**1.** Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0,$$

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

**(4)**

**2.**

Find $\int (3x^2 + 4x^5 - 7)\,\mathrm{d}x$ .

**(4)**

**3.** Given that $f(x) = \frac{1}{x}, \quad x \neq 0,$

(a) sketch the graph of $y = f(x) + 3$ and state the equations of the asymptotes.

**(4)**

(b) Find the coordinates of the point where $y = f(x) + 3$ crosses a coordinate axis.

**(2)**

**4.** Given that the equation $kx^2 + 12x + k = 0$, where $k$ is a positive constant, has equal roots, find the value of $k$.

**(4)**

**(Total 4 marks)**

**5.**

The curve $C$ has equation $y = 4x^2 + \frac{5-x}{x}$, $x \neq 0$. The point $P$ on $C$ has $x$-coordinate 1.

(a) Show that the value of $\frac{dy}{dx}$ at $P$ is 3.

**(5)**

(b) Find an equation of the tangent to $C$ at $P$.

**(3)**

This tangent meets the $x$-axis at the point $(k, 0)$.

(c) Find the value of $k$.

**(2)**

**6.** The equation

$$x^2 + kx + 8 = k$$

has no real solutions for $x$.

(a) Show that $k$ satisfies $k^2 + 4k - 32 < 0$.

**(3)**

(b) Hence find the set of possible values of $k$.

**(4)**

**7.** The curve $C$ has equation

$$y = (x+3)(x-1)^2 .$$

(a) Sketch $C$ showing clearly the coordinates of the points where the curve meets the coordinate axes.

**(4)**

(b) Show that the equation of $C$ can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where $k$ is a positive integer, and state the value of $k$.

**(2)**

There are two points on $C$ where the gradient of the tangent to $C$ is equal to 3.

(c) Find the $x$-coordinates of these two points.

**(6)**

**8.** The gradient of the curve $C$ is given by

$$\frac{dy}{dx} = (3x-1)^2.$$

The point $P(1, 4)$ lies on $C$.

(a) Find an equation of the normal to $C$ at $P$.

**(4)**

(b) Find an equation for the curve $C$ in the form $y = f(x)$.

**(5)**

(c) Using $\frac{dy}{dx} = (3x-1)^2$, show that there is no point on $C$ at which the tangent is parallel to the line $y = 1 - 2x$.

**(2)**

**9.** The curve $C$ has equation $y = f(x)$, $x > 0$, and $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on $C$,

(a) find $f(x)$ and simplify your answer.

**(6)**

(b) Find an equation of the normal to $C$ at the point $P(4, 1)$.

**(4)**

**10.** (a) On the same axes sketch the graphs of the curves with equations

(i) $y = x^2(x - 2)$,

**(3)**

(ii) $y = x(6 - x)$,

**(3)**

and indicate on your sketches the coordinates of all the points where the curves cross the $x$-axis.

(b) Use algebra to find the coordinates of the points where the graphs intersect.

**(7)**

**11.** The curve $C$ has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0 .$$

The point $P$ on $C$ has $x$-coordinate equal to 2.

(a) Show that the equation of the tangent to $C$ at the point $P$ is $y = 1 - 2x$.

**(6)**

(b) Find an equation of the normal to $C$ at the point $P$.

**(3)**

The tangent at $P$ meets the $x$-axis at $A$ and the normal at $P$ meets the $x$-axis at $B$.

(c) Find the area of triangle $APB$.

**(4)**