Quick Sort – Part 1

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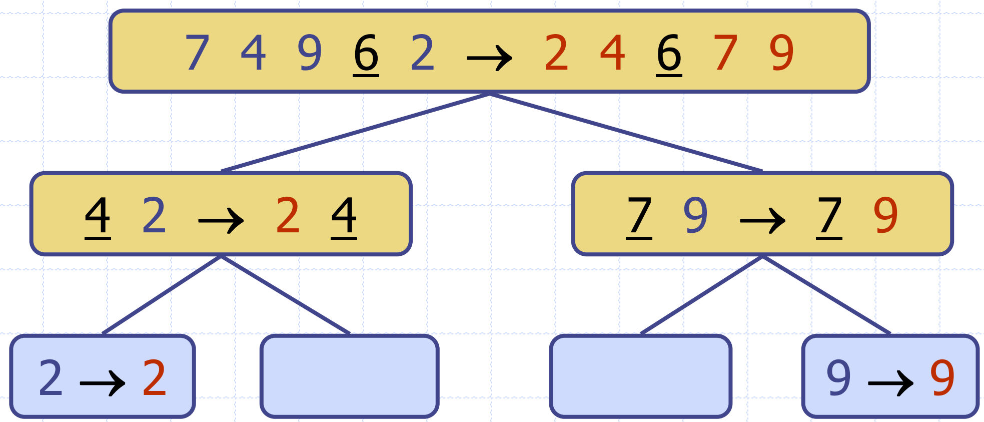
Slides from *Algorithm Design & Applications* by Michael T. Goodrich and Roberto Tamassia, © 2015 John Wiley & Sons, Inc.Goodrich and Tamassia, ISBN: 978-1118335918.

Reading Material

- *Algorithm Design & Applications* by Michael T. Goodrich and Roberto Tamassia
  - **Chapter 8 Section 8.2**

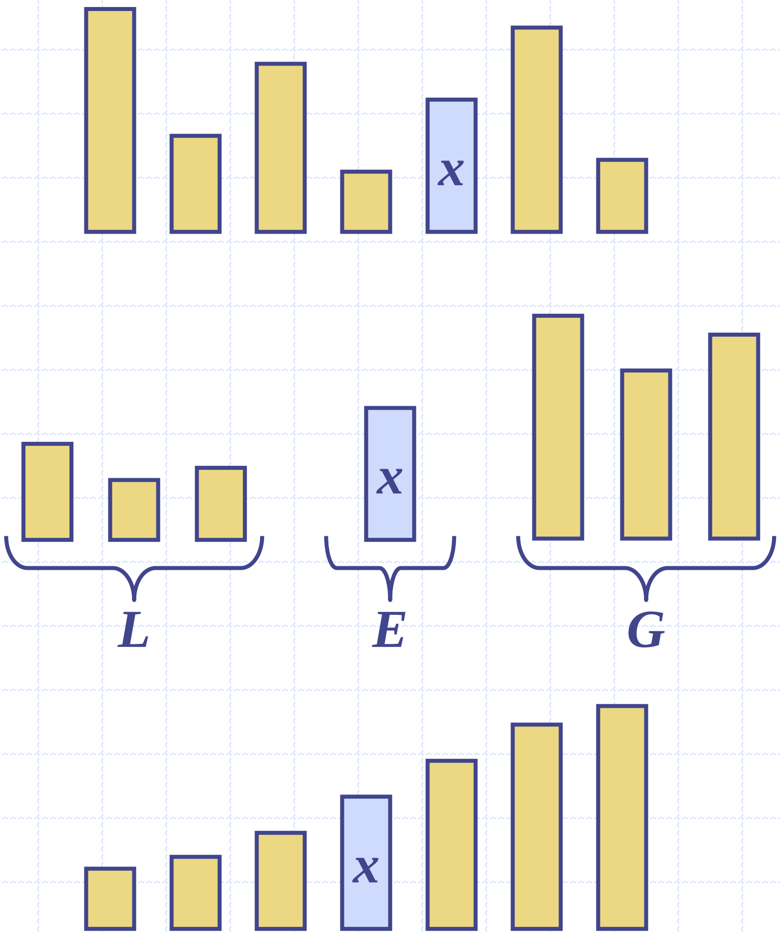
Presentation for use with the textbook, Algorithm Design and Applications, by M. T. Goodrich and R. Tamassia, Wiley, 2015

Quick-Sort



Quick-Sort

- Quick-sort is a randomized sorting algorithm based on the divide-and-conquer paradigm:
  - Divide: pick a random element x (called pivot) and partition S into
    - L elements less than x
    - E elements equal x
    - G elements greater than x
  - Recur: sort L and G
  - Conquer: join L, E and G



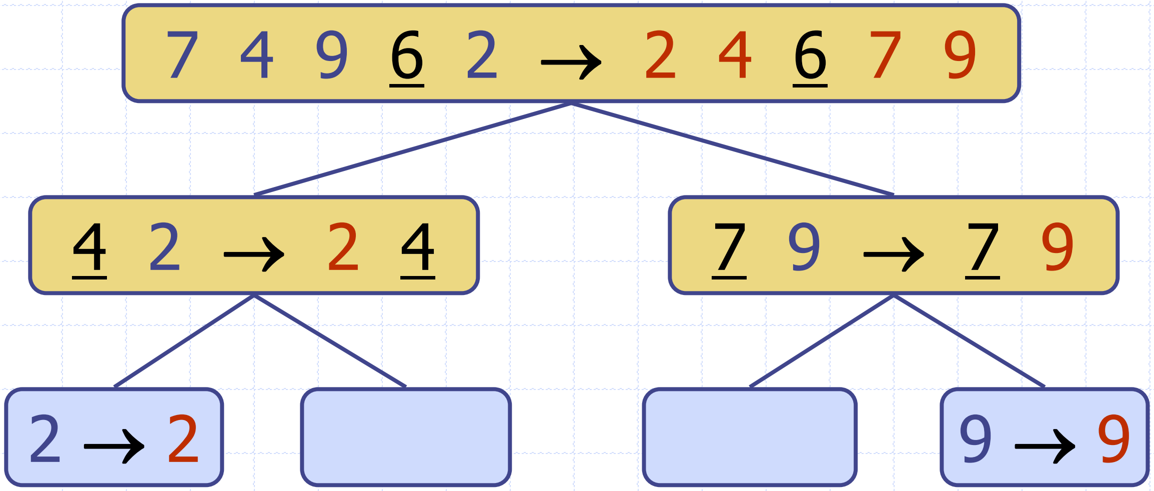
Partition

- We partition an input sequence as follows:
  - We remove, in turn, each element y from S and
  - We insert y into L, E or G, depending on the result of the comparison with the pivot x
- Each insertion and removal is at the beginning or at the end of a sequence, and hence takes $O(1)$ time
- Thus, the partition step of quick-sort takes $O(n)$ time

Algorithm *partition*(S, p)
    Input sequence S, position p of pivot
    Output subsequences L, E, G of the
        elements of S less than, equal to,
        or greater than the pivot, resp.
    $L, E, G \leftarrow$ empty sequences
    $x \leftarrow S.remove(p)$
    while $\neg S.isEmpty()$
        $y \leftarrow S.remove(S.first())$
        if $y < x$
            $L.addLast(y)$
        else if $y = x$
            $E.addLast(y)$
        else $\{ y > x \}$
            $G.addLast(y)$
    return L, E, G

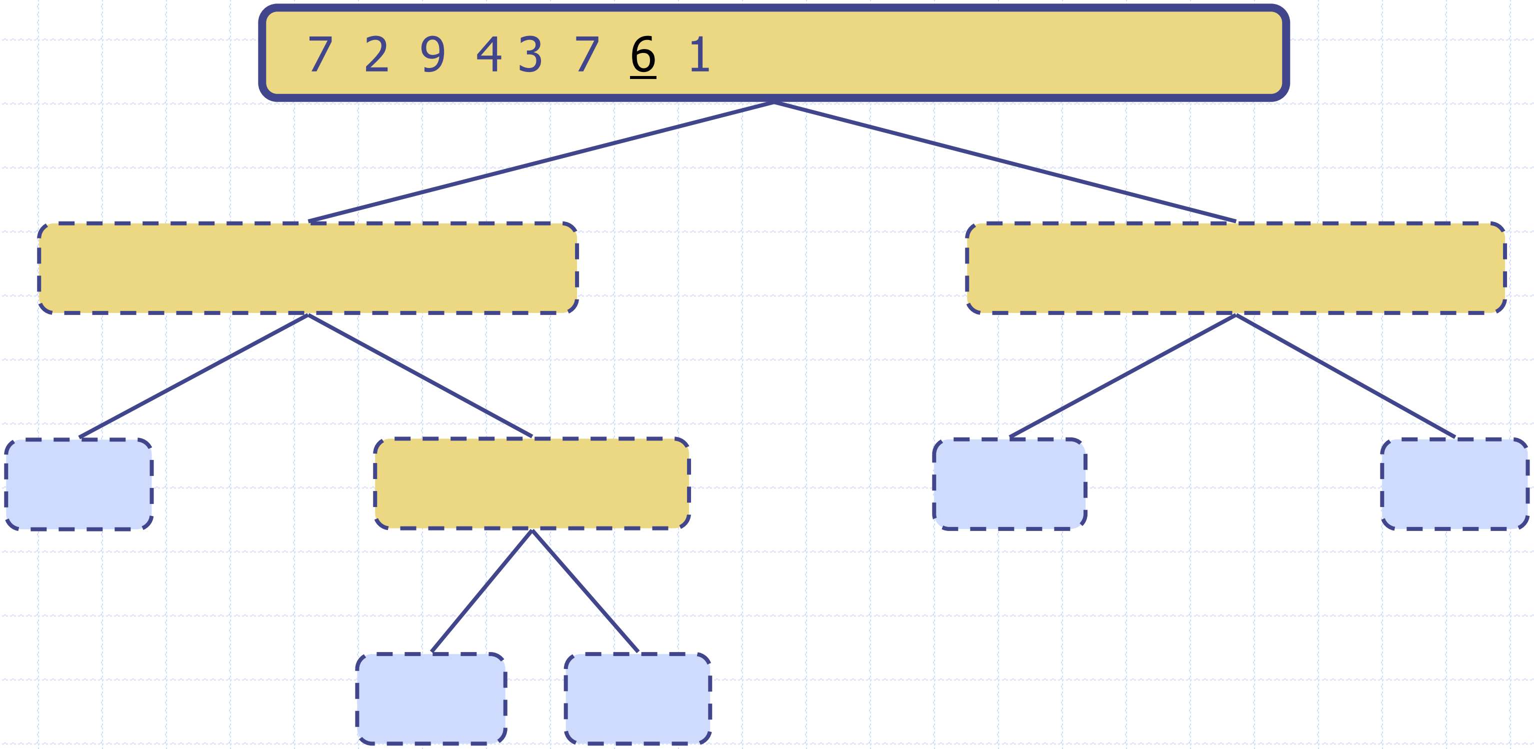
Quick-Sort Tree

- An execution of quick-sort is depicted by a binary tree
  - Each node represents a recursive call of quick-sort and stores
    - Unsorted sequence before the execution and its pivot
    - Sorted sequence at the end of the execution
  - The root is the initial call
  - The leaves are calls on subsequences of size 0 or 1



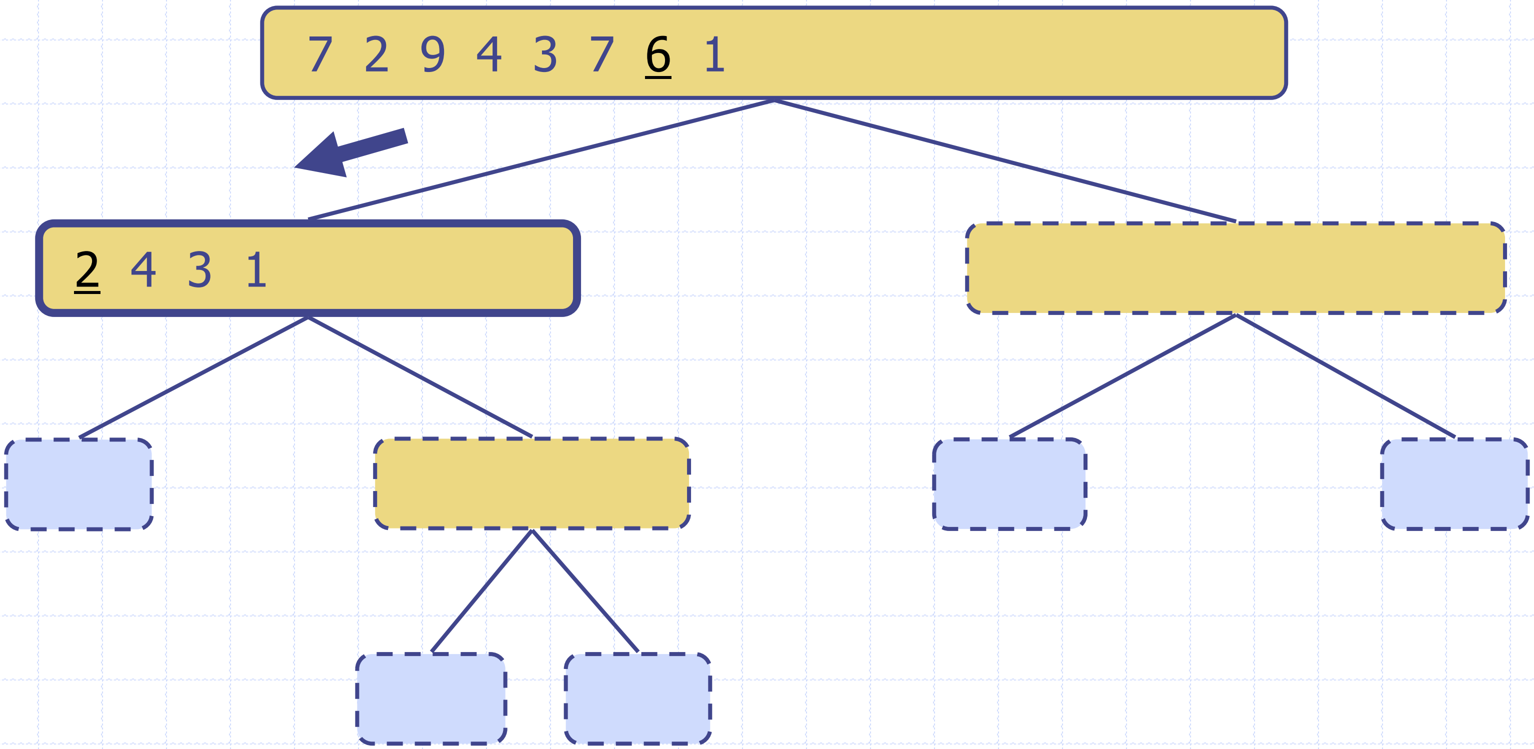
Execution Example

- Pivot selection



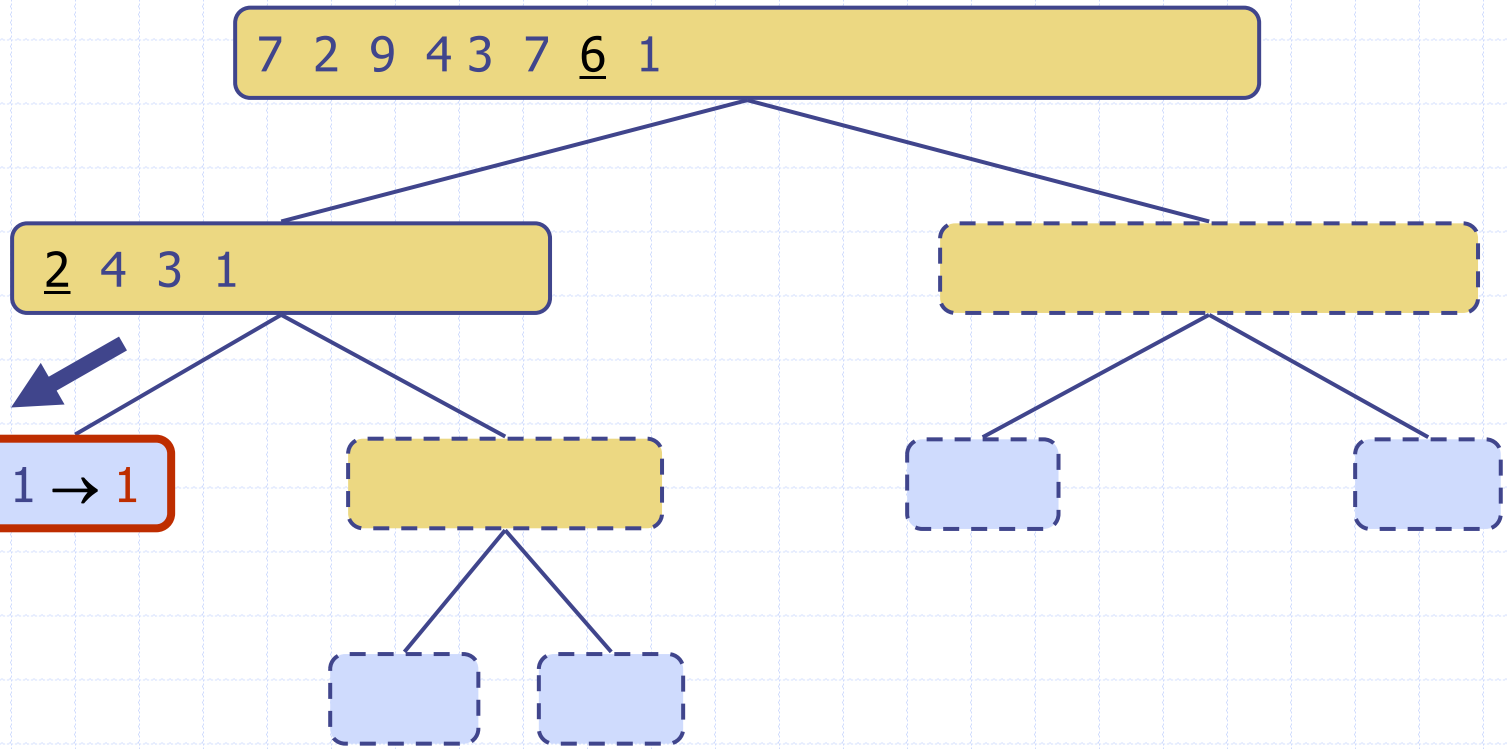
Execution Example (cont.)

- Partition, recursive call, pivot selection



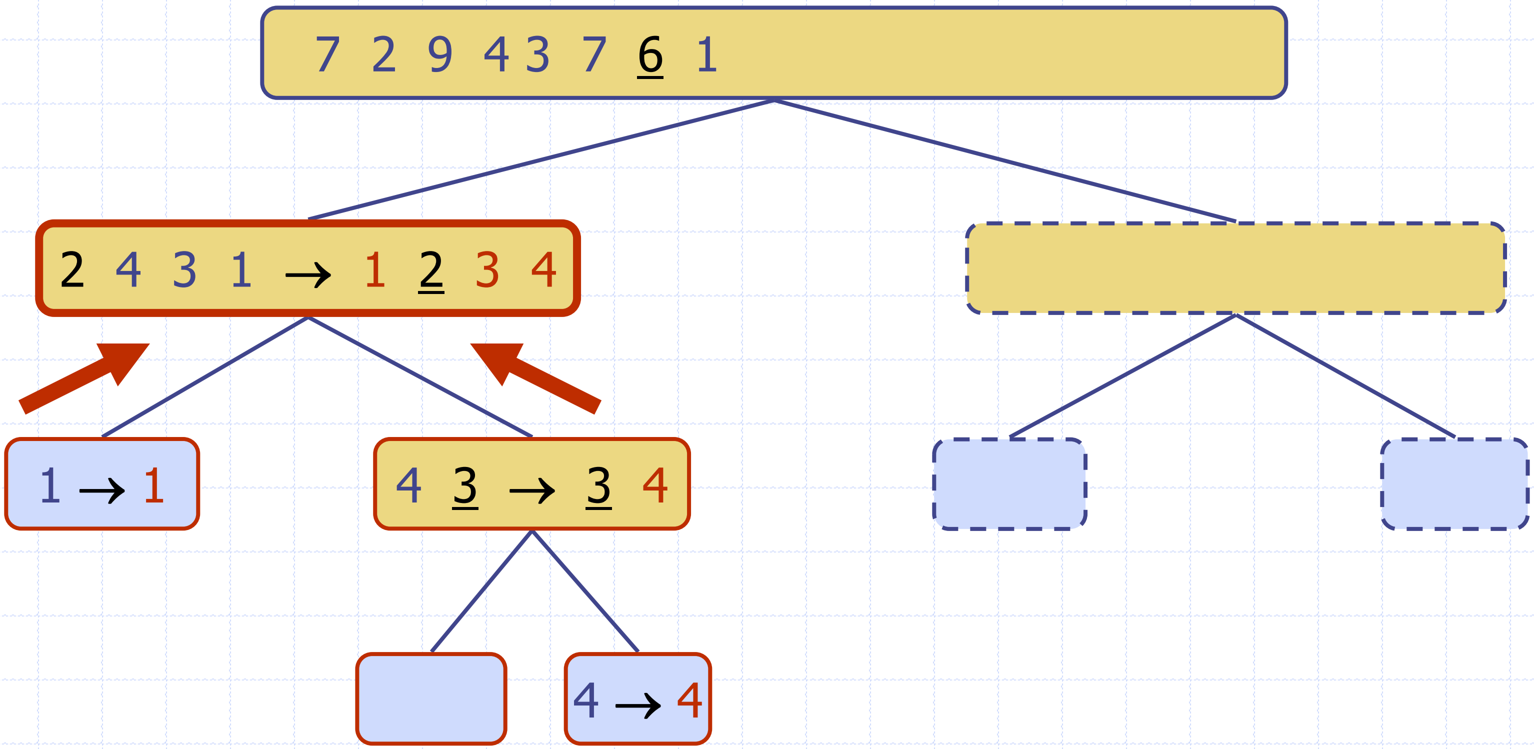
Execution Example (cont.)

- Partition, recursive call, base case



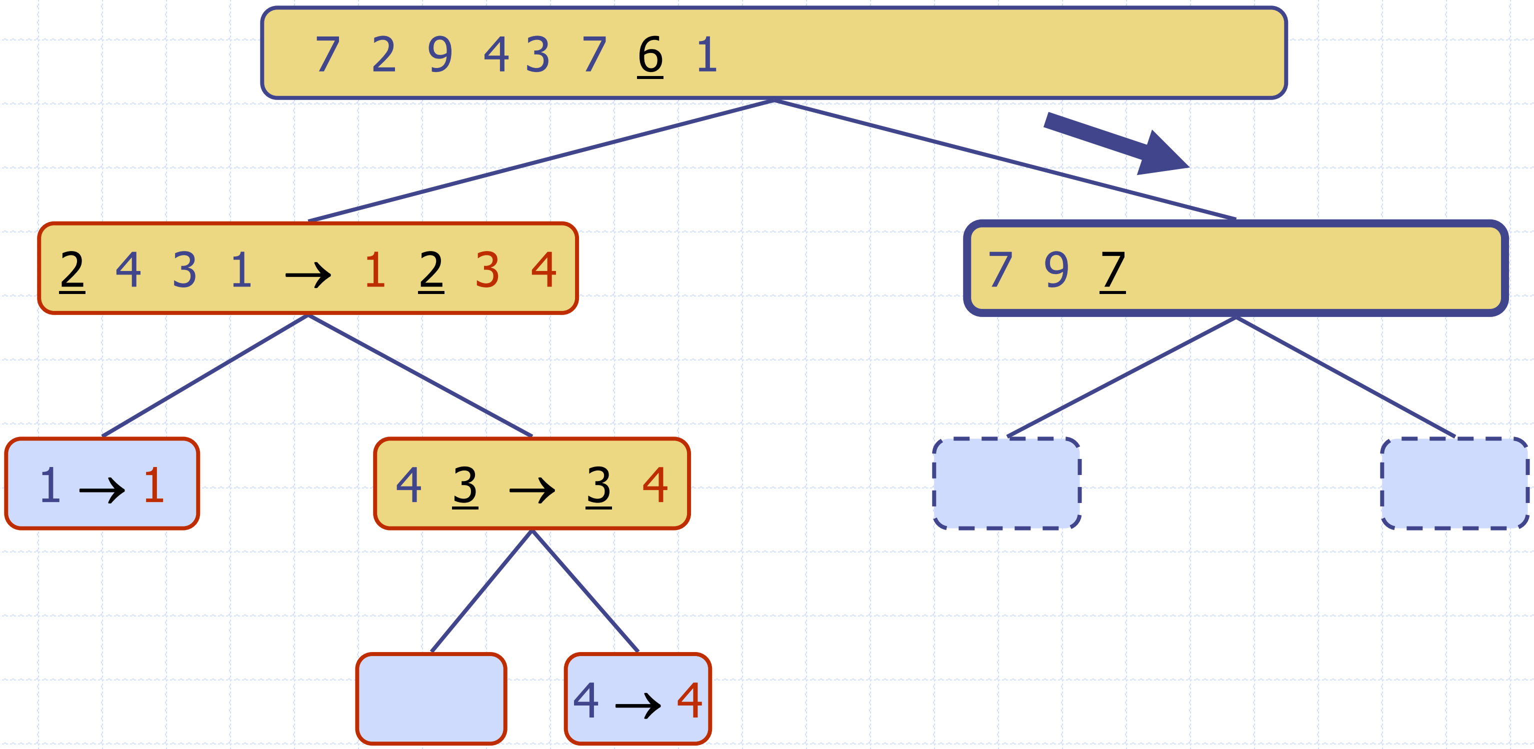
Execution Example (cont.)

- Recursive call, …, base case, join



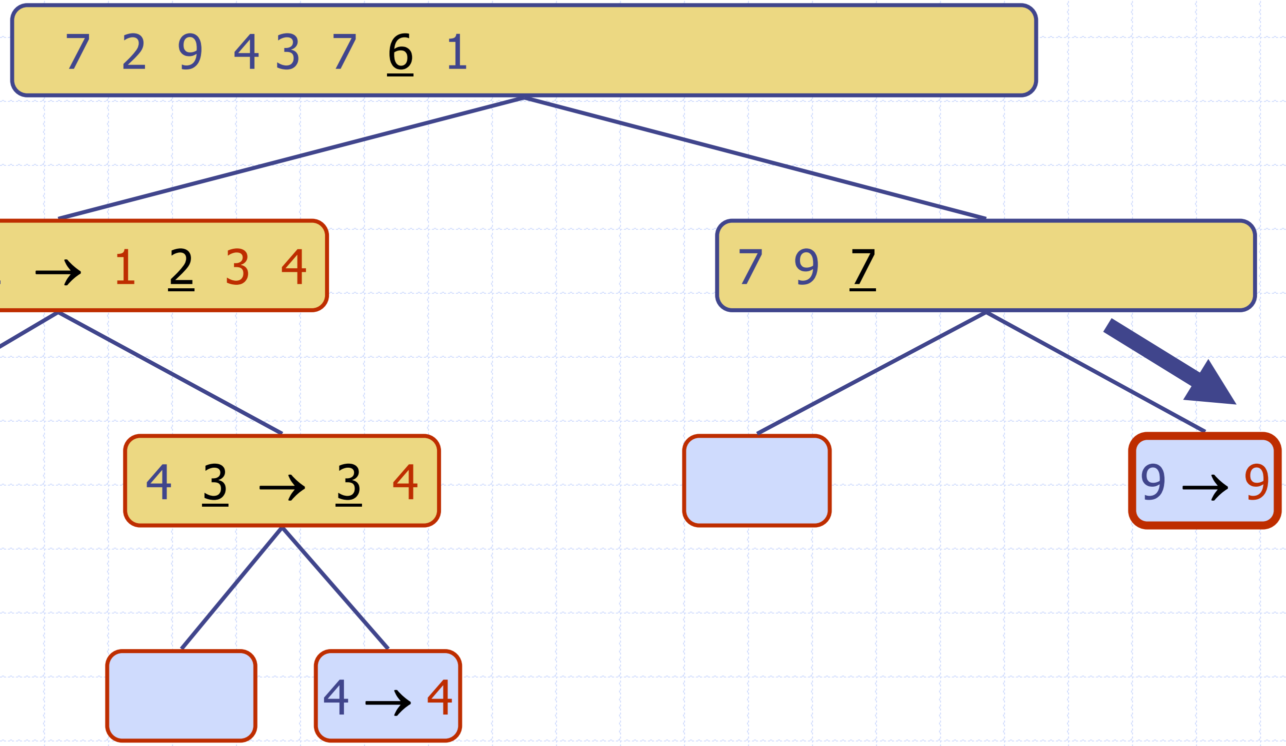
Execution Example (cont.)

- Recursive call, pivot selection



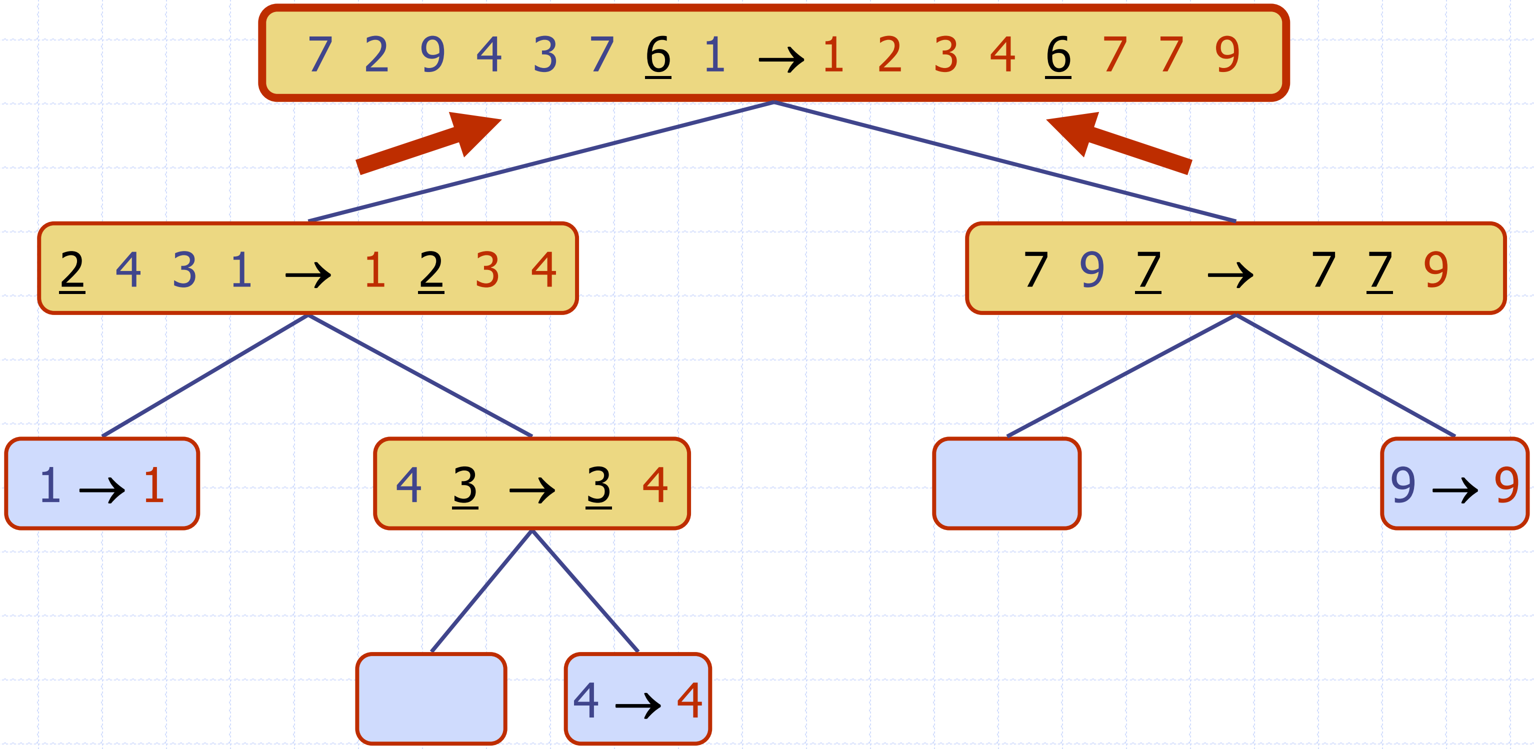
Execution Example (cont.)

- Partition, …, recursive call, base case



Execution Example (cont.)

- Join, join



Worst-case Running Time

- The worst case for quick-sort occurs when the pivot is the unique minimum or maximum element
- One of $\boldsymbol{L}$ and $\boldsymbol{G}$ has size $\boldsymbol{n} - 1$ and the other has size 0
- The running time is proportional to the sum

$$\boldsymbol{n} + (\boldsymbol{n} - 1) + \ldots + 2 + 1$$

- Thus, the worst-case running time of quick-sort is $\boldsymbol{O}(\boldsymbol{n}^2)$

depth	time
0	$\boldsymbol{n}$
1	$\boldsymbol{n} - 1$
…	…
$\boldsymbol{n} - 1$	1