1. Find the first three terms, in ascending powers of x, of the binomial expansion of $(3 + 2x)^5$, giving each term in its simplest form.

(4)

2. The points A and B have coordinates $(5, -1)$ and $(13, 11)$ respectively.

(a) Find the coordinates of the mid-point of AB.

(2)

Given that AB is a diameter of the circle C,

(b) find an equation for C.

(4)

3. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(a) $3^x = 5$,

(3)

(b) $\log_2(2x + 1) - \log_2 x = 2$.

(4)

4. (a) Show that the equation

$$5 \cos^2 x = 3(1 + \sin x)$$

can be written as

$$5 \sin^2 x + 3 \sin x - 2 = 0.$$

(2)

(b) Hence solve, for $0 \leqslant x < 360°$, the equation

$$5 \cos^2 x = 3(1 + \sin x),$$

giving your answers to 1 decimal place where appropriate.

(5)

5. $$f(x) = x^3 + 4x^2 + x - 6.$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)

(c) Write down all the solutions to the equation

$$x^3 + 4x^2 + x - 6 = 0.$$

(1)

6. (a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) $x - 3$,

(ii) $x + 2$.

(3)

(b) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0.$$

(4)

7.

Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3-2x)^5$, giving each term in its simplest form.

(4)

8. Given that $0 < x < 4$ and

$$\log_5(4 - x) - 2\log_5 x = 1,$$

find the value of x.

(6)

9.

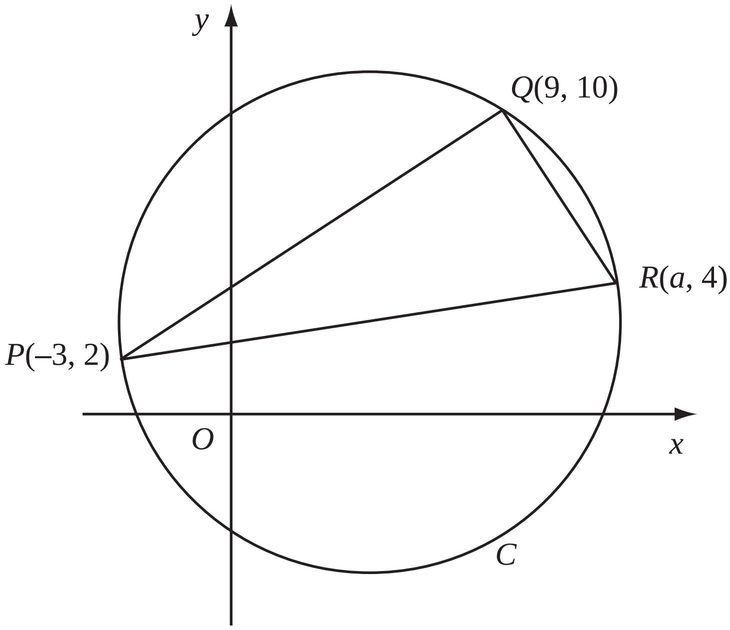


Figure 2

The points $P(-3, 2)$, $Q(9, 10)$ and $R(a, 4)$ lie on the circle C, as shown in Figure 2.
Given that PR is a diameter of C,

(a) show that $a = 13$,

(3)

(b) find an equation for C.

(5)

10. $f(x) = x^3 - 2x^2 + ax + b$, where a and b are constants.

When $f(x)$ is divided by $(x - 2)$, the remainder is 1.

When $f(x)$ is divided by $(x + 1)$, the remainder is 28.

(a) Find the value of a and the value of b.

(6)

(b) Show that $(x - 3)$ is a factor of $f(x)$.

(2)

11. (a) Find all the values of θ, to 1 decimal place, in the interval $0° \leqslant \theta < 360°$ for which

$$5\sin(\theta + 30°) = 3.$$

(4)

Question 11 continued

(b) Find all the values of θ, to 1 decimal place, in the interval $0° \leqslant \theta < 360°$ for which

$$\tan^2\theta = 4.$$

(5)

12. The curve C has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

(a) Find $\dfrac{dy}{dx}$.

(2)

(b) Using the result from part (a), find the coordinates of the turning points of C.

(4)

(c) Find $\dfrac{d^2y}{dx^2}$.

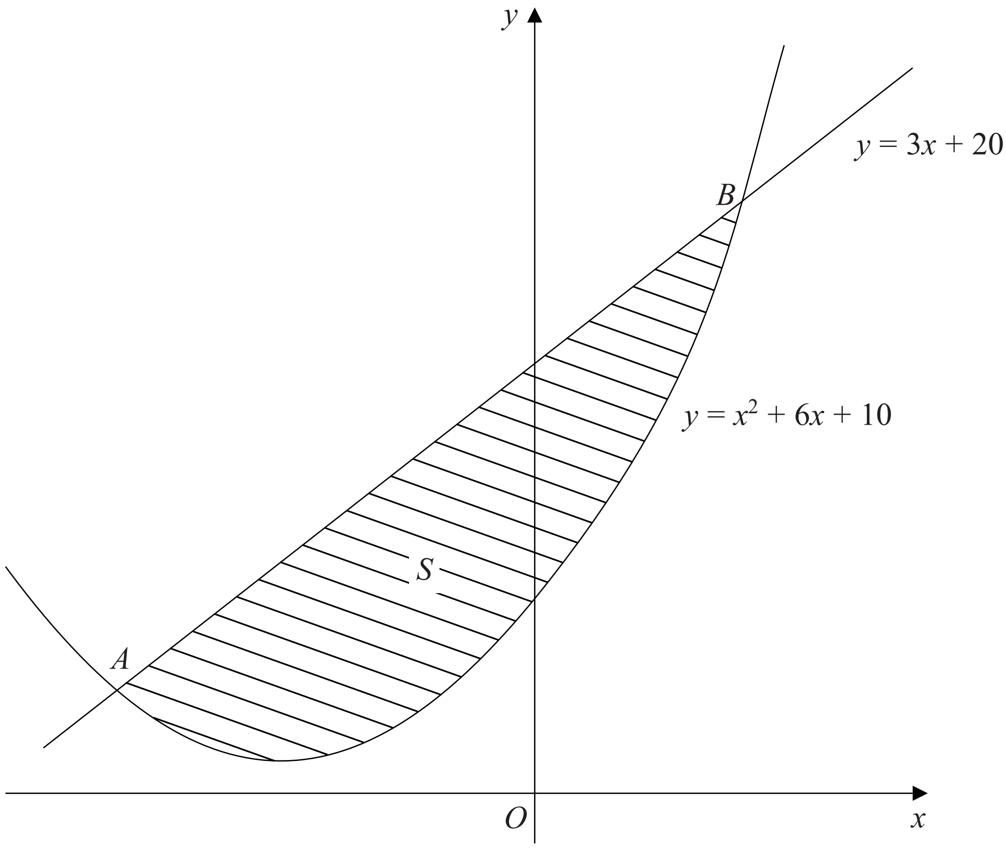
(2)

(d) Hence, or otherwise, determine the nature of the turning points of C.

(2)

13.

Figure 2



The line with equation $y = 3x + 20$ cuts the curve with equation $y = x^2 + 6x + 10$ at the points A and B, as shown in Figure 2.

(a) Use algebra to find the coordinates of A and the coordinates of B.

(5)

The shaded region S is bounded by the line and the curve, as shown in Figure 2.

(b) Use calculus to find the exact area of S.

(7)

14.

Figure 3

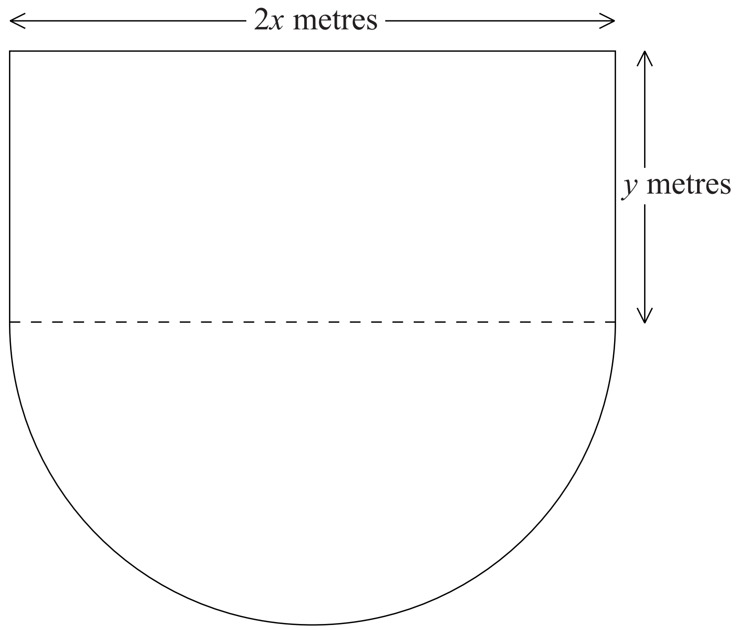


Figure 3 shows the plan of a stage in the shape of a rectangle joined to a semicircle. The length of the rectangular part is $2x$ metres and the width is y metres. The diameter of the semicircular part is $2x$ metres. The perimeter of the stage is 80 m.

(a) Show that the area, A m^2, of the stage is given by

$$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2.$$

(4)

(b) Use calculus to find the value of x at which A has a stationary value.

(4)

(c) Prove that the value of x you found in part (b) gives the maximum value of A.

(2)

(d) Calculate, to the nearest m^2, the maximum area of the stage.

(2)

15.

Figure 1

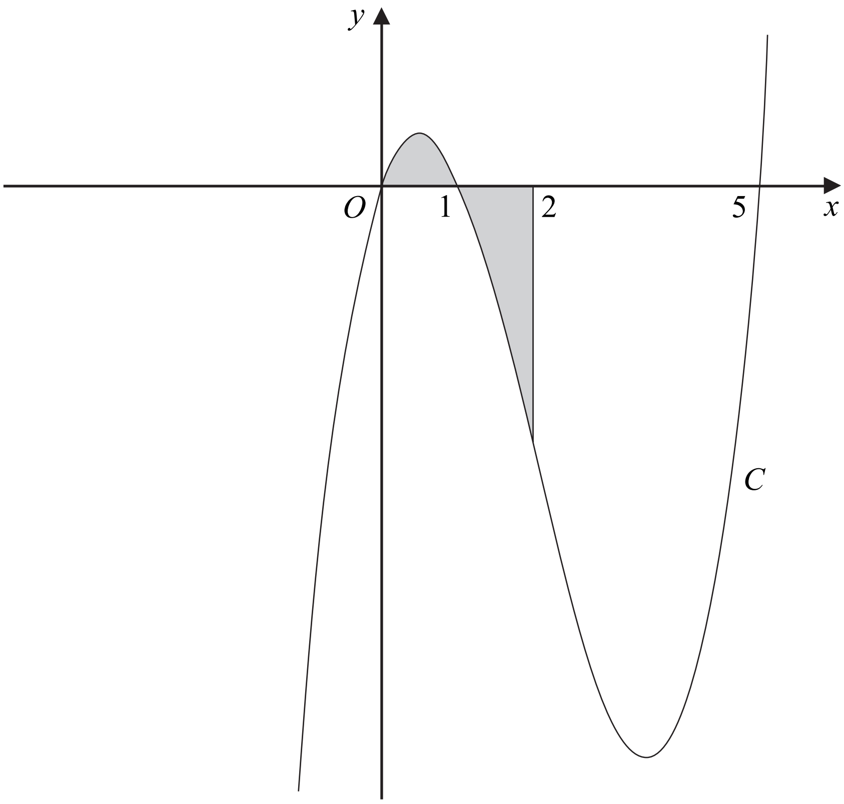


Figure 1 shows a sketch of part of the curve C with equation

$$y = x(x - 1)(x - 5).$$

Use calculus to find the total area of the finite region, shown shaded in Figure 1, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(9)

16.

A circle C has centre M (6, 4) and radius 3.

(a) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$

(2)

Figure 3

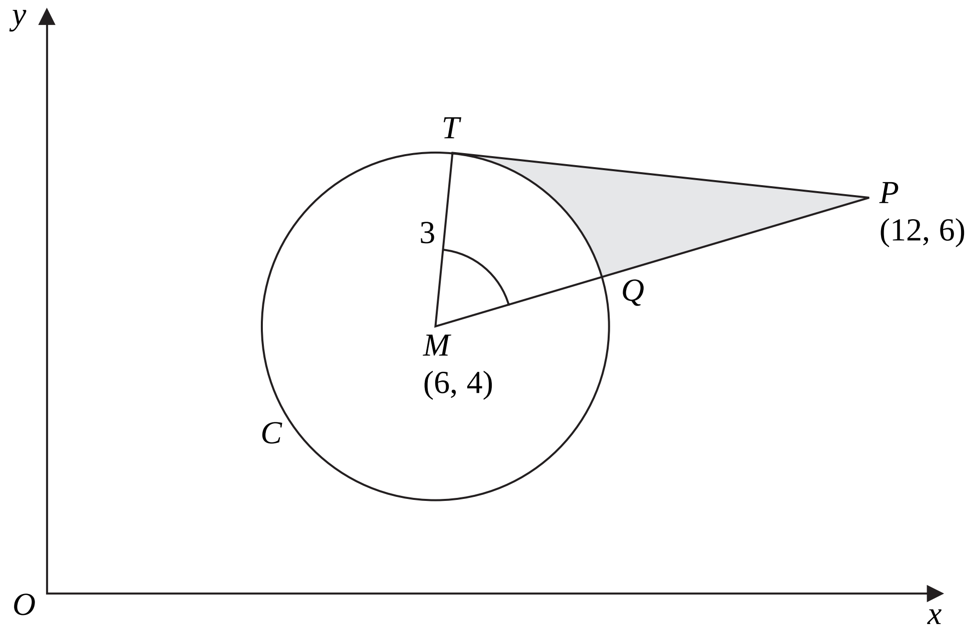


Figure 3 shows the circle C. The point T lies on the circle and the tangent at T passes through the point P (12, 6). The line MP cuts the circle at Q.

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ, as shown in Figure 3.

(c) Find the area of the shaded region TPQ. Give your answer to 3 decimal places.

(5)

17. (a) Show that the equation

$$4\sin^2 x + 9\cos x - 6 = 0$$

can be written as

$$4\cos^2 x - 9\cos x + 2 = 0.$$

(2)

(b) Hence solve, for $0 \leqslant x < 720°$,

$$4\sin^2 x + 9\cos x - 6 = 0,$$

giving your answers to 1 decimal place.

(6)

18.

A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(a) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that r varies,

(b) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(c) Justify that the value of V you have found is a maximum.

(2)