1. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P is moving with velocity $4\mathbf{i}\,\text{m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})\,\text{m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

2. A particle P moves with acceleration $(4\mathbf{i} - 5\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, P passes through the origin O.

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A.

The position vector of A is $(\lambda\mathbf{i} - 4.5\mathbf{j})$m relative to O, where λ is a constant.

(b) Find the value of T.

(4)

(c) Hence find the value of λ

(2)

3. [*In this question*, **i** *and* **j** *are horizontal unit vectors*.]

A particle P of mass 4 kg is at rest at the point A on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})\text{N}$ and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})\text{N}$, where λ and μ are constants, are applied to P

Given that P moves in the direction of the vector $(3\mathbf{i} + \mathbf{j})$

(a) show that

$$\lambda - 3\mu + 7 = 0$$

(4)

At time $t = 4$ seconds, P passes through the point B.

Given that $\lambda = 2$

(b) find the length of AB.

(5)

4. [*In this question,* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin O*]

A particle P is moving on a smooth horizontal plane.

The particle has constant acceleration $(2.4\mathbf{i} + \mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P passes through the point A.

At time $t = 5\,\text{s}$, P passes through the point B.

The velocity of P as it passes through A is $(-16\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$

(a) Find the speed of P as it passes through B.

(4)

The position vector of A is $(44\mathbf{i} - 10\mathbf{j})\,\text{m}$.

At time $t = T$ seconds, where $T > 5$, P passes through the point C.

The position vector of C is $(4\mathbf{i} + c\mathbf{j})\,\text{m}$.

(b) Find the value of T.

(3)

(c) Find the value of c.

(3)

5.

A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, the particle is at the point A and is moving with velocity $(-\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(a) Find the value of T.

(4)

At time $t = 4$ seconds, P is at the point B.

(b) Find the distance AB.

(4)

6.

A particle P is moving with constant velocity $(-3\mathbf{i} + 2\mathbf{j})\ \text{m s}^{-1}$. At time $t = 6\ \text{s}$ P is at the point with position vector $(-4\mathbf{i} - 7\mathbf{j})\ \text{m}$. Find the distance of P from the origin at time $t = 2\ \text{s}$.

(5)

7. [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is **s** km. When $t = 0$, $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j}$. When $t = 4$, $\mathbf{s} = 21\mathbf{i} + 10\mathbf{j}$. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that $\mathbf{s} = (3t + 9)\,\mathbf{i} + (4t - 6)\,\mathbf{j}$.

(2)

A lighthouse L is located at the point with position vector $(18\mathbf{i} + 6\mathbf{j})$ km. When $t = T$, the ship S is 10 km from L.

(d) Find the possible values of T.

(6)

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O.*]

A particle P moves with constant acceleration.
At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$
At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j})\,\text{m}$.

(a) Show that the magnitude of the acceleration of P is $2.5\,\text{m s}^{-2}$

(4)

At the instant when P leaves the point A, the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j})\,\text{m s}^{-2}$

At the instant when P reaches the point B, the direction of motion of P is north east.

(b) Find the time it takes for P to travel from A to B.

(4)

9.

A particle P of mass 2 kg is moving under the action of a constant force **F** newtons. The velocity of P is $(2\mathbf{i} - 5\mathbf{j})$ m s^{-1} at time $t = 0$, and $(7\mathbf{i} + 10\mathbf{j})$ m s^{-1} at time $t = 5$ s.

Find

(a) the speed of P at $t = 0$,

(2)

(b) the vector **F** in the form $a\mathbf{i} + b\mathbf{j}$,

(5)

(c) the value of t when P is moving parallel to **i**.

(4)

10.

[*In this question, the unit vectors* **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.*]

A boat P is moving with constant velocity $(-4\mathbf{i}+8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i}-8\mathbf{j})$ km. At time t hours, the position vector of P is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i}+8\mathbf{j})$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

11.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

(b) Find **r** in terms of t.

(2)

(c) Find the value of t when the boat is north-east of O.

(3)

(d) Find the value of t when the boat is moving in a north-east direction.

(3)

12.

[*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship sets sail at 9 am from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 am the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(a) Find the speed of the ship in km h^{-1}.

(4)

(b) Show that the position vector $\mathbf{r}$ km of the ship, t hours after 9 am, is given by $\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$.

(2)

At 10 am, a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 am, the passenger observes that L is now south-west of the ship.

(c) Find the position vector of L.

(5)

13. A particle P is moving with constant velocity. The position vector of P at time t seconds ($t \geqslant 0$) is $\mathbf{r}$ metres, relative to a fixed origin O, and is given by

$$\mathbf{r} = (2t - 3)\mathbf{i} + (4 - 5t)\mathbf{j}$$

(a) Find the initial position vector of P.

(1)

The particle P passes through the point with position vector $(3.4\mathbf{i} - 12\mathbf{j})$ m at time T seconds.

(b) Find the value of T.

(3)

(c) Find the speed of P.

(4)

14.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two cars P and Q are moving on straight horizontal roads with constant velocities. The velocity of P is $(15\mathbf{i} + 20\mathbf{j})$ m s^{-1} and the velocity of Q is $(20\mathbf{i} - 5\mathbf{j})$ m s^{-1}

(a) Find the direction of motion of Q, giving your answer as a bearing to the nearest degree.

(3)

At time $t = 0$, the position vector of P is $400\mathbf{i}$ metres and the position vector of Q is $800\mathbf{j}$ metres. At time t seconds, the position vectors of P and Q are $\mathbf{p}$ metres and $\mathbf{q}$ metres respectively.

(b) Find an expression for

(i) $\mathbf{p}$ in terms of t,

(ii) $\mathbf{q}$ in terms of t.

(3)

(c) Find the position vector of Q when Q is due west of P.

(4)

15.

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle P.

The force $\mathbf{F}_1$ is given by $\mathbf{F}_1 = (-\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2$ acts in the direction of the vector $(\mathbf{i} + \mathbf{j})$.

Given that the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ acts in the direction of the vector $(\mathbf{i} + 3\mathbf{j})$,

(a) find $\mathbf{F}_2$

(7)

The acceleration of P is $(3\mathbf{i} + 9\mathbf{j})$ m s^{-2}. At time $t = 0$, the velocity of P is $(3\mathbf{i} - 22\mathbf{j})$ m s^{-1}

(b) Find the speed of P when $t = 3$ seconds.

(4)

16. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.*]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})$ km h^{-1} and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1}.

(a) Find, to the nearest degree, the bearing on which Q is moving.

(2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

At time t hours after 2 pm, the position vector of P is $\mathbf{p}$ km and the position vector of Q is $\mathbf{q}$ km.

(b) Write down expressions, in terms of t, for

(i) $\mathbf{p}$,

(ii) $\mathbf{q}$,

(iii) $\overrightarrow{PQ}$.

(5)

(c) Find the time when

(i) Q is due north of P,

(ii) Q is north-west of P.

(4)

17.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O.*]

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j})$ km.

(a) Find the position vector of S at time t hours.

(2)

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j})$ km. The two ships meet at the point P.

(b) Find the value of n.

(5)

(c) Find the distance OP.

(4)

18.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving with constant velocity $(-12\mathbf{i} + 7.5\mathbf{j})$ km h^{-1}.

(a) Find the direction in which S is moving, giving your answer as a bearing.

(3)

At time t hours after noon, the position vector of S is $\mathbf{s}$ km. When $t = 0$, $\mathbf{s} = 40\mathbf{i} - 6\mathbf{j}$.

(b) Write down $\mathbf{s}$ in terms of t.

(2)

A fixed beacon B is at the point with position vector $(7\mathbf{i} + 12.5\mathbf{j})$ km.

(c) Find the distance of S from B when $t = 3$

(4)

(d) Find the distance of S from B when S is due north of B.

(4)

19. *[In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively.]*

The velocity, **v** m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$

(3)

(b) Find the bearing on which P is moving when $t = 2$

(2)

(c) Find the value of t when P is moving

(i) parallel to **j**,

(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

(6)

20.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given relative to a fixed origin O.*]

Two ships, P and Q, are moving with constant velocities.
The velocity of P is $(9\mathbf{i} - 2\mathbf{j})$ km h^{-1} and the velocity of Q is $(4\mathbf{i} + 8\mathbf{j})$ km h^{-1}

(a) Find the direction of motion of P, giving your answer as a bearing to the nearest degree.

(3)

When $t = 0$, the position vector of P is $(9\mathbf{i} + 10\mathbf{j})$ km and the position vector of Q is $(\mathbf{i} + 4\mathbf{j})$ km. At time t hours, the position vectors of P and Q are $\mathbf{p}$ km and $\mathbf{q}$ km respectively.

(b) Find an expression for

(i) $\mathbf{p}$ in terms of t,

(ii) $\mathbf{q}$ in terms of t.

(3)

(c) Hence show that, at time t hours,

$$\overrightarrow{QP} = (8 + 5t)\mathbf{i} + (6 - 10t)\mathbf{j}$$

(2)

(d) Find the values of t when the ships are 10 km apart.

(6)