1. Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, \quad x > 0$$

(6)

2. A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(5)

3

A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{\mathrm{d}y}{\mathrm{d}x}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

4. The curve C has equation $y = 6 - 3x - \dfrac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\dfrac{d^2y}{dx^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.

(3)

5. The curve with equation

$$y = x^2 - 32\sqrt{(x)} + 20, \qquad x > 0$$

has a stationary point P.

Use calculus

(a) to find the coordinates of P,

(6)

(b) to determine the nature of the stationary point P.

(3)

6

A curve has equation $y = \mathrm{g}(x)$.

Given that

- $\mathrm{g}(x)$ is a cubic expression in which the coefficient of x^3 is equal to the coefficient of x
- the curve with equation $y = \mathrm{g}(x)$ passes through the origin
- the curve with equation $y = \mathrm{g}(x)$ has a stationary point at $(2, 9)$

(a) find $\mathrm{g}(x)$,

(7)

(b) prove that the stationary point at $(2, 9)$ is a maximum.

(2)

7. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point (2, 10) lies on C
- the gradient of the curve at (2, 10) is -3

(a) (i) show that the value of a is -2

(ii) find the value of b.

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write $\mathrm{f}(x)$ in the form $(x - 4)\mathrm{Q}(x)$ where $\mathrm{Q}(x)$ is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = \mathrm{f}(0.2x)$$

and the coordinate axes.

(2)

8. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

9.

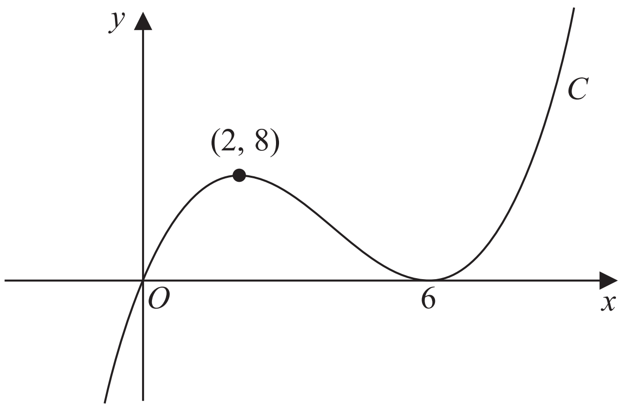

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$\mathrm{f}'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation.

(2)

(c) Find the equation of C. You may leave your answer in factorised form.

(3)

10.

The curve C has equation

$$y = 5x^4 - 24x^3 + 42x^2 - 32x + 11 \qquad x \in \mathbb{R}$$

(a) Find

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) (i) Verify that C has a stationary point at $x = 1$

(ii) Show that this stationary point is a point of inflection, giving reasons for your answer.

(4)

11.

$$f(x) = x^3 + 2x^2 - 8x + 5$$

(a) Find $f''(x)$

(2)

(b) (i) Solve $f''(x) = 0$

(ii) Hence find the range of values of x for which $f(x)$ is concave.

(2)

12. The curve C has equation $y = \mathrm{f}(x)$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$

(2)

(b) Hence find the coordinates of the point where C crosses the y-axis.

(3)

13.

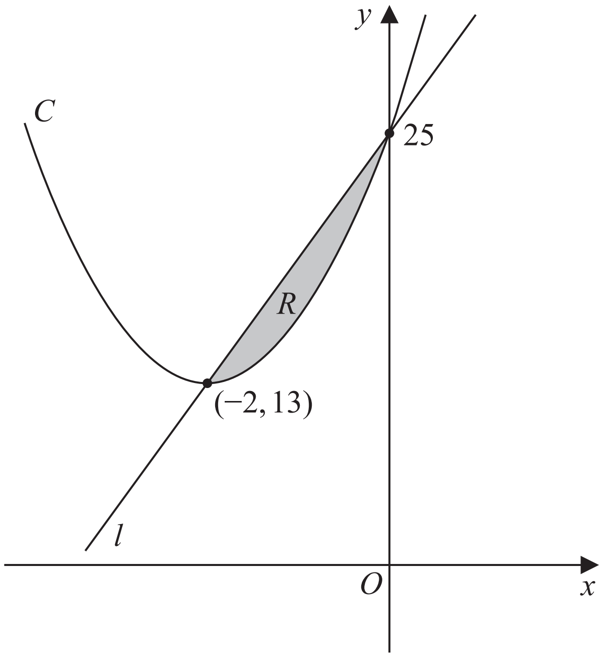

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ and a straight line l.

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $\mathrm{f}(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = \mathrm{f}(x)$

use inequalities to define R.

(5)

14.

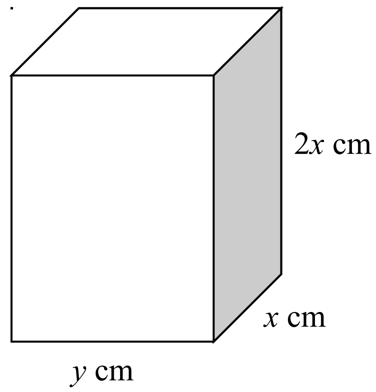

Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(5)

(c) Justify that the value of V you have found is a maximum.

(2)

15.

A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

16.

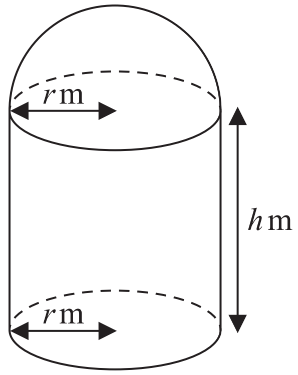

Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is $6\,\text{m}^3$.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

17.

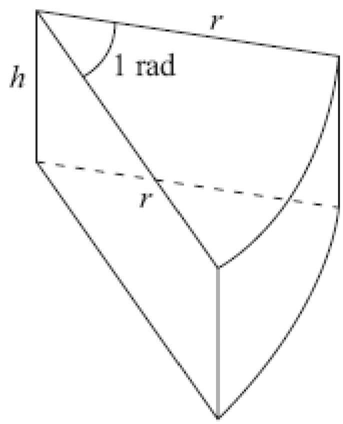

Figure 2

Figure 2 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(*a*) Show that the surface area of the box, S cm^2, is given by

$$S = r^2 + \frac{1800}{r}.$$

(5)

(*b*) Use calculus to find the value of r for which S is stationary.

(4)

(*c*) Prove that this value of r gives a minimum value of S.

(2)

(*d*) Find, to the nearest cm^2, this minimum value of S.

(2)

18.

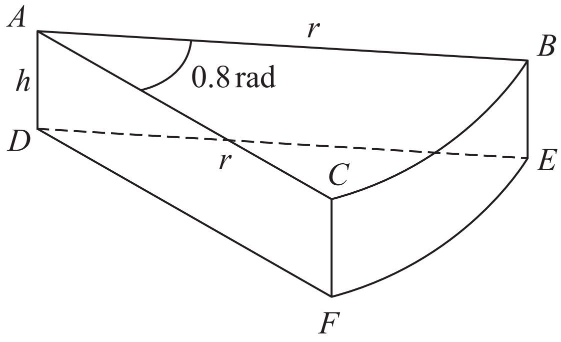

Figure 5

A company makes toys for children.

Figure 5 shows the design for a solid toy that looks like a piece of cheese.

The toy is modelled so that

- face ABC is a sector of a circle with radius r cm and centre A
- angle $BAC = 0.8$ radians
- faces ABC and DEF are congruent
- edges AD, CF and BE are perpendicular to faces ABC and DEF
- edges AD, CF and BE have length h cm

Given that the volume of the toy is $240\,\text{cm}^3$

(a) show that the surface area of the toy, $S\,\text{cm}^2$, is given by

$$S = 0.8r^2 + \frac{1680}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(b) find the value of r for which S has a stationary point.

(4)

(c) Prove, by further differentiation, that this value of r gives the minimum surface area of the toy.

(2)