Sets

Set Notation

Roster Notation: $A = \{a, b, c\}$ or $A = \{a, b, c, ..., z\}$
Set Builder Notation: $A = \{\ x \mid x$ is a lowercase character in the Latin alphabet $\}$

Terminology and implications

Given sets...

$$A = \{a, b, c\}$$
$$B = \{a, b, c, ..., z\}$$
$$C = \{a, e, i, o, u\}$$
$$D = \{a, i, u, e, o\}$$
$$E = \{a, e, i\}$$

We know

$a \in A$	a is an element of A
$e \notin A$	e is not an element of A
$A \notin A$	A set cannot be an element of a set
$\emptyset = \{\}$	
$U =$ All elements of interest	
$C = D$	
$C \neq E$	
$E \subset C$	E is a proper subset of C
$E \subseteq C$	E is a subset of C
$A \cup E = \{a, b, c, e, i\}$	A union E equals everything in A or E
$A \cap E = \{a\}$	A join E equals everything in A and E
$A^c = \{d, e, f, ..., z\}$	The compliment of A is all elements in the universal set and not in A

Laws and Properties

Commutative

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Associative

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

Distributive

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

De Morgans Laws

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

Combinatorics

$$\begin{aligned}
n(S) &= \text{Number of unique items in set S} \\
n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\
n(A \cup B \cup C) &= n(A) + n(B) + n(C) \\
&\quad - n(A \cap B) - n(A \cap C) \\
&\quad - n(B \cap C) + n(A \cap B \cap C)
\end{aligned}$$

Fundamental Counting Principal

$$\begin{aligned}
m \text{ ways of performing task } T_1 \\
n \text{ ways of performing task } T_2 \\
\therefore m * n \text{ ways of performing } T_1 \text{ followed by } T_2
\end{aligned}$$

Permutations & Combinations

Permutations (Order)

Permutations of a *distinct set* is an arrangement of those objects in a *definite* order.

$$\begin{aligned}
P(n, n) &= n! \\
P(n, r) &= \frac{n!}{(n-r)!} \\
P(n, r) &= n \text{ nPr } r
\end{aligned}$$

Permutations of a *non-distinct set*.

$$P(n, r) = \frac{n!}{n_1! * n_2! \ldots n_n!}$$

An example...

ATLANTA

A: 3, T: 2, N: 1, L: 1, Total: 7

$\frac{7!}{3!*2!*1!*1!}$

Combinations (Unordered)

$$\begin{aligned}
C(n, r) &= \frac{n!}{r!(n-r)!} \\
C(n, r) &= n \text{ nCr } r
\end{aligned}$$