Case Western Reserve University
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Graph Terminology & Representations

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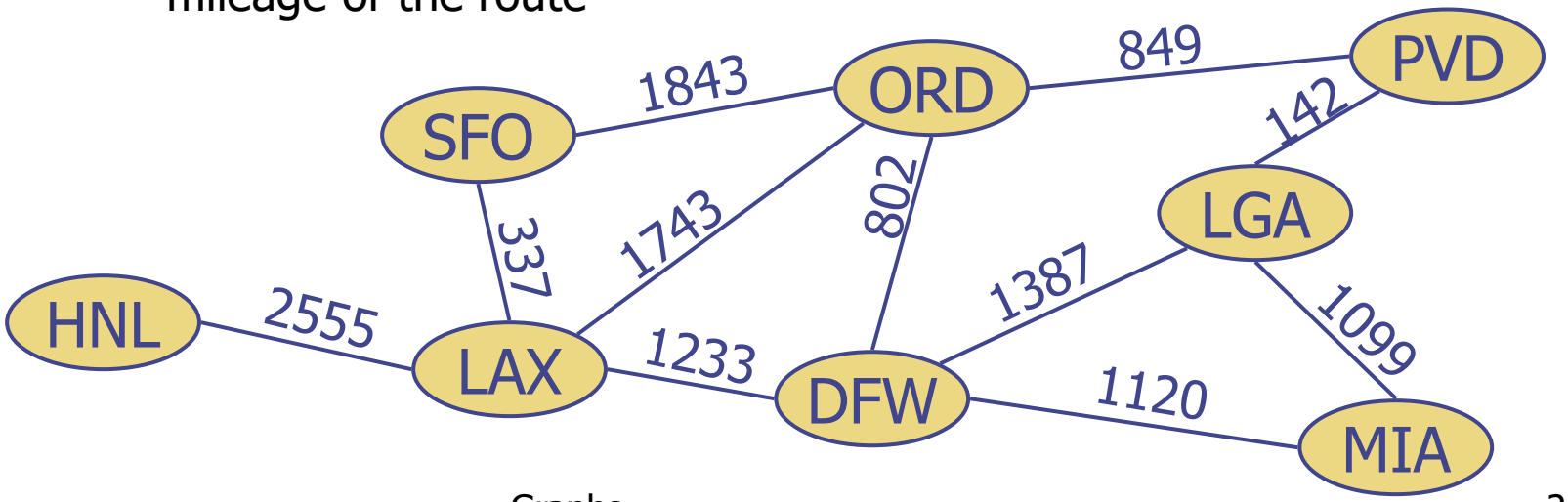
Information on slides originating from *Algorithm Design & Applications* by Michael T. Goodrich and Roberto Tamassia, © 2015 John Wiley & Sons, Inc.Goodrich and Tamassia, ISBN: 978-1118335918.

Reading Material

- ***Algorithm Design & Applications*** **by Michael T. Goodrich and Roberto Tamassia**
  - **Chapter 13 Section 13.1**

Graphs

- A graph is a pair (V, E), where
  - V is a set of nodes, called vertices
  - E is a collection of pairs of vertices, called edges
  - Vertices and edges are positions and store elements
- Example:
  - A vertex represents an airport and stores the three-letter airport code
  - An edge represents a flight route between two airports and stores the mileage of the route

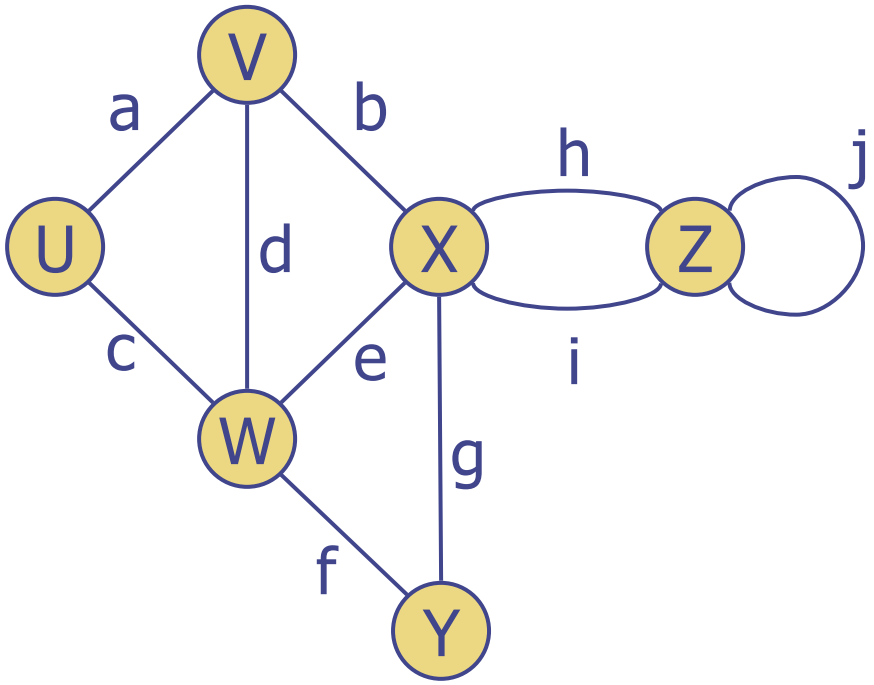


Edge Types

- Directed edge
  - ordered pair of vertices (u,v)
  - first vertex u is the origin
  - second vertex v is the destination
  - e.g., a flight
- Undirected edge
  - unordered pair of vertices (u,v)
  - e.g., a flight route
- Directed graph
  - all the edges are directed
  - e.g., route network
- Undirected graph
  - all the edges are undirected
  - e.g., flight network

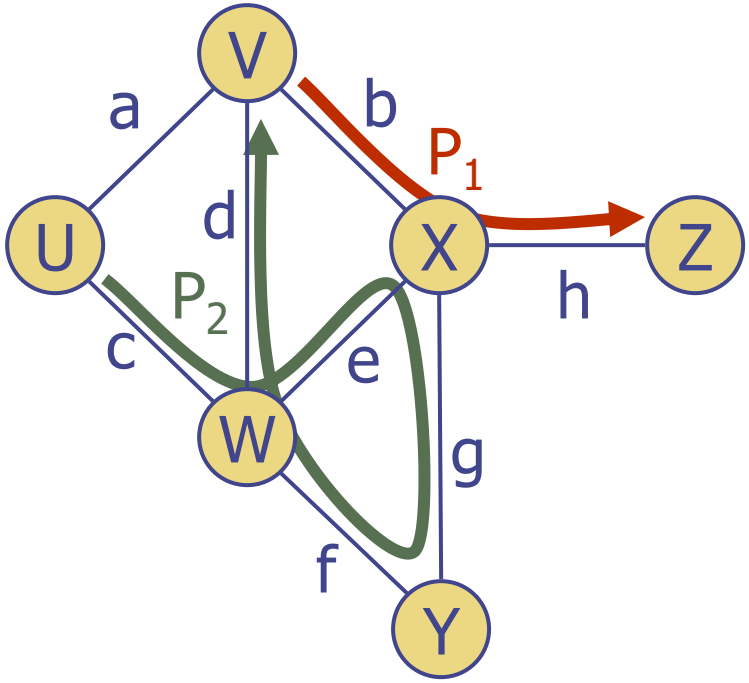
Terminology

- End vertices (or endpoints) of an edge
  - U and V are the endpoints of a
- Edges incident on a vertex
  - a, d, and b are incident on V
- Adjacent vertices
  - U and V are adjacent
- Degree of a vertex
  - X has degree 5
- Parallel edges
  - h and i are parallel edges
- Self-loop
  - j is a self-loop



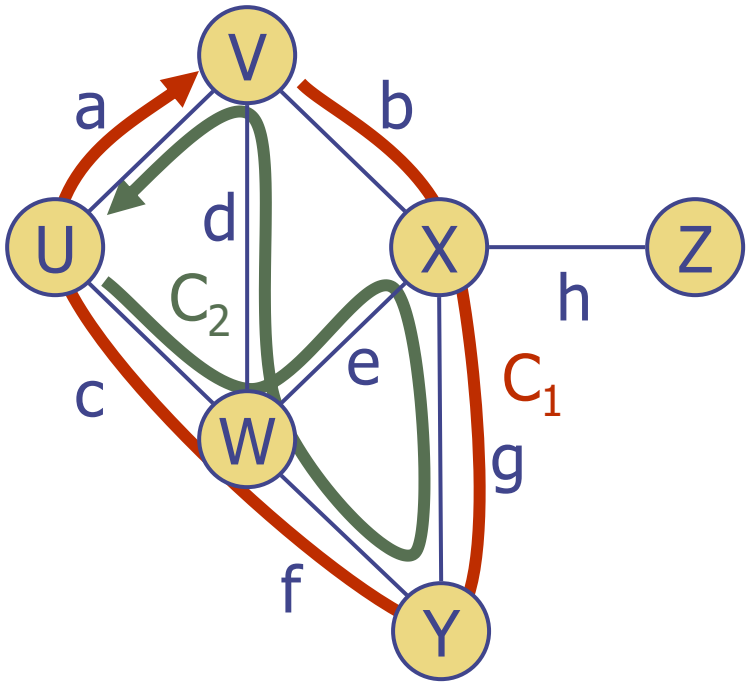
Terminology (cont.)

- Path
  - sequence of alternating vertices and edges
  - begins with a vertex
  - ends with a vertex
  - each edge is preceded and followed by its endpoints
- Simple path
  - path such that all its vertices and edges are distinct
- Examples
  - P_1=(V,b,X,h,Z) is a simple path
  - P_2=(U,c,W,e,X,g,Y,f,W,d,V) is a path that is not simple



Terminology (cont.)

- Cycle
  - circular sequence of alternating vertices and edges
  - each edge is preceded and followed by its endpoints
- Simple cycle
  - cycle such that all its vertices and edges are distinct
- Examples
  - C_1=(V,b,X,g,Y,f,W,c,U,a,↵) is a simple cycle
  - C_2=(U,c,W,e,X,g,Y,f,W,d,V,a,↵) is a cycle that is not simple



Properties for Undirected Graph

Property 1

$\sum_v \deg(v) = 2m$

Proof: each edge (u,v) is counted twice. One by u and one by v.

Property 2

In an undirected graph with no self-loops and no multiple edges

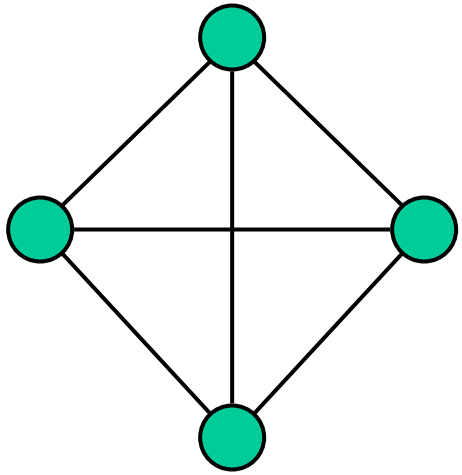
$m \leq n\,(n-1)/2$

Proof: each vertex has degree at most $(n-1)$

What is the bound for a directed graph?

Notation

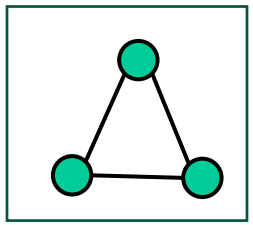
n	number of vertices
m	number of edges
$\deg(v)$	degree of vertex v



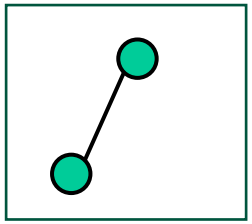
Example

- $n = 4$
- $m = 6$
- $\deg(v) = 3$

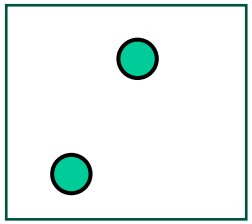
Yet Even More Terminology



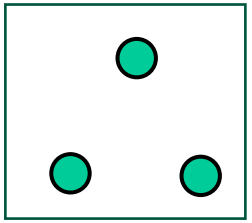
Graph G



- subgraph
- not spanning
- tree



- subgraph
- not spanning
- not a tree



- subgraph
- spanning
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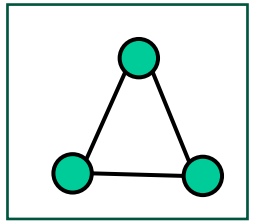
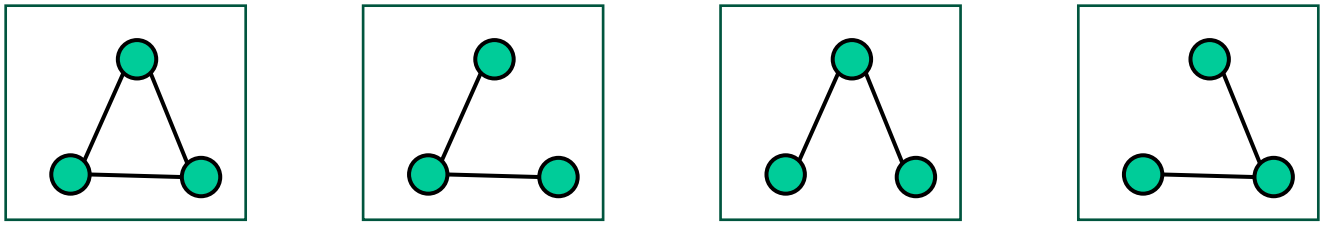


- subgraph
- spanning
- not a tree

- subgraph $G'=(V', E')$ of graph $G=(V, E)$ is a graph with $V' \subseteq V$ and $E' \subseteq E$
- spanning subgraph is a subgraph with $V'=V$
  - "spanning" since it "spans" all vertices of the graph
- tree is a connected graph with no cycle

Spanning Tree

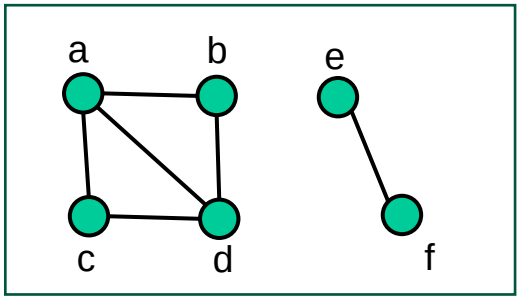


Graph G

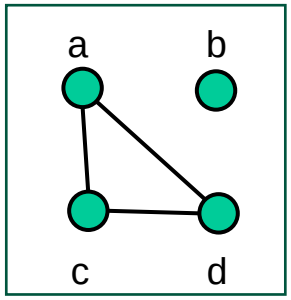
Spanning Trees of Graph G

- spanning tree is a spanning subgraph that is also a tree

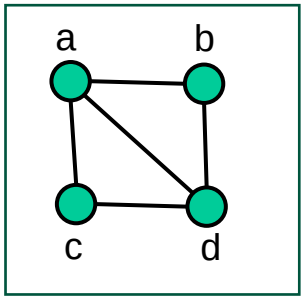
Connected Component



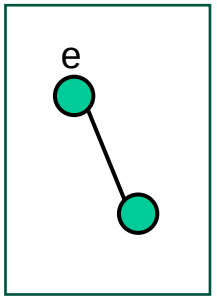
Graph G



not a conn comp of G



conn comp of G



conn comp of G

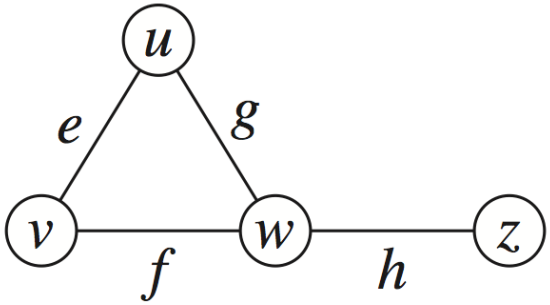
- graph is connected if for every 2 vertices u and v, there is a path from u to v
- connected component of a graph G=(V,E) is a maximal connected subgraph
  - maximal if no vertex can be added to preserve the connected component of the graph

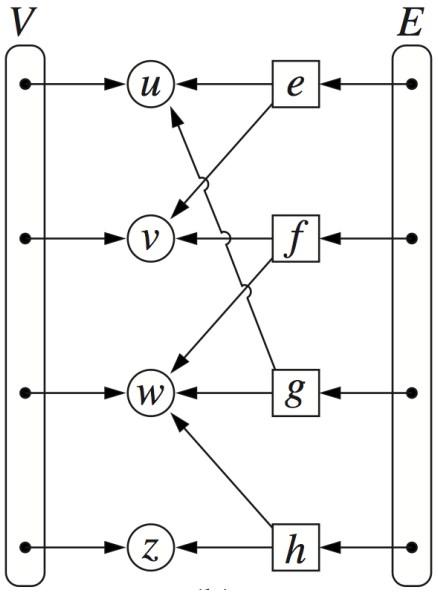
Vertices and Edges

- A **graph** is a collection of **vertices** and **edges**.
- A **Vertex** is can be an abstract unlabeled object or it can be labeled (e.g., with an integer number or an airport code) or it can store other objects
- An **Edge** can likewise be an abstract unlabeled object or it can be labeled (e.g., a flight number, travel distance, cost), or it can also store other objects.

Edge List Structure

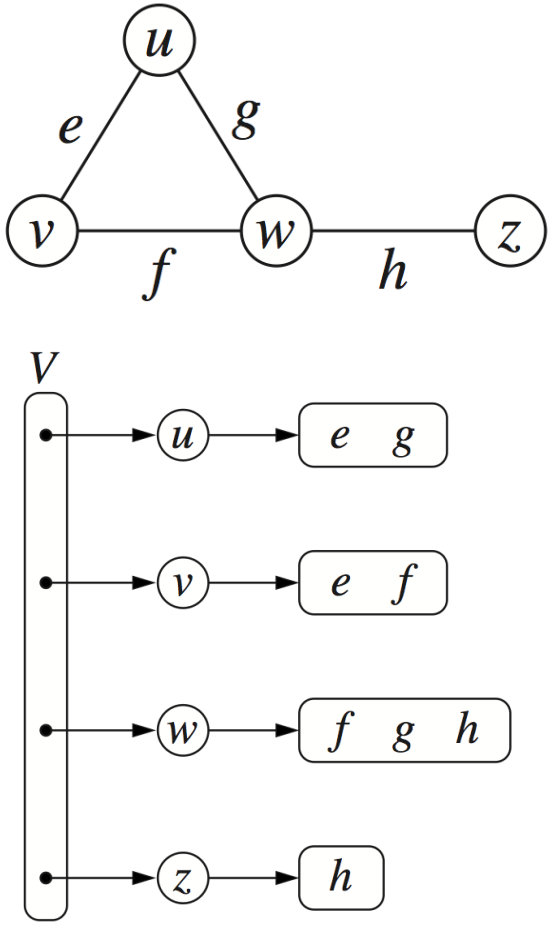
- Vertex object
  - element
  - reference to position in vertex sequence
- Edge object
  - element
  - origin vertex object
  - destination vertex object
  - reference to position in edge sequence
- Vertex sequence
  - sequence of vertex objects
- Edge sequence
  - sequence of edge objects





Adjacency List Structure

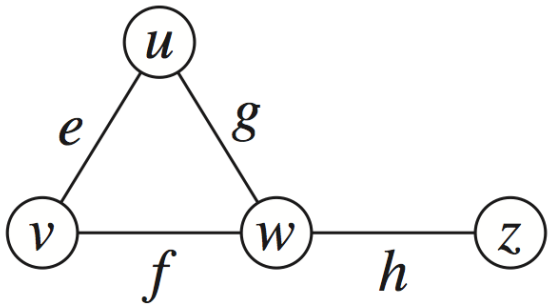
- Incidence sequence for each vertex
  - sequence of references to edge objects of incident edges
- Augmented edge objects
  - references to associated positions in incidence sequences of end vertices



Example of Incidence sequence for each vertex

Adjacency Matrix Structure

- Edge list structure
- Augmented vertex objects
  - Integer key (index) associated with vertex
- 2D-array adjacency array
  - Reference to edge object for adjacent vertices
  - Null for non nonadjacent vertices
- The "old fashioned" version just has 0 for no edge and 1 for edge



		0	1	2	3
u	0		e	g	
v	1	e		f	
w	2	g	f		h
z	3			h	

A digraph G and adjacency list structure of G

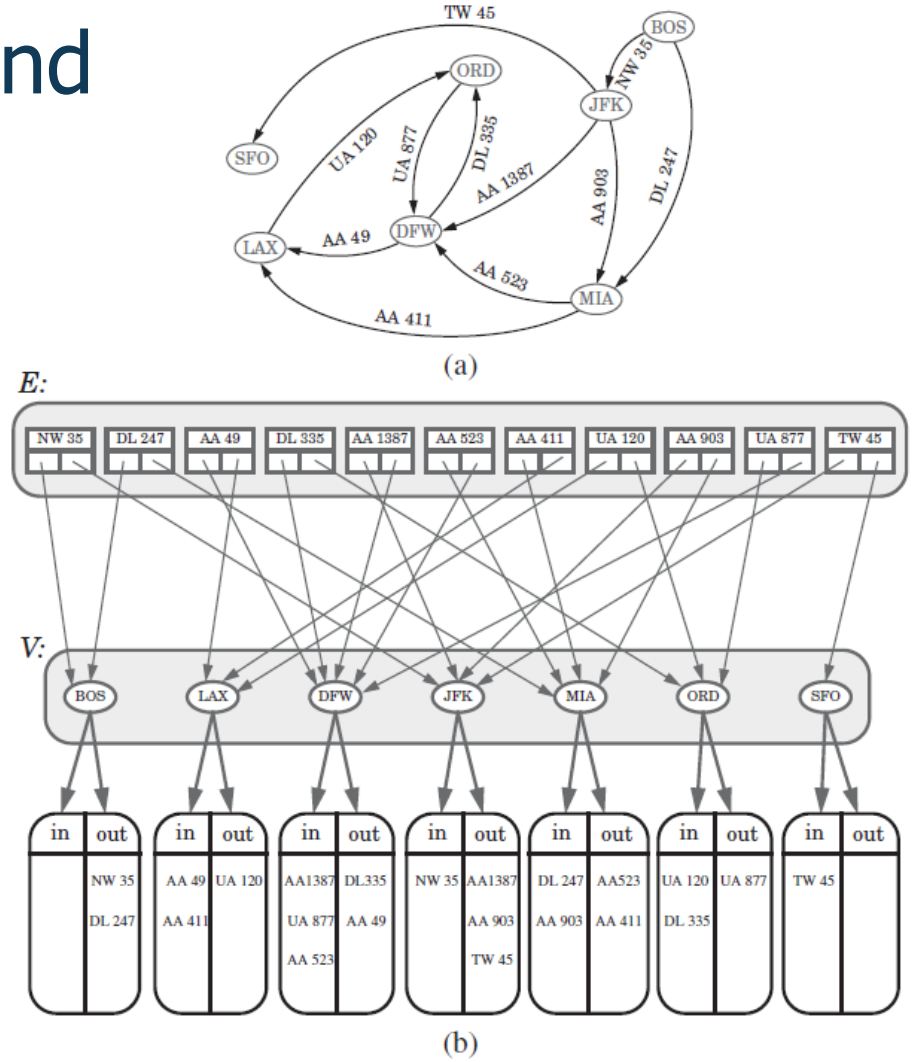


Figure 13.3: (a) A directed graph G; (b) a schematic representation of the adjacency list structure of G. In this example, we have a set of vertex objects and set of edge objects. Each edge object has pointers to its two end vertices and each vertex object has pointers to the two parts of its adjacency list, which store references to incident edges, one part for incoming edges and one for outgoing edges.

Graph Operations

- Return the number, n, of vertices in G.
- Return the number, m, of edges in G.
- Return a set or list containing all n vertices in G.
- Return a set or list containing all m edges in G.
- Return some vertex, v, in G.
- Return the degree, deg(v), of a given vertex, v, in G.
- Return a set or list containing all the edges incident upon a given vertex, v, in G.
- Return a set or list containing all the vertices adjacent to a given vertex, v, in G.
- Return the two end vertices of an edge, e, in G; if e is directed, indicate which vertex is the origin of e and which is the destination of e.
- Return whether two given vertices, v and w, are adjacent in G.

Graph Operations, Continued

- Indicate whether a given edge, e, is directed in G.
- Return the in-degree of v, inDegree(v).
- Return a set or list containing all the incoming (or outgoing) edges incident upon a given vertex, v, in G.
- Return a set or list containing all the vertices adjacent to a given vertex, v, along incoming (or outgoing) edges in G.

- Insert a new directed (or undirected) edge, e, between two given vertices, v and w, in G.
- Insert a new (isolated) vertex, v, in G.
- Remove a given edge, e, from G.
- Remove a given vertex, v, and all its incident edges from G.

Performance

(All bounds are big-oh running times, except for "Space")

▪ n vertices, m edges ▪ no parallel edges ▪ no self-loops	Edge List	Adjacency List	Adjacency Matrix
Space	$n + m$	$n + m$	n^2
incidentEdges(v)	m	$\deg(v)$	n
areAdjacent (v, w)	m	$\min(\deg(v), \deg(w))$	1
insertVertex(o)	1	1	n^2
insertEdge(v, w, o)	1	1	1
removeVertex(v)	m	$\deg(v)$	n^2
removeEdge(e)	1	1	1

Thank You !

Questions ?