Probability and Statistics

Example Problems

Example 1

Three balls are selected at random without replacement from an urn containing four green balls and six red balls. Let the random variable X denote the number of green balls drawn.
(a) List the outcomes of the experiment.
{GGG, GGR, GRG, RGG, GRR, RGR, RRG, RRR}
(b) Find the value assigned to each outcome of the experiment by the random variable X.
{3, 2, 2, 2, 1, 1, 1, 0}
(c) Find the event consisting of the outcomes to which the value of 0 has been assigned by X.
{RRR}

Example 2

Let X denote the random variable that gives the sum of the faces that fall uppermost when two fair dice are rolled. Find $P(X = 2)$.
We know that there are 36 total outcomes and only 1 of those results in $X = 2$ (a roll of 1 and 1).

$$\frac{1}{36} = 0.03$$

Example 3

Determine whether the table gives the probability distribution of the random variable X. Explain your answer.

x	-2	-1	0	1	2
P(X=x)	0.1	0.2	0.3	0.1	0.2

No, because the sum of the probabilities is less than 1.

Example 4

Find the expected value $E(X)$ of a random variable X having the following probability distribution.

x	-2	2	6	10	14	18
P(X=x)	0.18	0.09	0.19	0.09	0.12	0.33

$$E(X) = -2(0.18) + 2(0.09) + 6(0.19) + 10(0.09) + 14(0.12) + 18(0.33) = 9.48$$

Example 5

Use the formula $C(n, x)p^x q^{n-x}$ to determine the probability of the given event.
The probability of exactly **zero** successes in **nine** trials of a binomial experiment in which $p = \frac{1}{2}$

$$C(9, 0) * (\frac{1}{4})^0 * (\frac{3}{4})^9 = 0.0751$$

Example 6

The scores on an economics examination are normally distributed with a mean of **68** and a standard deviation of **14**. If the instructor assigns a grade of A to **12%** of the class, what is the lowest score a student may have and still obtain an A?

$$100\% - 12\% = 88\%$$

Then, find 88% on the *Appendix of Tables* which ends up being ≈ 1.17
Next, add the multiply by the standard deviation and add the mean.

$$68 + (1.175 * 14) = 84.45$$

Distribution of Random Variables

Flip a coin three times and let X denote the number of heads.

Outcome	HHH	HHT	HTH	HTT	THH	THT	TTH	TTT
Value(x)	3	2	2	1	2	1	1	0

Binomial Distribution

$$C(n, x) * p^x * q^{n-x}$$

where...
    n: Number of trials
    x: Number of successes
    p: Chance of success
    q: Chance of failure (1 - p)

Calculator Info

Given the *mean*, the *standard deviation*, find the percent in range *min-max*
    2nd → DISTR → (2) normalcdf
    lower, upper, μ (mean), σ (standard deviation)

Find the mean, standard deviation, mode, and median
    STAT → 1 (Edit)
    Fill in L1 with list and L2 with frequency list (if applicable, otherwise blank)
    STAT → (Right Arrow) CALC → (1) 1-Var Stats
    Set List: to L1
    2nd → LIST → (1) L1
    Repeat with FreqList and L2 if applicable
    Mean: $\bar{x}$
    Standard Deviation: σx
    Median: Med

Matrix Information

Matrix... 2nd → x^{-1} (Matrix)
To solve a system of equations...

$$1x + 2y = 5 \text{ and } 3x + 4y = 6$$

$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix} \text{ in general form is } A \begin{bmatrix} x \\ y \end{bmatrix} = B$$

$$A^{-1}B = \begin{bmatrix} x \\ y \end{bmatrix}$$