Interest

1 Preface

For the remainder of this paper, the following variables will be as set forth, unless specified otherwise.

A: Accumulated Amount (*Future Value*)

P: Principal (*Present Value*)

r: Nominal Interest Rate Per Year

m: Yearly Number of Conversion Periods

t: Term (Number of Years)

As well as...

i: Interest Rate Per Period

$$\frac{r}{m} \tag{1}$$

n: Total Number of Conversion Periods

$$m * t \tag{2}$$

2 Simple Interest

The value of an investment after a given period of time with a given rate of interest (non-compounding).

$$A = P(1 + rt) \tag{3}$$

3 Compound Interest

Like simple interest, but you earn interest on your interest.

- Interest that is periodically added to the principal
- Earns interest on itself

$$A = P(1 + i)^n \tag{4}$$

4 Continuous Compounding Interest

Compound Interest that is compounding constantly.

$$A = Pe^{rt} \tag{5}$$

5 Effective Rate of Interest

The yearly interest rate that would be the same as compounding m times a year at rate r.
The **effective rate of interest** is the **annual rate** which would yield the **same accumulated amount** as the **nominal rate** (r) compounded m times over the term (t). It can also be called the **annual percentage yield**.

$$r_{eff} = (1 + i)^m - 1 \tag{6}$$

where:

r_{eff}: Effective Rate of Interest

6 Present Value

The amount of money you would have to put in now to get A out.

6.1 Compound Interest

$$P = A(1 + i)^{-n} \tag{7}$$

6.2 Continuous Interest

$$P = Ae^{-rt} \tag{8}$$

Annuity

1 Preface

For the remainder of this paper, the following variables will be as set forth, unless specified otherwise.

R: Periodic Payment

P: Present Value

S: Future Value

r: Nominal Interest Rate Per Year

t: Term (*Number of Years*)

m: Yearly Payment Periods (*Same as number of times compounded per year*)

As well as...

n: Total Payment Periods

$$m * t \tag{9}$$

i: Interest Rate Per Period

$$\frac{r}{m} \tag{10}$$

2 Future Value "S"

How much you will have total.

$$S = R[\frac{(1 + i)^n - 1}{i}] \tag{11}$$

3 Present Value "P"

How much you would have to invest now to match a given annuities final value.

$$P = R[\frac{1 - (1 + i)^{-n}}{i}] \tag{12}$$

4 Amortization Formula

Paying off a loan with period payments, interest will be working against you.

The periodic payment R on a loan of P dollars to be amortized over n periods with interest charged at the rate of i per period.

$$R = \frac{Pi}{1-(1+i)^{-n}} \tag{13}$$

5 Calculate R when saving up to a value (not paying off)

$$R = \frac{Pi}{(1+i)^{n}-1} \tag{14}$$

6 Equity

- Find payment per period for loan using the amortization formula (13)
- Plug that R into present value formula (12) with $n =$ number of periods remaining, save result as *current*.
- Solve for Total $-$ current

1 Sets

Set Notation

Roster Notation: $A = \{a, b, c\}$ or $A = \{a, b, c, ..., z\}$
Set Builder Notation: $A = \{\, x \mid x$ is a lowercase character in the Latin alphabet $\}$

Terminology and implications

Given sets...

$$A = \{a, b, c\}$$
$$B = \{a, b, c, ..., z\}$$
$$C = \{a, e, i, o, u\}$$
$$D = \{a, i, u, e, o\}$$
$$E = \{a, e, i\}$$

We know

$a \in A$	a is an element of A
$e \notin A$	e is not an element of A
$A \notin A$	A set cannot be an element of a set
$\emptyset = \{\}$	
$U =$ All elements of interest	
$C = D$	
$C \neq E$	
$E \subset C$	E is a proper subset of C
$E \subseteq C$	E is a subset of C
$A \cup E = \{a, b, c, e, i\}$	A union E equals everything in A or E
$A \cap E = \{a\}$	A join E equals everything in A and E
$A^c = \{d, e, f, ..., z\}$	The compliment of A is all elements in

Laws and Properties

Commutative

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Associative

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

Distributive

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

De Morgans Laws

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

Combinatorics

$$n(S) = \text{Number of unique items in set S}$$
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Fundamental Counting Principal

$$m \text{ ways of performing task } T_1$$
$$n \text{ ways of performing task } T_2$$
$$\therefore m * n \text{ ways of performing } T_1 \text{ followed by } T_2$$

Permutations & Combinations

Permutations (Order)

Permutations of a *distinct set* is an arrangement of those objects in a *definite* order.

$$P(n,n) = n!$$
$$P(n,r) = \frac{n!}{(n-r)!}$$
$$P(n,r) = n \text{ nPr } r$$

Permutations of a *non-distinct set.*

$$P(n,r) = \frac{n!}{n_1! * n_2! \dots n_n!}$$

An example...

ATLANTA
A: 3, T: 2, N: 1, L: 1, Total: 7
$\frac{7!}{3!*2!*1!*1!}$

Combinations (Unordered)

$$C(n,r) = \frac{n!}{r!(n-r)!}$$
$$C(n,r) = n \text{ nCr } r$$

2 Probability Examples

Symbols

$$n \text{ choose } x = \binom{n}{x}$$

Children

In a **four-child** family, what are the odds of the following?

$$\text{Total} = 2^4 = 16$$

(a) Three girls and a boy in the family?

$$\frac{4!}{3!1!} = \frac{24}{6} = 4$$

(b) A youngest child in the family who is a girl?

$$1 * 2^3 = 8$$

(c) An oldest child and a youngest child in the family who are both boys

$$1 * 2^2 * 1 = 4$$

Cards

$$\text{Total} = \binom{52}{\text{Number drawn}}$$

When drawing **one card** what are the odds it is a **club** or **jack**?

$$13 + 4 - 1 = 16$$

When drawing **two cards** What are the odds it is a **pair**?

$$13 * \binom{4}{2}$$

Coin

A coin is tossed **six** times. What are the odds of the following?

$$\text{Total} = 2^6 = 64$$

(a) What are the odds the coin lands on heads more than one?

$$1 - \frac{\binom{6}{0} + \binom{6}{1}}{\text{Total}} = 1 - \frac{7}{64}$$

(b) The coin lands on heads exactly 2 times?

$$\binom{6}{2}$$

Defection

Lots of 36. Sample of 8. Any defective = rejection. Contains 2 defective. What are the odds of shipping?

$$\text{Total} = \binom{36}{8}$$

$$\binom{34}{8}$$

3 Probability and Stats

Example Problems

Example 1

Three balls are selected at random without replacement from an urn containing four green balls and six red balls. Let the random variable X denote the number of green balls drawn.
(a) List the outcomes of the experiment.
{GGG, GGR, GRG, RGG, GRR, RGR, RRG, RRR}
(b) Find the value assigned to each outcome of the experiment by the random variable X.
{3, 2, 2, 2, 1, 1, 1, 0}
(c) Find the event consisting of the outcomes to which the value of 0 has been assigned by X.
{RRR}

Example 2

Let X denote the random variable that gives the sum of the faces that fall uppermost when two fair dice are rolled. Find P(X = 2).
We know that there are 36 total outcomes and only 1 of those results in X = 2 (a roll of 1 and 1).

$$\frac{1}{36} = 0.03$$

Example 3

Determine whether the table gives the probability distribution of the random variable X. Explain your answer.

x	-2	-1	0	1	2
P(X=x)	0.1	0.2	0.3	0.1	0.2

No, because the sum of the probabilities is less than 1.

Example 4

Find the expected value E(X) of a random variable X having the following probability distribution.

x	-2	2	6	10	14	18
P(X=x)	0.18	0.09	0.19	0.09	0.12	0.33

$$E(X) = -2(0.18) + 2(0.09) + 6(0.19) + 10(0.09) + 14(0.12) + 18(0.33) = 9.48$$

Example 5

Use the formula $C(n,x)p^x q^{n-x}$ to determine the probability of the given event.

The probability of exactly **zero** successes in **nine** trials of a binomial experiment in which $p = \frac{1}{2}$

$$C(9,0) * (\frac{1}{4})^0 * (\frac{3}{4})^9 = 0.0751$$

Example 6

The scores on an economics examination are normally distributed with a mean of **68** and a standard deviation of **14**. If the instructor assigns a grade of A to **12%** of the class, what is the lowest score a student may have and still obtain an A?

$$100\% - 12\% = 88\%$$

Then, find 88% on the *Appendix of Tables* which ends up being ≈ 1.17
Next, add the multiply by the standard deviation and add the mean.

$$68 + (1.175 * 14) = 84.45$$

Distribution of Random Variables

Flip a coin three times and let X denote the number of heads.

Outcome	HHH	HHT	HTH	HTT	THH	THT	TTH	TTT
Value(x)	3	2	2	1	2	1	1	0

Binomial Distribution

$$C(n,x) * p^x * q^{n-x}$$

where...
- n: Number of trials
- x: Number of successes
- p: Chance of success
- q: Chance of failure (1 - p)

Calculator Info

Given the *mean*, the *standard deviation*, find the percent in range *min-max*

2nd → DISTR → (2) normalcdf
lower, upper, μ (mean), σ (standard deviation)

Find the mean, standard deviation, mode, and median

STAT → 1 (Edit)
Fill in L1 with list and L2 with frequency list (if applicable, otherwise blank)
STAT → (Right Arrow) CALC → (1) 1-Var Stats
Set List: to L1
2nd → LIST → (1) L1
Repeat with FreqList and L2 if applicable
Mean: $\overline{x}$
Standard Deviation: σx
Median: Med

Matrix Information

Matrix... 2nd → x^{-1} (Matrix)
To solve a system of equations...
(Find the identity of the variables)

$$1x + 2y = 5 \text{ and } 3x + 4y = 6$$

$$\begin{bmatrix}1 & 2\\4 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3\\6\end{bmatrix} \text{ in general form is } A\begin{bmatrix}x\\y\end{bmatrix} = B$$

$$A^{-1}B = \begin{bmatrix}x\\y\end{bmatrix}$$