1 Here are the first four terms of a sequence of fractions.

$$\frac{1}{1} \qquad \frac{2}{3} \qquad \frac{3}{5} \qquad \frac{4}{7}$$

The numerators of the fractions form the sequence of whole numbers 1 2 3 4 …
The denominators of the fractions form the sequence of odd numbers 1 3 5 7 …

(a) Write down an expression, in terms of n, for the nth term of this sequence of fractions.

(2)

(b) Using algebra, prove that when the square of any odd number is divided by 4 the remainder is 1

(3)

(Total for Question 1 is 5 marks)

2 Prove algebraically that the product of any two odd numbers is always an odd number.

(Total for Question 2 is 4 marks)

3 Prove that the difference between two consecutive square numbers is always an odd number.
Show clear algebraic working.

(Total for Question 3 is 3 marks)

4 The table gives information about the first six terms of a sequence of numbers.

Term number	1	2	3	4	5	6
Term of sequence	$\frac{1 \times 2}{2}$	$\frac{2 \times 3}{2}$	$\frac{3 \times 4}{2}$	$\frac{4 \times 5}{2}$	$\frac{5 \times 6}{2}$	$\frac{6 \times 7}{2}$

Prove algebraically that the sum of any two consecutive terms of this sequence is always a square number.

(Total for Question 4 is 4 marks)

5 Using algebra, prove that, given any 3 consecutive even numbers, the difference between the square of the largest number and the square of the smallest number is always 8 times the middle number.

(Total for Question 5 is 3 marks)

6 Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n.

(Total for Question 6 is 3 marks)

7 (a) Expand and simplify $(y-2)(y-5)$

....

(2)

(b) Prove algebraically that

$(2n+1)^2 - (2n+1)$ is an even number

for all positive integer values of n.

(3)

(Total for Question 7 is 5 marks)

8 Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

(Total for Question 8 is 4 marks)

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9 Using algebra, prove that, given any 3 consecutive whole numbers, the sum of the square of the smallest number and the square of the largest number is always 2 more than twice the square of the middle number.

(Total for Question 9 is 3 marks)

10 (a) Show that $x(x - 1)(x + 1) = x^3 - x$

(1)

(b) Prove that the difference between a whole number and the cube of this number is always a multiple of 6

(3)

(Total for Question 10 is 4 marks)

11 N is a multiple of 5

$A = N + 1$
$B = N - 1$

Prove, using algebra, that $A^2 - B^2$ is always a multiple of 20

(Total for Question 11 is 3 marks)

12 Prove that when the sum of the squares of any two consecutive odd numbers is divided by 8, the remainder is always 2
Show clear algebraic working.

(Total for Question 12 is 3 marks)

13 $(2x + 23)$, $(8x + 2)$ and $(20x - 52)$ are three consecutive terms of an arithmetic sequence.

Prove that the common difference of the sequence is 12

(Total for Question 13 is 4 marks)