Spring 2023 CSDS 410 Analysis of Algorithms
Prof. Dianne Foreback Assignment 3

Due Sunday, Oct 22, 2023 before 11:59 p.m. in Canvas
Possible 106/100 points

Directions:
Please upload your Assignment 3 into Canvas and make certain that the quality of the upload is clear. Also, enumerate your answers; e.g. before your answer for problem 1, preceded it with 1a, 1b, 1c, etc.

Please double check that your assignment is properly submitted into Canvas and is visible. Since file dates can be modified, and out of fairness to all, assignments not uploaded into Canvas by the final due date will not be accepted. That is, if your file is on a Google Drive but you forgot to place it into Canvas, it will not be accepted even if the file date shows an acceptable modify date.

Note that this assignment **may** be handwritten providing your writing is easily read.

Assignment Material:
This assignment covers graphs. Submit your work on canvas. Always check canvas for updates and corrections. Unless otherwise stated, whenever a question asks you to **describe an algorithm**, you should:

a. Explain the main concept of your algorithm.
b. Give pseudo-code.
c. Present an example of running your algorithm.
d. Prove/justify its correctness and its running time.

Please label your problem answers with 1a. 1b. 1c. 1d as appropriate.

Problems:

1. [12 points] Consider the DirectedDFS algorithm 13.11 from the book or as given in class. It does not distinguish between forward and cross edges.
a. Modify this algorithm to differentiate between these edges; that is, give its pseudocode. Thus, edges can be one of four types: tree, back, cross or forward.
b. Give an example of running your algorithm.
c. Prove its correctness for labeling these four edges and its running time.

2. [10 points, 4 points for a and 6 points for b]

Bob loves foreign languages and wants to plan his course schedule to take the following nine language courses: LA15, LA16, LA22, LA31, LA32, LA126, LA127, LA141, and LA169. The course prerequisites are:

- LA15: (none)
- LA16: LA15
- LA22: (none)
- LA31: LA15
- LA32: LA16, LA31
- LA126: LA22, LA32
- LA127: LA16
- LA141: LA22, LA16
- LA169: LA32.

a. Draw a directed graph to model the course and prerequisites.

b. Find the sequence of courses that allows Bob to satisfy all the prerequisites using the topological sort algorithm explained in class. Consider the prerequisites as an adjacency list in the order that

prerequisites are given. Thus, there will be a distinct topological sort ordering based on the adjacency list. Example, for LA32, the adjacency list is LA16, LA31. LA16 is first in the list followed by LA31.

3. [22 points, 4 points for a, 4 points for b, 7 points for c, 7 points for d]

Let G be a graph whose vertices are the integers 1 through 8, and let the adjacent vertices of each vertex be given by the table below:

vertex	adjacent vertices
1	(2, 3, 4)
2	(1, 3, 4)
3	(1, 2, 4)
4	(1, 2, 3, 6)
5	(6, 7, 8)
6	(4, 5, 7)
7	(5, 6, 8)
8	(5, 7)

a. Draw G.

b. Draw a schematic representation of the adjacency list structure of G like Figure 13.3.

c. Order the vertices as they are visited in a DFS traversal starting at vertex 1. Also, draw the spanning tree that represents this.

d. Order the vertices as they are visited in a BFS traversal starting at vertex 1. Also, draw the shortest path tree that represents this.

4. [30 points – 10 points for each part a, b and c] Justify Theorem 13.11. You will need to prove each part a, b and c, of the theorem. Consider using induction or a contradiction.

Theorem 13.11: Let G be an undirected graph with n vertices and m edges. Then we have the following:

a. If G is connected, then $m \geq n - 1$

b. If G is a tree, then $m = n - 1$

c. If G is a forest, then $m \leq n - 1$

(continued on the next page)

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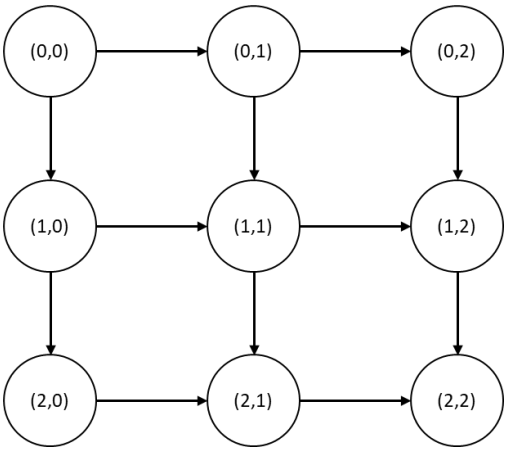
5. [16 pts] This question pertains to topological sorting on a grid graph.

a. Give four distinct orderings of the nodes in the following directed acyclic graph.

b. Generalize your four orderings to the case of the graph with the same connectivity pattern on an $n \; x \; n$ grid.



6. [16 pts] Consider an undirected graph G=(V, E). Describe an O(n+m) time algorithm based on BFS that returns the path between 2 vertices of a tree.

a. Explain the main concept of your algorithm.
b. Give pseudo-code.
c. Present an example of running your algorithm.
d. Prove/justify its correctness and its running time.