

Graph of Equations

Set Notation

Roster Notation: $A = \{a, b, c\}$ or $A = \{a, b, c, \dots, z\}$

Set Builder Notation: $A = \{x \mid x \text{ is a lowercase character in the Latin alphabet}\}$

Terminology and implications

Given sets...

$$A = \{a, b, c\}$$

$$B = \{a, b, c, \dots, z\}$$

$$C = \{a, e, i, o, u\}$$

$$D = \{a, i, u, e, o\}$$

$$E = \{a, e, i\}$$

We know

$$a \in A$$

a is an element of A

$$e \notin A$$

e is not an element of A

$$A \notin A$$

A set cannot be an element of a set

$$\emptyset = \{\}$$

$$U = \text{All elements of interest}$$

$$C = D$$

$$C \neq E$$

$$E \subset C$$

E is a proper subset of C

$$E \subseteq C$$

E is a subset of C

$$A \cup E = \{a, b, c, e, i\}$$

A union E equals everything in A or E

$$A \cap E = \{a\}$$

A join E equals everything in A and E

$$A^c = \{d, e, f, \dots, z\}$$

The compliment of A is all elements in the universal set and not in A

Laws and Properties

Commutative

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

Associative

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Distributive

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

De Morgans Laws

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

Combinatorics

$$\begin{aligned}n(S) &= \text{Number of unique items in set } S \\n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\n(A \cup B \cup C) &= n(A) + n(B) + n(C) \\&\quad - n(A \cap B) - n(A \cap C) \\&\quad - n(B \cap C) + n(A \cap B \cap C)\end{aligned}$$

Fundamental Counting Principal

$$\begin{aligned}&m \text{ ways of performing task } T_1 \\&n \text{ ways of performing task } T_2 \\&\therefore m * n \text{ ways of performing } T_1 \text{ followed by } T_2\end{aligned}$$

Permutations & Combinations

Permutations (Order)

Permutations of a *distinct set* is an arrangement of those objects in a *definite* order.

$$\begin{aligned}P(n, n) &= n! \\P(n, r) &= \frac{n!}{(n-r)!} \\P(n, r) &= {}_n \text{Pr } r\end{aligned}$$

Permutations of a *non-distinct set*.

$$P(n, r) = \frac{n!}{n_1! * n_2! \dots n_n!}$$

An example...

$$\begin{aligned}&\text{ATLANTA} \\&\text{A: 3, T: 2, N: 1, L: 1, Total: 7} \\&\frac{7!}{3! * 2! * 1! * 1!}\end{aligned}$$

Combinations (Unordered)

$$\begin{aligned}C(n, r) &= \frac{n!}{r!(n-r)!} \\C(n, r) &= {}_n \text{Cr } r\end{aligned}$$