Consider the heating of army tents using on-off, constant flow, heaters.

Given:

Z = # of tents $(z \in [1, Z])$
G = # of generators $(g \in [1, G])$
n = planning horizon for optimization
i = single time period index with interval time t $(i \in [1, n])$
$T_{z,i}$ = tent temperature
$\dot{Q}_z$ = tent heat provided by heater

$u_{z,i}$ = Status of heater for tent z at time index i.

$u_{g,i}$ = Status of generator g at time index i.

F_g = Gallons of fuel consumed by generator g.

P_g = Power provided by generator g.

P_z = Power consumed by heater z.

η_z = Efficiency of heater z (Power consumption to heat output).

R_z = resistance parameter of tent z

C_z = capacitance parameter of tent z

$\dot{q}_{z,i}$ = Loads on tent (Σ solar + internal). ⟵ * Look at Sol-air and/or q_{conv}/q_{rad} split.

(I) States:

$u_{z,i} \in [0,1]$

$u_{g,i} \in [0,1]$

$T_{z,i} \in \mathbb{R}$

(II) Constraints:

Consider Physics:

$$C_z \frac{dT_{z,i}}{dt} = -\frac{\overbrace{(T_{z,i} - T_{\infty i})}^{\theta_{z,i}}}{R_z} + \underbrace{\dot{q}_{z,i} + u_{z,i}\dot{Q}_z}_{\phi_z}$$

(a)
$$\boxed{\frac{d\theta_{z,i}}{dt} + \frac{\theta_{z,i}}{C_z R_z} = \frac{\phi_{z,i}}{C_z} \quad \text{with } \theta_{z,0} = T_{z,i-1} - T_{\infty i}}$$

$$\cancel{\theta_{z,i}(t=0) = \theta_{z,0}}$$

The general solution to (G) is:

$$\theta_{z,i}\, e^{+\frac{t}{RC}} = \int \frac{\phi_{z,i}}{C_z} e^{+\frac{t}{RC}}\, dt + D$$

$$\theta_{z,i}\, e^{+\frac{t}{RC}} = +\phi_{z,i} R\, e^{+t/RC} + D$$

$$\theta_{z,0} = +\phi_{z,i} R + D$$

$$D = \theta_{z,0} - \phi_{z,i} R$$

$$\theta_{z,i}\, e^{-t/RC} = +\phi_{z,i} R\, e^{+t/RC} + \theta_{z,0} - \phi_{z,i} R$$

$$(T_{z,i} - T_{\infty,i})\, e^{+t/RC} = +\phi_{z,i} R\, e^{+t/RC} + T_{z,i-1} - T_{\infty i} - \phi_{z,i} R$$

$$T_{z,i} = T_{\infty,i} + \phi_{z,i} R + (T_{z,i-1} - T_{\infty i} - \phi_{z,i} R)\, e^{-t/RC}$$

And so we have:

(1) $$T_{z,i} - T_{z,i-1} e^{-t/RC} - u_{z,i} \dot{Q}_z R(1 - e^{-t/RC}) = T_{\infty,i}(1 - e^{-t/RC}) + \dot{q}_{z,i} R(1 - e^{-t/RC}) \quad \forall z, i$$

Temperature Constraints

(2) $$T_{z,i} \geq T^{ll}_{z,i} \quad \forall z, i$$

(3) $$T_{z,i} \leq T^{ul}_{z,i} \quad \forall z, i$$

Generator Constraints

(4) $$\sum_z u_{z,i} P_{z,i} \leq \sum_g u_{g,i} P_{g,i} \quad \forall i$$

(III) Objective Function

$$\min \sum_{i=1}^{n} \sum_{g=1}^{G} u_{g,i} F_g$$ minimize fuel consumption over planning horizon.