

# **Workshop 7**

# Recap: Classical Planning Problem

Not every problem belongs to classical planning problem

**Deterministic action:**  $S - a \rightarrow S'$

- Every action only has a certain outcome, and you know what that outcome will be
- Counterexample: coin toss  $\rightarrow$  probabilistic actions
- Single-agent
- Static environment
- .....

# Other action types

- **Probabilistic:** We could possibly end up in more than one state, and we know the probability distribution of these states (Example: Toss a fair coin)
- **Non-deterministic:** We know all possible outcome, but not the probability distribution
- **Stochastic:** limited info about possible outcomes

# MDP problem

- Still use model-based approach to solve it

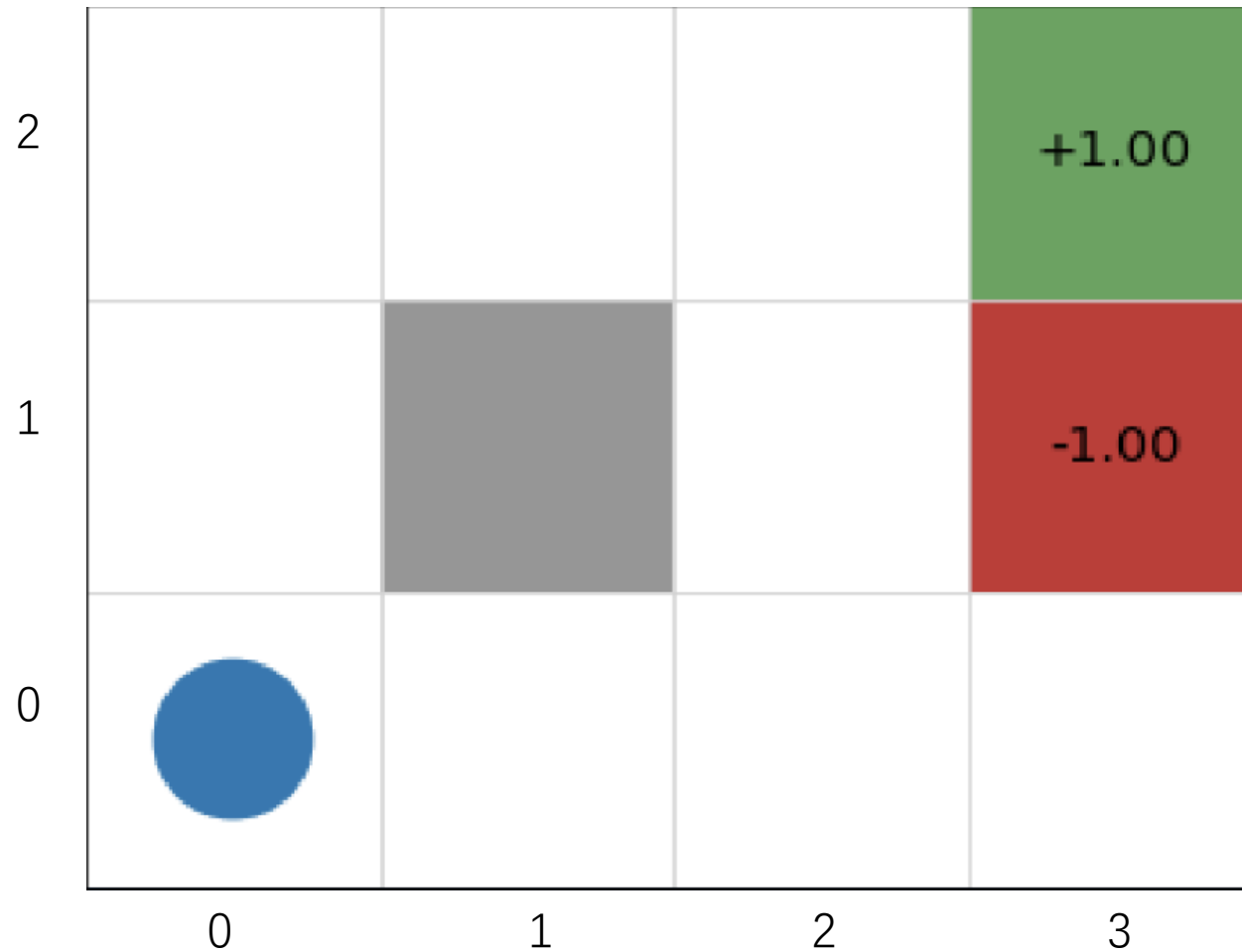
## 2 Models:

- **Goal-cost MDP model:** with a set of specific goal state, intend to achieve some goals, objective: minimize our cost to the goal
- **Discounted reward MDP model:** don't have goal state, have terminal state instead, objective: maximize the reward

## Solvers:

- Policy Iteration

# Lecture Example



0.8 succ  
0.1 slip left  
0.1 slip right

# Representations

$S = \{ \langle x, y \rangle \mid x \text{ belongs to } (0,3), y \text{ belong to } \{0,2\} \} \cup \{s_t\} \setminus (1,1)$

$s_0 = \langle 0, 0 \rangle \quad S_T = \{s_t\}$

## Action function:

$A(s_t) = \{ \}$

$A(s) = \{N, W, E, S\}$

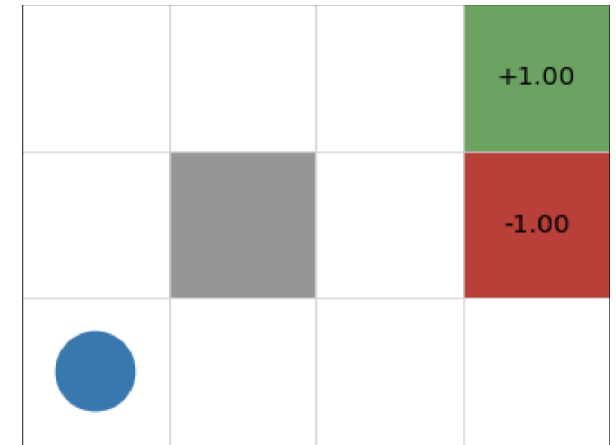
**except**  $A((3,2)) = A((3,1)) = \{\text{exit}\}$

## Reward function:

$r(s, a) = 0$  for any  $s$ , belong to  $S$ ,  $a$  belongs to  $A$

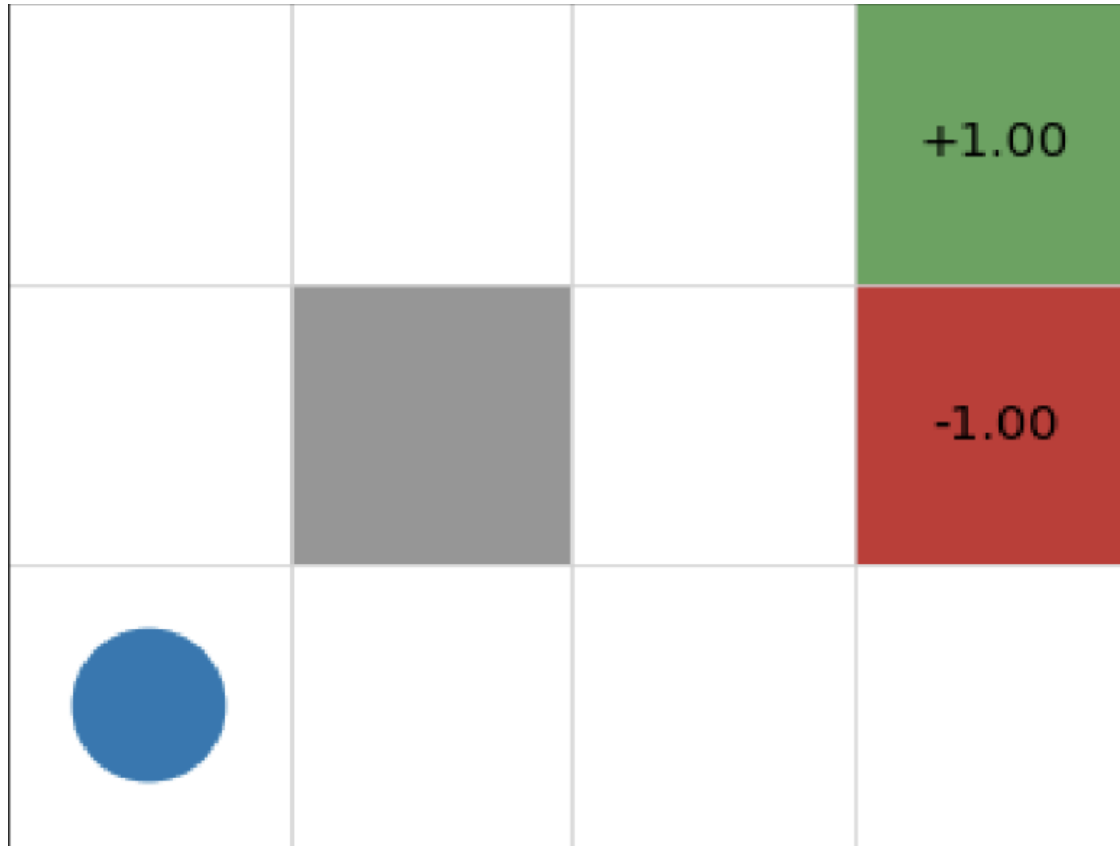
**Except**  $r((3,2), \text{exit}) = +1$

**And**  $r((3,1), \text{exit}) = -1$



**Discount factor  $0 < \gamma < 1$**

# Probability Distribution



**Probability distribution for exit action**

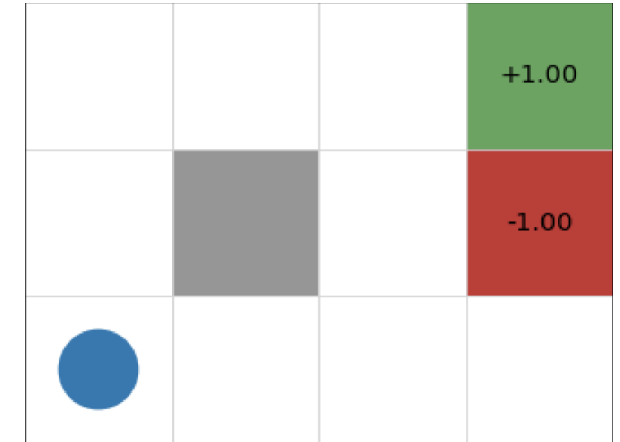
- $P_{\text{exit}}(s_t \mid (3, 2)) = 1$
- $P_{\text{exit}}(s_t \mid (3, 1)) = 1$
- $P_{\text{exit}}(s' \mid \text{any } s \text{ except above 2 state}) = 0$

# Probability Distribution for North action

$$P_N((x', y') \mid (x, y)) =$$

## Common case

- Successful: If  $x', y' == x, \min(2, y+1)$  then  $p = 0.8$
- Slip Right: If  $x', y' == \min(3, x+1), y$  then  $p = 0.1$
- Slip Left: If  $x', y' == \max(0, x-1), y$  then  $p = 0.1$



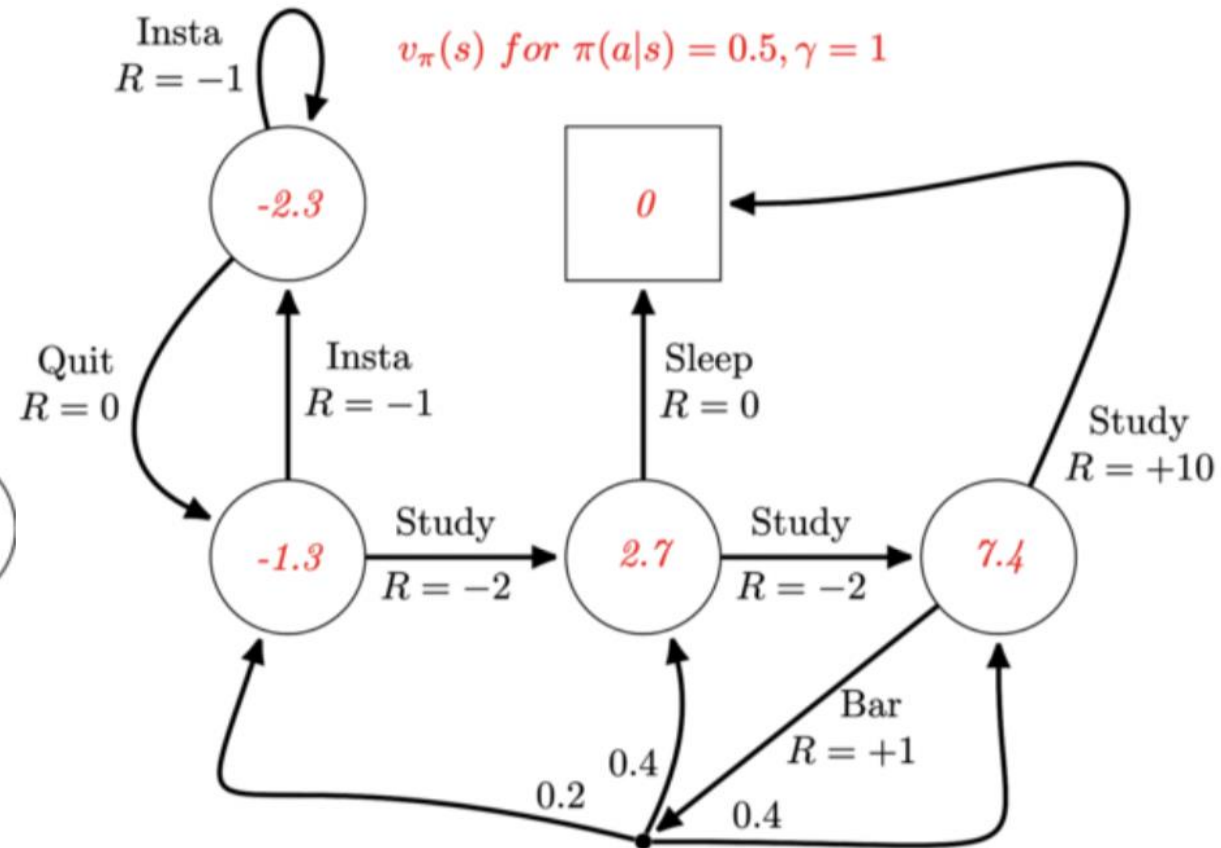
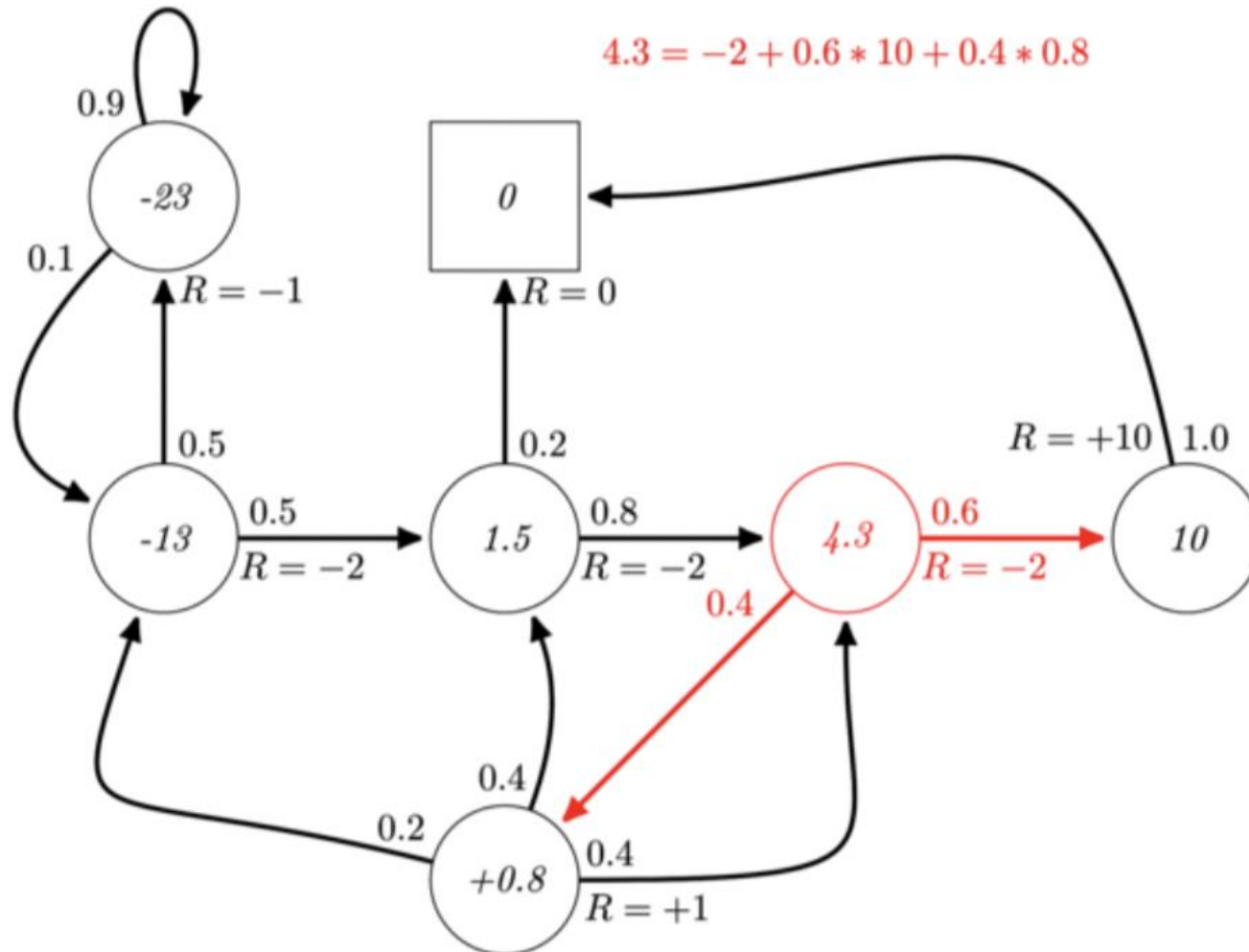
## Special Case: Wall

- Do North and Successful: If  $x, y == x', y' == (1,0)$  then  $p = 0.8$
- Do North but Slip Left: If  $x, y == x', y' == (2,1)$  then  $p = 0.1$
- Do North but Slip Right: If  $x, y == x', y' == (0,1)$  then  $p = 0.1$

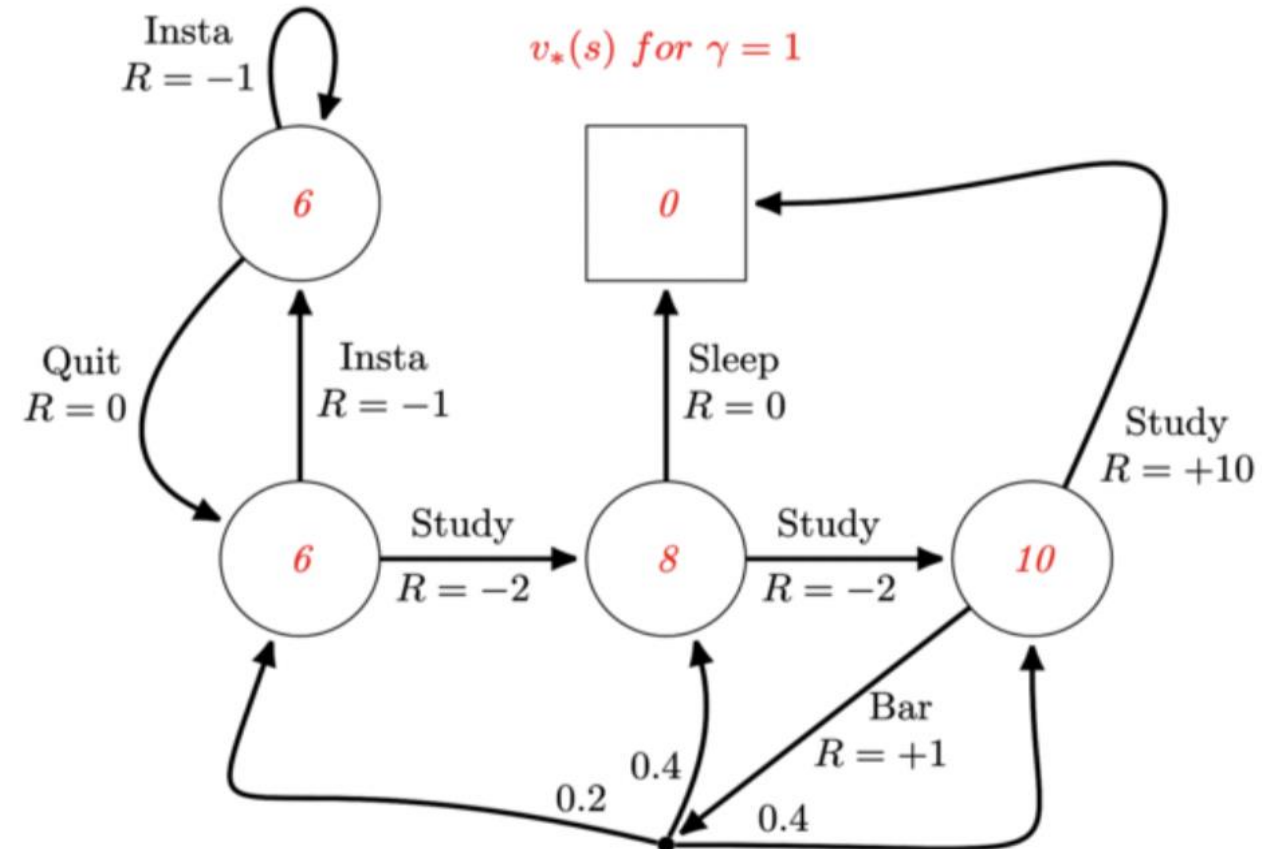
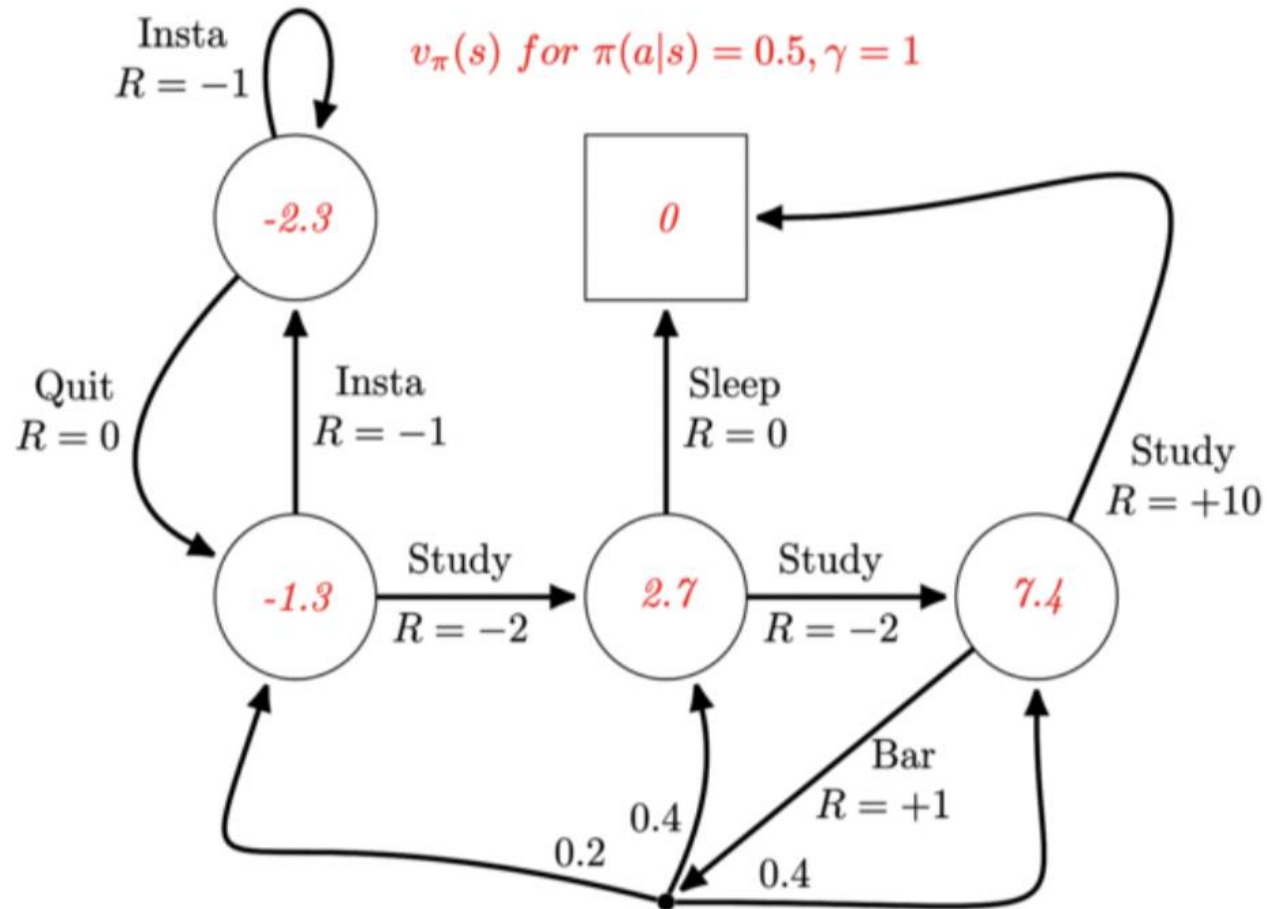
0.8 succ  
0.1 slip left  
0.1 slip right



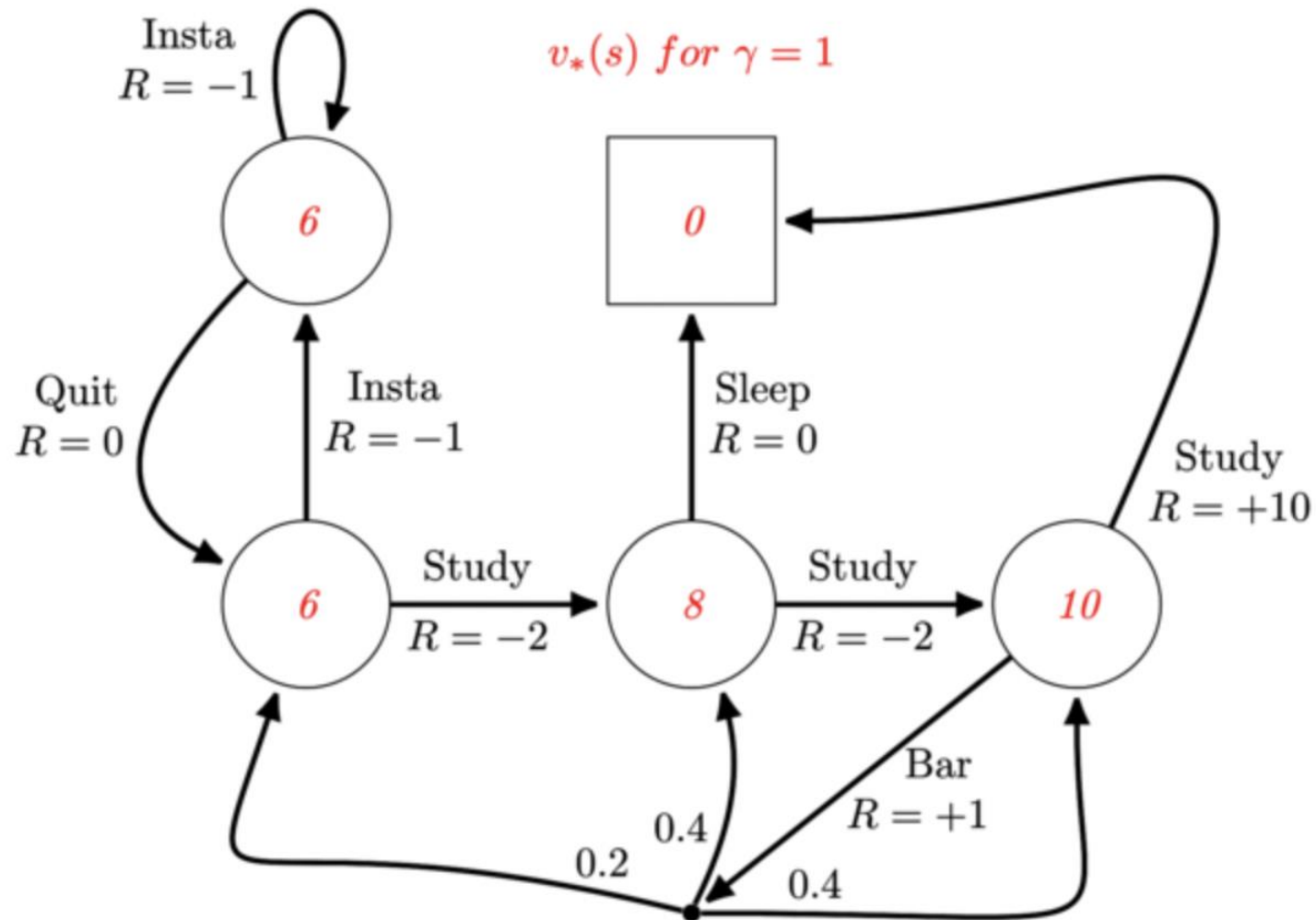
**Problem 2.A** Compare the value functions for the Markov Reward Process and Markov Decision process shown below. Why are they different? Is there a different policy for the MDP which would result in the same values as shown in the MRP?



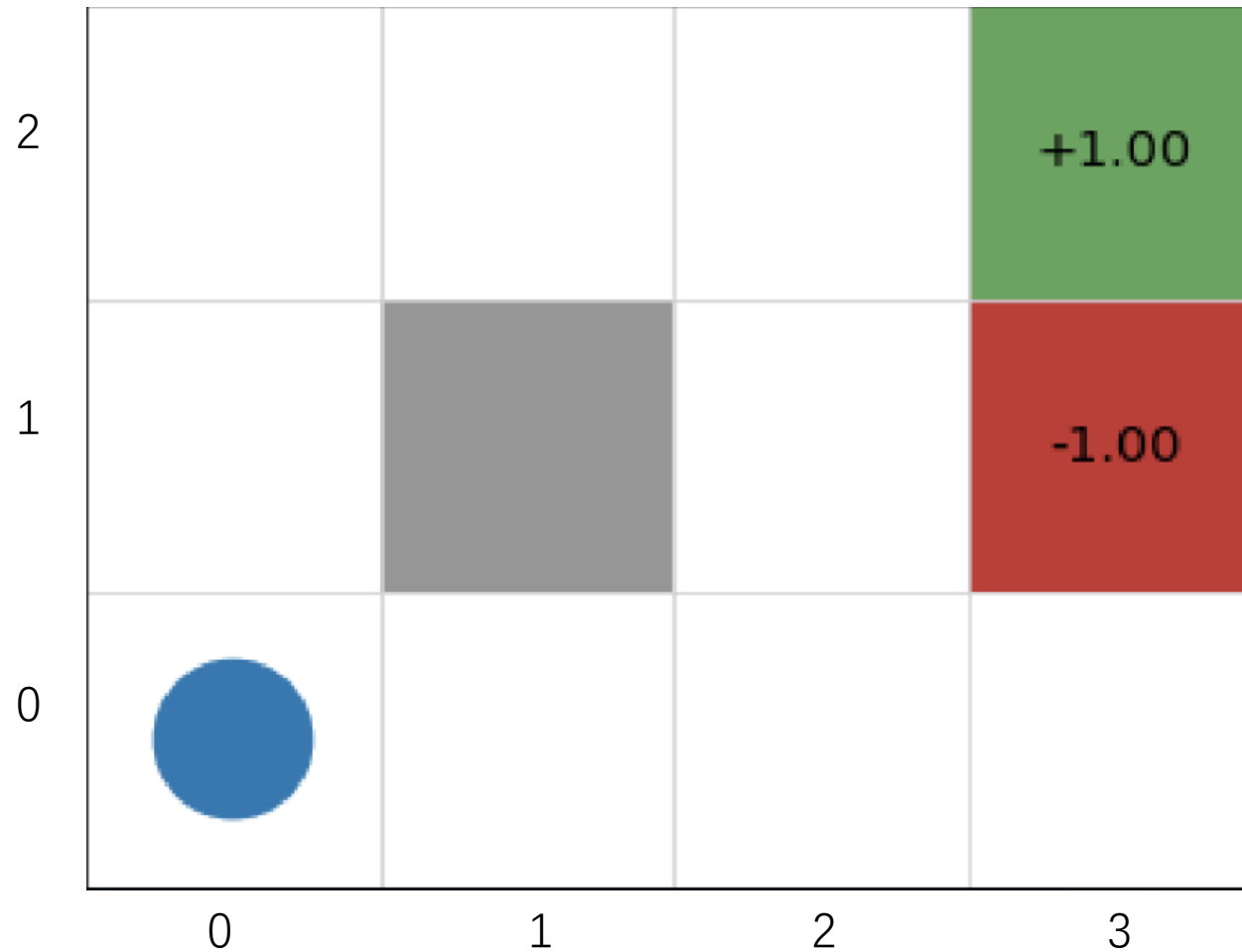
**Problem 2.B** Now compare the values to the optimal value function. Why are they different?



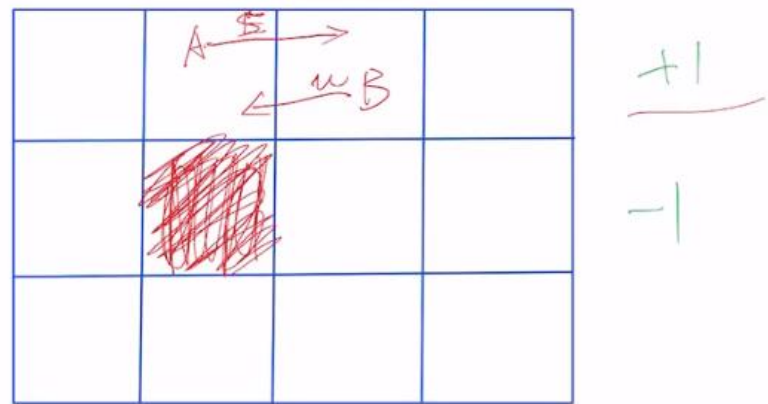
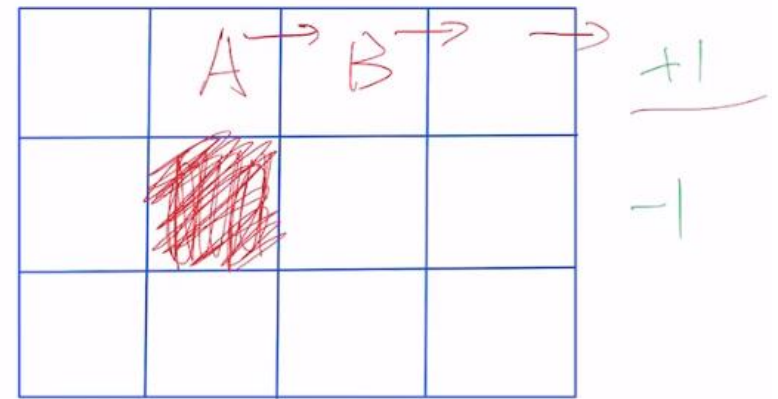
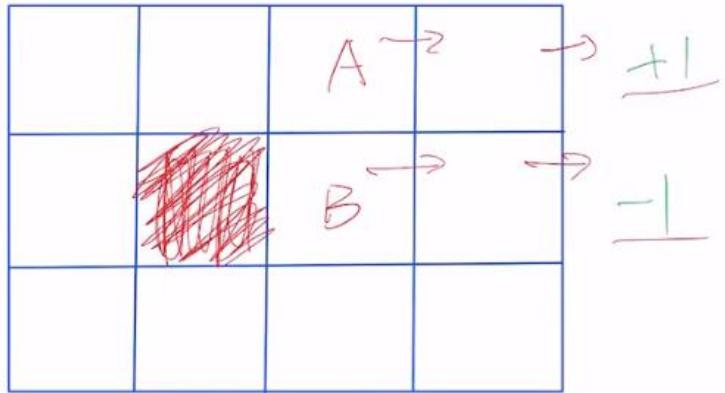
**Problem 3** Given the optimal state value function above, what is the optimal action to take in the bottom left state? What about the rightmost state? How can you tell?



# Lecture Example

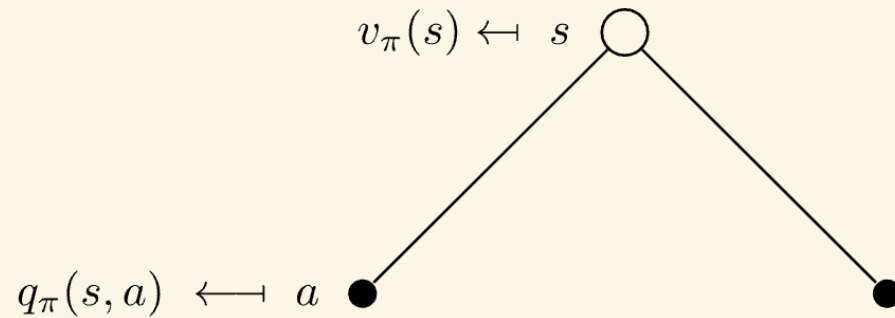


0.8 succ  
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# Formula for $V(s)$

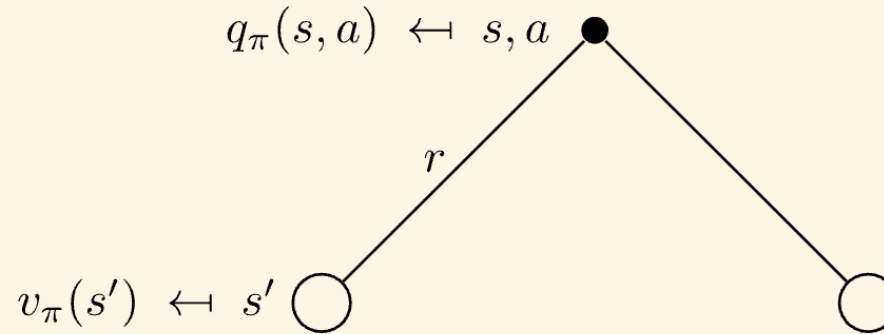
**Bellman Expectation Equation for  $V^\pi$  (look ahead)**



$$v_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a | s) q_\pi(s, a)$$

# Formula for Q

**Bellman Expectation Equation for  $Q^\pi$  (look ahead)**



$$q_\pi(s, a) = \mathcal{R}_s^a + \gamma \sum_{s' \in \mathcal{S}} \mathcal{P}_{ss'}^a v_\pi(s')$$