1.

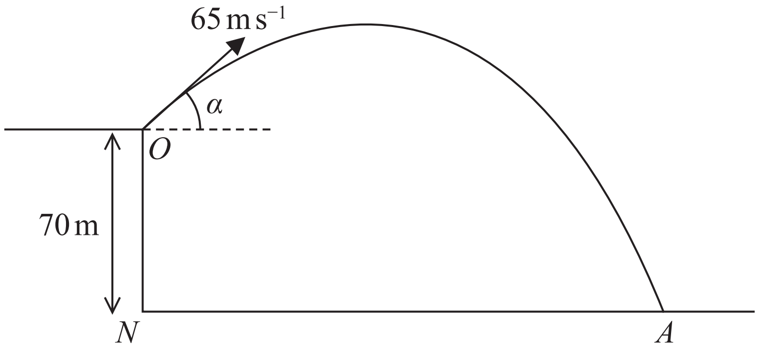

Figure 3

A small stone is projected with speed $65\,\text{m s}^{-1}$ from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m s}^{-2}$

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

2.

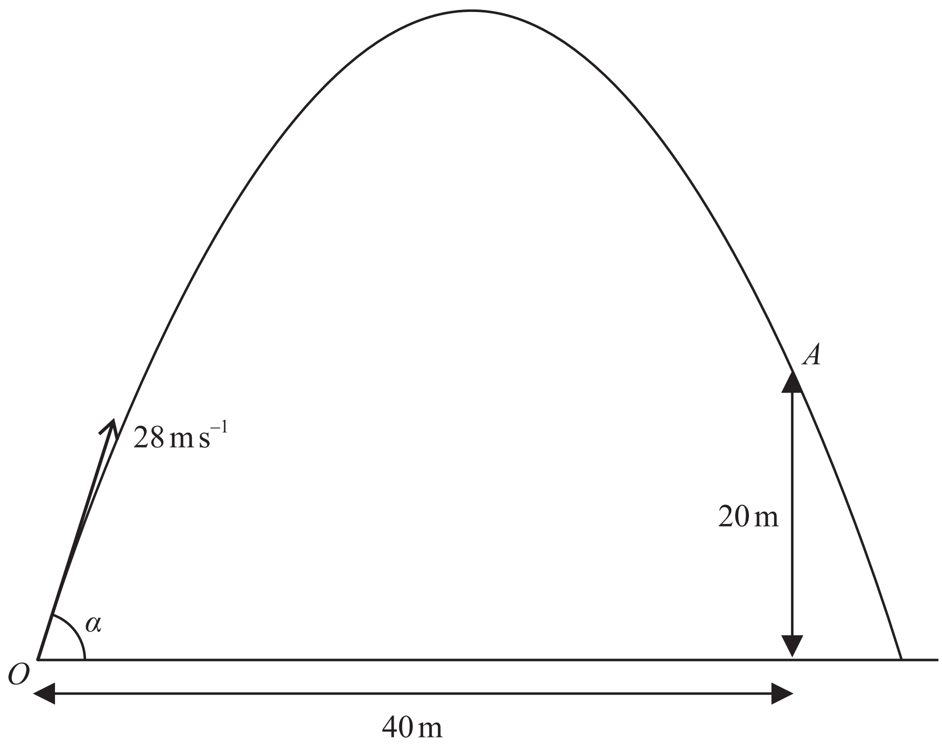

Figure 2

A small ball is projected with speed 28 m s^{-1} from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$

(2)

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$

(5)

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A.

(3)

The model does not include air resistance.

(d) State one other limitation of the model.

(1)

3.

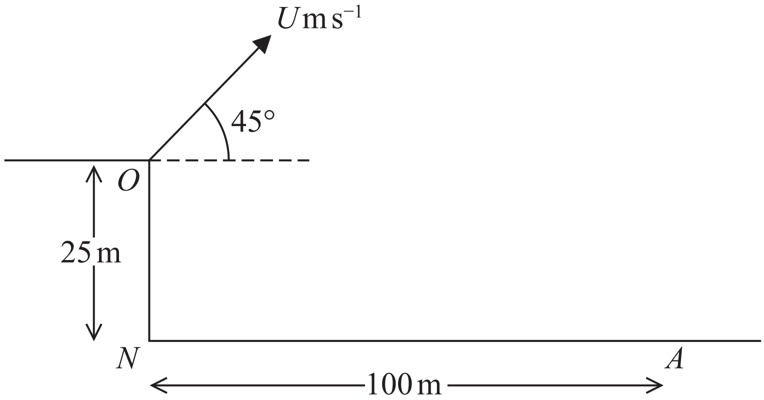

Figure 2

A small ball is projected with speed U m s^{-1} from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A, where $AN = 100$ m, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 28$

(6)

(b) find the greatest height of the ball above the horizontal ground NA.

(3)

In a refinement to the model of the motion of the ball from O to A, the effect of air resistance is included.

This refined model is used to find a new value of U.

(c) How would this new value of U compare with 28, the value given in part (a)?

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

4.

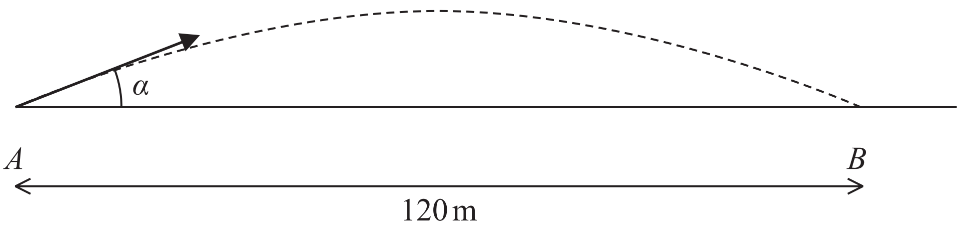

Figure 3

A golf ball is at rest at the point A on horizontal ground.

The ball is hit and initially moves at an angle α to the ground.

The ball first hits the ground at the point B, where $AB = 120$ m, as shown in Figure 3.

The motion of the ball is modelled as that of a particle, moving freely under gravity, whose initial speed is $U\,\text{m s}^{-1}$

Using this model,

(a) show that $U^2 \sin\alpha \cos\alpha = 588$

(6)

The ball reaches a maximum height of 10 m above the ground.

(b) Show that $U^2 = 1960$

(4)

In a refinement to the model, the effect of air resistance is included.

The motion of the ball, from A to B, is now modelled as that of a particle whose initial speed is $V\,\text{m s}^{-1}$

This refined model is used to calculate a value for V

(c) State which is greater, U or V, giving a reason for your answer.

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

5.

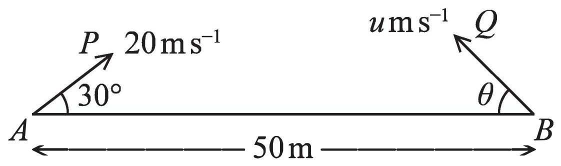

Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed $20\,\mathrm{m\,s^{-1}}$ at $30°$ to AB.

Ball Q is projected from B with speed $u\,\mathrm{m\,s^{-1}}$ at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

(i) the size of angle θ,

(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

6.

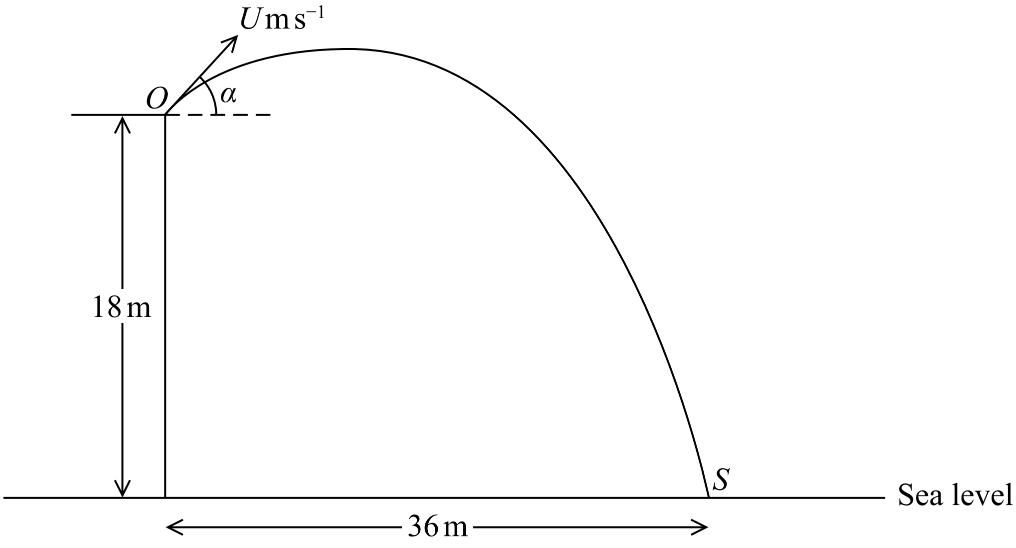

Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10$ m s^{-2}

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)

7.

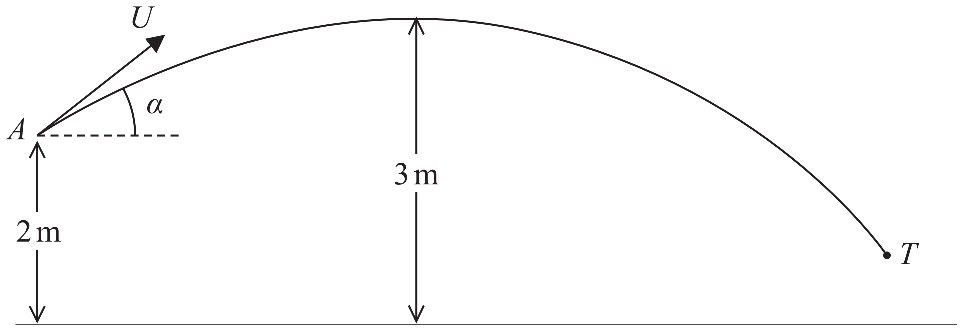

Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground. The target is modelled as being the point T, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2 \alpha}$.

(2)

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α

(9)

(c) State one limitation of the model that could affect your answer to part (b).

(1)

(d) Find the time taken for the ball to travel from A to T.

(3)

8.

[*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

At time $t = 0$, a particle P is projected with velocity $(4\mathbf{i} + 9\mathbf{j})$ m s^{-1} from a fixed point O on horizontal ground. The particle moves freely under gravity. When P is at the point H on its path, P is at its greatest height above the ground.

(a) Find the time taken by P to reach H.

(2)

At the point A on its path, the position vector of P relative to O is $(k\mathbf{i} + k\mathbf{j})$ m, where k is a positive constant.

(b) Find the value of k.

(4)

(c) Find, in terms of k, the position vector of the other point on the path of P which is at the same vertical height above the ground as the point A.

(3)

At time T seconds the particle is at the point B and is moving perpendicular to $(4\mathbf{i} + 9\mathbf{j})$

(d) Find the value of T.

(4)

9.

[*In this question the unit vectors* **i** *and* **j** *are in a vertical plane*, **i** *being horizontal and* **j** *being vertically upwards*.]

At $t = 0$ a particle P is projected from a fixed point O with velocity $(7\mathbf{i} + 7\sqrt{3}\mathbf{j})\ \mathrm{m\,s^{-1}}$. The particle moves freely under gravity. The position vector of a point on the path of P is $(x\mathbf{i} + y\mathbf{j})$ m relative to O.

(a) Show that

$$y = \sqrt{3}x - \frac{g}{98}x^2$$

(5)

(b) Find the direction of motion of P when it passes through the point on the path where $x = 20$

(4)

At time T seconds P passes through the point with position vector $(2\lambda\mathbf{i} + \lambda\mathbf{j})$ m where λ is a positive constant.

(c) Find the value of T.

(4)

10. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

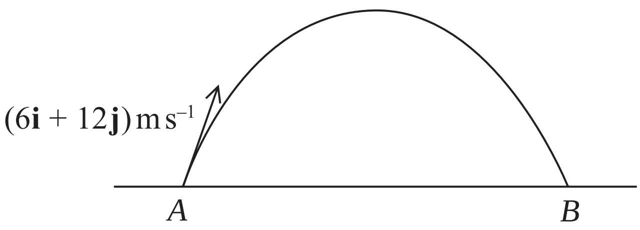

Figure 5

A small ball is projected with velocity $(6\mathbf{i} + 12\mathbf{j})\,\text{m s}^{-1}$ from a fixed point A on horizontal ground. The ball hits the ground at the point B, as shown in Figure 5. The motion of the ball is modelled as a particle moving freely under gravity.

(a) Find the distance AB.

(4)

When the height of the ball above the ground is more than h metres, the speed of the ball is less than $10\,\text{m s}^{-1}$

(b) Find the smallest possible value of h.

(4)

When the ball is at the point C on its path, the direction of motion of the ball is perpendicular to the direction of motion of the ball at the instant before it hits the ground at B.

(c) Find, in terms of **i** and **j**, the velocity of the ball when it is at C.

(3)

11. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane, with* **i** *being horizontal and* **j** *being vertically upwards*.]

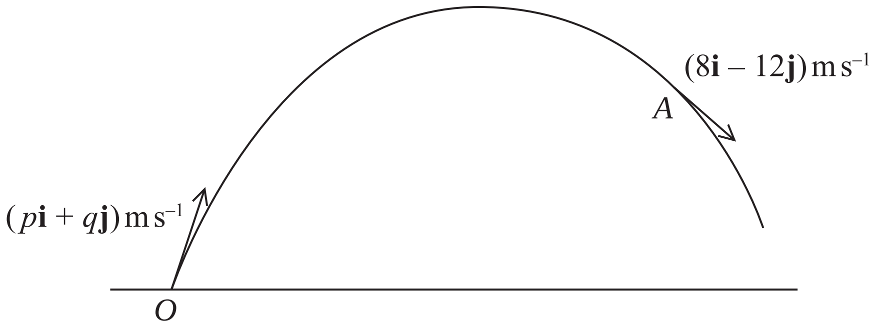

Figure 4

At time $t = 0$, a small ball is projected from a fixed point O on horizontal ground. The ball is projected from O with velocity $(p\mathbf{i} + q\mathbf{j})\,\mathrm{m\,s^{-1}}$, where p and q are positive constants. The ball moves freely under gravity.

At time $t = 3$ seconds, the ball passes through the point A with velocity $(8\mathbf{i} - 12\mathbf{j})\,\mathrm{m\,s^{-1}}$, as shown in Figure 4.

(a) Find the speed of the ball at the instant it is projected from O.

(5)

For an interval of T seconds the speed, $v\,\mathrm{m\,s^{-1}}$, of the ball is such that $v \leqslant 10$

(b) Find the value of T.

(4)

At the point B on the path of the ball, the direction of motion of the ball is perpendicular to the direction of motion of the ball at A.

(c) Find the vertical height of B above A.

(4)