1. (a) Sketch the graph of the curve with equation

$$y = \left|\ln(2x + 5)\right| \qquad x > -\frac{5}{2}$$

On your sketch you should clearly state the equations of any asymptotes and mark the coordinates of points where the curve meets the coordinate axes.

(3)

(b) Solve the equation $\left|\ln(2x + 5)\right| = \ln 9$

(3)

(Total 6 marks)

2. (a) By writing $u = \log_4 r$, where $r > 0$, show that

$$\log_4 r = \frac{1}{2}\log_2 r$$

(2)

(b) Solve the equation

$$\log_4\left(5x^2 - 11\right) = \log_2\left(3x - 5\right)$$

(5)

3. Given that $\phi = \frac{1}{2}\left(\sqrt{5}+1\right)$,

(a) show that

(i) $\phi^2 = \phi + 1$

(ii) $\frac{1}{\phi} = \phi - 1$

(4)

(b) The equations of two curves are

$$y = \frac{1}{x} \qquad x > 0$$

and $$y = \ln x - x + k \qquad x > 0$$

where k is a positive constant.
The curves touch at the point P.
Find in terms of ϕ

(i) the coordinates of P,

(ii) the value of k.

(6)

(+S1)

4.

(a) Solve the equation

$$\sqrt{(3x+16)} = 3 + \sqrt{(x+1)}$$

(5)

(b) Solve the equation

$$\log_3(x-7) - \tfrac{1}{2}\log_3 x = 1 - \log_3 2$$

(7)

(Total 12 marks)

5.

Given that $x > y > 0$,

(a) by writing $\log_y x = z$, or otherwise, show that $\log_y x = \dfrac{1}{\log_x y}$.

(2)

(b) Given also that $\log_x y = \log_y x$, show that $y = \dfrac{1}{x}$.

(2)

(c) Solve the simultaneous equations

$$\log_x y = \log_y x,$$

$$\log_x (x - y) = \log_y (x + y).$$

(7)

6. (a) Given that $x > 0$, $y > 0$, $x \neq 1$ and $n > 0$, show that

$$\log_x y = \log_{x^n} y^n$$

(2)

(b) Solve the following, leaving your answers in the form 2^p, where p is a rational number.

(i) $\log_2 u + \log_4 u^2 + \log_8 u^3 + \log_{16} u^4 = 5$

(ii) $\log_2 v + \log_4 v + \log_8 v + \log_{16} v = 5$

(iii) $\log_4 w^2 + \dfrac{3\log_8 64}{\log_2 w} = 5$

(9)

(Total 11 marks)

7. (i) Anna, who is confused about the rules for logarithms, states that

$$\left(\log_3 p\right)^2 = \log_3\left(p^2\right)$$

and $$\log_3(p + q) = \log_3 p + \log_3 q.$$

However, there is a value for p and a value for q for which both statements are correct.

Find the value of p and the value of q.

(7)

(ii) Solve

$$\frac{\log_3(3x^3 - 23x^2 + 40x)}{\log_3 9} = 0.5 + \log_3(3x - 8).$$

(7)

(Total 14 marks)

8. [In this question the values of a, x, and n are such that a and x are positive real numbers, with $a > 1$, $x \neq a$, $x \neq 1$ and n is an integer with $n > 1$]

Sam was confused about the rules of logarithms and thought that

$$\log_a x^n = \left(\log_a x\right)^n \qquad (1)$$

(a) Given that x satisfies statement (1) find x in terms of a and n.

(3)

Sam also thought that

$$\log_a x + \log_a x^2 + \ldots + \log_a x^n = \log_a x + \left(\log_a x\right)^2 + \ldots + \left(\log_a x\right)^n \qquad (2)$$

(b) For $n = 3$, x_1 and x_2 ($x_1 > x_2$) are the two values of x that satisfy statement (2).

(i) Find, in terms of a, an expression for x_1 and an expression for x_2.

(ii) Find the exact value of $\log_a\left(\dfrac{x_1}{x_2}\right)$.

(5)

(c) Show that if $\log_a x$ satisfies statement (2) then

$$2\left(\log_a x\right)^n - n\left(n+1\right)\log_a x + \left(n^2 + n - 2\right) = 0$$

(6)

(Total 14 marks)