

1. A sequence of terms a_1, a_2, a_3, \dots is defined by

$$\begin{aligned} a_1 &= 3 \\ a_{n+1} &= 8 - a_n \end{aligned}$$

- (a) (i) Show that this sequence is periodic.

- (ii) State the order of this periodic sequence.

(2)

- (b) Find the value of

$$\sum_{n=1}^{85} a_n$$

(2)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

3. A sequence $u_1, u_2, u_3 \dots$ is defined by

$$u_1 = 35$$

$$u_{n+1} = u_n + 7 \cos\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

- (a) (i) Show that $u_2 = 40$

- (ii) Find the value of u_3 and the value of u_4

(3)

Given that the sequence is periodic with order 4

- (b) (i) write down the value of u_5

- (ii) find the value of $\sum_{r=1}^{25} u_r$

(3)

4. A sequence of numbers a_1, a_2, a_3, \dots is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \quad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0 \tag{3}$$

(b) For this sequence explain why $k \neq 1$ (1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r \tag{3}$$