1

(a) Use the iteration formula $x_{n+1} = \sqrt[3]{10 - 2x_n}$ to find the values of x_1, x_2 and x_3
Start with $x_0 = 2$

$x_1 =$ ....

$x_2 =$ ....

$x_3 =$ ....

(3)

The values of x_1, x_2 and x_3 found in part (a) are estimates of the solution of an equation of the form $x^3 + ax + b = 0$ where a and b are integers.

(b) Find the value of a and the value of b.

$a =$ ....

$b =$ ....

(1)

(Total for Question 1 is 4 marks)

2 Using $x_{n+1} = -2 - \dfrac{4}{x_n^2}$

with $x_0 = -2.5$

(a) find the values of x_1, x_2 and x_3

$x_1 =$ ....

$x_2 =$ ....

$x_3 =$ ....

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 + 2x^2 + 4 = 0$

....

....

....

....

(2)

(Total for Question 2 is 5 marks)

3 (a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

(2)

(b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

(1)

(c) Starting with $x_0 = 2$,
use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

....

(3)

(Total for Question 3 is 6 marks)

4 (a) Show that the equation $x^3 + 7x - 5 = 0$ has a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^3 + 7x - 5 = 0$ can be arranged to give $x = \dfrac{5}{x^2 + 7}$

(2)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \dfrac{5}{x_n^{\,2} + 7}$ three times to find an estimate for the solution of $x^3 + 7x - 5 = 0$

....

(3)

(d) By substituting your answer to part (c) into $x^3 + 7x - 5$,
comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$

....

....

(2)

(Total for Question 4 is 9 marks)

5 The number of animals in a population at the start of year t is P_t
The number of animals at the start of year 1 is 400

Given that

$$P_{t+1} = 1.01P_t$$

work out the number of animals at the start of year 3

....

(Total for Question 5 is 2 marks)

6 At time $t = 0$ hours a tank is full of water.

Water leaks from the tank.
At the end of every hour there is 2% less water in the tank than at the start of the hour.

The volume of water, in litres, in the tank at time t hours is V_t

Given that

$$V_0 = 2000$$
$$V_{t+1} = kV_t$$

write down the value of k.

$k =$ ....

(Total for Question 6 is 1 mark)

7 At the start of year n, the number of animals in a population is P_n

At the start of the following year, the number of animals in the population is P_{n+1} where

$$P_{n+1} = kP_n$$

At the start of 2017 the number of animals in the population was 4000
At the start of 2019 the number of animals in the population was 3610

Find the value of the constant k.

....

(Total for Question 7 is 3 marks)

8 A hot air balloon is descending.
The height of the balloon n minutes after it starts to descend is h_n metres.

The height of the balloon $(n + 1)$ minutes after it starts to descend, h_{n+1} metres, is given by

$$h_{n+1} = K \times h_n + 20 \quad \text{where } K \text{ is a constant.}$$

The balloon starts to descend from a height of 1200 metres at 09 15
At 09 16 the height of the balloon is 1040 metres.

Work out the height of the balloon at 09 18

.... m

(Total for Question 8 is 4 marks)

9 The profit made by a shop increases each year.

The profit made by the shop in year n is £P_n

Given that the profit made by the shop in the next year is £P_{n+1} then

$$P_{n+1} = aP_n + 800 \text{ where } a \text{ is a constant.}$$

The table shows the profit made by the shop in 2018 and in 2019

Year	2018	2019
Profit	£24 000	£29 600

Work out the profit predicted to be made by the shop in 2021

£..

(Total for Question 9 is 4 marks)

10 The number of rabbits on a farm at the end of month n is P_n
The number of rabbits at the end of the next month is given by $P_{n+1} = 1.2P_n - 50$

At the end of March there are 200 rabbits on the farm.

(a) Work out how many rabbits there will be on the farm at the end of June.

....

(3)

(b) Considering your results in part (a), suggest what will happen to the number of rabbits on the farm after a long time.

....

....

(1)

(Total for Question 10 is 4 marks)