1. In the binomial expansion of

$$\left(1+\frac{12n}{5}x\right)^n$$

the coefficients of x^2 and x^3 are equal and non-zero.

(a) Find the possible values of n.

(4)

(b) State, giving a reason, which value of n gives a valid expansion when $x = \frac{1}{2}$

(2)

(Total 6 marks)

2. (*a*) For $|y| < 1$, write down the binomial series expansion of $(1 - y)^{-2}$ in ascending powers of y up to and including the term in y^3.

(1)

(*b*) Hence, or otherwise, show that

$$1 + \frac{2x}{1+x} + \frac{3x^2}{(1+x)^2} + \ldots + \frac{rx^{r-1}}{(1+x)^{r-1}} + \ldots$$

can be written in the form $(a + x)^n$. Write down the values of the integers a and n.

(4)

(*c*) Find the set of values of x for which the series in part (*b*) is convergent.

(3)

3. (a) (i) Write down the binomial series expansion of

$$\left(1+\frac{2}{n}\right)^n \qquad n \in \mathbb{N}, n > 2$$

in powers of $\left(\frac{2}{n}\right)$ up to and including the term in $\left(\frac{2}{n}\right)^3$

(ii) Hence prove that, for $n \in \mathbb{N}$, $n \geqslant 3$

$$\left(1+\frac{2}{n}\right)^n \geqslant \frac{19}{3}-\frac{6}{n}$$

(3)

(b) Use the binomial series expansion of $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$ to show that $\sqrt{3} < \frac{7}{4}$

(4)

$$\mathrm{f}(x) = \left(1+\frac{2}{x}\right)^x - 3^{\frac{x}{6}} \qquad x \in \mathbb{R}, x > 0$$

Given that the function $\mathrm{f}(x)$ is continuous and that $\sqrt[6]{3} > \frac{6}{5}$

(c) prove that $\mathrm{f}(x) = 0$ has a root in the interval $[9, 10]$

(5)

(+S1)

4. (a) Find the binomial series expansion for $(4 + y)^{\frac{1}{2}}$ in ascending powers of y up to and including the term in y^3. Simplify the coefficient of each term.

(3)

(b) Hence show that the binomial series expansion for $(4+5x+x^2)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3 is

$$2 + \frac{5x}{4} - \frac{9x^2}{64} + \frac{45x^3}{512}$$

(3)

(c) Show that the binomial series expansion of $(4+5x+x^2)^{\frac{1}{2}}$ will converge for $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$

(6)

(d) Use the result in part (b) to estimate

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{4+5x+x^2}\ \mathrm{d}x$$

Give your answer as a single fraction.

(3)

(Total 15 marks)