**1.** Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\dfrac{dy}{dx}$.

**(3)**

2. Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, \quad x > 0$$

**(6)**

**3.**

Given that $y = x^4 + 6x^{\frac{1}{2}}$, find in their simplest form

(a) $\frac{\mathrm{d}y}{\mathrm{d}x}$

**(3)**

(b) Find $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$

**(3)**

**4.** Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(a) $\frac{\mathrm{d}y}{\mathrm{d}x}$,

**(3)**

(b) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

**(4)**

**5.**

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$$

(a) Find $\dfrac{dy}{dx}$ giving each term in its simplest form.

**(4)**

(b) Find $\dfrac{d^2y}{dx^2}$

**(2)**

**6.** The curve $C$ has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

**(3)**

(b) Verify that $C$ has a stationary point when $x = 2$

**(2)**

(c) Determine the nature of this stationary point, giving a reason for your answer.

**(2)**

**7.** The curve $C$ has equation $y = 6 - 3x - \dfrac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point $P$ when $x = \sqrt{2}$

**(4)**

(b) Find the $x$-coordinate of the other turning point $Q$ on the curve.

**(1)**

(c) Find $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

**(1)**

(d) Hence or otherwise, state with justification, the nature of each of these turning points $P$ and $Q$.

**(3)**

**8.** The curve with equation

$$y = x^2 - 32\sqrt{(x)} + 20, \qquad x > 0$$

has a stationary point $P$.

Use calculus

(a) to find the coordinates of $P$,

**(6)**

(b) to determine the nature of the stationary point $P$.

**(3)**

**9.** A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where $m$ and $c$ are integers to be found.

**Solutions relying on calculator technology are not acceptable.**

**(5)**

**10.** Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0$$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

**(6)**

**11.** The curve $C$ has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form.

**(4)**

(b) Find an equation of the tangent to $C$ at the point where $x = 2$

**(4)**

**12.**

A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

**(3)**

(b) Hence find the exact range of values of $x$ for which the curve is increasing.

**(2)**

**13.** The curve $C_1$ has equation

$$y = x^2(x+2)$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

**(2)**

(b) Sketch $C_1$, showing the coordinates of the points where $C_1$ meets the $x$-axis.

**(3)**

(c) Find the gradient of $C_1$ at each point where $C_1$ meets the $x$-axis.

**(2)**

The curve $C_2$ has equation

$$y = (x-k)^2(x-k+2)$$

where $k$ is a constant and $k > 2$

(d) Sketch $C_2$, showing the coordinates of the points where $C_2$ meets the $x$ and $y$ axes.

**(3)**

**14.**

(a) Factorise completely $x^3 - 4x$

**(3)**

(b) Sketch the curve $C$ with equation

$$y = x^3 - 4x,$$

showing the coordinates of the points at which the curve meets the $x$-axis.

**(3)**

The point $A$ with $x$-coordinate $-1$ and the point $B$ with $x$-coordinate 3 lie on the curve $C$.

(c) Find an equation of the line which passes through $A$ and $B$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants.

**(5)**

(d) Show that the length of $AB$ is $k\sqrt{10}$, where $k$ is a constant to be found.

**(2)**

**15.** The curve $C$ has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

**(4)**

(b) Show that the point $P(4, -8)$ lies on $C$.

**(2)**

(c) Find an equation of the normal to $C$ at the point $P$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(6)**

**16.** The curve $C$ has equation

$$y = (x+1)(x+3)^2$$

(a) Sketch $C$, showing the coordinates of the points at which $C$ meets the axes.

**(4)**

(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$.

**(3)**

The point $A$, with $x$-coordinate $-5$, lies on $C$.

(c) Find the equation of the tangent to $C$ at $A$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants.

**(4)**

Another point $B$ also lies on $C$. The tangents to $C$ at $A$ and $B$ are parallel.

(d) Find the $x$-coordinate of $B$.

**(3)**

**17.**

The curve $C$ has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$$

(a) Find

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

**(3)**

(b) (i) Verify that $C$ has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

**(4)**

**18.**

$$f(x) = x^3 + 2x^2 - 8x + 5$$

(a) Find $f''(x)$

**(2)**

(b) (i) Solve $f''(x) = 0$

(ii) Hence find the range of values of $x$ for which $f(x)$ is concave.

**(2)**

**19.**

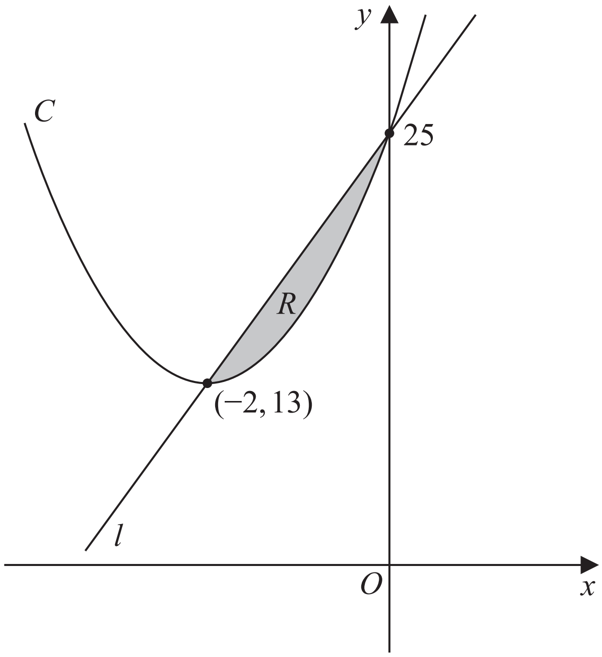


**Figure 1**

Figure 1 shows a sketch of a curve $C$ with equation $y = \mathrm{f}(x)$ and a straight line $l$.

The curve $C$ meets $l$ at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region $R$ is bounded by $C$ and $l$ as shown in Figure 1.

Given that

- $\mathrm{f}(x)$ is a quadratic function in $x$
- $(-2, 13)$ is the minimum turning point of $y = \mathrm{f}(x)$

use inequalities to define $R$.

**(5)**

**20.** The curve $C$ has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and $a$ and $b$ are constants.

Given

- the point (2, 10) lies on $C$
- the gradient of the curve at (2, 10) is $-3$

(a) (i) show that the value of $a$ is $-2$

(ii) find the value of $b$.

**(4)**

(b) Hence show that $C$ has no stationary points.

**(3)**

(c) Write $f(x)$ in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

**(2)**

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = f(0.2x)$$

and the coordinate axes.

**(2)**

**21.**

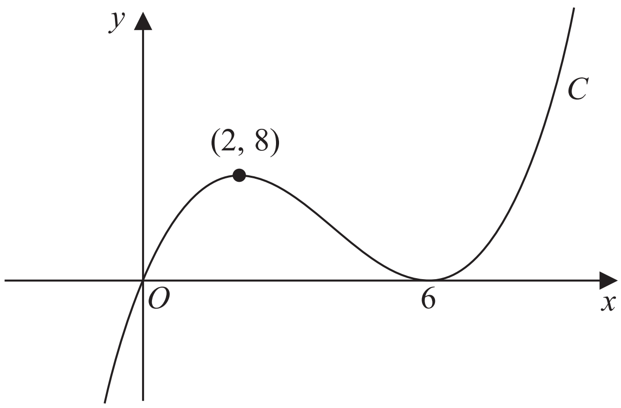


**Figure 1**

Figure 1 shows a sketch of a curve $C$ with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a cubic expression in $x$.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of $x$ for which

$$\mathrm{f}'(x) < 0$$

**(1)**

The line with equation $y = k$, where $k$ is a constant, intersects $C$ at only one point.

(b) Find the set of values of $k$, giving your answer in set notation.

**(2)**

(c) Find the equation of $C$. You may leave your answer in factorised form.

**(3)**

**22.**

A curve has equation $y = \mathrm{g}(x)$.

Given that

- $\mathrm{g}(x)$ is a cubic expression in which the coefficient of $x^3$ is equal to the coefficient of $x$
- the curve with equation $y = \mathrm{g}(x)$ passes through the origin
- the curve with equation $y = \mathrm{g}(x)$ has a stationary point at $(2, 9)$

(a) find $\mathrm{g}(x)$,

**(7)**

(b) prove that the stationary point at $(2, 9)$ is a maximum.

**(2)**