Spring 2023 CSDS 410 Analysis of Algorithms
Prof. Dianne Foreback Assignment 4

Due Sunday, Nov 5th before 11:59 p.m. in Canvas
Possible 100 points

Directions:
Please upload your Assignment 4 into Canvas and make certain that the quality of the upload is clear. Also, enumerate your answers; e.g. before your answer for problem 1, preceded it with 1a, 1b, 1c, etc.

Please double check that your assignment is properly submitted into Canvas and is visible. Since file dates can be modified, and out of fairness to all, assignments not uploaded into Canvas by the final due date will not be accepted. That is, if your file is on a Google Drive but you forgot to place it into Canvas, it will not be accepted even if the file date shows an acceptable modify date.

Note that this assignment **may** be handwritten providing your writing is easily read.

Assignment Material:
This assignment covers Dynamic Programming and Greedy Algorithms.

Please label your problem answers with 1a. 1b. 1c. 1d as appropriate.

Problems:

1. [24 = 2+14+4+4] Suppose that in an instance of the coins-in-a-line game the coins have the following values in the line:
   {5, 1, 7, 5, 10, 9, 7, 8}

   a. What is the maximum that the first player, Alice, can win, assuming that the second player, Bob, plays optimally?
   b. Using the algorithm from class, **show the matrix** with correct values for each matrix element, when appropriate, for this example. The matrix is an $n\ x\ n$ matrix. The pseudo-code is also available in the presentation for this problem. Read part c of this problem to help with this.
   c. Aside from the two base cases, there is a pattern for determining the value stored in the entry M[i,j] in the matrix. What is this pattern? That is, rewrite the following **wrong** pattern for M[i, j].

      M[i,j] = max { C(i) + min {3 cells down from M[i,j], 1 cell up from M[i,j] },
      C(j) + min{1 cell diagonal down left from M[i,j], 2 cells right} }

   d. List in order the coins that Alice picks.

2. [20 = 12+4+4] This is a 0-1 Knapsack Problem. Let S = {item1, item2, item3, item4, item5} be a collection of objects with benefit-weight values item1:($100, 20 lbs), item2:($20,10 lbs), item3:($30,5 lbs), item4:($160, 40 lbs), item5:($90, 30 lbs). This is the order that must be considered when answer questions on this problem. Notice, this is the example given in class except that collection of objects are ordered differently. You must keep the ordering provided in this example. You must show work for part a to receive points for any part of this question.

   a. What is the maximum **value** obtained with a knapsack that can hold 60 lbs? You must show your work by presenting an the matrix with values according to the algorithm.

You can give W in increments of 5. That is your matrix will look something like this. Put the correct values in this matrix.

	0	5	10	15	20	25	30	35	40	45	50	55	60
0													
1													
2													
3													
4													
5													

b. What items are selected?
c. In class, the items in the set were in non-decreasing by weight. Does the order in which the items in the set produce different answers? Briefly explain why/why not?

3. [8] Exercise R-12.6 from the book. Your work must be shown to receive points otherwise no points are earned.

4. [8] Exercise R-12.9 from the book. **Hint**: Let n represent the weight of the sack. You must explain the variables for the knapsack problems (both the 0-1 and fractional). You must also explain what conditions must be assumed for the 0-1 Knapsack problem and then the conditions that must be assumed for the fractional knapsack problem with respect to this Internet auction that Sally is hosting. Do not give general information.

5. [8] Show that, in the coins-in-a-line-game, a greedy strategy of having the first player, Alice, always choose the available coin with the highest value will not necessarily result in an optimal solution (or even a winning solution) for her. Do this by giving an example with 6 coins.

6. [8] Show that, in the coins-in-a-line game, a greedy-denial strategy of having the first player, Alice, always choose the available coin that minimizes the maximum value of the coin available to Bob will not necessarily result in an optimal solution for her. You can do this by giving an example with coins.

7. [8] Exercise R-10.1 from the book. Show your work.

8. [8] Exercise R-10.3 from the book. Show your work.

9. [8] Exercise C-10.2 from the book.