Workshop 11

Game Theory

- **Game form**: normal/extensive
- **Number of player**

Strategy of each player:

- Pure: only one strategy, each turn choose one action for 100%
- Mixed: choose different strategy with different probability distribution to choose one action

Profiles & Utility function U_i(a)

Motivation Example: Auctions

- Winner payment determination: highest / 2nd Highest
- Knowledge of the bid: open-cry, sealed-bid
- Order of the bid: ascending, descending
- Number of goods: 1 in this example

- Common Values
- Private Values: $U_i = V_i - P_i$
- V_i = how much the bidder think the item worth
- P_i = how much the bidder pay for the item

Auction exercise

Auction types

British, English auction:

- Starting from a relatively low price
- No one make higher offer, auction stop
- Bidder with highest offer win this auction
- Highest price, open cry, ascending, number of item == 1

Dutch auction

- Start with a high price, twice of common price
- Each Time auctioneer might drop the price a little bit
- Whoever accept the price, make the offer first, will win the auction,
- Highest-price, open cry, Descending, number of item == 1

Vickrey auction

- The winner will still be the one offer the highest price, but they will pay the second highest price
- Sealed-bid auction
- One-time auction

Problem 1

Consider a Vickrey auction between two bidders, Bidder A and Bidder B, who each value the item at \$50 and \$51 respectively. Assume that the buyer has no ‘reserve’ price.

a) Are there (weakly or strictly) dominant strategies for these two bidders? If so, what are they? To simplify your analysis, assume that bids can only be whole numbers between 49 and 52.

		Bidder B (val = 51)			
		49	50	51	52
	49				
Bidder A	50				
(val = 50)	51				
	52				

Problem 1

Consider a Vickrey auction between two bidders, Bidder A and Bidder B, who each value the item at \$50 and \$51 respectively. Assume that the buyer has no 'reserve' price.

a) Are there (weakly or strictly) dominant strategies for these two bidders? If so, what are they? To simplify your analysis, assume that bids can only be whole numbers between 49 and 52.

		Bidder B (val = 51)			
		49	50	51	52
Bidder A (val = 50)	49	$\frac{1}{2}$, 1	0, 2	0, 2	0, 2
	50	1, 0	0, $\frac{1}{2}$	0, 1	0, 1
	51	1, 0	0, 0	-$\frac{1}{2}$, 0	0, 0
	52	1, 0	0, 0	-1, 0	-1, -$\frac{1}{2}$

Problem 1

		Bidder B (val = 51)			
		49	50	51	52
Bidder A (val = 50)	49	$\frac{1}{2}$, 1	0, 2	0, 2	0, 2
	50	1, 0	0, $\frac{1}{2}$	0, 1	0, 1
	51	1, 0	0, 0	-$\frac{1}{2}$, 0	0, 0
	52	1, 0	0, 0	-1, 0	-1, -$\frac{1}{2}$

Each bidder has a *weakly* dominant strategy. Consider Bidder A's reasoning process first:

i. If Bidder B plays 49, then I should play 50, 51, or 52.

   If Bidder B plays 50, then I can play any strategy.

   If Bidder B plays 51, then I should play 49 or 50.

   If Bidder B plays 52, then I should play 49, 50, or 51.

   Thus, strategy 50 is the only (pure) strategy that is not dominated by some other (pure) strategy, so that is my weakly-dominant strategy.

Problem 1

Bidder A (val = 50) \ Bidder B (val = 51)	49	50	51	52
49	$\frac{1}{2}$, 1	0, 2	0, 2	0, 2
50	1, 0	0, $\frac{1}{2}$	0, 1	0, 1
51	1, 0	0, 0	-$\frac{1}{2}$, 0	0, 0
52	1, 0	0, 0	-1, 0	-1, -$\frac{1}{2}$

ii. Bidder B then reasons in the same way to demonstrate that 51 is weakly dominant strategy for B.

Note however, that Bidder B cannot start with the premise that Bidder A will choose 50 and then reason from here, because Bidder B does not know Bidder A's private value. This is an example of a game with *imperfect information*, which we will not consider any further in this subject.

Problem 1

Consider a Vickrey auction between two bidders, Bidder A and Bidder B, who each value the item at \$50 and \$51 respectively. Assume that the buyer has no 'reserve' price.

a) Are there (weakly or strictly) dominant strategies for these two bidders? If so, what are they? To simplify your analysis, assume that bids can only be whole numbers between 49 and 52.

		Bidder B (val = 51)			
		49	50	51	52
Bidder A (val = 50)	49				
	50				
	51				
	52				

b) What are the equilibrium states in this simplified game?

Problem 1

	49	50	51	52
49	0.5, 1	0, 2	0, 2	0, 2
50	1, 0	0, 0.5	0, 1	0, 1
51	1, 0	0, 0	-0.5, 0	0, 0
52	1, 0	0, 0	-1, 0	-1, -0.5

Problem 1

		Bidder B (val = 51)			
		49	50	51	52
	49	$\frac{1}{2}$, 1	0, 2	0, 2	0, 2
Bidder A	50	1, 0	0, $\frac{1}{2}$	0, 1	0, 1
(val = 50)	51	1, 0	0, 0	-$\frac{1}{2}$, 0	0, 0
	52	1, 0	0, 0	-1, 0	-1, -$\frac{1}{2}$

b) Clearly, the combination of the two weakly dominant strategies is one equilibrium, but are their others? Yes!

The equilibria are at: (51, 49), (52, 49), (49, 50), (49, 51), (49, 52), (50, 51), (50, 52), (51, 50), (51, 52), (52, 50).

Problem 2

Assume row player will choose T with x probability,
And Column player will choose L with y probability

Game 1:

		Column player	
		L	R
Row	T	320, 40	40, 80
player	B	40, 80	80, 40

Note that the only difference between the two games is the payoff 320 vs. 44 in the top-left cell.

a) What strategy would you choose in each game?

b) Calculate the mixed strategy that both players should play in both of these games. How close was your intuition to the mixed strategy that you calculated?

Problem 2

Assume row player will choose T with x probability,
And Column player will choose L with y probability

Game 1:

$E(U\_column) = E(L) * Y + E(R) * (1-Y)$
$E(L) = x * 40 + (1-x)*80$
$E(R) = x * 80 + (1-x)*40$
$E(L) = E(R)$

Column player

		L	R
Row	T	320, 40	40, 80
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Problem 2

Assume row player will choose T with x probability,
And Column player will choose L with y probability

Game 1:

$E(U\_row) = E(T) * x + E(B) * (1-x)$

$E(T) = y * 320 + (1-y)*40$
$E(B) = y * 40 + (1-y)*80$
$E(T) = E(B)$

		Column player	
		L	R
Row player	T	320, 40	40, 80
	B	40, 80	80, 40

Note that the only difference between the two games is the payoff 320 vs. 44 in the top-left cell.

a) What strategy would you choose in each game?

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Problem 2

Game 2:

		Column player	
		L	R
Row	T	44, 40	40, 80
player	B	40, 80	80, 40

Note that the only difference between the two games is the payoff 320 vs. 44 in the top-left cell.

a) What strategy would you choose in each game?

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Game 2:

E(U_column) = E(L)* Y + E(R)* (1-Y)

E(L) = x * 40 + (1-x)*80
E(R) = x * 80 + (1-x)*40
E(L) = E(R)

Column player

Row player		L	R
	T	44, 40	40, 80
	B	40, 80	80, 40

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Game 2:

$E(U\_row) = E(T) * x + E(B) * (1-x)$

$E(T) = y * 44 + (1-y)*40$
$E(B) = y * 40 + (1-y)*80$
$E(T) = E(B)$

Column player

		L	R
Row	T	44, 40	40, 80
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