x^3 + 7x - 5 = 0$ has a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^3 + 7x - 5 = 0$ can be arranged to give $x = \frac{5}{x^2 + 7}$

(2)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \frac{5}{x_n^2 + 7}$ three times to find an estimate for the solution of $x^3 + 7x - 5 = 0$

(3)

- (d) By substituting your answer to part (c) into $x^3 + 7x - 5$,
comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$

(2)

(Total for Question 4 is 9 marks)