Graph of Equations

1 Review

Assume...

P_1: (x_1, y_1)

P_2: (x_2, y_2)

1.1 Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where:

d: Distance between P_1 and P_2

1.2 The Midpoint Formula

$$m = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

where:

m: Midpoint between P_1 and P_2

2 Equations of Circles

You can draw a circle using an **relationship** not a function.

$$(x - h)^2 + (y - k)^2 = r^2$$

where:

(h, k): Center Point

r: Radius

3 Symmetry

3.1 Y-Axis

- Called an "**Even Function**"
- Looks the same after reflection over Y-Axis
- Has to meet the following requirement(s)...

$$f(x) = f(-x)$$

One example of such a function is $y = x^2$.

$$f(4) = 16$$
$$f(-4) = 16$$
$$16 = 16$$

3.2 X-Axis

- **Not a function**, doesn't pass vertical line test
- Called a **relationship**
- Has to meet the following requirement(s)...

$$x \mapsto \{-y, y\}$$

One example of such a equation is $x = y^2$ but **not** $y = \sqrt{x}$ because that would only allow positive x values.

$$9^2 = 81$$
$$(-9)^2 = 81$$

3.3 Origin

- Called an "**Odd Function**"
- Visually the same after 180° rotation about $(0, 0)$
- Has to meet the following requirement(s)...

$$f(x) = y$$
$$f(-x) = -y$$

One example of such a function is $y = x^3$

$$f(2) = 8$$
$$f(-2) = -8$$

4 Equations of Lines

Assume...

m: Slope

4.1 Slope

You can use the slope formula to find the rate of change between two points.

$$m = \frac{\text{"Ryse"}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

4.2 Forms

4.2.1 Slope-Intercept Form

$$y = mx + b$$

where:

b: x-intercept

4.2.2 Point Slope Form

If you need point-slope form, just sub out values. However, if you need to find slope-intercept form you can solve for y.

$$y - y_1 = m(x - x_1)$$

4.2.3 Intercept Form

$$\frac{x}{a} + \frac{y}{b} = 1$$

where:

a: x-intercept, point (a, 0) falls on the line

b: y-intercept, point (0, b) falls on the line

This form can be converted into **General Form** through the multiplication of the least common multiple of a and b. Then subtracting the value on the right side of the equation.

4.2.4 General Form

$$Ax + By + C = 0$$

where:

A is non-negative

A, B, and C are all *integers*

4.3 Relationships of Lines

4.3.1 Parallel Lines

– **same slopes**.

4.3.2 Perpendicular Lines

– **opposite reciprocal slopes**.
Consider the following where lines t_1 and t_2 are perpendicular.

$$t_1 = 3/8$$
$$t_2 = -8/3$$

5 Functions and Equations

5.1 Is it a function?

– Each x only maps to one y

6 Domain & Range

6.1 Formatting

Example...

$$D : (-1, 2]$$
$$R : (-\infty, 12)$$

– "(", ")" means exclusive

– "[", "]" means inclusive

– **Never** use [] with ∞

6.2 Zeros

Solve for when $y = 0$
They are x-intercepts

6.3 Increasing and Decreasing

Never use "[", "]", always "(", ")"
Always Least → Greatest

6.4 Relative Maximum and Minimum

A **point** on a line where the line is either above on both sides (*Minimum*) or below on both sides (*Maximum*). **Cannot** be an **end point**.

6.5 New Functions

6.5.1 Greatest Integer Function (Floor)

Represented by

$$f(x) = [[x]]$$

Left side solid (*Included*), right side empty (*Excluded*)

6.5.2 Peace-wise Function

An equation, but with conditionals
Example...

$$f(x) = \begin{cases} x^2 - 3 & \text{if } x \geq 3 \\ -2x^4 + 9x^3 & \text{if } x < 3 \end{cases}$$

Plug it into calculator by multiplying things and conditions

$$f(x) = (x^2 - 3)(x \geq 3) + (-2x^4 + 9x^3)(x < 3)$$

6.6 Algebra of functions

Assume...

$f(x)$: 3x + 1

$g(x)$: 4x - 1

Can be done in 2 different ways

– Do the algebra on the function

– Do the algebra on the return from the function

– Only one example will be shown, but it works on them all

6.6.1 Addition of functions

Algebra on the functions...

$$\begin{aligned} h(x) &= (f + g)(x) \\ &= 3x + 3 + 4x - 1 \\ &= 7x + 2 \end{aligned}$$

Algebra on the return values... (*Only example*)

$$\begin{aligned} (f + g)(2) &= f(2) + g(2) \\ &= (2(2) + 3) + (4(2) - 1) \\ &= 16 \end{aligned}$$

6.6.2 Subtraction of functions

$$\begin{aligned} h(x) &= (f - g)(x) \\ &= (3x + 1) - (4x - 1) \\ &= 3x + 1 - 4x + 1 \\ &= -x + 2 \end{aligned}$$

6.6.3 Multiplication of functions

$$\begin{aligned}h(x) &= (f * g)(x)\\ &= (3x+1)(4x-1)\\ &= 12x^2 - 3x + 4x - 1\\ &= 12x^2 + x - 1\end{aligned}$$

6.6.4 Division of functions

$$\begin{aligned}h(x) &= (\frac{f}{g})(x)\\ &= \frac{3x+1}{4x-1}\end{aligned}$$

6.6.5 Composition of functions

$$\begin{aligned}h(x) &= (f \circ g)(x) = f(g(x))\\ &= 3(4x-1)+1\\ &= 12x - 3 + 1\\ &= 12x - 2\end{aligned}$$

6.6.6 Inverse of functions

Swap the x/y values and then solve for y;

$$y = 3x + 1$$

Swap the x and y

$$\begin{aligned}x &= 3y + 1\\ 3y &= x - 1\\ y &= \frac{x-1}{3}\end{aligned}$$