

# A new feedback narrowband active noise control system with online secondary-path modeling based on adaptive notch filtering

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**Abstract**—Narrowband active noise control (ANC, NANC) systems have been developed for attenuating noises that are generated by rotating machines such as motors, cutting machines, diesel engines, fans, etc. The conventional feedforward NANC systems adopt a non-acoustic sensor to detect and extract the noise signal frequencies for creating the reference sine and cosine waves, leading to high implementation cost as well as performance degradation due to frequency mismatch (FM). In this paper, we propose a new efficient feedback NANC system where an adaptive notch filter bank is utilized to construct the reference sinusoidal signals for the ANC system controller that consists of multiple two-weight magnitude/phase adjusters (MPA). The proposed system is also equipped with an online secondary-path modeling (SPM) subsystem that is capable of compensating the secondary-path drift. Extensive simulations are conducted to confirm the effectiveness of the proposed system.

**Index Terms**—Narrowband active noise control (NANC), feedback NANC, adaptive notch filter bank, normalized gradient algorithm, online secondary-path modeling, noise reduction performance (NRP)

## I. INTRODUCTION

Active noise control (ANC) has been widely applied to attenuate undesirable noises in various real-world applications, because it enjoys many advantages in size, weight, performance, and sometimes even implementation cost, as compared to passive noise control technologies. Usually, ANC can be divided into two types: the broadband ANC (BANC) and the narrowband ANC (NANC) [1], [2]. In this work, we focus on the latter. The NANC is developed to mitigate annoying noises generated by rotating machines such as motors, cutting machines, diesel engines, fans, etc.

The feedforward NANC systems have been extensively investigated in the past decades to improve their performance, see [3]-[11] and references therein. A complete parallel NANC system is proposed by using of a delayless bandpass filter bank to feed individual error components to the corresponding subcontroller [3]. In [4], a variable step-size (VSS) FxLMS algorithm is developed to improve the performance of the NANC. A gain scheduling technique and a VSS are introduced into an NANC system with online feedback-path modeling [5]. In [6], an efficient NANC system with online secondary-path modeling (SPM) is presented that provides excellent noise reduction performance (NRP) for nonstationary primary noises and/or time-varying secondary path. Note that most of the

above-mentioned studies assume that the frequencies specified by the synchronization signal are identical to those of the primary noise. That is, there is no frequency mismatch (FM) between the primary noise frequencies and the frequencies extracted from the synchronization signal. However, the FM usually exists in practice, because the non-acoustic sensor used to acquire the synchronization signal inevitably suffers from aging and fatigue. The performance of the NANC system significantly degrades because of the FM [7]-[11]. To mitigate the influence of FM, many effective techniques have been developed, see, e.g., [7]-[11] and references therein. The direct source of FM is the non-acoustic sensor such as tachometer.

If the target primary noise is of sinusoidal or periodic nature, the feedback NANC (FBNANC) may also be used [1], [2]. In an FBNANC system, only a single microphone is equipped to detect the residual noise and no reference sensor is required. Therefore, there is no FM problem with the FBNANC. In recent years, a number of developments have been witnessed in FBNANC [12]-[17].  $H_\infty$  optimization based controller,  $H_2/H_\infty$  feedback controller, generalized leaky FxLMS algorithm, etc. have been used to improve the performance of FBNANC systems, see [16] and references therein. In particular, a parallel adaptive linear enhancer based on a second-order bandpass filter derived from a second-order notch filter is recently used to establish a new FBNANC system that provides excellent NRP [17]. However, how to set the initial bandpass filter weights might pose a troublesome issue if a piece of information on the primary noise frequencies, coarse or fine, is not available in advance. Furthermore, the secondary-path drift is not considered in [17].

In this paper, a new efficient FBNANC system is proposed that an adaptive notch filter bank (ANFB) is utilized to extract reference sinusoidal signals for the ANC controller which is composed of multiple two-weight magnitude/phase adjusters (MPA). In addition, an online SPM subsystem is also included to compensate the secondary-path drift. The proposed FBNANC system enjoys two major advantages. First, no a priori information on the primary noise frequencies is needed. Second, the online SPM subsystem can deal with the secondary-path drift in an efficient and effective way. Extensive simulations are conducted to confirm the effectiveness of our proposed system.

幅值相位  
调节器

二次路径建模

## II. A NEW FEEDBACK NARROWBAND ANC SYSTEM

To overcome the drawbacks of the existing FBNANC systems, especially the recent one that is proposed in [17], a new efficient FBNANC system is proposed that an ANFB is utilized to synthesize the reference sinusoidal signals. The controller is composed of multiple two-weight magnitude/phase adjusters (MPA). Figure 1 shows a block diagram of the proposed FBNANC system which consists of three subsystems, namely: 1) an ANFB that is introduced to extract the individual reference sinusoid, 2) a controller that consists of multiple two-weight MPAs, and 3) an online SPM subsystem that is designated to estimate the secondary path and compensate its drift. These three subsystems are seamlessly integrated in a way such that the entire FBNANC system can efficiently and effectively provide attractive NRP in the presence of both primary noise non-stationarity (magnitude and/or frequency) and secondary-path drift.

提出了一种利用ANFB来合成参考正弦信号的新的效率FBNANC系统。

简称ANFB  
主动带陷  
滤波器组

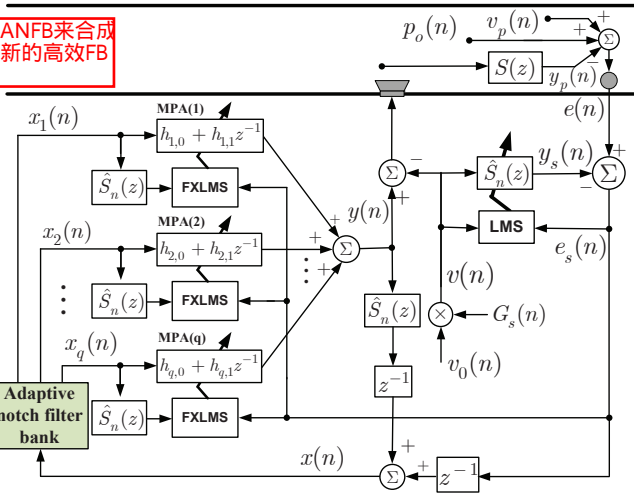


Fig. 1. Proposed feedback narrowband ANC system with online SPM.

### A. ANFB for reference signals generation

The primary noise signal in Fig. 1 is expressed by

$$p(n) = p_o(n) + v_p(n) \quad (1)$$

$$= \sum_{i=1}^q A_i \cos(\omega_{p,i}n + \theta_i) + v_p(n)$$

where  $A_i$ ,  $\omega_{p,i}$  and  $\theta_i$  are the amplitude, angular frequency, and phase of the  $i$ th frequency component, respectively,  $v_p(n)$  is a white Gaussian noise with zero mean and variance  $\sigma_{v_p}^2$ .

The input signal  $x(n)$  to the ANFB is a primary noise estimate  $\hat{p}(n-1)$  that is constructed from the ANC residual noise  $e(n)$  and the secondary source  $y(n)$ .

$$x(n) = \hat{p}(n-1) = e_s(n-1) + \hat{y}(n-1) \quad (2)$$

$$\hat{y}(n) = \sum_{m=0}^{\hat{M}-1} \hat{s}_m(n)y(n-m) \quad (3)$$

where  $\{\hat{s}_m(n)\}_{m=0}^{\hat{M}-1}$  are coefficients of the FIR secondary-path estimate  $\hat{S}_n(z)$  with order  $\hat{M}-1$ ,  $e_s(n)$  is the SPM subsystem error that will be given later on.

As shown in Fig. 2, an ANFB is introduced to extract the sinusoids  $x_1(n), x_2(n), \dots, x_q(n)$  that are contained in the primary noise estimate. Several ANFBs may be selected to serve this purpose [18]. Here, the cascaded ANFB is chosen, because it has less complexity, enjoys good performance, and never requires any a priori information on the primary noise frequencies [18]. Notch filter cells in this ANFB are based on a second-order IIR notch filter having the following transfer function [18], [19]

$$H_i(z) = \frac{1 + a_i z^{-1} + z^{-2}}{1 + \rho a_i z^{-1} + \rho^2 z^{-2}}, \quad i = 1, 2, \dots, q \quad (4)$$

where  $a_i (= -2 \cos \omega_i)$  is defined by the notch frequency  $\omega_i$ , constant  $\rho (\in [0, 1))$  is called pole attraction parameter which determines the notch filter bandwidth, i.e., the closer is the  $\rho$  to unit, the narrower is the notch in frequency domain. The  $i$ th ANFB can completely remove a sinusoid with frequency  $\omega_i$ , while passing other sinusoids almost unchanged if  $\rho$  is set very close to unit. Usually, a large  $\rho$  like 0.975, 0.980, 0.985, is selected when the target frequencies are closely spaced.

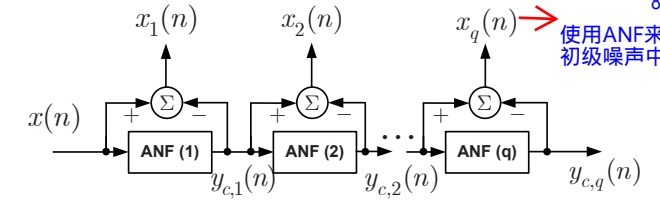


Fig. 2. An ANFB for synthesizing reference sinusoids.

The normalized gradient (NG) algorithm is used to tune the parameter  $a_i$  so as to estimate and track the primary noise frequencies  $\omega_i$ .

$$a_i(n+1) = a_i(n) - \mu_a y_{b,i}(n) \frac{g_i(n)}{\varepsilon + G_i(n)} \quad (5)$$

$$G_i(n) = \alpha G_i(n-1) + (1-\alpha)g_i^2(n) \quad (6)$$

where  $\mu_a$  is a step size that usually takes a small positive value,  $\alpha \in (0, 1)$  is a forgetting factor that is usually set quite close to unit, say, 0.98, 0.99,  $\varepsilon$  is a small positive constant that prevents division by zero. The output  $y_{c,i}(n)$ , the bandpass filter output  $x_i(n)$ , and the gradient signal  $g_i(n)$  of the  $i$ th active notch filter are respectively given by

$$y_{c,i}(n) = -\rho a_i(n)y_{c,i}(n-1) - \rho^2 y_{c,i}(n-2) \quad (7)$$

$$+ y_{c,i-1}(n) + a_i(n)y_{c,i-1}(n-1) + y_{c,i-1}(n-2)$$

$$x_i(n) = y_{c,i-1}(n) - y_{c,i}(n) \quad (8)$$

$$g_i(n) = y_{c,i-1}(n-1) - \rho y_{c,i}(n-1) \quad (9)$$

$$y_{c,0}(n) = x(n). \quad (10)$$

### B. MPA-based controller

The ANC controller consists of  $q$  two-weight MPAs that are utilized to generate the secondary source. The output of the  $i$ th channel or MPA is expressed by

$$y_i(n) = h_{0,i}(n)x_i(n) + h_{1,i}(n)x_i(n-1) \quad (11)$$

每个ANF中的滤波器的传递函数是下面这个式子

例如

越接近于unit

通常目标频率间隔很近时选择0.98左右的

使用ANF来提取包含在初级噪声中的正弦波

通过使用归一化梯度算法(NG)来更新ai得值, 达到追踪初级噪声中的频率变化

where  $\{h_{0,i}(n), h_{1,i}(n)\}$  are the two weights of  $i$ th MPA. The FXLMS algorithm is adopted to update the two weights

$$h_{0,i}(n+1) = h_{0,i}(n) + \mu_c e_s(n) \hat{x}_i(n) \quad (12)$$

$$h_{1,i}(n+1) = h_{1,i}(n) + \mu_c e_s(n) \hat{x}_i(n-1) \quad (13)$$

where

$$\hat{x}_i(n) = \sum_{m=0}^{\hat{M}-1} \hat{s}_m(n) x_i(n-m) \quad (14)$$

$$e_s(n) = e(n) - y_s(n) \quad (15)$$

$$e(n) = p(n) - y_p(n) \quad (16)$$

$$y_s(n) = \sum_{m=0}^{\hat{M}-1} \hat{s}_m(n) v(n-m) \quad (17)$$

$$y_p(n) = \sum_{m=0}^{M-1} s_m(n) [y(n-m) - v(n-m)] \quad (18)$$

$$v(n) = v_o(n) G_s(n) \quad (19)$$

$$G_s(n) = \beta G_s(n-1) + (1-\beta) e^2(n-1), \quad (20)$$

constant  $\mu_c$  is another step size,  $v_o(n)$  is an auxiliary white Gaussian noise (AWGN) with variance  $\sigma_o^2$  that is intentionally injected into the ANC system to facilitate the SPM,  $G_s(n)$  is a scaling factor and is a function of the residual noise power,  $\beta \in (0, 1)$  is a constant that is very close to unit, taking values like 0.999, 0.9995 [20].

### C. Online SPM subsystem

To estimate the secondary path and cope with its drift in real time, an online SPM subsystem is incorporated into the proposed FBNANC system. Transfer function  $S(z)$  denotes the true secondary path and is usually considered and treated as an FIR filter with coefficients  $\{s_m\}_{m=0}^{M-1}$  and length  $M$ . Its estimate is denoted by  $\hat{S}_n(z)$  having FIR coefficients  $\{\hat{s}_m(n)\}_{m=0}^{\hat{M}-1}$  that are updated by the least mean square (LMS) algorithm as follows:

$$\text{系数更新 } \hat{s}_m(n+1) = \hat{s}_m(n) + \mu_s e_s(n) v(n-m) \quad (21)$$

where  $\mu_s$  is another step size.

Note that the AWGN  $v_o(n)$  is scaled by a factor or a function of one-sample-delayed residual noise  $e(n-1)$  before injected into the secondary source [20], as shown in (19) and (20). Because of this scaling, the contribution of the AWGN to the residual noise power will become smaller and smaller as the entire FBNANC system approaches its steady state.

## III. SIMULATIONS

Extensive simulations are conducted to confirm and verify the effectiveness of the proposed FBNANC system. Two cases are simulated. In the first case (Case A), the primary noise frequencies show sudden changes during the FBNANC operation. The secondary path presents some drift in the second case (Case B). Simulation conditions are provided in

TABLE I  
SIMULATION CONDITIONS AND USER PARAMETERS.

	Case A	Case B
fre	<b>1st half:</b> $\omega_{p,i} = \{0.10, 0.20, 0.30\}\pi$ <b>2nd half:</b> $\omega_{p,i} = \{0.15, 0.25, 0.40\}\pi$	$\omega_{p,i} = \{0.15, 0.25, 0.40\}\pi$
Amp	$A_i = \{1.00, 0.50, 0.25\}$	$A_i = \{1.00, 0.50, 0.25\}$
Phase	$\theta_i = \{0.10, 0.20, 0.30\}\pi$	$\theta_i = \{0.10, 0.20, 0.30\}\pi$
$\sigma_p^2$	0.10	0.10
$\sigma_o^2$	1.00	1.00
$\beta$	0.9995	0.9995
$S(z)$	Lowpass FIR $M = 21$ , cutoff = $0.50\pi$ $s = \text{fir1}(M-1, 0.5)$	<b>1st half:</b> Lowpass FIR $M = 21$ , cutoff = $0.50\pi$ $s_1 = \text{fir1}(M-1, 0.5)$ <b>2nd half:</b> Lowpass FIR $s_2 = s_1$ $+ \text{randn}(1, M) \times 0.25$
$\hat{S}_n(z)$	FIR, $\hat{M} = 31$	FIR, $\hat{M} = 41$
ANF	$\rho = 0.975$ $\alpha = 0.98$ , $\epsilon = 0.01$	$\rho = 0.975$ $\alpha = 0.98$ , $\epsilon = 0.01$
step sizes	$\mu_a = 0.0001$ , $\mu_c = 0.002$ $\mu_s = 0.0002$	$\mu_a = 0.0001$ , $\mu_c = 0.002$ $\mu_s = 0.0002$
runs	40	40

Table I. The NRP and SPM MSE are respectively defined as follows:

$$NRP(n) = 10 \log_{10} \frac{E[e^2(n)]}{E[p^2(n)]}, \quad (dB) \quad (22)$$

$$MSE(n) = 10 \log_{10} \frac{\|s(n) - \hat{s}(n)\|^2}{\|s(n)\|^2}, \quad (dB) \quad (23)$$

where  $s(n)$  and  $\hat{s}(n)$  denote FIR coefficient vector of the true and estimated secondary path at time instant  $n$ , respectively.

The simulation results for Case A and Case B are provided in Figs. 3 and 4, respectively. From these figures and many other simulation results that are not provided here due to space limitation, we have the following remarks:

- 1) The ANFB could effectively target and extract the sinusoids whose frequencies are time-varying (Fig. 3(a), Fig. 4(a)). The ANF coefficients were all initially set to  $-1.99$  to allow the ANFs to search and target from the lowest to the highest frequency.
- 2) In our simulations, the number of active ANF cells used was the same as the number of sinusoids in the primary noise. Our proposed FBNANC system still worked well even though more or less active ANF cells are included. This property is very nice in real applications where the number of frequencies is unknown in advance.
- 3) The MPAs could efficiently and effectively generate the secondary source. As the MPA only has two weights, the FBNANC controller is significantly smaller in size as compared to the long FIR controller that is adopted in previous works [12]-[16].
- 4) The SPM strategy also worked well in both cases. However, our FBNANC system can only handle moderate secondary-path drift (Fig. 4(d)). If the secondary-path variation is large or abrupt, our proposed system will present very poor stability, rendering itself useless.

Improving the SPM robustness should be an interesting future research topic.

- 5) Our proposed system was also compared with an ideal version of itself that has no SPM subsystem and the secondary-path estimate is identical to the truth. Both systems indicated quite similar convergence rate. In Case A, the NRP of the ideal version was about  $-8.42$  (dB), whereas our proposed system produced an NRP of  $-8.01$  (dB). Their corresponding NRP values for Case B were about  $-8.45$  (dB) and  $-8.02$  (dB), respectively. The NRP gap between the two systems is not significant in both cases, but more effort should be made to make it smaller.
- 6) In feedforward NANC systems, our observation is that the accuracy of the SPM does not have significant impact on the overall system NRP as long as the resultant secondary-path estimate is not very poor [6]. However, the online SPM accuracy presented great impact on both the NRP and the stability in our proposed FBNANC system. This may be due to the role the secondary-path estimate  $\hat{S}_n(z)$  plays in reconstructing the primary noise  $x(n)(= \hat{p}(n-1))$ . In contrast, the secondary-path estimate  $\hat{S}_n(z)$  is only used to filter the controller input waves that are used to update the controller weights in the feedforward NANC systems.

#### IV. CONCLUSIONS

In this paper, a new FBNANC system with online SPM has been proposed. An ANFB is incorporated into the FBNANC system to extract reference sinusoidal signals for the controller that consists of two-weight MPAs. An online SPM subsystem is also included to estimate the secondary path and deal with its drift. The use of MPAs and ANFB has significantly reduced the complexity of the proposed system. Extensive simulations are conducted to confirm the system effectiveness in the presence of non-stationarity and secondary-path drift. Future research topics include: 1) performing statistical analysis of the proposed system, 2) applying the proposed system to real noises and real secondary paths, 3) introducing additional subsystem(s) to improve the SPM accuracy and robustness against secondary-path variation, and so on.

#### ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China (No.62003149), the Natural Science Foundation of Jiangsu Province (BK20200612), China, as well as the JSPS Grant-in-Aid for Scientific Research (C) (15K06117, 18K04175), Japan. Cordial thanks are also given to Mr. Genki Kono for his assistance in coding some of the MATLAB programs used in our simulations.

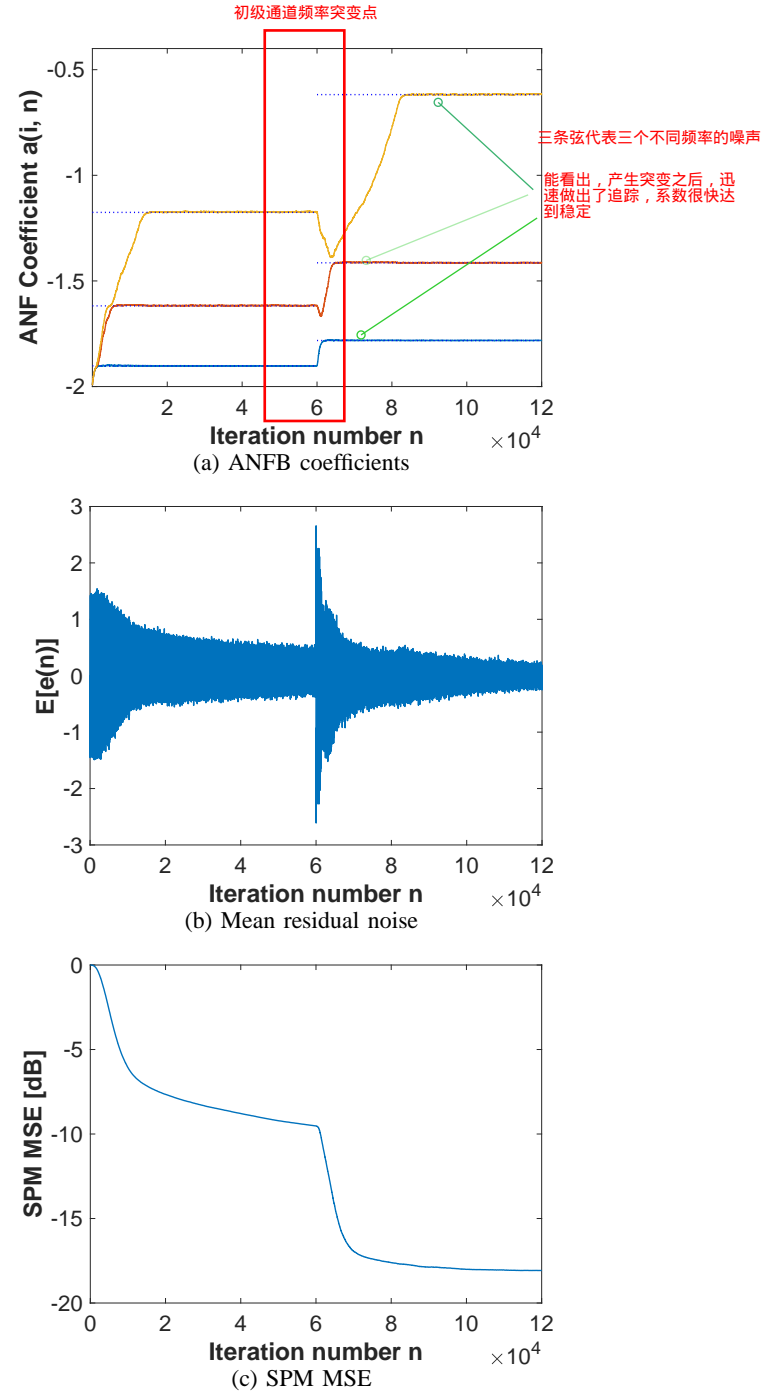


Fig. 3. Simulation results of Case A.



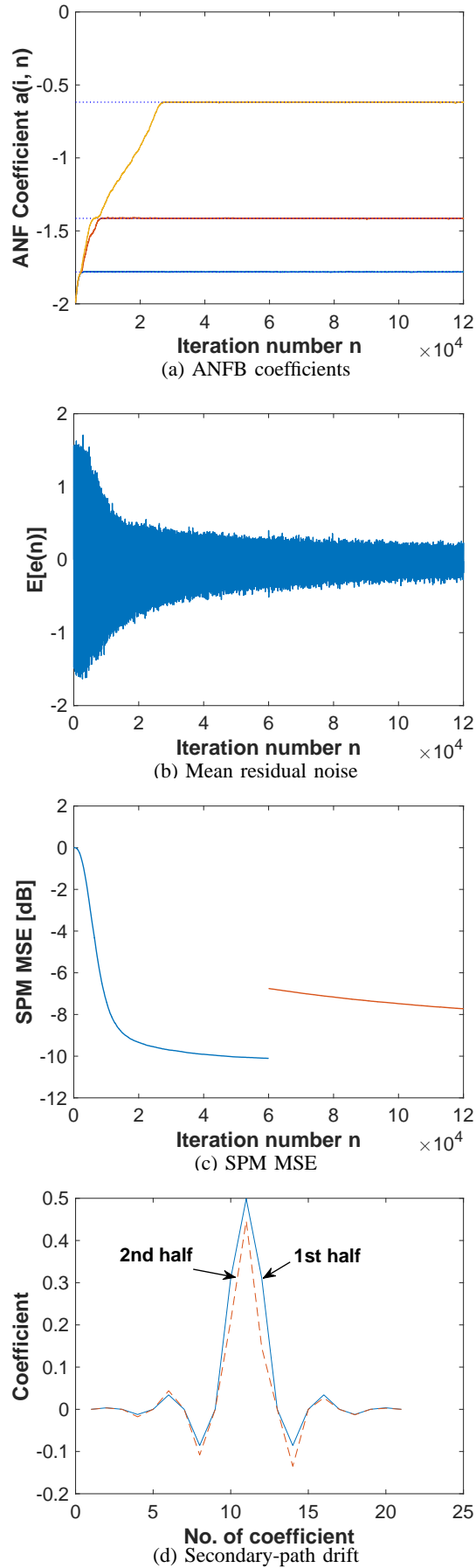


Fig. 4. Simulation results of Case B.

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