Probability and Statistics

Example Problems

Example 1

Three balls are selected at random without replacement from an urn containing four green balls and six red balls. Let the random variable X denote the number of green balls drawn.
(a) List the outcomes of the experiment.
{GGG, GGR, GRG, RGG, GRR, RGR, RRG, RRR}
(b) Find the value assigned to each outcome of the experiment by the random variable X.
{3, 2, 2, 2, 1, 1, 1, 0}
(c) Find the event consisting of the outcomes to which the value of 0 has been assigned by X.
{RRR}

Example 2

Let X denote the random variable that gives the sum of the faces that fall uppermost when two fair dice are rolled. Find P(X = 2).
We know that there are 36 total outcomes and only 1 of those results in X = 2 (a roll of 1 and 1).

$$\frac{1}{36} = 0.03$$

Example 3

Determine whether the table gives the probability distribution of the random variable X. Explain your answer.

x	-2	-1	0	1	2
P(X=x)	0.1	0.2	0.3	0.1	0.2

No, because the sum of the probabilities is less than 1.

Example 4

Find the expected value E(X) of a random variable X having the following probability distribution.

x	-2	2	6	10	14	18
P(X=x)	0.18	0.09	0.19	0.09	0.12	0.33

$$E(X) = -2(0.18) + 2(0.09) + 6(0.19) + 10(0.09) + 14(0.12) + 18(0.33) = 9.48$$

Example 5

Use the formula $C(n, x)p^x q^{n-x}$ to determine the probability of the given event.
The probability of exactly **zero** successes in **nine** trials of a binomial experiment in which $p = \frac{1}{2}$

$$C(9, 0) * (\frac{1}{4})^0 * (\frac{3}{4})^9 = 0.0751$$

Example 6

The scores on an economics examination are normally distributed with a mean of **68** and a standard deviation of **14**. If the instructor assigns a grade of A to **12%** of the class, what is the lowest score a student may have and still obtain an A?

$$100\% - 12\% = 88\%$$

Then, find 88% on the *Appendix of Tables* which ends up being ≈ 1.17
Next, add the multiply by the standard deviation and add the mean.

$$68 + (1.175 * 14) = 84.45$$

Distribution of Random Variables

Flip a coin three times and let X denote the number of heads.

Outcome	HHH	HHT	HTH	HTT	THH	THT	TTH	TTT
Value(x)	3	2	2	1	2	1	1	0

Binomial Distribution

$$C(n, x) * p^x * q^{n-x}$$

where...
- n: Number of trials
- x: Number of successes
- p: Chance of success
- q: Chance of failure (1 - p)