Rectangular Coordinates

1 The Distance Formula

This distance formula is used to find the **euclidean distance** between two points on the **Cartesian plane**. The distance formula has a strong connection to the Pythagorean theorem.

$$c^2 = b^2 + a^2 \tag{1}$$

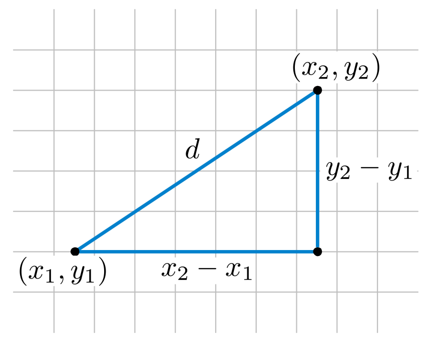
The distance formula is essentially the Pythagorean theorem (1) after replacing the a and b values with *change in* x and *change in* y.

Consider the following for $(x_1, y_1), (x_2, y_2)$.

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2 \tag{2}$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \tag{3}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{4}$$



1.1 Example Problem

Find the distance between $(-2, 5)$ and $(4, -3)$ using the distance formula (4).

$$\begin{aligned} d &= \sqrt{(4 - -2)^2 + (-3 - 5)^2} \\ &= \sqrt{36 + 64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

2 Verify right triangle

Given a, b, and c prove that a right triangle can be created with those side lengths. If the largest value is not equal to the sum of the two smaller values a right triangle cannot be created

2.1 Example Problem

Prove that a triangle can(not) be created with side lengths $\sqrt{5}$, $\sqrt{45}$, and $\sqrt{50}$.
$\sqrt{50}$ is the longest side length, so we have to see if it is the sum of the smaller sides.

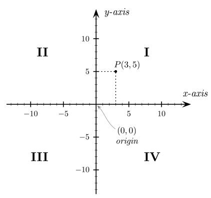
$$\begin{aligned} \sqrt{50} &\stackrel{?}{=} \sqrt{5} + \sqrt{45} \\ \sqrt{50} &= \sqrt{50} \end{aligned}$$

3 Homework

1 Determine the quadrant(s) in which (x, y) could be located. (Select all that apply.)

$$x < 0 \text{ and } y > 0$$

This pair could only be located in the 2nd quadrant.



2 Determine the quadrant(s) in which (x, y) could be located. (Select all that apply.)

$$x < 0 \text{ and } y < 0$$

This pair could only be located in the 3rd quadrant.

3 Determine the quadrant(s) in which (x, y) could be located. (Select all that apply.)

$$x < 0 \text{ and } y = 6$$

This pair could only be located in the 2nd quadrant.

4 Determine the quadrant(s) in which (x, y) could be located. (Select all that apply.)

$$xy > 0$$

This pair could be located in either the 1st or the 3rd quadrant.

6 (a) Find the length of each side of the right triangle

distance between $(1, 2)$ and $(13, 2)$

$$\begin{aligned} d_1 &= \sqrt{(1-13)^2 + (2-2)^2} \\ &= 12 \end{aligned}$$

distance between $(13, 2)$ and $(13, 11)$

$$\begin{aligned} d_2 &= \sqrt{(13-13)^2 + (2-11)^2} \\ &= 9 \end{aligned}$$

distance between $(1, 2)$ and $(13, 11)$

$$\begin{aligned} d_3 &= \sqrt{(1-13)^2 + (11--13)^2} \\ &= 15 \end{aligned}$$

(b) Show that these lengths satisfy the Pythagorean Theorem.

$$\begin{aligned} d_1^2 &= 12^2 \\ &= 144 \\ d_2^2 &= 9^2 \\ &= 81 \\ d_1^2 + d_2^2 &= 255 = d_3^2 \end{aligned}$$

7 Consider the following. $(5, 3), (8, 3)$

(a) Plot the points

(b) Find the distance between the points.

$$\begin{aligned} d &= \sqrt{(5-8)^2 + (3-3)^2} \\ &= 3 \text{ units} \end{aligned}$$

(c) Find the midpoint of the line segment joining the points.

$$(x, y) = (\frac{a}{b}, \frac{c}{d})$$