1. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

(a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

(b) With reference to the model, interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find, according to the model,

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

(d) State one reason why this may not be a realistic model in the long term.

(1)

2.

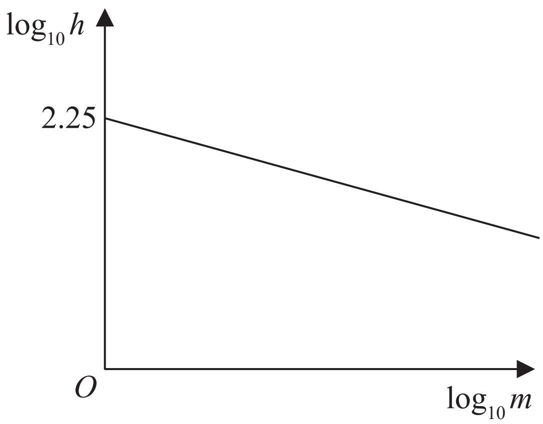

Figure 2

The resting heart rate, h, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of -0.235

(a) Find, to 3 significant figures, the value of p and the value of q.

(3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

3. An advertising agency is monitoring the number of views of an online advert.

The equation

$$\log_{10} V = 0.072t + 2.379 \qquad 1 \leqslant t \leqslant 30,\ t \in \mathbb{N}$$

is used to model the total number of views of the advert, V, in the first t days after the advert went live.

(a) Show that $V = ab^t$ where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(4)

(b) Interpret, with reference to the model, the value of ab.

(1)

Using this model, calculate

(c) the total number of views of the advert in the first 20 days after the advert went live. Give your answer to 2 significant figures.

(2)

4.

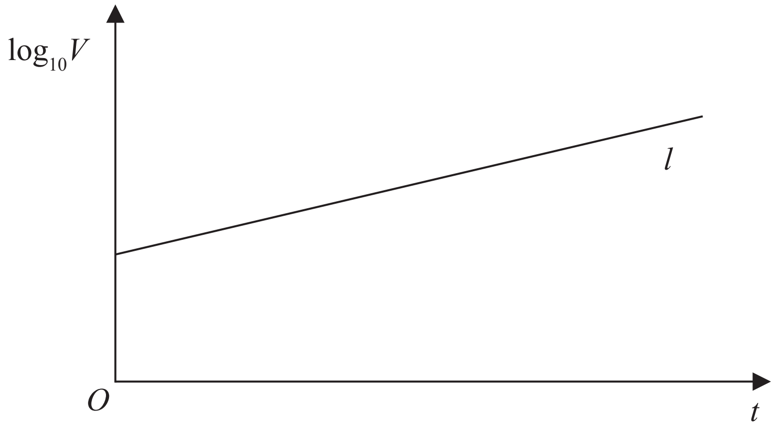

Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10}V$ since 1st January 1980.

The equation of line l is $\log_{10}V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

5. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of $V\,\mathrm{km\,h^{-1}}$.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

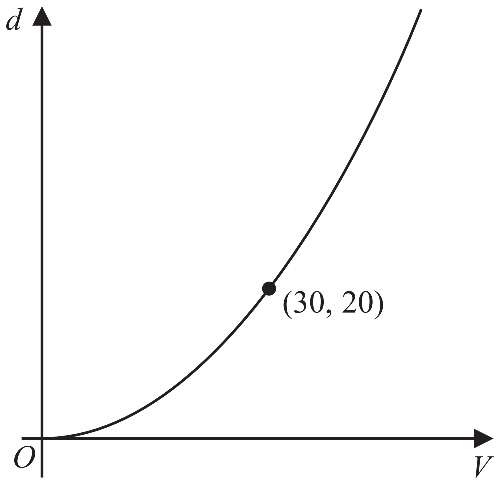

Figure 5

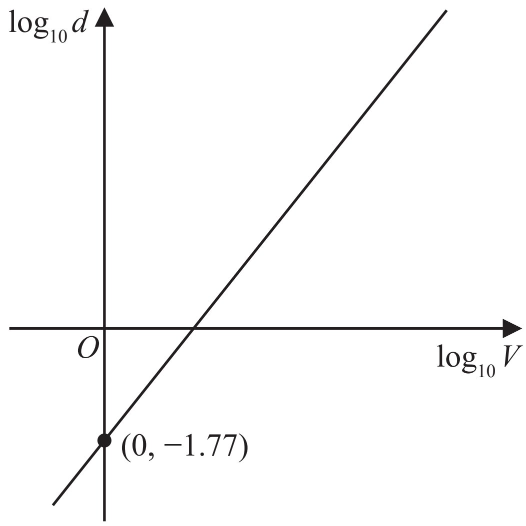

Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at $60\,\mathrm{km\,h^{-1}}$ in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

Question 5 continued

6. The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10} T = b\log_{10} l + \log_{10} a$$

(2)

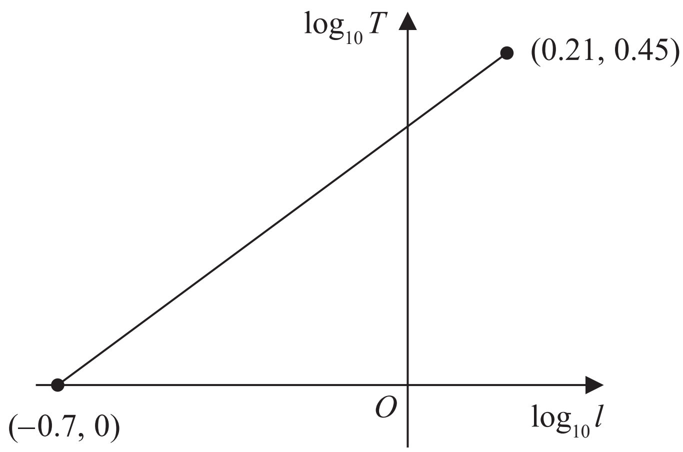

Figure 3

A student carried out an experiment to find the values of the constants a and b.

The student recorded the value of T for different values of l.

Figure 3 shows the linear relationship between $\log_{10} l$ and $\log_{10} T$ for the student's data.
The straight line passes through the points (–0.7, 0) and (0.21, 0.45)

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b, each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant a.

(1)

Question 6 continued

7.

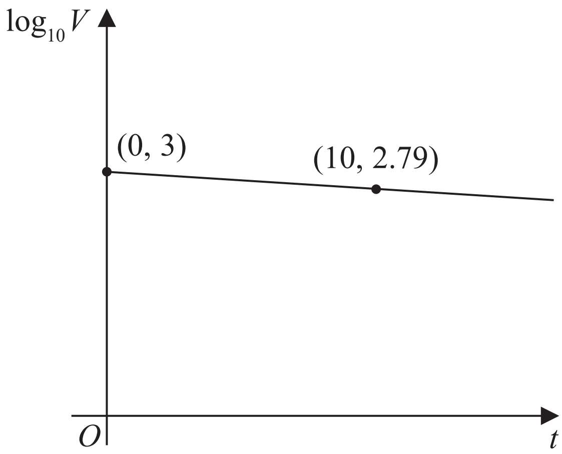

Figure 2

The value, V pounds, of a mobile phone, t months after it was bought, is modelled by

$$V = ab^t$$

where a and b are constants.

Figure 2 shows the linear relationship between $\log_{10} V$ and t.

The line passes through the points (0, 3) and (10, 2.79) as shown.

Using these points,

(a) find the initial value of the phone,

(2)

(b) find a complete equation for V in terms of t, giving the exact value of a and giving the value of b to 3 significant figures.

(3)

Exactly 2 years after it was bought, the value of the phone was £320

(c) Use this information to evaluate the reliability of the model.

(2)

Question 7 continued

8. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

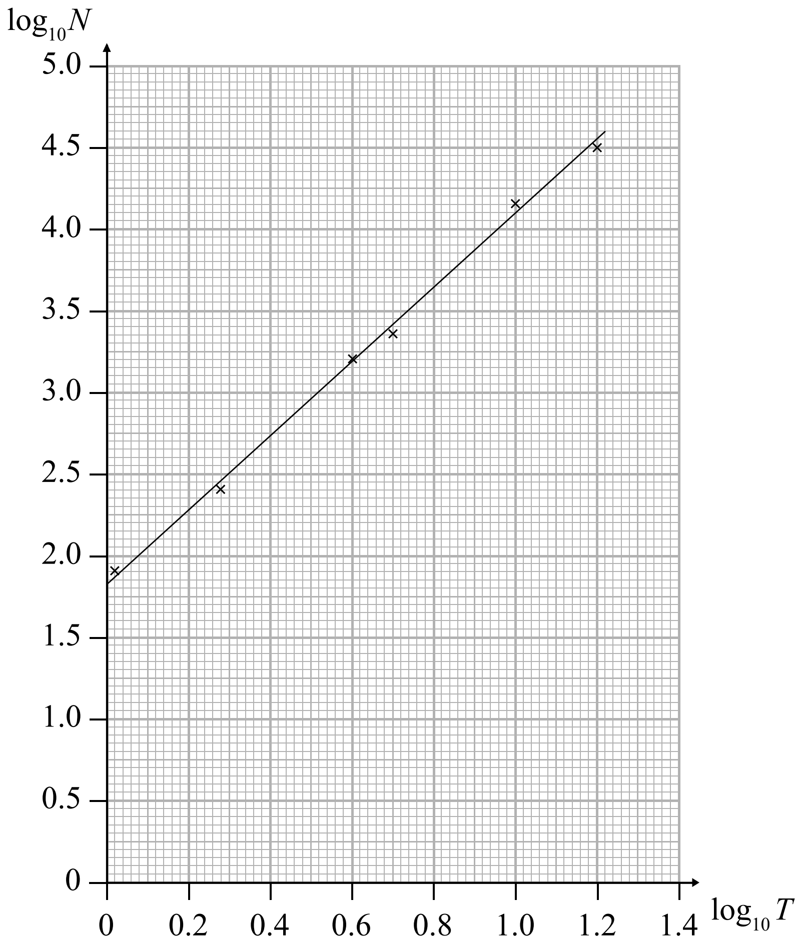

Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

Question 8 continued