Problem introduction

Using the monthly sales data (in millions of dollars) of a manufacturing company over last eight years to predict sales data each month in 2022, and find the best Exponential Smoothing model to predict.

Problem 1

Use first two years of data to estimate initial (time 0) Level (L0), Trend (T0), and

the additive seasonal factors for each month of the year.

L_0 = 101.311595

T_0 = 1.5863627

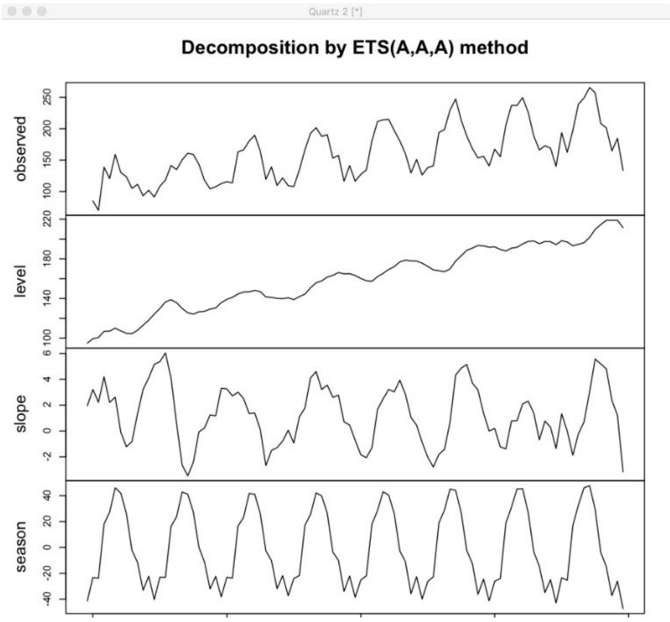
Additive seasonal factors:

Dec	Nov	Oct	Sep	Aug	Jul	Jun	May	Apr	Mar	Feb	Jan
0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11
-14.981	-19.176	25.34098	11.28012	36.7052	26.05441	20.05031	1.37636	-9.68444	-27.1614	-22.6714	-27.1332

Problem 2

Using smoothing constants α=0.2, β=0.1, and γ=0.1 apply additive trend-seasonal model to your data and compute Mean % Error, Mean Absolute % Error, RMSE (Root Mean Square Error), and other indicators. Discuss what information these indicators provide. Use your model to forecast next 12 months.

<1> Additive trend-seasonal model

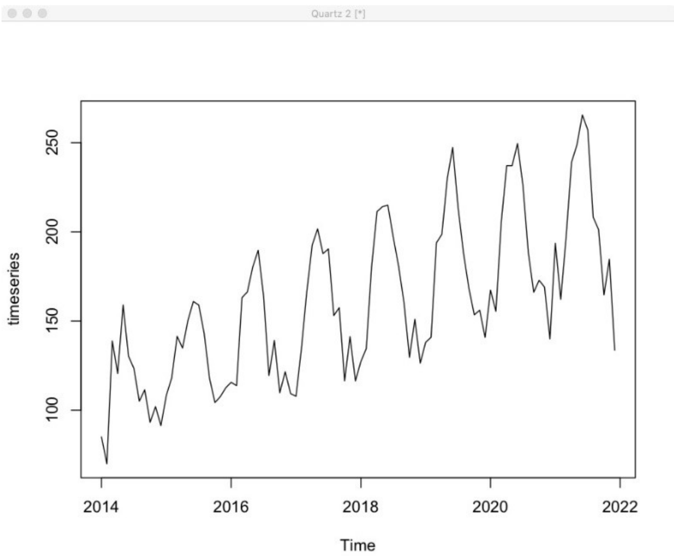


Level component = L_t

Slope = T_t

Season = S_t

<2>Time series



The sales data observed between 2014 and 2022 in the manufacturing company, is shown in this picture.

<3> The Indicators of additive trend-seasonal model

```
> accuracy(hw2_forecast)
                     ME     RMSE      MAE        MPE     MAPE      MASE      ACF1
Training set -0.5316714 15.27477 12.28863 -0.8857627 8.203792 0.7922201 0.2586142
```

<4>What are Mean % Error, Mean Absolute % Error and RMSE?

MPE: The **mean percentage error (MPE)** is the computed average of percentage errors by which forecasts of a model differ from actual Sales value of the quantity being forecast, which shows our forecast on average is off 0.8857627%.

MAPE: Mean Absolute Percentage Error (MAPE) is the mean of all absolute percentage errors between the predicted sales and actual sales values, which shows our forecast on average is off 8.203792%.

MASE: It is the mean absolute error of the forecast sales values, divided by the mean absolute error of the in-sample one-step naive forecast, which is equal to 0.7922201.

RMSE: It is difference between prediction sales and truth sales each month, which is related to the best model we will choose.

<5> Forecast of Sales data in 2022 for additive trend-seasonal model

```
> forecast(hw2_forecast, level=0.95, h=12)
         Point Forecast     Lo 95    Hi 95
Jan 2022       184.8032 152.00773 217.5986
Feb 2022       179.7021 145.46265 213.9415
Mar 2022       218.9038 182.23742 255.5702
Apr 2022       231.5653 191.39929 271.7314
May 2022       241.7543 197.02726 286.4813
Jun 2022       240.2469 189.97246 290.5214
Jul 2022       218.8701 162.16145 275.5787
Aug 2022       182.4644 118.53435 246.3945
Sep 2022       168.0413  96.19011 239.8925
Oct 2022       142.5685  62.16954 222.9675
Nov 2022       150.5748  61.06072 240.0889
Dec 2022       126.2701  27.12150 225.4186
```

Problem 3

Repeat Problem (1) and problem (2) for a multiplicative trend-seasonal model. Compare this model to the additive trend-seasonal model obtained in problem (2). Use your model to forecast next 12 months.

<1> estimate initial (time 0) Level (L0), Trend (T0), and the multiplicative seasonal factors

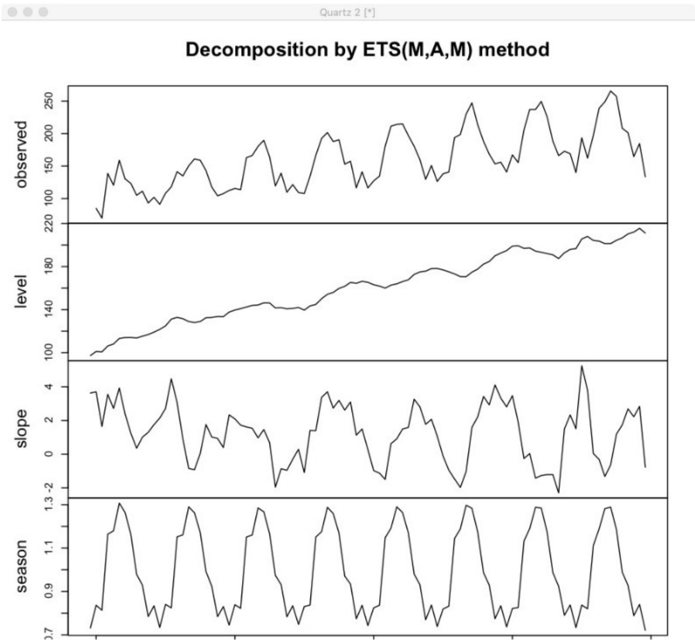
$L_0 = 101.311595$

$T_0 = 1.5863627$

multiplicative seasonal factors

Dec	Nov	Oct	Sep	Aug	Jul	Jun	May	Apr	Mar	Feb	Jan
0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11
0.860069	0.81569	1.223991	1.095571	1.318348	1.211609	1.157615	1.002281	0.922847	0.783218	0.822838	0.785924

<2> multiplicative trend-seasonal model



Level component = L_t

Slope = T_t

Season = S_t

<3> The Indicators of multiplicative trend-seasonal model

```
> accuracy(hw2_forecast1)
                     ME    RMSE      MAE        MPE     MAPE    MASE       ACF1
Training set -0.4882295 13.1621 10.04798 -0.8662507 6.530475 0.64777 0.02913443
```

<4> What are Mean % Error, Mean Absolute % Error and RMSE?

MPE: The **mean percentage error (MPE)** is the computed average of percentage errors by which forecasts of a model differ from actual Sales value of the quantity being forecast, which shows our forecast on average is off 0.8662507%.

MAPE: Mean Absolute Percentage Error (MAPE) is the mean of all absolute percentage errors between the predicted sales and actual sales values, which shows our forecast on average is off 6.530475%.

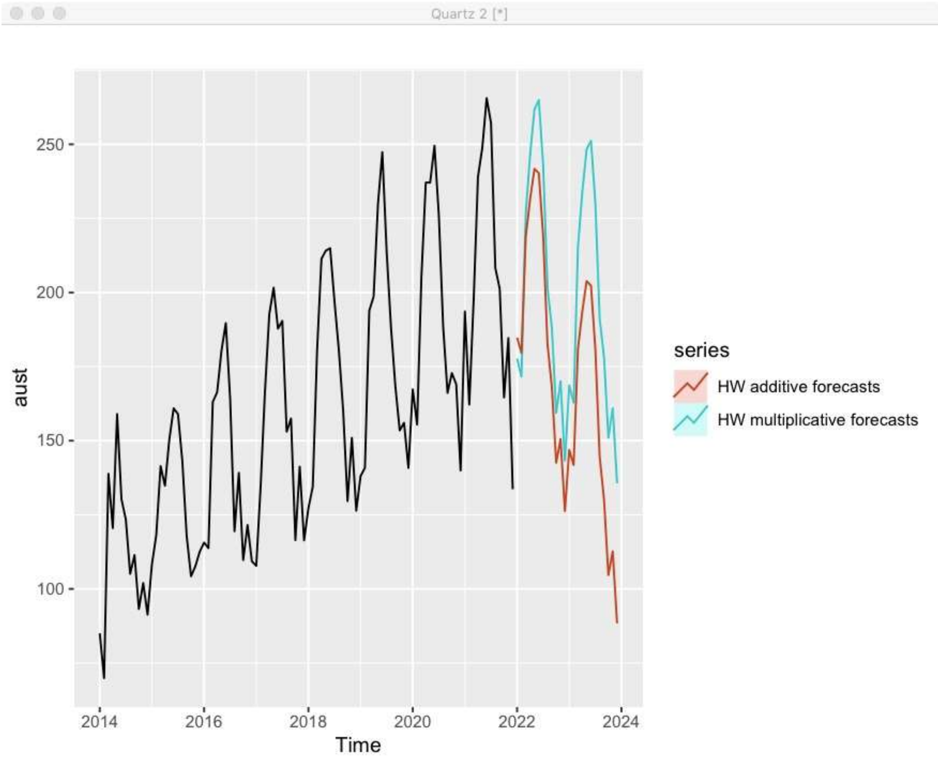
MASE: It is the mean absolute error of the forecast sales values, divided by the mean absolute error of the in-sample one-step naive forecast, which is equal to 0.64777.

RMSE: It is difference between prediction sales and truth sales each month, which is related to the best model we will choose.

<5> Forecast of Sales data in 2022 for multiplicative trend-seasonal model

```
> forecast(hw2_forecast1, level=0.95, h=12)
         Point Forecast     Lo 95    Hi 95
Jan 2022       176.1066 144.49244 207.7207
Feb 2022       171.8844 139.64854 204.1203
Mar 2022       232.1978 185.49664 278.8990
Apr 2022       247.1218 192.54938 301.6943
May 2022       265.5627 200.06035 331.0651
Jun 2022       266.0651 192.03970 340.0905
Jul 2022       243.7999 166.99864 320.6011
Aug 2022       202.7430 130.46093 275.0250
Sep 2022       189.4457 113.23558 265.6558
Oct 2022       160.3592  87.88890 232.8295
Nov 2022       170.2344  84.23792 256.2309
Dec 2022       145.5619  63.78907 227.3347
```

<6> Compare multiplicative trend-seasonal model to the additive trend-seasonal model



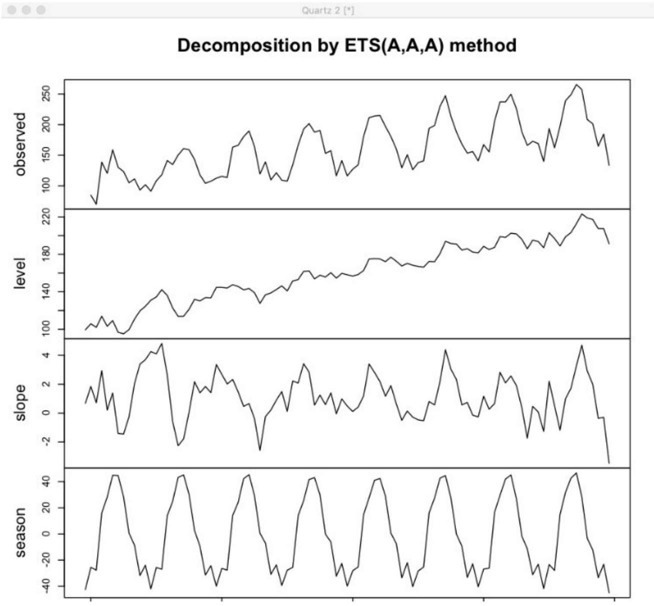
Problem 4

Use R to find the best additive trend-seasonal model as well as best multiplicative trend-seasonal model (restrict all smoothing constants to be between 0.1 and 0.5). Compare accuracy measures (RMSE, etc.) of these models to one obtained in part (b) and part (c). What is your forecast for next 12 months based on this model?

<1> The best additive trend-seasonal model

α=0.4999, β=0.1001, and γ=0.1008

```
ETS(A,A,A)

Call:
 ets(y = timeseries, model = "AAA", damped = F, lower = c(0.1,

 Call:
     0.1, 0.1, 0.8), upper = c(0.5, 0.5, 0.5, 0.9))

  Smoothing parameters:
    alpha = 0.4999
    beta  = 0.1001
    gamma = 0.1008

  Initial states:
    l = 99.3771
    b = 0.6808
    s = -42.4718 -24.1736 -33.129 -10.8566 -0.5956 28.4514
           47.5136 43.7002 30.9561 13.8411 -26.557 -26.6788

  sigma:  15.5915

     AIC     AICc      BIC
 982.0465 989.8926 1025.6404
```



Level component = L_t

Slope = T_t

Season = S_t

RMSE = 14.23307

```
> accuracy(hw2_forecast4) ### RMSE = 14.23307
                    ME     RMSE     MAE        MPE     MAPE      MASE        ACF1
Training set -0.431841 14.23307 11.7514 -0.5199155 7.765724 0.7575859 -0.08722734
```

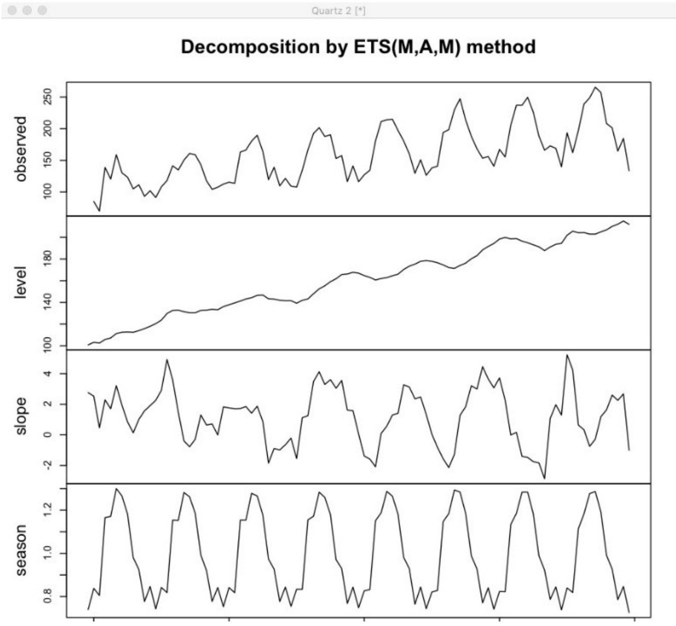
Forecast of Sales data in 2022 with best additive trend-seasonal model

```
> fcast1
         Point Forecast       Lo 95    Hi 95
Jan 2022       164.4524 133.89348 195.0112
Feb 2022       156.4580 120.82127 192.0947
Mar 2022       195.7624 154.19767 237.3272
Apr 2022       209.1885 160.96484 257.4122
May 2022       216.5917 161.07262 272.1108
Jun 2022       217.1515 153.77261 280.5304
Jul 2022       195.3125 123.56441 267.0605
Aug 2022       160.0820  79.49805 240.6660
Sep 2022       147.7970  57.94385 237.6502
Oct 2022       123.1420  23.61350 222.6705
Nov 2022       130.0194  20.43151 239.6073
Dec 2022       104.7004 -15.31222 224.7131
```

<2> The best multiplicative trend-seasonal model

α=0.1571, β=0.1, and γ=0.1

```
Call:
 ets(y = timeseries, model = "MAM", damped = F, lower = c(0.1,

 Call:
     0.1, 0.1, 0.8), upper = c(0.5, 0.5, 0.5, 0.9))

  Smoothing parameters:
    alpha = 0.1571
    beta  = 0.1
    gamma = 0.1

  Initial states:
    l = 100.8498
    b = 2.7606
    s = 0.7403 0.844 0.7732 0.9181 0.9863 1.1927
           1.2799 1.2822 1.1774 1.1452 0.8212 0.8396

  sigma:  0.0926

      AIC      AICc       BIC
 966.4976  974.3438 1010.0916
```



Level component = L_t

Slope = T_t

Season = S_t

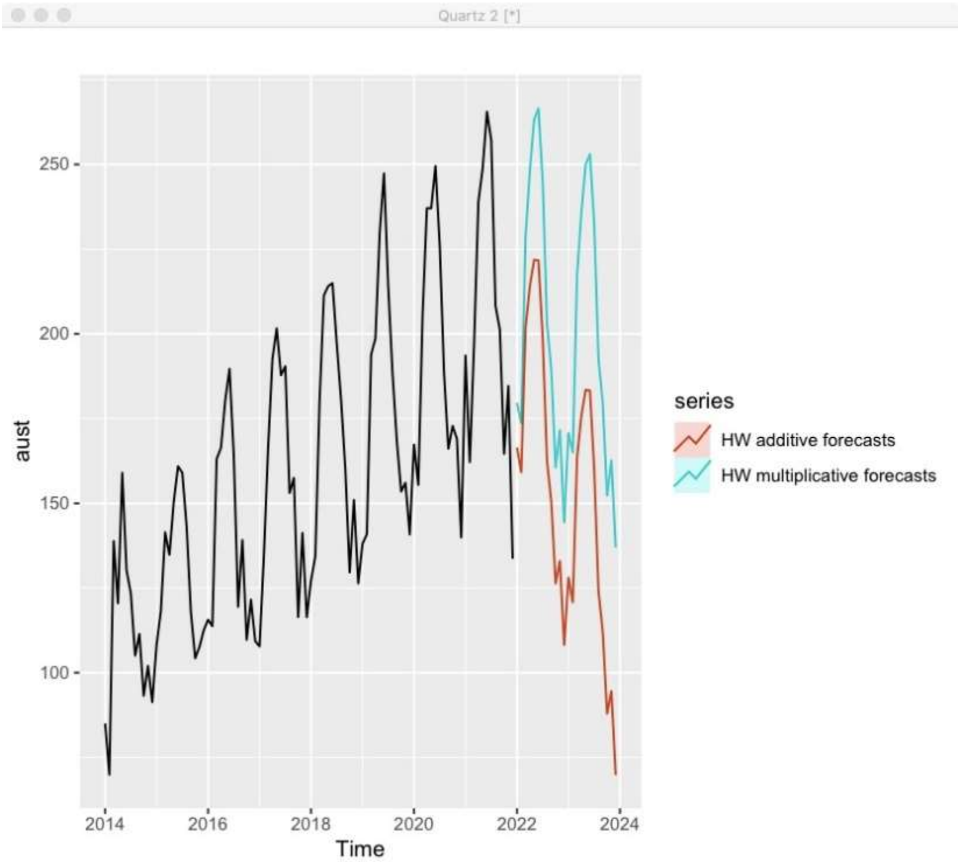
RMSE = 13.21017

```
> accuracy(hw2_forecast2) ### RMSE = 13.21017
                     ME     RMSE      MAE        MPE     MAPE      MASE       ACF1
Training set -0.4470279 13.21017 9.967699 -0.8027637 6.469812 0.6425947 0.06873661
```

Forecast of Sales data in 2022 with best multiplicative trend-seasonal model

```
> fcast
         Point Forecast     Lo 95    Hi 95
Jan 2022       177.1754 145.02676 209.3240
Feb 2022       171.8146 139.60648 204.0228
Mar 2022       232.8046 186.55976 279.0495
Apr 2022       246.0147 192.90458 299.1249
May 2022       264.1462 200.90027 327.3921
Jun 2022       264.8728 193.60010 336.1454
Jul 2022       244.1723 169.85631 318.4882
Aug 2022       201.9925 132.35801 271.6269
Sep 2022       188.1115 114.79761 261.4253
Oct 2022       158.6430  89.00680 228.2792
Nov 2022       170.0390  86.37064 253.7073
Dec 2022       145.1784  65.50448 224.8523
```

<3> Compare the best additive trend-seasonal model and best multiplicative trend-seasonal model in Time Series



<4> Find the best model

According to the lowest RMSE (13.21017), MPE and MAPE, we choose the multiplicative trend-seasonal model with α=0.1571, β=0.1, and γ=0.1 as the best model to predict the Sales.

Summary

The forecasts generated by the method with the additive seasonality display larger and increasing seasonal variation as the level of the forecasts increases compared to the forecasts generated by the method with multiplicative seasonality. The multiplicative seasonal component sums to approximately m = 12, The smoothing parameters and initial estimates for the components have been estimated by minimizing RMSE. With the additive method, the seasonal component is expressed in absolute terms in the scale of the observed series, and in the level equation the series is seasonally adjusted by subtracting the seasonal component. Within each year, the seasonal component will add up to approximately zero. With the multiplicative method, the seasonal component is expressed in relative terms (percentages), and the series is seasonally adjusted by dividing through by the seasonal component. Within each year, the seasonal component will sum up to approximately m. We apply Holt-Winters' method with both additive and multiplicative seasonality to forecast monthly sales in a manufacturing company.