2024-12-28 复习 · 设 $x(t)$ 为确定信号

$X(t)=A\cos(\omega_0 t+\Theta)$ 求 FT:

$$X(\omega)=\int_{-\infty}^{+\infty}X(t)\cdot e^{-j\omega t}dt=\int_{-\infty}^{+\infty}A\cos(\omega_0 t+\Theta)e^{-j\omega t}dt$$

拆

$$=A\int_{-\infty}^{+\infty}\{\cos(\omega_0 t)\cos\Theta-\sin(\omega_0 t)\sin\Theta\}\cdot e^{-j\omega t}dt$$

$$=A\cos\Theta\int_{-\infty}^{+\infty}\cos\omega_0 t\, e^{-j\omega t}dt-A\sin\Theta\int_{-\infty}^{+\infty}\sin\omega_0 t\, e^{-j\omega t}dt$$

$$\frac{1}{2}(e^{j\omega_0 t}+e^{-j\omega_0 t})\qquad\qquad \frac{1}{2}(e^{j\omega_0 t}-e^{-j\omega_0 t}).$$

$$=\frac{A\pi}{2}\cos\Theta\cdot(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))-\frac{A\pi}{2}\sin\Theta(\delta(\omega-\omega_0)-\delta(\omega+\omega_0))$$

这说明任意一个余弦信号，在频谱上总是只有两个点处的质量

① $\omega-\omega_0$ 处质量：$\frac{A\pi}{2}\cos\Theta-\frac{A\pi}{2}\sin\Theta$

② $\omega+\omega_0$ 处质量：$\frac{A\pi}{2}\cos\Theta+\frac{A\pi}{2}\sin\Theta$

因此，任意给定一个频谱图，我们可以将其分解成 $\sum_i P_i\delta_i$ 的形式

其中 $\delta_i=\delta(\omega-\omega_i)$，成对出现，可以分解成无数个 $\cos(\omega_i t+\Theta)$ 叠加的形式

设 $X(t)=A\cos(\omega_0 t+\Theta)$ 是随机信号，A 为常量，$\Theta\in[0,2\pi]$ 均匀

$$R_X(\tau)=E\{A\cos(\omega_0\tau+\Theta)\cdot A\cos(\Theta)\}=A^2\cdot E\{\frac{1}{2}[\cos(\omega_0\tau+2\Theta)+\cos\omega_0\tau]\}$$

$$=\frac{1}{2}A^2\cos\omega_0\tau$$

故 $$S_X(\omega)=\int_{-\infty}^{+\infty}R_X(\tau)\cdot e^{-j\omega\tau}d\tau=\frac{1}{2}A^2\int_{-\infty}^{+\infty}\cos\omega_0\tau\, e^{-j\omega\tau}d\tau=\frac{2\pi}{4}A^2\cdot(\delta(\omega+\omega_0)+\delta(\omega-\omega_0))$$

$$=\frac{A^2\pi}{2}\cdot(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$$

2024-12-28 复习

选择：设 $n(t) = \sum_k A_k \cos(\omega_0 t + \Delta\omega_k t + \Theta_k)$.

其中 $\Delta\omega_k$ 是一个小频率偏移（常数）.

其中 Θ_k 是均匀分布独立同分布

常数或随机变量

$$n(t) = \sum_k A_k \cdot \{\cos\omega_0 t \cos(\Delta\omega_k t + \Theta_k) - \sin\omega_0 t \cdot \sin(\Delta\omega_k t + \Theta_k)\}$$

$$= \underbrace{\left(\sum_k A_k \cos(\Delta\omega_k t + \Theta_k)\right)}_{n_c(t)\text{：同相分量}} \cdot \cos\omega_0 t - \underbrace{\left(\sum_k \sin(\Delta\omega_k t + \Theta_k)\right)}_{n_s(t)\text{：正交分量}} \cdot \sin\omega_0 t$$

对任意信号，我们都可以这样拆分，但对于带信号选取合适的 ω_0，能使得 $\Delta\omega_k$ 往往很小，这样我们在分析时才能将其近似为常量.

分析 $n_c(t)$ 和 $n_s(t)$ 的性质：关键在于 Θ_k 独立同均匀分布

① 低频特性：ω_0 选定合适时，可以让 $\Delta\omega_k t$ 很小，故 $n_c(t)$ 为低频信号

② $E[n_c(t)] = E[n_s(t)] = 0$. 为什么？因为 Θ_k 均匀分布独立同.

③ $\sigma_{n_c}^2 = \sigma_{n_s}^2 = \sigma_n^2$.

$$\sigma_{n_c}^2 = E[n_c^2(t)] - (E[\cancel{n_c(t)}]^{\,0})^2 = \left(\sum_k A_k \cos(\Delta\omega_k t + \Theta_k)\right)^2 \text{ 独立同.}$$

$$= \sum_k A_k^2 \cos^2(\Delta\omega_k t + \Theta_k)$$

$2\cos^2\theta - 1 = \cos 2\theta$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$$= \sum_k A_k^2 \cdot \frac{1}{2}(1 + \cos(2\Delta\omega_k t + 2\Theta_k)) = \sum_k \frac{1}{2} A_k^2.$$

2024-12-28 复习

$$S_{n_c}(\omega)=S_{n_s}(\omega)=\begin{cases}S(\omega+\omega_0)+S(\omega-\omega_0), & \omega<|B|\\ 0, & 其他\end{cases}$$

设 $X(t)=A\cos(\omega_0 t+\Theta)$ 则

$$R_X(\tau)=E\{X(\tau)\cdot X(0)\}=\frac{1}{2}A^2\cos(\omega_0\tau)\Leftrightarrow \overbrace{\frac{A^2\pi}{2}(\delta(\omega+\omega_0)+\delta(\omega-\omega_0))}^{S_X(\omega)}$$

对原信号做变换：$R_N(\tau)=\sum_k \frac{1}{2}A_k^2\cdot\cos\{(\omega_0+\Delta\omega_k)\tau\}$

$$\Updownarrow$$

$$S_N(\omega)=\sum_k\frac{A_k^2\pi}{2}\cdot\{\delta(\omega+(\omega_0+\Delta\omega_k))+\delta(\omega-(\omega_0+\Delta\omega_k))\}$$

右移　　左移ω_0

而 $S_{N_c}(\omega)=\sum_k\frac{1}{2}A_k^2\pi\{\delta(\omega+\Delta\omega_k)+\delta(\omega-\Delta\omega_k)\}$ 得证.

$E[n_c(t)n_s(t)]=0$：

$$E[n_c(t)n_s(t)]=E\{\sum_k A_k\cos(\Delta\omega_k t+\Theta_k)\cdot\sum_k A_k\sin(\Delta\omega_k t+\Theta_k)\}$$

$$=E\{\sum_k A_k^2\underbrace{\cos(\alpha_k)\sin(\alpha_k)}_{=\frac{1}{2}\sin(2\alpha_k)\xrightarrow{E}0}\}=0.$$

信号的复表示：设 $s(t)=\sum_k A_k\cos[\omega_k t+\varphi_k]$　$\omega_k>0$

则 $\hat{s}(t)=\sum_k A_k\sin[\omega_k t+\varphi_k]$ 为 $s(t)$ 的希尔伯特变换

而 $\tilde{s}(t)=s(t)+j\hat{s}(t)$ 则为实信号 $s(t)$ 的复表示、

$$=\sum_k A_k\cdot e^{j(\omega_k t+\varphi_k)}$$ 求频率域信号

$$\tilde{S}(\omega)=\sum_k(A_k\cdot e^{j\varphi_k})\cdot\{2\pi\delta(\omega-\omega_k)\}=2S(\omega)\cdot U(\omega).$$

$\boxed{单边双折.}$

①如何根据 $x(t)$ 计算 $\hat{x}(t)$ 即把 cos 换成 sin.

②对不是写成 cos 和 sin 信号?

$\left(* \frac{1}{\pi t}\right)$ 即可

$$x(t)=A(t)\cos(w_0 t+\varphi(t)) = A(t)\{\underbrace{\cos w_0 t}_{快变}\cos\varphi(t)-\underbrace{\sin w_0 t}_{快变}\sin\varphi(t)\}$$

$$a_k\cos(w_0 t+\theta_k) * \frac{1}{\pi t} = \int_{-\infty}^{+\infty} a_k\cdot\cos(w_0\xi+\theta_k)\cdot\frac{-1}{\pi(\xi-\tau)}d\xi$$

$$\cos(w_0\xi+\theta_k) \to \frac{1}{2}\left(e^{j(w_0\xi+\theta_k)}+e^{-j(w_0\xi+\theta_k)}\right)$$

$$= \frac{1}{2}\cdot e^{j\theta_k}\cdot a_k\boxed{\int_{-\infty}^{+\infty}\frac{1}{\pi(\tau-\xi)}\cdot e^{jw_0\xi}d\xi} + \frac{1}{2}e^{-j\theta_k}\cdot a_k\int_{-\infty}^{+\infty}\frac{1}{\pi(\tau-\xi)}\cdot e^{-jw_0\xi}d\xi$$

$$\frac{1}{\pi\xi}e^{jw_0(\tau-\xi)}d\xi \qquad\qquad \frac{1}{\pi\xi}e^{-jw_0(\tau-\xi)}$$

$$e^{jw_0\tau}\cdot\boxed{\int_{-\infty}^{+\infty}\frac{1}{\pi\xi}\cdot e^{-jw_0\xi}d\xi} \qquad ? \quad \boxed{-2j\,u(w-w_0)} \qquad e^{-jw_0\tau}\boxed{\int_{-\infty}^{+\infty}\frac{1}{\pi\xi}\cdot e^{jw_0\xi}} \to 2j\,u(w-w_0).$$

对 w_0 求导

对 w_0 积分

$$\int_{-\infty}^{+\infty}\frac{-j\xi}{\pi\xi}\cdot e^{-jw_0\xi}d\xi. \qquad -2j\,\delta(w-w_0)$$

$$= \frac{-j}{\pi}\int_{-\infty}^{+\infty}e^{-jw_0\xi}d\xi = \frac{-j}{\pi}\cdot 2\pi\,\delta(w-w_0)$$

证明 $\frac{1}{\pi t} \leftrightarrow -j\,\mathrm{sgn}(\omega)$

$$\int_{-\infty}^{+\infty} \frac{1}{\pi t} e^{-j\omega t} dt \xRightarrow{\text{对称性}} \int_{-\infty}^{+\infty} \frac{-jt}{\pi t} e^{-j\omega t} dt = \int_{-\infty}^{+\infty} -\frac{j}{\pi} e^{-j\omega t} dt$$

$$= -\frac{j}{\pi} \cdot 2\pi\delta(\omega) = -2j\delta(\omega) \xRightarrow{\text{对ω积分}} \underbrace{\boxed{-2j\,u(\omega)}}_{\text{我的理解}} +C \xrightarrow{\text{整体相位}} \underbrace{\boxed{-j\,\mathrm{sgn}(\omega)}}_{\text{书上答案}}$$

如何确定 $-2ju(\omega)+C$ 的 C 值. 看一个指定 ω 处的值.

比如 $\omega=0$ $\int_{-\infty}^{+\infty} \frac{1}{\pi t} e^{0} dt = 0$. (奇函数对称极限积分)

故 $C=-1$ 合理.

$$\boxed{\begin{aligned} &\delta(t) - \frac{1}{j\pi t} \xleftrightarrow{FT} 2u(\omega) = \mathrm{sgn}(\omega)+1 \\ &\frac{1}{\pi t} \xleftrightarrow{FT} -j\,\mathrm{sgn}(\omega) \\ &\delta(t) \xleftrightarrow{FT} 1 \end{aligned}}$$

$\therefore \boxed{u(t) \xleftrightarrow{FT} \pi\delta(\omega) + \frac{1}{j\omega}}$

$$\cancel{\int_{-\infty}^{+\infty} u(t) e^{-j\omega t} dt \xRightarrow{\text{对t求导}} \int_{-\infty}^{+\infty} \delta(t) e^{-j\omega t} + u(t) e^{-j\omega t} (-j\omega) dt}$$

$$\cancel{= \text{2个部分} \ 1 + (-j\omega) \int_{-\infty}^{+\infty} u(t) e^{-j\omega t} dt}$$

2024-12-28 复习

$$\int_{-\infty}^{+\infty} u(t) e^{-j\omega t} dt = \underbrace{\int_{0-}^{0+} u(t) e^{-j\omega t} dt}_{} + \underbrace{\int_{0}^{+\infty} e^{-j\omega t} dt}_{}$$

$$\left[-\frac{1}{-j\omega} e^{-j\omega t}\right]_{0}^{+\infty}$$

$$=$$

$$\frac{-1}{j\pi t} \leftrightarrow \operatorname{sgn}(\omega).$$

$$\boxed{\frac{-j\sin(\omega t)}{-j\omega}}$$

$$= +\underbrace{\frac{1}{j\omega}}_{} + \pi\delta(\omega)$$

$$\boxed{\frac{1}{\pi t} \leftrightarrow -j\operatorname{sgn}(\omega)}$$

$$\boxed{\delta(t) \leftrightarrow 1}$$

$$\boxed{u(t) \leftrightarrow \frac{1}{j\omega} + \pi\delta(\omega)}$$

$$\left[\frac{e^{-j\omega t}}{-j\omega}\right]^{+\infty} =$$

$t=$ $\pi\delta(\omega).$

已知: $\boxed{\frac{1}{2\pi}\int_{-\infty}^{+\infty} 2u(\omega) \cdot e^{j\omega t} d\omega = \delta(t) - \frac{1}{j\pi t}}$

求 $\int_{-\infty}^{+\infty} u(t) e^{-j\omega t} dt = -\int_{-\infty}^{+\infty} \underbrace{u(-t)}_{} \cdot e^{-j\omega(-t)} d(-t)$

$\#$ ~~$-\int_{-\infty}^{+\infty} u(t) e^{j\omega}$~~ $= -\int_{-\infty}^{+\infty} (1-u(t)) \cdot e^{j\omega t} d(-t)$

$u(-t) = 1 - u(t)$ $= \int_{-\infty}^{\infty} (1-u(t)) \cdot e^{j\omega t} dt$

$$= \underbrace{\int_{-\infty}^{+\infty} e^{j\omega t} dt}_{2\pi\delta(\omega)} - \underbrace{\int_{-\infty}^{+\infty} u(t) e^{j\omega t} dt}_{-\left\{\pi\delta(\omega) - \frac{1}{j\omega}\right\}} = \pi\delta(\omega) + \frac{1}{j\omega}.$$

2024-12-23

设 $F(\omega)=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$ 求 $\boxed{\int_{-\infty}^{+\infty} F(t)\cdot e^{-j\omega t}dt}$

$\updownarrow$

$$\boxed{f(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{j\omega t}d\omega}$$

$\downarrow$ 交换 w和 t

$$f(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(t)\cdot e^{j\omega t}dt$$

$\downarrow$

$$f(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(-t)\cdot e^{j\omega(-t)}d(-t)$$

$$=-\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(-t)\cdot e^{-j\omega t}dt$$

$$\boxed{\therefore \begin{cases} F(-t)\leftrightarrow -2\pi f(\omega) \\ f(t)\leftrightarrow F(\omega) \end{cases}}$$

$$\delta(t)-\frac{1}{j\pi t}\leftrightarrow 2u(\omega)$$

$$2u(-t)\leftrightarrow 2\pi\delta(\omega)-\frac{2}{j\omega}$$

$$u(-t)\leftrightarrow \pi\delta(\omega)-\frac{1}{j\omega}$$

$$u(t)=\boxed{1-u(-t)}=2\pi\delta(\omega)-\left\{\pi\delta(\omega)-\frac{1}{j\omega}\right\}$$

$$=\pi\delta(\omega)+\frac{1}{j\omega}$$

只记住三条最基本的原则
① $FT\{1\}=2\pi\delta(\omega)$
② 若 $FT\{f(t)\}=F(\omega)$ 则 $FT\{F(-t)\}=\overset{2\pi}{\cancel{-2\pi}}f(\omega)$
③ 若 $FT\{f(t)\}=F(\omega)$ 则 $FT\{f'(t)\}=(j\omega)F(\omega)$

2024-12-28

1、求

$FT\{\delta(t)\}$？ $\because FT\{1\}=2\pi\delta(\omega)$ 故 $\frac{1}{2\pi}\cdot FT\{1\}=\delta(\omega)$.

用② 故 $FT\{\delta(-t)\}=\frac{2\pi}{2\pi}\times\frac{1}{2\pi}=+1$ 得 $FT\{\delta(t)\}=1$

2. 求 $FT\{u(t)\}$

由于 $FT\{\delta(t)\}=1$ $\therefore FT\{u(t)\}=\frac{1}{j\omega}+\pi\delta(t)$

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① $FT\{u(t)\}=\frac{1}{j\omega}+\pi\delta(t)$

② ~~$FT\{f(t)\}=F(\omega)\Leftrightarrow FT\{\delta(t)\}=(j\omega)F(\omega)$~~

③ $FT\{f(t)\}=F(\omega)\Leftrightarrow FT\{F(-t)\}=-2\pi f(\omega)$.

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(1) 求 $FT\{\delta(t)\}$： $FT\{u(t)\}=\frac{1}{j\omega}+\pi\delta(\omega)$

$\Updownarrow$

$FT\{\delta(t)\}=1+\pi j\omega\delta(\omega)$ $\boxed{不对}$.

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① $\int_{-\infty}^{+\infty}\delta(\xi)f(\xi)d\xi=f(0)$ 性质.

② $\int_{-\infty}^{t}\delta(\xi)d\xi=u(t)$ 性质.

求 $\int_{-\infty}^{+\infty}u(t)e^{-j\omega t}dt=\int_{-\infty}^{+\infty}\int_{-\infty}^{t}\delta(\xi)d\xi\cdot e^{-j\omega t}dt$

$=\int_{-\infty}^{+\infty}\int_{-\infty}^{t}\delta(\xi)e^{-j\omega t}d\xi\,dt$

$=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\delta(\xi)e^{-j\omega t}\cdot\Gamma[\xi\le t]\,dt\,d\xi$

$$= \int_{-\infty}^{+\infty}\int_{\xi}^{+\infty} \delta(\xi)\, e^{-j\omega t}\, dt\, d\xi$$

$$= \int_{-\infty}^{+\infty} \delta(\xi)\cdot\left[\frac{1}{-j\omega}e^{-j\omega t}\right]_{\xi}^{+\infty} d\xi = \left[\frac{1}{-j\omega}e^{-j\omega t}\right]_{0}^{+\infty} = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$\downarrow$$

$$\cancel{\frac{1}{-j\omega}\,[\dots\delta(\omega)]}$$

$$求 \lim_{R\to+\infty}\left(\frac{e^{-j\omega R}}{-j\omega}\right) \xrightarrow{R等} e^{-j\omega R}.$$

$$\int_{-\infty}^{+\infty} e^{-j\omega t}\, dt = 2\pi\delta(\omega)$$

$$\downarrow$$

$$\left[-\frac{1}{j\omega}e^{-j\omega t}\right]_{-\infty}^{+\infty} = \overbrace{\left(\frac{e^{-j\omega R}}{-j\omega}\right)}^{\pi\delta(\omega)} - \overbrace{\left(\frac{e^{j\omega R}}{-j\omega}\right)}^{-\pi\delta(\omega)} = 2\pi\delta(\omega).$$

$$\boxed{\lim_{T\to\infty}\frac{e^{\pm j\omega T}}{-j\omega} = \mp\pi\delta(\omega)}$$

※：在约式：

$$\boxed{\frac{e^{\pm j\omega\infty}}{j\omega} = \pm\pi\delta(\omega)}$$

(ω)

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} u(\omega)\, e^{j\omega t}\, d\omega = \frac{1}{2\pi}\int_{0}^{+\infty} e^{j\omega t}\, d\omega = \frac{1}{2\pi}\left[\frac{1}{jt}e^{j\omega t}\right]_{0}^{+\infty}.$$

$$= \frac{1}{2\pi}\left\{\underbrace{\frac{e^{jt\infty}}{jt}}_{\pi\delta(t)} - \frac{1}{jt}\right\} = \frac{1}{2}\delta(t) - \frac{1}{2\pi j t}$$

求FT$\{\frac{1}{\pi t}\}$

$$\int_{-\infty}^{+\infty}\frac{1}{\pi t}e^{-j\omega t}dt \xrightarrow{对\omega求导} \int_{-\infty}^{+\infty}\frac{-jt}{\pi t}e^{-j\omega t}dt = -\frac{j}{\pi}\int_{-\infty}^{+\infty}e^{-j\omega t}dt = -\frac{j}{\pi}\cdot 2\pi\delta(\omega)$$

$$-2j\delta(\omega)$$

$$\xrightarrow{对\omega积分} \boxed{-2j\,u(\omega)+C} \xrightarrow{令\omega=0} \boxed{-j\,\mathrm{sgn}(\omega)}$$

$$\int_{-\infty}^{+\infty}\frac{1}{\pi t}e^{-j\omega t}dt \longrightarrow \int_{-\infty}^{+\infty}\frac{1}{\pi(-t)}e^{j\omega t}d(-t) = -\int_{-\infty}^{+\infty}\frac{1}{\pi t}e^{j\omega t}dt$$

$$= -\frac{j}{\pi}\int_{-\infty}^{+\infty}\frac{1}{jt}e^{j\omega t}dt = -\frac{j}{\pi}\int_{-\infty}\frac{1}{j^2\omega t}e^{j\omega t}d(j\omega t)$$

$$= -\frac{1}{j\pi}\int\frac{1}{\omega t}e^{j\omega t}d(j\omega t) = -\frac{1}{j\pi}\left(\int\frac{1}{\omega t}de^{j\omega t}\right)$$

$$= -\frac{1}{j\pi}\left\{\left[e^{j\omega t}\frac{1}{\omega t}\right]_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty}e^{j\omega t}d\frac{1}{\omega t}\right\}$$

$$\left[e^{j\omega t}\frac{1}{\omega t}\right]_{-\infty}^{+\infty} \to 0$$

$$= +\frac{1}{j\pi}\int_{-\infty}^{+\infty}-\frac{1}{\omega t^2}e^{j\omega t}dt$$ 不行

证明 $\frac{1}{\pi t} * (-\frac{1}{\pi t}) = \delta(t)$

$$-j\,\mathrm{sgn}(\omega) \cdot j\,\mathrm{sgn}(\omega) = \mathrm{sgn}^2(\omega) = 1$$

$\delta(t) \leftrightarrow 1$

证明 $\cos(\omega_0 t+\varphi) * \frac{1}{\pi t} = \sin(\omega_0 t+\varphi)$

$$\int_{-\infty}^{+\infty}\cos(\omega_0 t+\varphi)\cdot e^{-j\omega t}dt = \int_{-\infty}^{+\infty}\frac{1}{2}\left(e^{j(\omega_0 t+\varphi)}+e^{-j(\omega_0 t+\varphi)}\right)\cdot e^{-j\omega t}dt$$

2024-12-28

$$= \frac{1}{2}e^{j\varphi}\cdot\underbrace{\int_{-\infty}^{+\infty} e^{j(\omega_0-\omega)t}\,dt}_{2\pi\delta(\omega-\omega_0)} + \frac{1}{2}e^{-j\varphi}\underbrace{\int_{-\infty}^{+\infty} e^{-j(\omega+\omega_0)t}\,dt}_{2\pi\delta(\omega+\omega_0)}$$

$$= \{\pi e^{j\varphi}\cdot\delta(\omega-\omega_0) + \pi e^{-j\varphi}\delta(\omega+\omega_0)\}$$

$+ \quad \omega \quad - \ldots$

然后 $\left(\pi e^{j\varphi}\delta(\omega-\omega_0) + \pi e^{-j\varphi}\delta(\omega+\omega_0)\right)\cdot \underline{-j\,\mathrm{sgn}(\omega)}$

$\omega=+\omega_0 \qquad \omega=-\omega_0$

$$\underline{-j\pi e^{j\varphi}\delta(\omega-\omega_0)} \longleftarrow \text{逆变换} \longrightarrow \boxed{j\pi e^{-j\varphi}\delta(\omega+\omega_0)}$$

$\cancel{-j\pi e^{j\varphi}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\delta(\omega-\omega_0)e^{j\omega t}\,d\omega}$ $\qquad -\frac{j}{2}e^{j\varphi}$

$$= -\frac{j}{2}e^{j\varphi}\int_{-\infty}^{+\infty}\delta(\omega)\,e^{j(\omega+\omega_0)t}\,d(\omega+\omega_0) = \cancel{\int_{-\infty}^{+\infty}} e^{j\omega_0 t}$$

$$= -\frac{j}{2}e^{j(\omega_0 t+\varphi)} \longleftrightarrow \frac{j}{2}e^{-j(\omega_0 t+\varphi)}$$

$$= -\frac{j}{2}\{\cancel{\cos(\omega_0 t+\varphi)} + j\sin(\omega_0 t+\varphi)\} + \frac{j}{2}\{\cancel{\cos(\omega_0 t+\varphi)} - j\sin(\omega_0 t+\varphi)\}$$

$$= \frac{1}{2}\sin(\omega_0 t+\varphi) + \frac{1}{2}\sin(\omega_0 t+\varphi) = \boxed{\sin(\omega_0 t+\varphi)}\ \text{对}$$

2024-11-28 复习

32

设 $a(t)$ 为低频信号 证明 $H\{a(t)\cos\omega_0 t\}$ = 棒 $a(t)\sin\omega_0 t$

$\because$

$a(t)\cdot\cos\omega_0 t \quad * \frac{1}{\pi t}$

$\updownarrow$

$\frac{1}{2\pi}A(\omega) * \pi\{\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\}$

$=\{\frac{1}{2}A(\omega-\omega_0)+\frac{1}{2}A(\omega+\omega_0)\}\cdot -j\,\mathrm{sgn}(\omega)$

$\omega>0. \qquad \omega<0.$

$-\frac{j}{2}A(\omega-\omega_0)+\frac{j}{2}A(\omega+\omega_0)$

$= -\frac{j}{2}\cdot A(\omega) * \{+\delta(\omega-\omega_0) \pm \delta(\omega+\omega_0)\}$

$\frac{1}{2\pi}A(\omega) * (-j\pi\{\cdots\})$

$= -\frac{j}{2}A(\omega) * \{\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\}$

一致得证.

$$\int_{-\infty}^{+\infty} \sin\omega_0 t\cdot e^{-j\omega t}\,dt = \int_{-\infty}^{+\infty}\frac{1}{2j}\{e^{j\omega_0 t}-e^{-j\omega_0 t}\}\cdot e^{-j\omega t}\,dt$$

$$= -j\pi\{\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\}$$