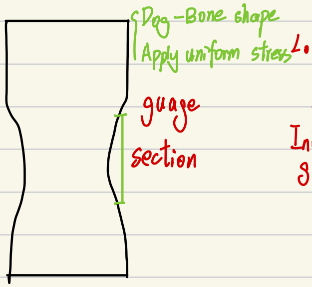
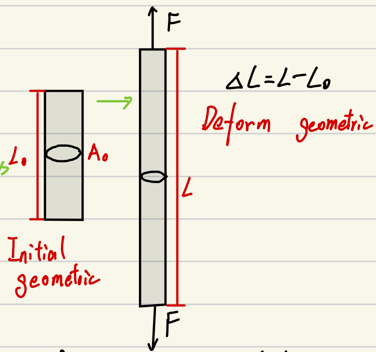
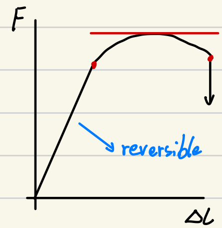
Materials 1

10.7, 2025

Uniaxal tensile test







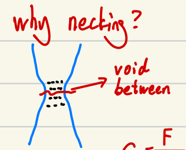
Assume A_0, L_0 are constants

$\sigma_n = \frac{F}{A_0}$ $\varepsilon_n = \frac{\Delta L}{L_0}$ (no unit)

nominal stress and strain

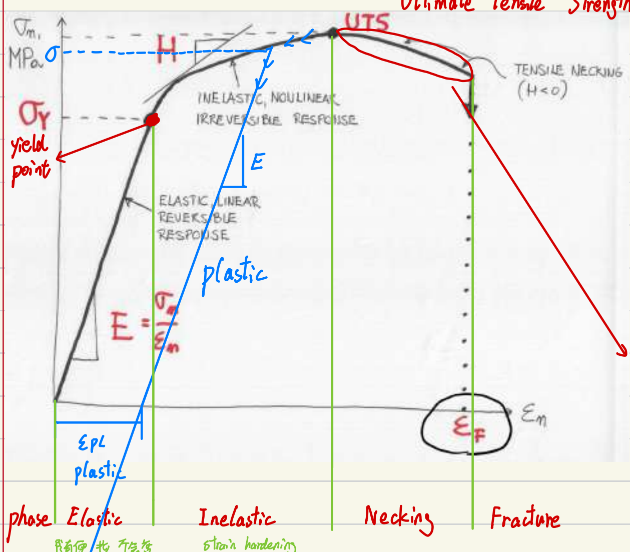
engineering

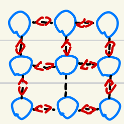
why necking?



$\sigma_n = \frac{F}{A_0}$

Ultimate tensile strength





$E = \sigma_y / \varepsilon_y$

tengent: Young's Modulus

$\sigma_{UTS} = \frac{F_{max}}{A_0}$

why the gragh go down?

Area go down due to void

For materials; Polymers and Ceramics may NOT display some of the phases

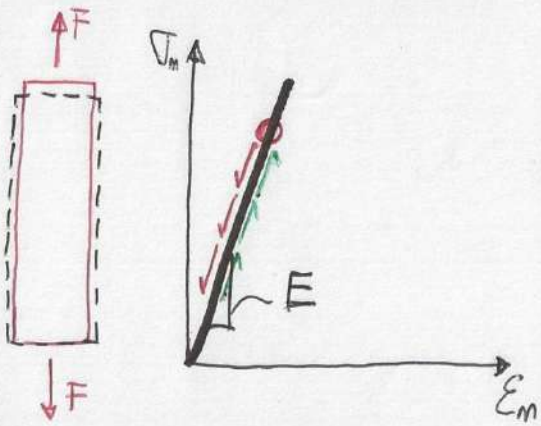
	Big	Small
E	Stiff	Soft/compliant
σ_0	Strong	weak
ε_0	Ductile 延展	Britle 易碎

Strain Hardening: (越拉越硬)

during plastic deformation, stronger force needed

Phases of tensile response

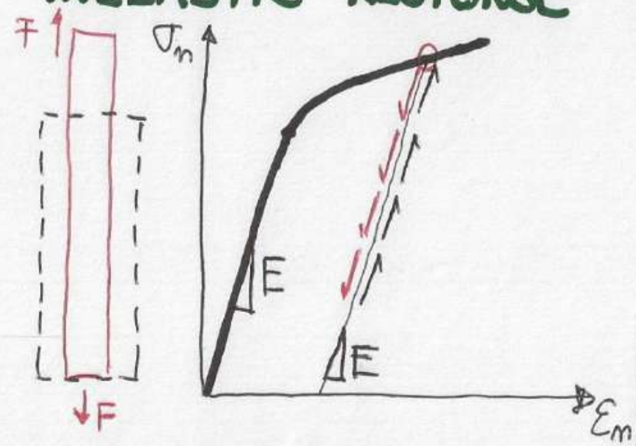
ELASTIC RESPONSE



- σ_m PROPORTIONAL TO ε_m
- UPON UNLOADING AND RELOADING, MEASUREMENTS FOLLOW INITIAL CURVE OF SLOPE E (REVERSIBLE RESPONSE, CONSERVATION OF ENERGY)
- TRANSVERSE STRAIN ε_m^T PROPORTIONAL TO ε_m

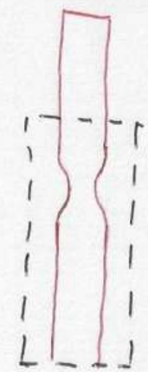
$$\varepsilon_m^T = -\nu\,\varepsilon_m$$

INELASTIC RESPONSE



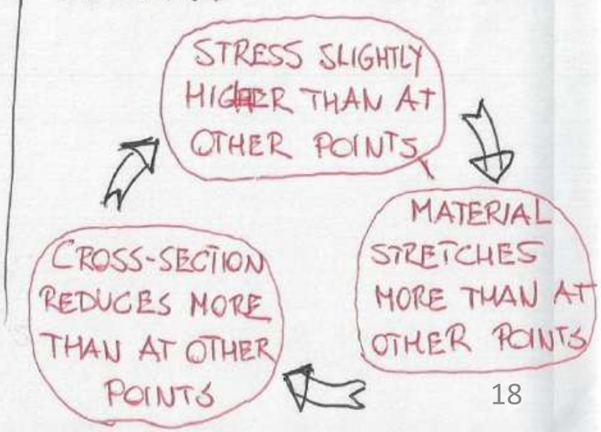
- $\sigma_m(\varepsilon_m)$ NON-LINEAR
- UPON UNLOADING - RELOADING, MEASUREMENTS FOLLOW STRAIGHT LINE OF SLOPE (E) (FOR METALS, BUT FOR ALL MATERIALS, UNLOADING NEVER FOLLOWS INITIAL LOADING CURVE)
- DEFORMATION IS IRREVERSIBLE, PERMANENT STRAIN, ENERGY IS DISSIPATED; MAT. MICROSTRUCTURE IS CHANGING

NECKING



AT LARGE STRAIN, NECKING OCCURS AND RESULTS IN FRACTURE

- INSTABILITY ORIGINATED BY GEOMETRIC / MATERIAL INHOMOGENEITY. E.G. SPECIMEN HAS SLIGHTLY SMALLER CROSS-SECTION AT 1 POINT:



Elastic and Inelastic strains;

$\varepsilon_n = \varepsilon^{el} + \varepsilon^{pl}$

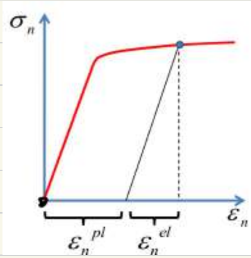
$\varepsilon_n^{el} = \frac{\sigma^n}{E}$

$\varepsilon_n^{pl} = \varepsilon_n - \frac{\sigma_n}{E}$

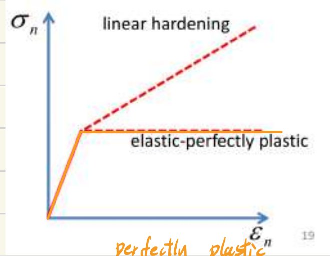
$\varepsilon_{max} = \varepsilon_y + H \cdot (\sigma_{app} - \sigma_y)$

hardness

可以理解为plastic时的E



hardening modulus



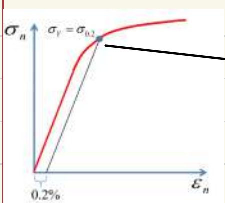
perfectly plastic

unload 后 elastic 部分恢复，只剩 $L + \Delta L_{plas}$

0.2% proof stress 量化何时从 elastic ⟶ inelastic

The stress which causes a permanent strain of 0.2% (Because the transition between elastic and inelastic is not sudden. hard to define)

also knowns plastic strain $\frac{\Delta L}{L} = 0.2\%$



⟶ unload at that point

why 0.2%?

for metals is roughly 0.2%

※ True stress and true strain

$\sigma_n = \frac{F}{A_0}$ $\qquad \sigma = \sigma_t = \frac{F}{A}$

Initial Area constant $\qquad$ changing area

$A_0 L_0 = AL \longrightarrow \frac{A_0}{A} = \frac{L}{L_0}$ $\qquad \sigma_n = \frac{\sigma_t}{1+\varepsilon_n}$

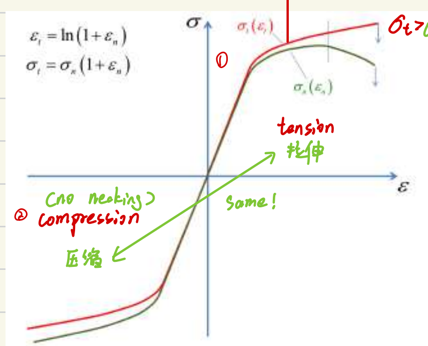
same volume

$$\int_0^{\varepsilon} d\varepsilon_t = \int_{L_0}^{L} \frac{1}{L} dL = \ln\left(\frac{L}{L_0}\right) = \ln\left(\frac{L_0 + \Delta L}{L_0}\right) = \ln(1+\varepsilon_n)$$

ε_t

$\varepsilon_t = \ln(1+\varepsilon_n)$ $\qquad$ relation between ε_t and ε_n

for metal 完全对称, for true graph, compression = tension



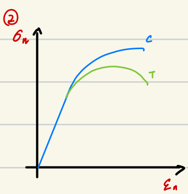
$\sigma_t > \sigma_n$

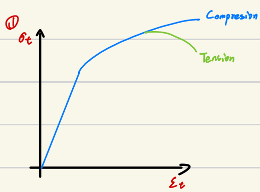
①

tension 拉伸

same!

(no necking) ② compression 压缩

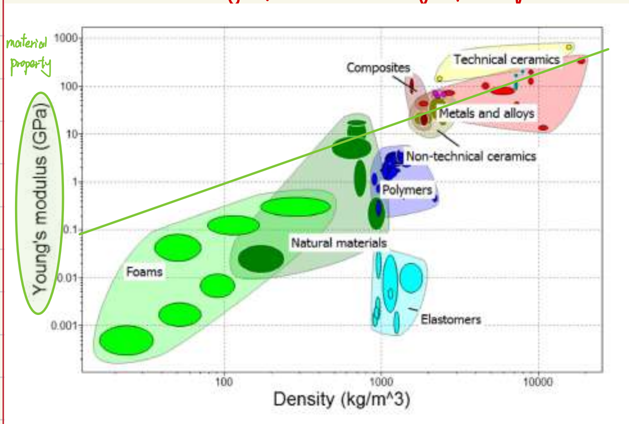




nominal is inaccurate because assumes geometry of object during test is same

e.g. { 拉伸中缩颈导致 Area↓
 压缩中 Area↑

Modulus vs density for different groups of materials



转换时记得用log

e.g. Find E/ρ maximum

let $E = k\rho$ Find maximum k

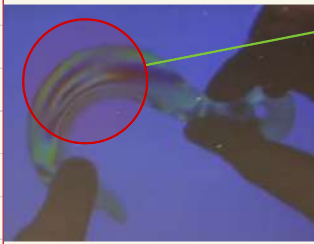
$\log E = \log k + \log \rho$ Find maximum $\log k$

$y = c + x$

$\log k$, x, increasing

Find max $\log k$ is max k

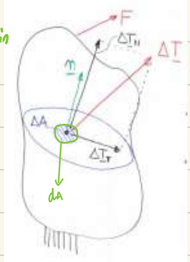
Normal and shear stress at a specific point (local stresses)



层数反映 stress/strain

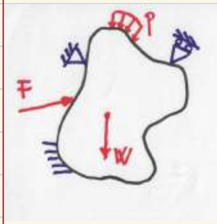
Use polymer

Stress and strain are field quantities



$$\sigma_m = \lim_{\Delta A \to 0} \frac{|\Delta T_N|}{\Delta A} = \frac{dT_N}{dA}$$

$$\tau = \lim_{\Delta A \to 0} \frac{|\Delta T_T|}{\Delta A} = \frac{dT_T}{dA}$$



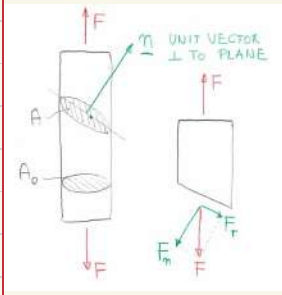
We need to consider 2 problems: $\longrightarrow$ need 3 equations

$\begin{cases} \text{How much it deforms ?} \\ \text{If it will fail ?} \end{cases}$

Stress Analysis

$\begin{cases} \text{Equilibrium} \\ \text{Compatibility} \\ \text{Constitutive} \end{cases}$

- Equilibrium — force balance
- Compatibility — Relate displacem to strain
- Constitutive — Material dependent relation Between [σ]-[ε]

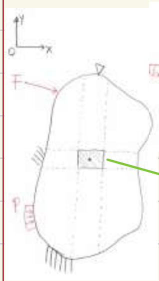


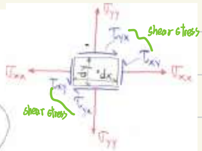
- Stress can be defined at any point in a body
- Stress can be defined at any plane passing through the point

$\sigma_n = \frac{F_m}{A}$ Normal or Direct stress

$\tau = \frac{F_T}{A}$ Shear stress

Stress tensor in 2D





take and analysis a rectangle

stress matrix

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$$

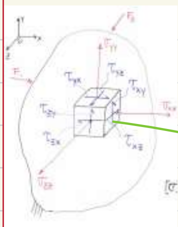
In equilibrium

$$\underbrace{\tau_{xy} \cdot dy\,dz}_{\text{force}}\, \overset{\text{arm}}{dx} - \underbrace{\tau_{yx}\, dx\,dz}_{\text{force}}\, \overset{\text{arm}}{dy} = 0$$

$$\tau_{xy} = \tau_{yx}$$

for 3D

Stress tensor in 3D



plane xx

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ & \sigma_{yy} & \tau_{yz} \\ \text{symm} & & \sigma_{zz} \end{bmatrix}$$

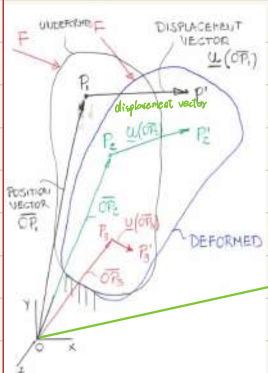
properties of the stress tensor

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \sigma_{xx} \\ \tau_{yx} \\ \tau_{zx} \end{pmatrix}$$

得到该面stress

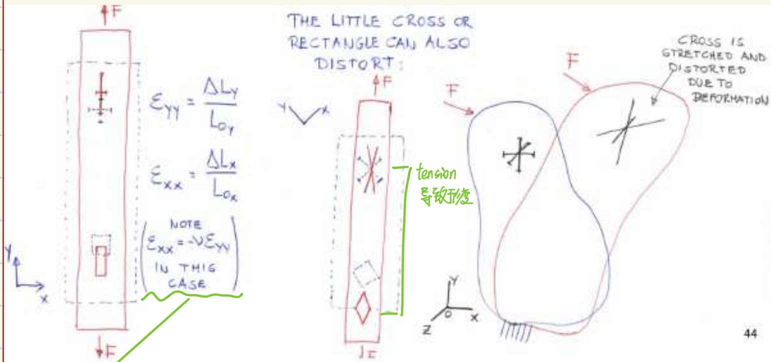
unit: typically Mpa $\longrightarrow$ N/mm^2

Strain measurement; material hardness.

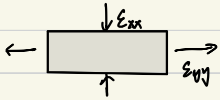


$$\underline{u}(\overrightarrow{OP}) = u(x,y,z) = \begin{pmatrix} u_x(x,y,z) \\ u_y(x,y,z) \\ u_z(x,y,z) \end{pmatrix}$$

undeformed coordinates



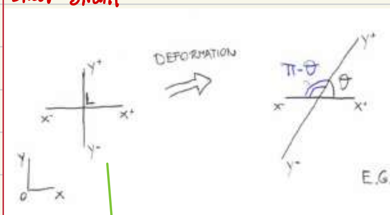
transition between ε_{xx} and ε_{yy}



$\varepsilon_{yy} = -\nu\varepsilon_{xx}$

ν: Poisson's Ratio

shear strain

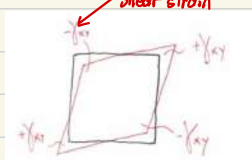


shear force 导致 stretch, deform

shear strain: $\gamma_{xy} = (\pi-\theta) - \frac{1}{2}\pi = \frac{1}{2}\pi - \theta$ } 即 change in angle

consider initial is perpendicular

shear strain



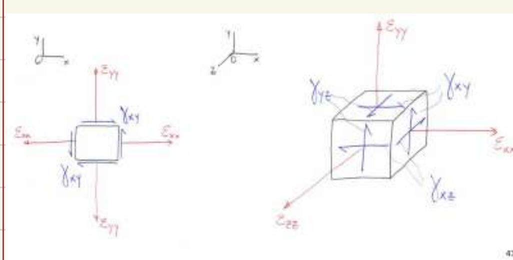
signs '+' '-' means slightly change

represent strain

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & \gamma_{xz} \\ \gamma_{yx} & \varepsilon_{yy} & \gamma_{yz} \\ \gamma_{zx} & \gamma_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & \gamma_{xz} \\ \gamma_{xy} & \varepsilon_{yy} & \gamma_{yz} \\ \gamma_{xz} & \gamma_{yz} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & \gamma_{xz} \\ & \varepsilon_{yy} & \gamma_{yz} \\ \text{SYMM} & & \varepsilon_{zz} \end{bmatrix}$$

ALTERNATIVELY AND EQUIVALENTLY, IN SOME TREATMENTS

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ & \varepsilon_{yy} & \varepsilon_{yz} \\ \text{SYMM} & & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \gamma_{xy}/2 & \gamma_{xz}/2 \\ & \varepsilon_{yy} & \gamma_{yz}/2 \\ \text{SYMM} & & \varepsilon_{zz} \end{bmatrix}$$

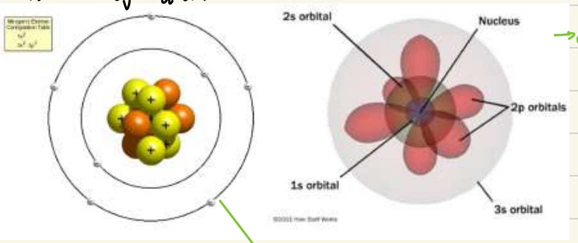


strain 和 stress 一样, 不反映任何力

只表示该点,物体 strain 和 stress 的状态.

Elasticity

Structure of atom



→ electron orbitals

orbitals are decrete with different energy level

electrons in continous motion

Atomic bonds

- primary bonds:
  - ionic
  - covalent (strongest)
  - metallic
- secondary bonds:
  - von der Waals
  - hydrogen bonds

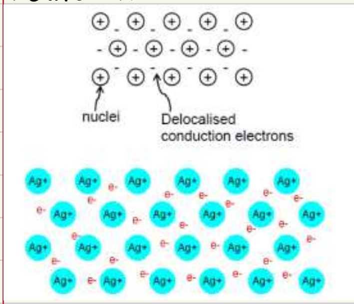
relatively weak

why solid can exert force ?

there are bonds betweens atoms

bonds impact properties of materials

Metallic bond



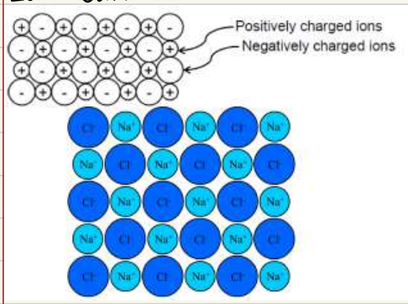
properties

- strong
- non-directional → good ductility → formability
- free electron → high electron conductivity

原子什么方向性都一样

atomic planes can slide.

Ionic bond

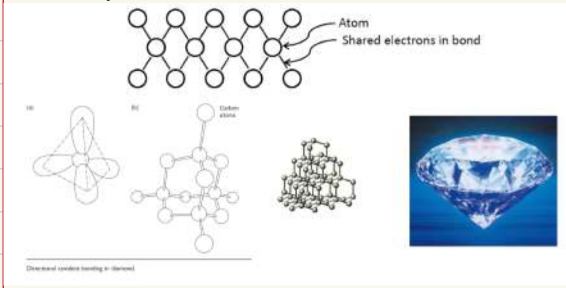


properties

- strong
- non-directional
- brittle

脆

Covalent bond



- extremely strong
- highly directional

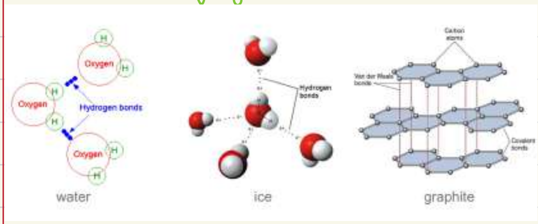
→ stiff, anisotrophic material

硬 各向异性

每个方向识别都不同

- limited slip → no conductivity

Van der Waals, hydrogen and similar bonds



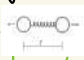
secondary bond 不共享,转移电子

- no share electrons but electrostatic attraction between 2 or more electrically neutral molecules

瞬时偶极
(temperary dipole)

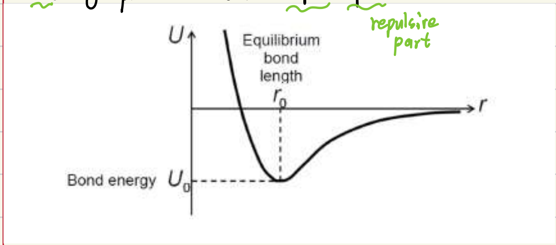
Van de waals	hydrogen
- weak	- strong
- short range 短程	- directional
- non-directional	

Potential energy

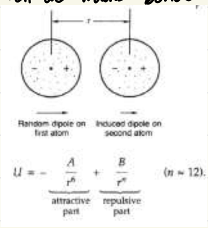


displacement

U(r) graph $\qquad U(r) = -\underbrace{\frac{A}{r^m}}_{\text{attractive part}} + \underbrace{\frac{B}{r^n}}_{\text{repulsive part}} \qquad n > m$



Von de waals bonds



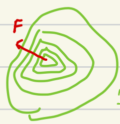
If potential energy V is a function of 1 coodinate only: $V = \int F\,dx \longrightarrow \underbrace{F = \frac{\partial V}{\partial x}}$

力=能量沿x得/失率

$$\underbrace{\vec{F} = \text{grad}(V) = \frac{\partial u}{\partial x}\vec{i} + \frac{\partial u}{\partial y}\cdot\vec{j} + \frac{\partial u}{\partial z}\cdot\vec{k}}$$

which means 力沿能量得失最快线

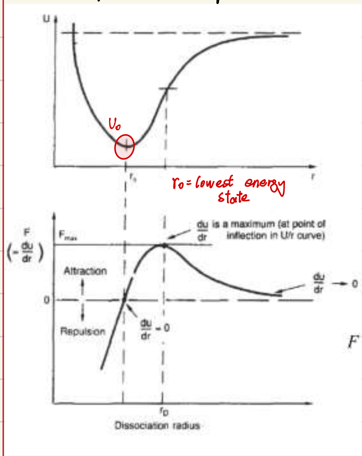
2D case



等energy线

universal physical principle:
A system will spontaneously evolve towards states that minimise the potential energy

Relationship between potential energy, Force and stiffness



$U(r) = -\frac{A}{r^m} + \frac{B}{r^n} \quad n>m$

Force $F(r) = \frac{dU}{dr}$

Stiffness $S(r) = \frac{dF}{dr} = \frac{d^2U}{dr^2} \quad S_0 = S(r_0)$

F

physical Origin of Young modulus

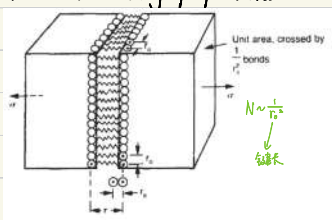
Hooke's law : for small strains, strain is nearly proportional to stress

$\sigma = E\varepsilon$ Linear Elastic

normal stress, Young modulus, normal strain

$\tau = G\gamma$

shear stress, Shear modulus, shear strain



stiffness of single bond

宏观 微观

$\sigma = N S_0 (r - r_0)$

伸长量

number of bonds/unit area $N = \frac{1}{r_0^2} k$

$\sigma = \left(\frac{S_0}{r_0}\right) \cdot \varepsilon_n \longrightarrow \left(\frac{r-r_0}{r_0}\right)$

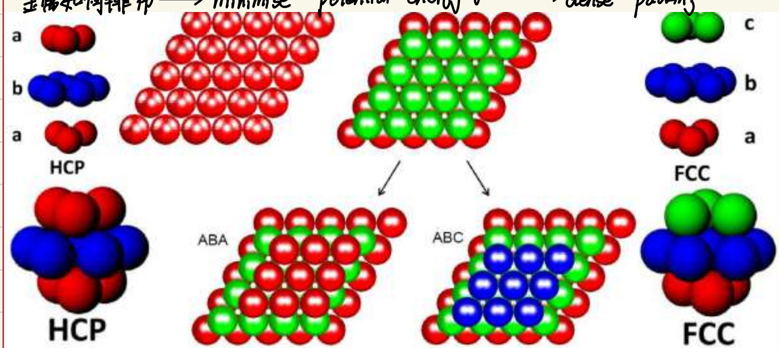
So $E = \frac{\sigma}{\varepsilon_n} = \frac{S_0}{r_0}$

Atomic packing in metals

金属如何排布 $\longrightarrow$ minimise potential energy $U \xrightarrow{\text{减小 r, U 越低}}$ dense packing

packing density 空间利用率

= volume Occupied / Total volume



一个 atom 和几个 atom 相连接

Coordinate number (Z)

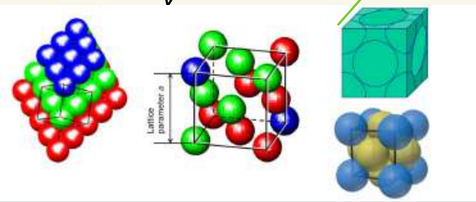
In crystal structure, number of atoms attach with a certain atom

Hexagonal Closed Packed crystals (HCP)

e.g. Mg, Ti ----

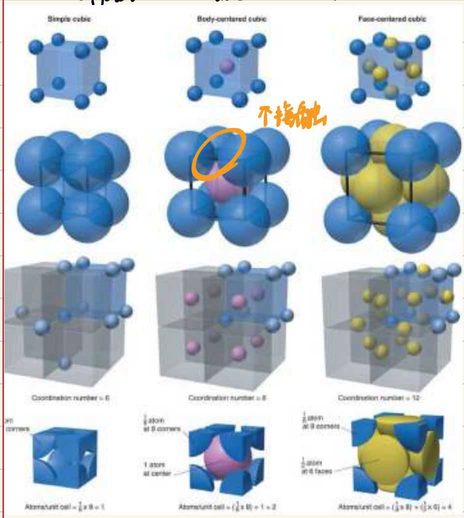
Face Centre Cubic (FCC)

For FCC crystals (e.g. Cu ...) a时八个角都要有一个atom顶点.

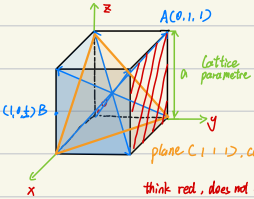


- $Z=12$
- Total number of atom $= 8\times\frac{1}{8}+6\times\frac{1}{2}=4$
- Packing density $= 4\times\frac{4}{3}\pi\times(\frac{1}{2}a)^3/(\sqrt{2}a)^3 = 0.74$

Other Crystal with Cubic unit cell



Crystallographic notation (定义 direction, plane) naming system 考试要标清轴+方向!



plane (1 1 1), cut {x=1, y=1, z=1}

think red, does not cut x, z, which {x=∞, y=1, z=∞} → naming plane, Miller index

定义轴+原点.

then 表示. position/direction

plane ⊥ direction!!!

naming plane, Miller index

$(\infty\ 1\ \infty) \xrightarrow{\text{use miller}} (0\ 1\ 0)$

可以通过平移origin变

然后取reciprocals 倒数

then use (0 1 0)

通常用cut 的 axis定义 (x y z)

若无, 则用 Miller's index 平移origin

定义不了就换 origin !!!

Family of direction

why? name Family of similar properties

e.g. body diagnose, totally 8 body diagnose

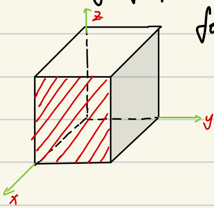
$[1\ 1\ 1] \xrightarrow{\text{use}} \langle 1\ 1\ 1\rangle$ 表示whole family

$\bar{1}$ means -1

$[1\ 1\ 1], [\bar{1}\ 1\ 1], [1\ \bar{1}\ 1], [1\ 1\ \bar{1}]$

$[\bar{1}\ \bar{1}\ 1], [\bar{1}\ 1\ \bar{1}], [1\ \bar{1}\ \bar{1}], [\bar{1}\ \bar{1}\ \bar{1}]$

Family of planes



for plane x y z

1 ∞ ∞

(1 0 0) direction of plane

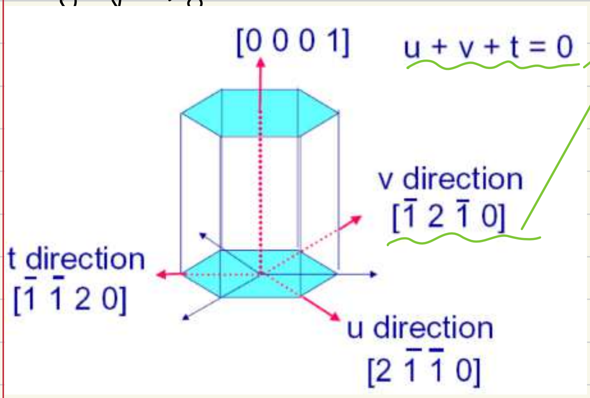
{1 0 0} use direction to define family

family name   对应6个 planes by moving origin

(0 0 1) (0 1 0) (1 0 0)

$(0\ 0\ \bar{1})$ $(0\ \bar{1}\ 0)$ $(\bar{1}\ 0\ 0)$

Naming of hexagonal



- First three digit sum=0

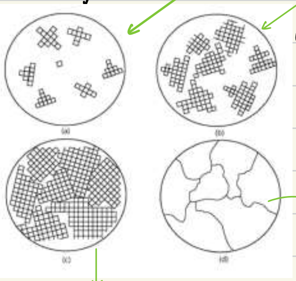
- 4 digit for hexagonal

plasticity

Grain formation

high Temperature

Temperature ↓



lower temperature

a. Nucleation of crystals 晶核形成

b. Crystal growth 晶体长大

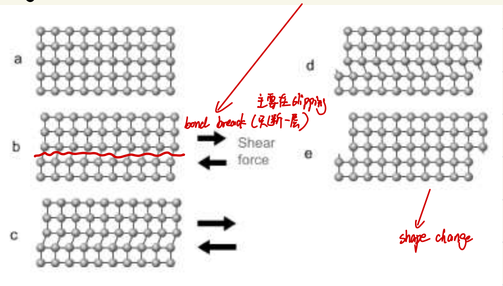
c. Irregular grains form as Crystals grow together 晶体互相挤压

d. Grain boundaries as seen in a microscope 晶界出现

grain

crystall structure

Why metals are ductile? (permanent deformation ← plastic)



Dislocation:

Caused by defect, appears extra half plane or half row in material

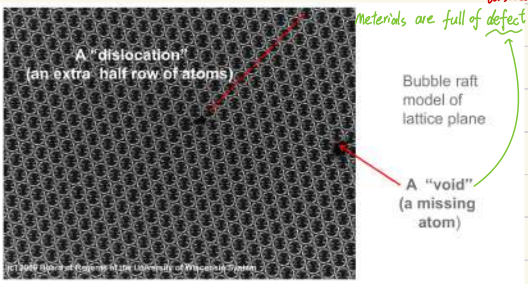
因位移造成的错位

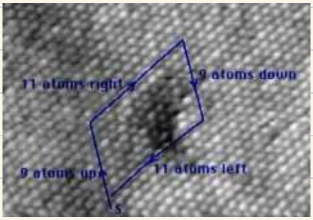
Dislocation (Observed by TEM)

transmission electron microscope.

dislocation "失位"

Meterials are full of defect ⟶ deform easier

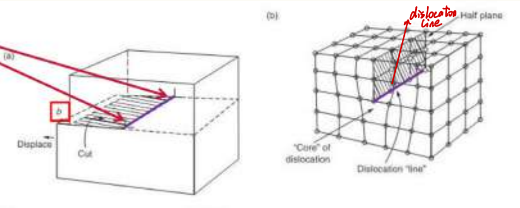


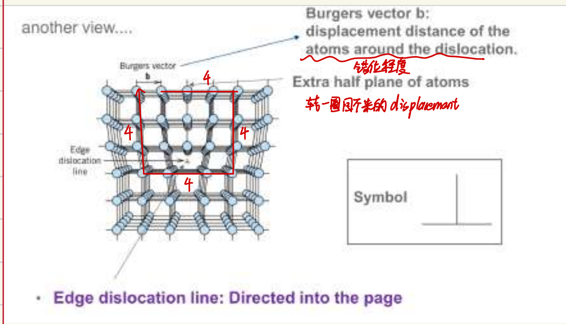


Edge dislocation

CONTINUUM APPROACH - NO ATOMS

1. cut a block of material to a certain point
2. move the top of the cut portion relative to the bottom by a distance **b** (in the range of the atom size) **normal** to the line ⊥-⊥



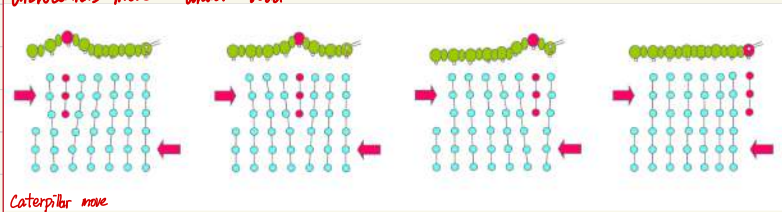


burger's location ⊥ dislocation line

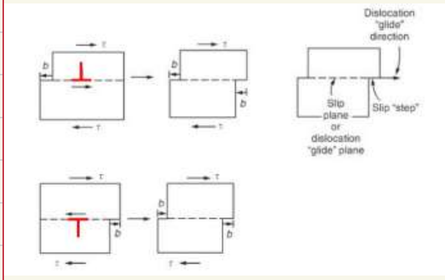
为何 shear force need < predict，因为存错位。

Why is the observed shear stress with dislocations present lower than pure shear prediction?

dislocations move under load



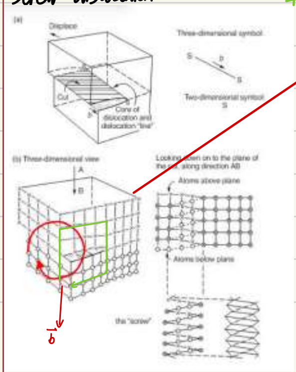
Caterpillar move



some terms to remember

- slip - movement of the dislocation, also called glide
- slip plane - crystallographic plane on which dislocation moves
- slip direction - crystallographic direction in the slip plane along dislocation moves
- slip system - slip plane + slip direction (direction (ie in the plane)
- slip step - the atomic displacement left in the crystal by the dislocation

Screw dislocation



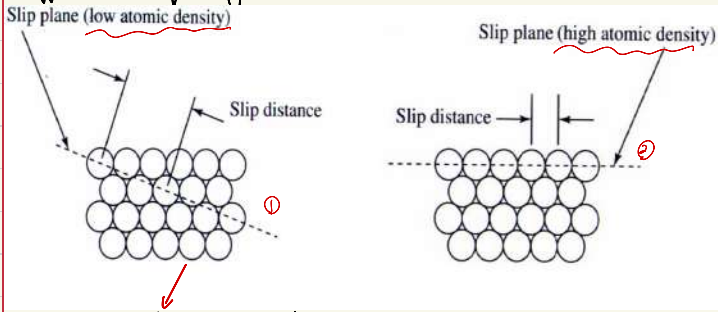
转一圈的 displacement

burger's vector // dislocation line

parallel

Mixed dislocation → both kinds of dislocations appear

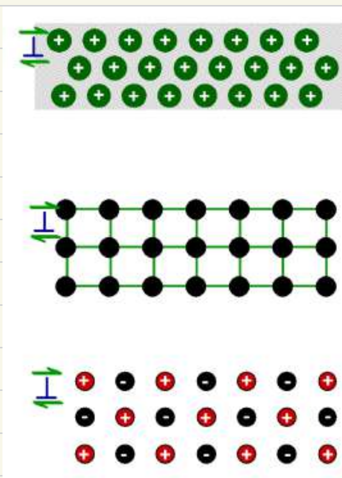
Different Slip system $E \sim Gb^2$



Atoms always choose direction that atoms easier to move!

①	②
not in contact to each other	in contact to each other
have go through the gaps	do not need to go through gaps
more energy needed	less energy needed
harder to move	easier to move
- slip distance is greater	slip distance is smaller

Dislocation motion and classes of materials



Metals — non-directional, ductile, soft

Covalent ceramics — directional, brittle, hard

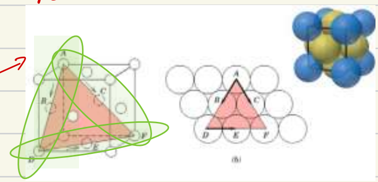
Ionic ceramics — non-directional, brittle, hard

Ductile or not ?

Slip Systems for Face-Centered Cubic, Body-Centered Cubic, and Hexagonal Close-Packed Metals

Metals		Slip Plane	Slip Direction		Number of Slip Systems
		Face-Centered Cubic			
Cu, Al, Ni, Ag, Au	Close packed planes →	{111}	$\langle 1\bar{1}0\rangle$	Many easy slip systems - ductile	12
		Body-Centered Cubic			
α-Fe, W, Mo	No close packed planes	{110}	$\langle \bar{1}11\rangle$	Many slip systems, but slip more difficult than fcc - fairly ductile	12
α-Fe, W		{211}	$\langle \bar{1}11\rangle$		12
α-Fe, K		{321}	$\langle \bar{1}11\rangle$		24
		Hexagonal Close-Packed			
Cd, Zn, Mg, Ti, Be	Close packed planes →	{0001}	$\langle 11\bar{2}0\rangle$	Limited slip Systems – less ductile	3
Ti, Mg, Zr		$\{10\bar{1}0\}$	$\langle 11\bar{2}0\rangle$		3
Ti, Mg		$\{10\bar{1}1\}$	$\langle 11\bar{2}0\rangle$		6

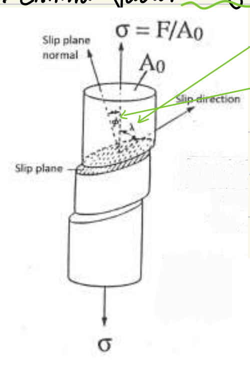
FCC



atom in contact to each other
less energy consume
So easy to move, ductile.

not for polycrystals

Schimid factor [single crystals] 纯几何推导

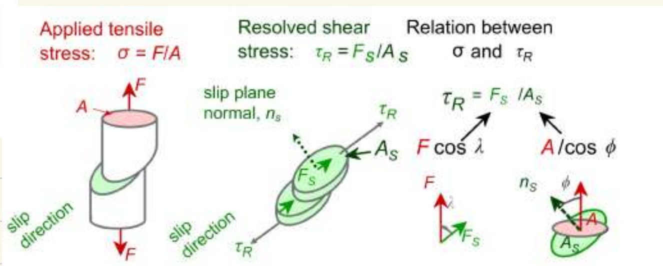


λ is angle between tensile stress and slip direction
ϕ is angle between slip plane normal and the tensile stress

$\tau = \sigma \cos\lambda \cos\phi > \tau_{crit}$

$S = \cos\lambda\cos\phi$ ... schmid-factor

τ_{crit} ... critical shear stress
shear needed to slip



In exams

※ Strengthening mechanics

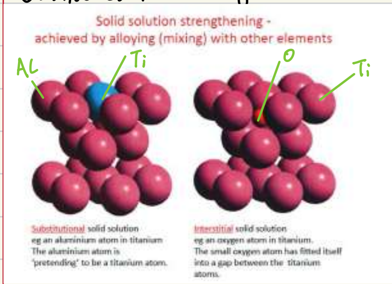
① Solid solution strengthen:
introduction of solute atoms into matrix

② Precipitation strengthen:
formation or introduction of particles into metal matrix

③ Grain size strengthen:
reduction of grain size and related increase grain boundary area

④ Work strengthen:
cold work increases dislocation density impeding each other

①Solid solution strengthen



2 types

被替换

①Substitutional: an atom substituted by another kind of (homogeneous) atom gap被夹住. (in unit cell)

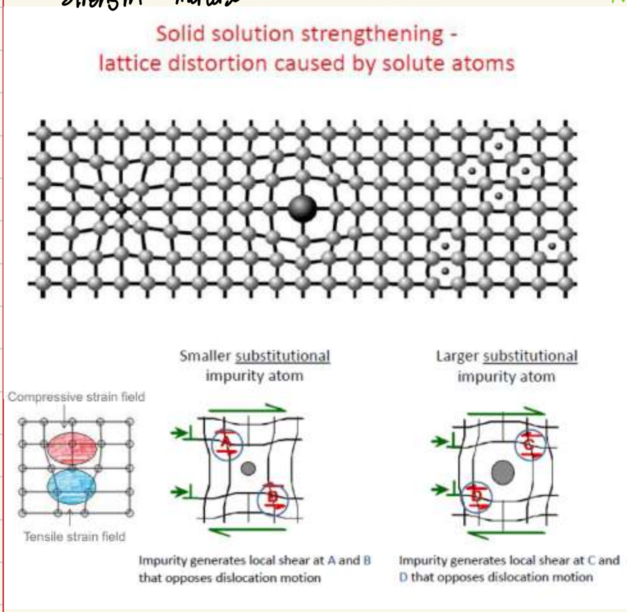
②Interstitial: an smaller atom in the gap of other atom (in unit cell)

Both in FCC structure

Why the method works?

- changing the atom arrangement
- the substituted/new atom will change the energy associated
- will create "stress-strain field", which distort the arrangement of pure atoms
  basicly change ro which equillibrium bond length original one
- dislocation also have "stress-strain field", then 2 "stress-strain field" Interact with each other.
- then the dislocation motion will be hinted (触发), and slow down the dislocation motion
- strength increase

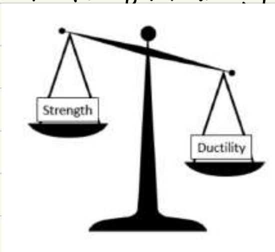
减缓 dislocation, strength 是 dislocation 的难易程度
dislocation 更慢 ⟶ strength 更强



for most materials
strength ↑ ductility ↓

why? - more dislocation motion.
the chance create cracks during dislocation motion is much higher

硬 脆
平衡 strength 和 ductility

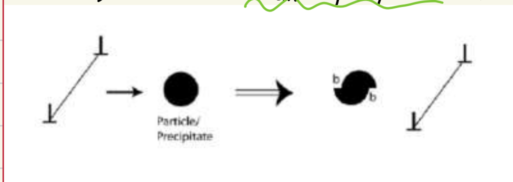


② Precipitation strengthening

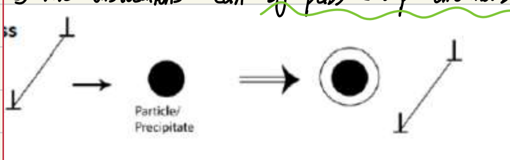
new material added into your host material

2 choices

① Dislocation can shear precipitates. But large precipitates are difficult to shear

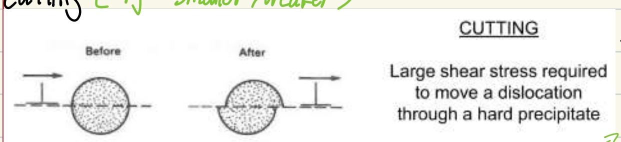


② The dislocations can by-pass (loop around) the precipitates



总是走消耗最少能量的路径，只能切 or 绕

Cutting (if smaller, weaker)

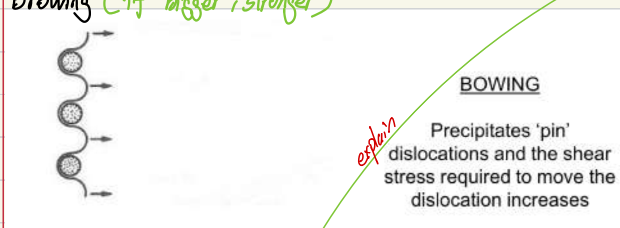


多，小，密，切消耗更少能量 why???

- Numerous Small, closely spaced particles must be cut by dislocations

Bigger particles are more difficult to cut

Browing (if bigger, stronger)

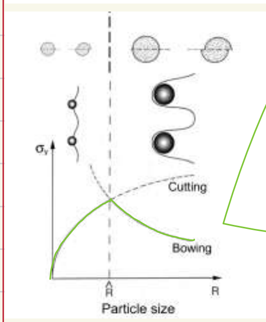


explain

少，大，稀，绕消耗更少能量 why???

- Fewer wide-spaced, Large particles can be browed around by dislocations

More widely-spaced particles easier to bow around



explain

(Interstitial)

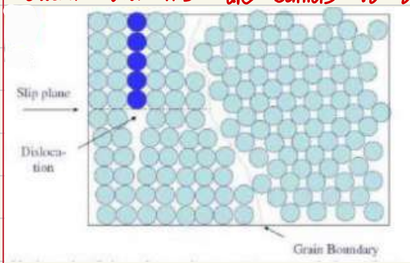
Solid solution 和 precipitation 区别

在 Solid solution, only few atom added whole system still as one (原来的仍占大头)

在 precipitation, second material added to the solution (第二种材料加很多 also not in FCC structure)

③ Grain refinement strengthening

Grain boundaries are barriers to slip



- smaller the grain, the more growing boundaries in system have, then stronger strength mechanics get

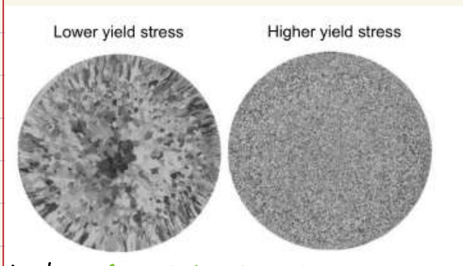
Hall-Petch equation:

constant given

$$\sigma_{yield} = \sigma_0 + k_y d^{-\frac{1}{2}}$$

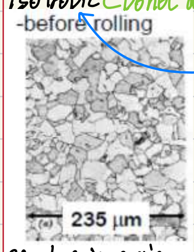
grain 越小，越强

Strength is inversely proportional to square root of grain size



each grain have same atom packing, same chemical properties but different orientation.

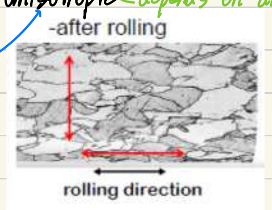
isotropic (Do not depends on directions)



equal-axis grain

two different property need to verify

anisotropic (depends on directions)



同样 Grain 在不同方向的 size 影响不同方向的 strength

*(At room temperature), strength ↑ ductility ↑

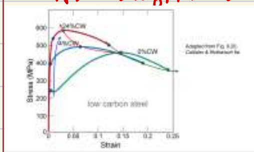
why? - small grains means more chance for dislocation of slip system then easier move on different directions. then ductility ↑

④ Work hardening

Basically Room Temperature deformation

- when work hardening, material deformed
- adding more dislocation, dislocation density ↑
- dislocation interact to each other, strength ↑

Also strength ↑ ductility ↓

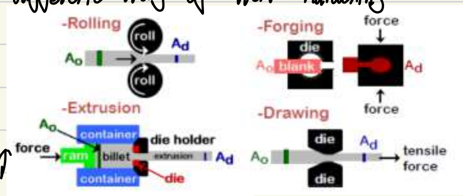


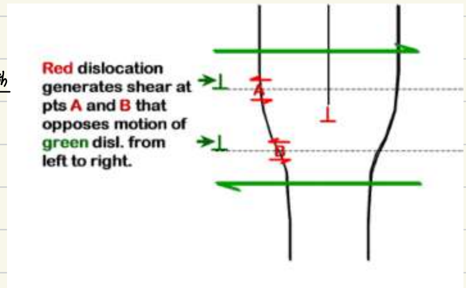
$$\text{dislocation density} = \frac{\text{Total dislocation length}}{\text{unit volume}}$$

more dislocation

higher shear stress

different way of work hardening

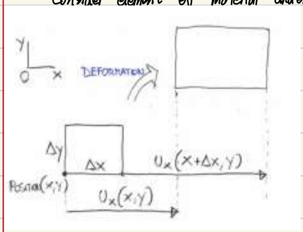




3 equations

①
Compatibility (strain-displacements) equations

Consider element of material undergoing only ε_{xx}, ε_{yy} normal strain



$$\varepsilon_{xx} = \lim_{\Delta x \to 0} \frac{\text{change in length}}{\text{original length}}$$

$$= \lim_{\Delta x \to 0} \frac{U_x(x+\Delta x, y) + \Delta x - U_x(x,y)}{\Delta x}$$

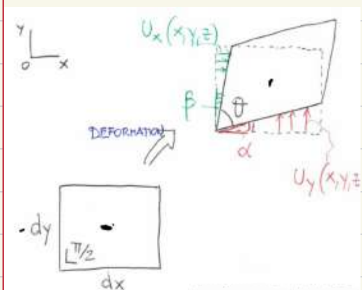
$$= \lim_{\Delta x \to 0} \frac{U_x(x+\Delta x, y) - U_x(x,y)}{\Delta x}$$

$$= \frac{\partial U_x}{\partial x}$$

repeating in y, z direction

then, $\varepsilon_{xx} = \frac{\partial U_x}{\partial x}$, $\varepsilon_{yy} = \frac{\partial U_y}{\partial y}$, $\varepsilon_{zz} = \frac{\partial U_z}{\partial z}$

Consider element of material undergoing only γ_{xy} shear strain



$\gamma_{xy} = \frac{\pi}{2} - \theta = \alpha + \beta$

if deformation are small

$\alpha \approx \tan\alpha$ $\quad \beta \approx \tan\beta$

$\gamma_{xy} = \tan\alpha + \tan\beta = \frac{\partial U_y}{\partial x} + \frac{\partial U_x}{\partial y}$

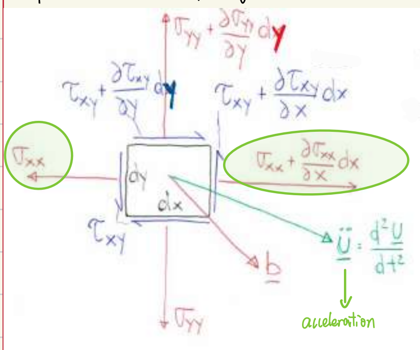
repeating in y, z direction

then, $\gamma_{xy} = \frac{\partial U_y}{\partial x} + \frac{\partial U_x}{\partial y}$, $\gamma_{xz} = \frac{\partial U_x}{\partial z} + \frac{\partial U_z}{\partial x}$, $\gamma_{yz} = \frac{\partial U_y}{\partial z} + \frac{\partial U_z}{\partial y}$

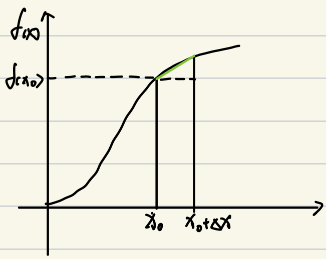
②
Indefinite equilibrium equations (2D)

Assume stresses can vary continuously in space

Define Vector $\underline{b}$ of "body Forces" (Include all the forces that can be expressed per unit volume.) N/m^3 内力密度



当dx足够小,两边差 $\frac{\partial \sigma_{xx}}{\partial x} dx$



当x small enough

$f(x_0 + \Delta x) = f(x_0) + \frac{\partial f}{\partial x} \cdot dx$

normal stress $\qquad$ shear stress $\qquad$ 内力

$$-\sigma_{xx}\,dy\,dz + \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx\right) dy\,dz - \tau_{xy}\,dx\,dz + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} dy\right) dx\,dz + b_x\,dx\,dy\,dz = \rho\,dx\,dy\,dz \cdot \ddot{U}_x$$

$$\Longrightarrow \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + b_x = \rho \ddot{U}_x$$

For 3D Indefinite equilibrium

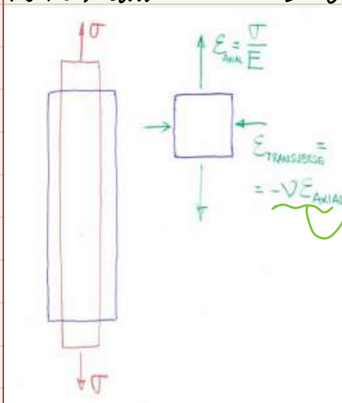
$$\begin{cases} \dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \tau_{xy}}{\partial y} + \dfrac{\partial \tau_{xz}}{\partial z} + b_x = \rho \ddot{u}_x \\ \dfrac{\partial \sigma_{yy}}{\partial y} + \dfrac{\partial \tau_{xy}}{\partial x} + \dfrac{\partial \tau_{yz}}{\partial z} + b_y = \rho \ddot{u}_y \\ \dfrac{\partial \sigma_{zz}}{\partial z} + \dfrac{\partial \tau_{yz}}{\partial y} + \dfrac{\partial \tau_{xz}}{\partial x} + b_z = \rho \ddot{u}_z \\ \begin{cases} \tau_{xy} = \tau_{yx} \\ \tau_{xz} = \tau_{zx} \\ \tau_{zy} = \tau_{yz} \end{cases} \end{cases} \Rightarrow \begin{cases} \overset{\nabla}{\mathrm{div}}[\sigma] + \underline{b} = \rho \underline{\ddot{u}} \\ \tau_{ij} = \tau_{ji} \end{cases} \quad \text{local equation of motion}$$

因为物体不转动，所以moment守恒

For quasi-static problems, where bodies are deformed slowly, accelerations vanish and

$$\underline{\ddot{u}} = 0 \quad \Rightarrow \quad \begin{cases} \mathrm{div}[\sigma] + \underline{b} = 0 \\ \tau_{ij} = \tau_{ji} \end{cases} \quad \text{static equilibrium}$$

Hooke's Law ⟶ $\varepsilon \propto \sigma$



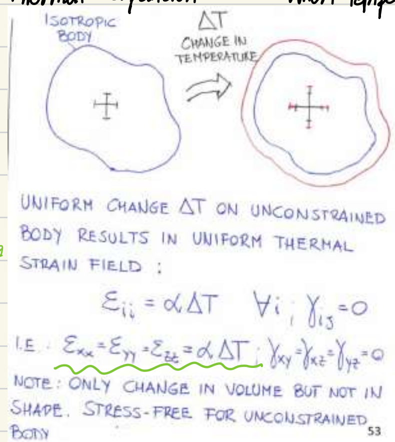
Poison construction

poison ratio

$\nu = -\dfrac{\text{strain变长}}{\text{strain变细}}$

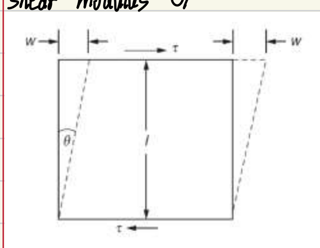
一面被拉长，另一面被拉细

Thermal expansion ⟶ when Temperature ↑ Volume ↑ but, shape no change



strain/expansion proportional to change in Temperature

shear modulus G



$\gamma = \theta = \arctan\left(\dfrac{w}{l}\right) \approx \dfrac{w}{l}$

shear stress: $\tau = \dfrac{F_{tangential}}{A}$

shear modulus: $\tau = G\gamma$ — shear stress and shear strain

in terms of E, ν: $G = \dfrac{E}{2(1+\nu)}$

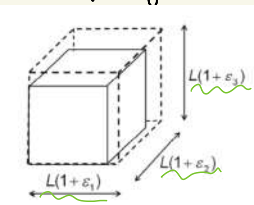
Bulk modulus K

why negative ? compress define as positive, so expansion take negative!

Define nominal volumetric strain ε_v or dilatation Δ as: $\varepsilon_v = \Delta = \frac{\Delta V}{V_0}$

just like hydrostatic

Define hydrostatic stress as: $\sigma_H = -p = \frac{1}{3}\sum_{i}^{3}\sigma_{ii} = \frac{\sigma_{11}+\sigma_{22}+\sigma_{33}}{3}$ (每个面 stress assume same)



知道 How much want to change the volume

bulk modulus K: $\sigma_H = K\cdot\Delta = K\cdot\varepsilon_v$ 体积压缩难易程度, K越大, 越难压缩

in terms of E, ν: $K = \frac{E}{3(1-2\nu)}$ → if $\nu \to 0.5$, $k \to \infty$

$$\varepsilon_v = \frac{\Delta V}{V} = \frac{\boxed{L^3(1+\varepsilon_1)(1+\varepsilon_2)(1+\varepsilon_3)} - L^3}{L^3}$$

V_0, V_0, V_{new}

$$= \varepsilon_1+\varepsilon_2+\varepsilon_3+O(\varepsilon^2)+O(\varepsilon^3)$$

$$\approx \varepsilon_1+\varepsilon_2+\varepsilon_3$$

因为 assume 形变很小, strain很小, so $\varepsilon^2, \varepsilon^3 \approx 0$

Constitutive equations for elasticity in 3D

- [σ]/[ε] RELATIONS ARE LINEAR → WE CAN APPLY THE PRINCIPLE OF SUPERPOSITION OF EFFECTS
- CONSIDER APPLICATION OF INDIVIDUAL STRESS COMPONENTS σ_{ij} AND CHANGE IN TEMPERATURE ΔT IN SEQUENCE, THEN SUM THE EFFECTS ON STRAINS.

	ε_{xx}	ε_{yy}	ε_{zz}	γ_{xy}	γ_{xz}	γ_{yz}
σ_{xx}	σ_{xx}/E	$-\frac{\nu\sigma_{xx}}{E}$	$-\frac{\nu\sigma_{xx}}{E}$	0	0	0
σ_{yy}	$-\frac{\nu\sigma_{yy}}{E}$	σ_{yy}/E	$-\frac{\nu\sigma_{yy}}{E}$	0	0	0
σ_{zz}	$-\frac{\nu\sigma_{zz}}{E}$	$-\frac{\nu\sigma_{zz}}{E}$	σ_{zz}/E	0	0	0
τ_{xy}	0	0	0	τ_{xy}/G	0	0
τ_{xz}	0	0	0	0	τ_{xz}/G	0
τ_{yz}	0	0	0	0	0	τ_{yz}/G
ΔT	$\alpha\Delta T$	$\alpha\Delta T$	$\alpha\Delta T$	0	0	0

56

Hooke's Law in 3D

3D以下不能直接 $\frac{\sigma}{\varepsilon}$, 因为维度不同

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu}{E}(\sigma_2+\sigma_3) + \alpha\Delta T$$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \frac{\nu}{E}(\sigma_1+\sigma_3) + \alpha\Delta T$$

$$\varepsilon_3 = \frac{\sigma_3}{E} - \frac{\nu}{E}(\sigma_1+\sigma_2) + \alpha\Delta T$$

$\gamma_{12} = \tau_{12}/G$ → shear modulus

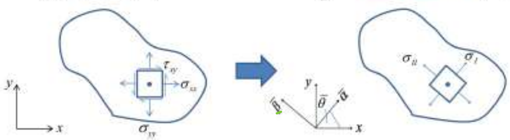
$\gamma_{13} = \tau_{13}/G$

$\gamma_{23} = \tau_{23}/G; \qquad G = E/[2(1+\nu)]$

Principal stresses and Principal directions

A special angle $\theta=\bar{\theta}$ exists such that no shear stresses act on the element sides.

$\tau_{\bar{\alpha}\bar{\beta}}\left(\sigma_{xx},\sigma_{yy},\tau_{xy},\bar{\theta}\right)=0$ $\quad\quad$ $\bar{\theta},\ \bar{\theta}+\frac{\pi}{2}$: **principal directions** $(\bar{\alpha},\bar{\beta})$



$$\begin{bmatrix}\sigma_{xx} & \tau_{xy}\\ \tau_{xy} & \sigma_{yy}\end{bmatrix} \Rightarrow \begin{bmatrix}\sigma_{I} & 0\\ 0 & \sigma_{II}\end{bmatrix}$$ σ_I,σ_{II} : **principal stresses**

shear=0

In 3D similarly
(with 3 rotations)

$$\begin{bmatrix}\sigma_{xx} & \tau_{xy} & \tau_{xz}\\ \tau_{xy} & \sigma_{yy} & \tau_{yz}\\ \tau_{xz} & \tau_{yz} & \sigma_{zz}\end{bmatrix} \Rightarrow \begin{bmatrix}\sigma_{I} & 0 & 0\\ 0 & \sigma_{II} & 0\\ 0 & 0 & \sigma_{III}\end{bmatrix}$$

- no shear stresses reference system

  分定 normal 达到极值

- the yiewling of material depends on principal stresses

判断是否 yield

yield criterion $f(\sigma_I, \sigma_{II}, \sigma_{III}, \sigma_Y) \gtrless 0$ $\begin{cases} > \text{plastic} \\ =0 \text{ 正好 yield} \\ < \text{elastic} \end{cases}$

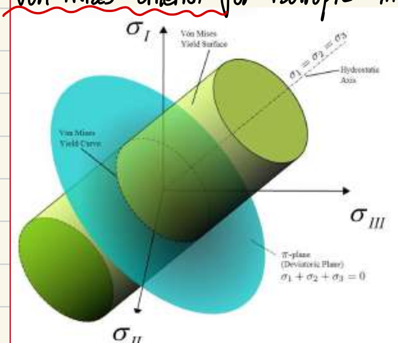
normal vector

$\tau=[\sigma]\underline{a}=\lambda\underline{a}$ eigenvalue

$([\sigma]-\lambda[I])a=0$ 在几何变换后.

eigenvector 方向不变

Von Mises criterion for isotropic metals



Mathematically:

$$\sigma_{eq}=\sqrt{\frac{1}{2}\left[\left(\sigma_I-\sigma_{II}\right)^2+\left(\sigma_I-\sigma_{III}\right)^2+\left(\sigma_{II}-\sigma_{III}\right)^2\right]}\geq\sigma_Y \quad \sigma_{eq}>\sigma_Y$$

material 自身性质

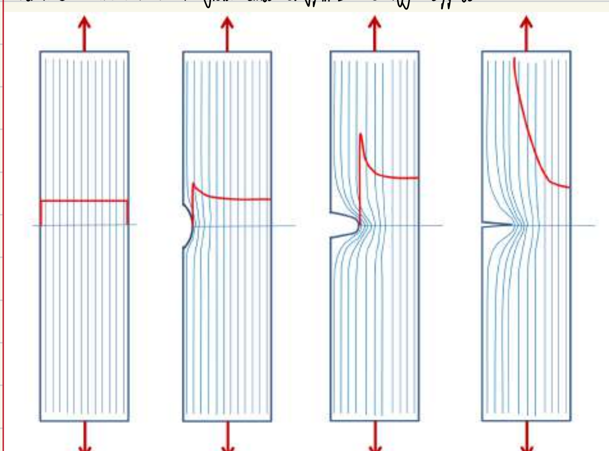
In a general Cartesian system:

$$\sigma_{eq}=\sqrt{\frac{1}{2}\left[\left(\sigma_{xx}-\sigma_{yy}\right)^2+\left(\sigma_{xx}-\sigma_{zz}\right)^2+\left(\sigma_{yy}-\sigma_{zz}\right)^2+6\left(\tau_{xy}^{\ 2}+\tau_{xz}^{\ 2}+\tau_{yz}^{\ 2}\right)\right]}$$

$$\sigma_{eq}=\sqrt{\frac{1}{2}\left[\left(\sigma_{xx}-\sigma_{yy}\right)^2+\left(\sigma_{xx}\right)^2+\left(\sigma_{yy}\right)^2+6\tau_{xy}^{\ 2}\right]} \quad \text{in plane stress}$$

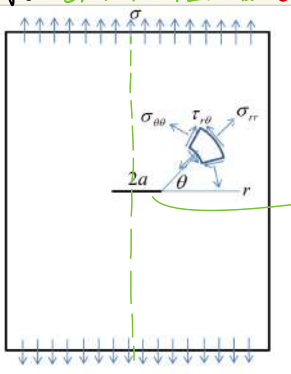
Von Mises

Fracture mechanics: K-field and Griffith's energy approach



stress distribution in different geometry

K-field (裂纹尖端应力场) Only for brittle materials



stress intensity factor

$$\sigma_{ij} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} f_{ij}(\theta) = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \qquad \sigma \propto \frac{1}{r}$$

到裂纹尖端距离 只与角度相关的函数

why use 2a, use symmetric geometric

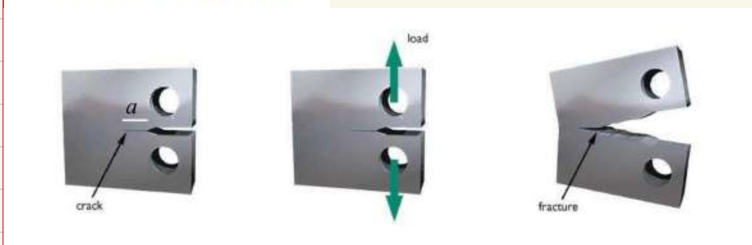
$$K_i = \sigma\sqrt{\pi a} \geq K_{ic}$$

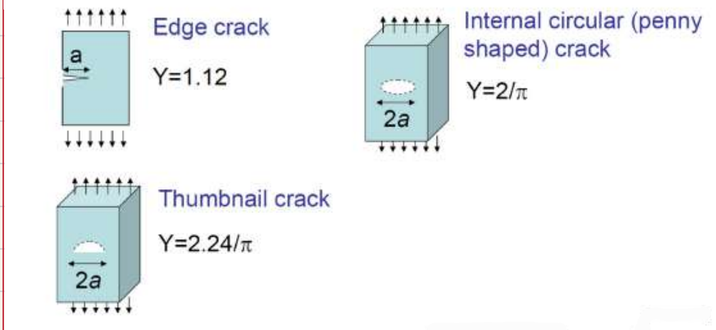
constant critical stress intensity factor

when K_i exceed constant $\sigma\sqrt{\pi a}$, crack

For every geometry
most general solution
In case, We always use
$$K_I = Y(\text{geometry}) \cdot \sigma\sqrt{\pi a}$$
use to explain geometry and boundary.

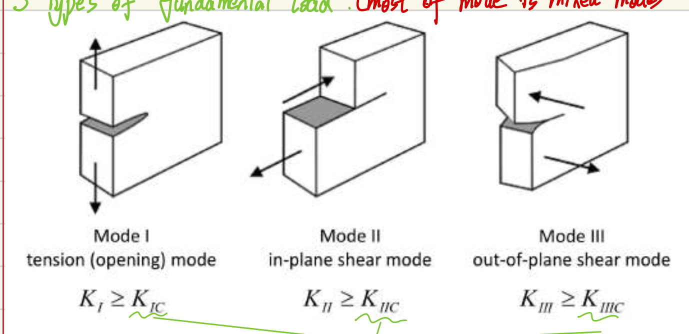
考虑非对称，非无限大板，等等.





Different facture modes and mixed-mode.

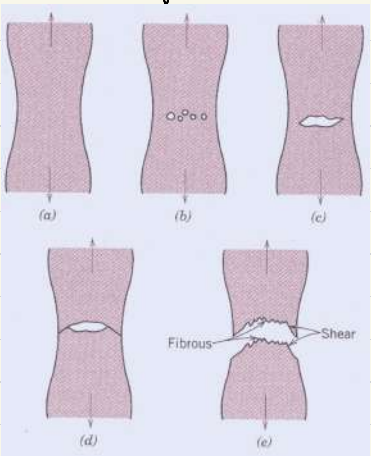
3 types of fundamental load (most of mode is mixed mode)



need consider which mode
but only mode I in exam

critical values

Mechanisms of crack initiation



How crack exist ?

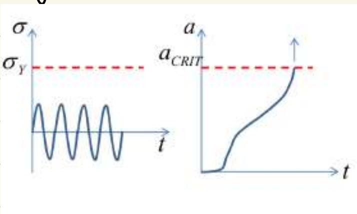
e.g. tension test

① hydrostatic loading in the region, cause impurities to detach
杂质 脱落

② void created

③ keep pulling. those void elongate, then finally crack.

Fatigue (疲劳)



repeated, cyclic straining of a material in elastic regime

在远低于 yield 的 σ 进行高频 stress, 也会造成 failure

Mechanical actions and machining

加工造成的表面划痕 or 孔洞也会造成 microcrack ⟶ 微观层面

Thermal Fatigue

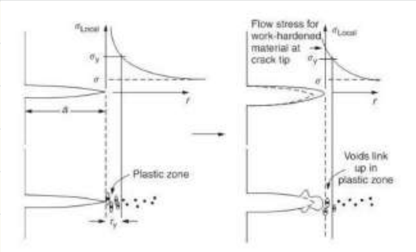
caused by periodic variation of component's temperature

chemical corrosion

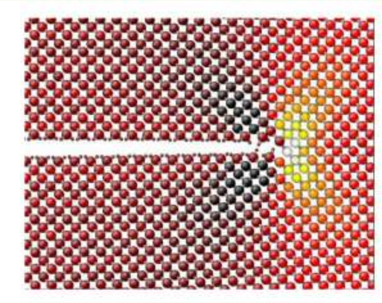
Material in aggresive enviroments

e.g. corrosion, oxidation in surface of metal

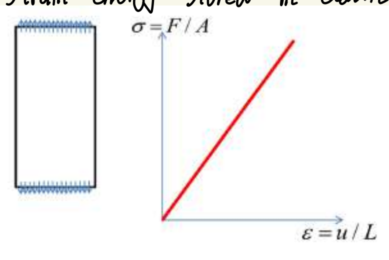
Ductile fracture { ① High plastic strain zone initiate void nucleation at impurities
② Crack tip is round
③ Crack proceeds by microvoid growth and coalescene



Brittle fracture { ① material cannot deform plastically by motion of dislocations or otherwise
② Atomic bonds are progressively brocken extending crack
③ Creation of new cracked surface necessitates energy spending (surface spending)



Strain energy stored in elastic body



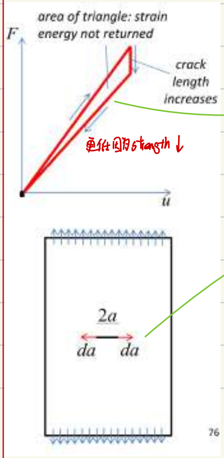
Griffith's

$$V = \int F\,du = AL\int \sigma\,d\varepsilon = V\int \sigma\,d\varepsilon = V\int_0^{\varepsilon} E\,d\varepsilon = VE\varepsilon^2/2 = V\frac{\sigma^2}{2E}$$

displacement

$F = \sigma \cdot A$
$u = \varepsilon \cdot L$

$\sigma = E \cdot \varepsilon$ in elastic phase



重新回到strength ↓

Energy is lost ⟶ where is the energy?

Energy must be spent to create the new crack surface of area 2dA

$\gamma_s = \frac{J}{m^2}$，生成单位面积裂纹所需能量

$-dV \geq 2\gamma_s dA$

断键吃的energy　　形成新表面所需的

why 2？每裂一次生成两个新表面

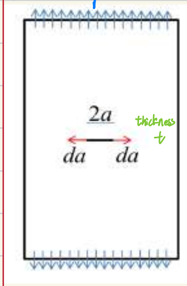
$G_1 = -\frac{dV}{dA} \geq 2\gamma_s$

能量守恒

strain energy release rate

e.g.

For a plate with a centre crack of length 2a and thickness t:



$$U = U_0 + U_{k\text{-plastic}} = V\frac{\sigma^2}{2E} - \frac{\pi\sigma^2 a^2 t}{E}$$

apply Griffith's hypothesis

$$dU = -\frac{2\pi\sigma^2 a t}{E}da \qquad A = 2at \qquad dA = 2t\,da$$

$$-\frac{dU}{dA} = \frac{\pi\sigma^2 a}{E} \geq 2\gamma_s$$

So $-dU \geq 2\gamma_s dA + \text{plastic work at crack tips} = G_{Ic}dA \Rightarrow G_I = -\frac{dU}{dA} \geq G_{Ic}$

Equivalent of K_{Ic} and G_{Ic}

$$G_I = -\frac{dU}{dA} = \frac{\pi\sigma^2 a}{E} \geq G_{Ic} \Rightarrow \sigma^2\pi a \geq E G_{Ic} \Rightarrow \sigma\sqrt{\pi a} \geq \sqrt{E G_{Ic}}$$

compare with K-field $\quad K_I = \sigma\sqrt{\pi a} \geq K_I$ $\quad$ compare Griffith's with Taylor's

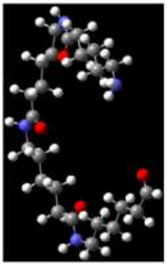
$$G_I = \frac{\pi\sigma^2 a}{E} = \frac{K_I^2}{E}$$

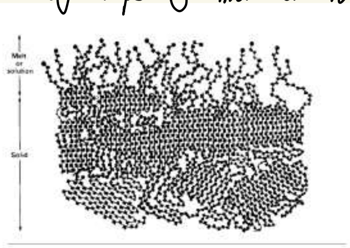
$$K_{Ic} = \sqrt{E G_{Ic}} \Rightarrow G_{Ic} = \frac{K_{Ic}^2}{E}$$

Polymers chain-like molecules by repeating monomer n times

compress 远强于 tensile

asymmetric structure 非对称、





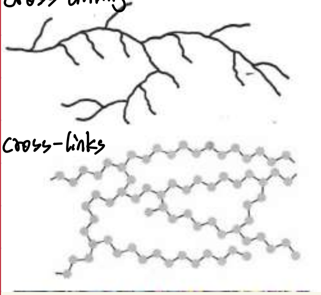
connect with each other by covelent bonds very strong.

Can be affected by temperature

$T > T_{cri}$, covelent $\quad$ melt in high T

$T < T_{cri}$, covelent and side-by-side

cross linking



cross-links

The degree of polymerisation

一条链中平均有多个重复单元 → 大 长链

→ 小 短链

Classification of polymers

Thermoplastic polymers or thermoplastics : 加热后熔化可再用

- Made by linear polymeric chains with no cross-linking and occasional branching.
- Soften when heated and can be formed as a viscous liquid.

Thermosetting polymers or thermosets : 加热后直接热分解or碳化，没有melting point

- Made by highly cross-linked chains, network polymers.
- Made by mixing a resin and a hardener either at room T or upon heating.
- When over-heated burn and decompose but do not lose shape due to cross-links

Elastomers : (little cross-linking) $T_G > T_{room}$, act like rubbers

- Made by almost-linear polymers with occasional cross-links
- At room temperature secondary bonds already melted ($T_G < T_{room}$).
- Cross-links give shape memory and material is elastic to large strains (rubbers)

Natural polymers

- Cellulose and lignin are the main components of wood and plants and have the molecular structure of a polymer;
- proteins also have the same structure

Crystallisation

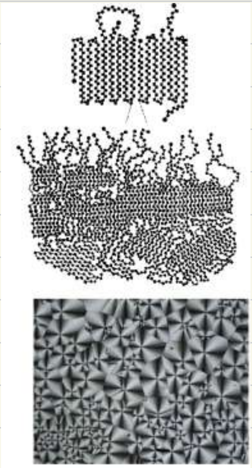
difficult for branched and cross-linked polymer

So Polymers hard to Crystallisation

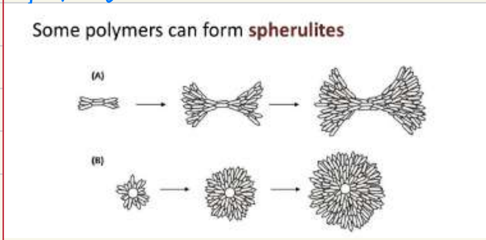
why? - polymers no orders to form.

结构无序，无法form crystall

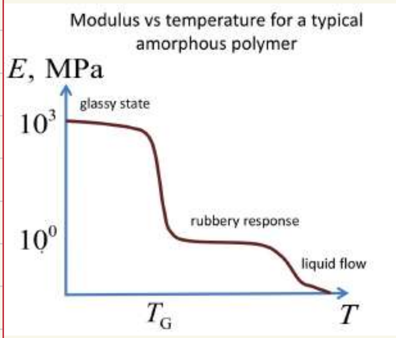
Crystallisation makes material more stable

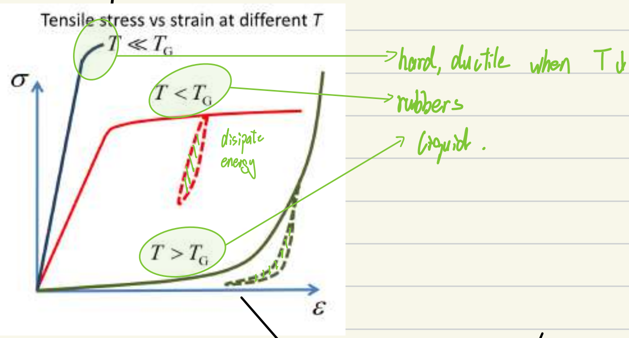


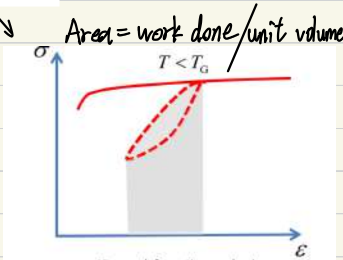
Spherulites

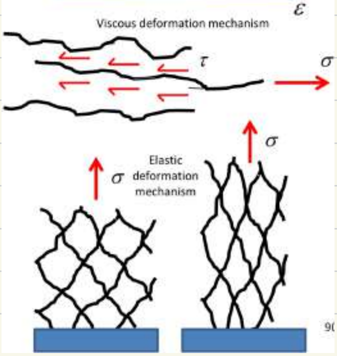


Glass transition and effect on tensile response.





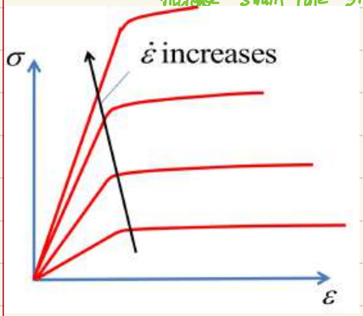




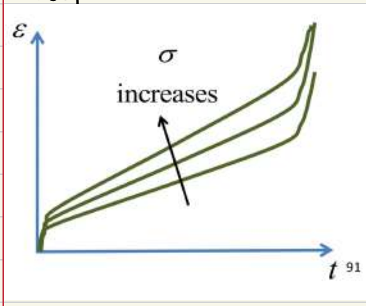
Viscous effects

- Strain rate sensitivity: ① stiffness and strength increase with strain rate $\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{(dL/dt)}{L} = \frac{v}{L}$
  ② ductility decrease with strain rate

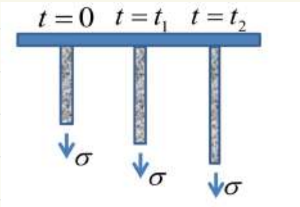
increase strain rate similar with decrease Temperature



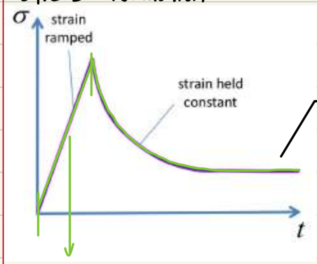
- creep



Stress keep constant, but strain still increase with time



- stress relaxation



viscoplastic

For polymer, It's called {viscoelastic / viscoplastic} rather than {elastic / plastic}

Ceramics

High melting point, hardness are advantages, large density

why packing not that effective, but large density?
{ionic bonds are strong / 2 differents kinds of atoms / elements are heavy}

Classification of ceramics

Glasses : Amorphous caremics based on SiO_2 and additives

无定形体

Vitreous ceramics : {Crystalline domains seperated by amorphous materials / Base on clays}

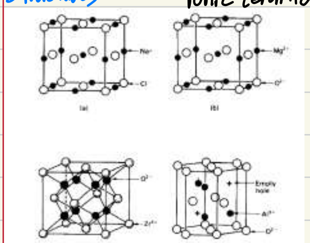
Engineering ceramics : Man-made crystalline ceramics made by heating non-metals
(C, artificial diamond) or oxides (e.g. Al_2O_3)

Cement and concrete : mixture of lime CaO, Silica SiO_2, Alumina Al_2O_3.
composite of sand/stone bonded by concrete

Natural ceramics : Natural diamond and stones

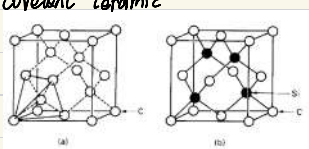
Structures

ionic ceramic

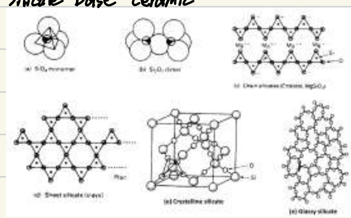


two ions with same charge cannot be together
so packing not that effective

Covelent ceramic

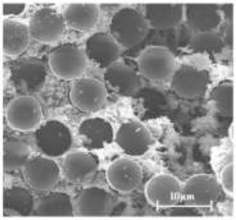


Silicate base ceramic

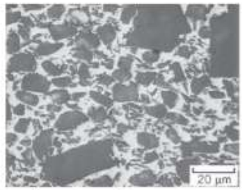


Ceramics Alloy and Composite

- Different compounds can mixed at high temperature to give ceramic alloys
- Why 变成合金就不那脆?



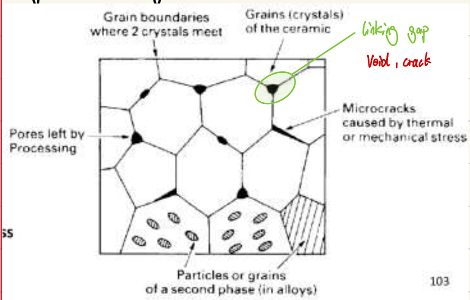
FRP:
glass and carbon fibres are ceramics



Metal matrix and ceramic particles

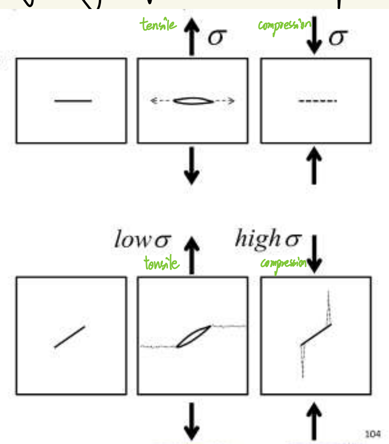
原本是 covelent/Ionic
directional, hard to slip.
加入金属
easier to slip

Defects in crystalline ceramics



ceramics always cracks !!!

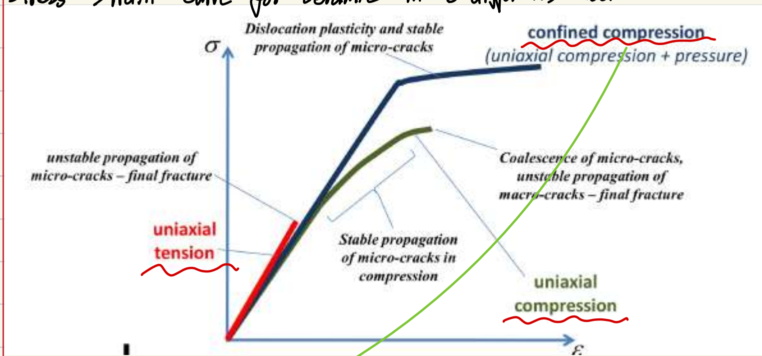
Asymmetry of the unixial response



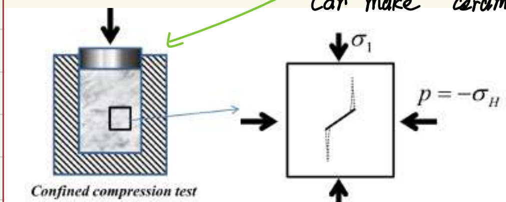
跟据 geometric, 有些抗 tensile test
有些抗 compression test.

Caused by asymetic of material, 同种材料在tensile 不一样
compression

Stress-Strain Curve for ceramic in 3 differents test



Can make ceramics yeild → why?



In presence of high confinement and consequently high hydrostatic stress (pressure), crack propagation is suppressed and the ceramic forced to deform by dislocation motion

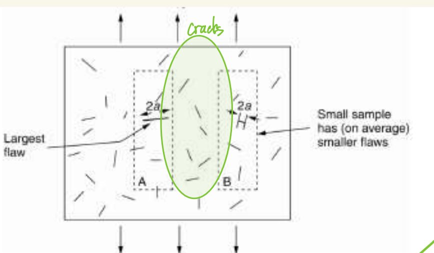
pressure in each directions keep cracks shut.

Stastical size effects

$$\sigma_{TS} = \frac{K_C}{\sqrt{\pi a_m}}.$$

$K_1 = \sigma\sqrt{\pi a_m} \approx K_c$

a max



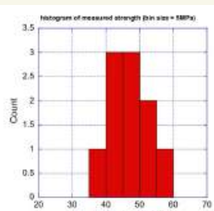
- Cracks are diluted
- So we use the worst crack which has biggest "a"

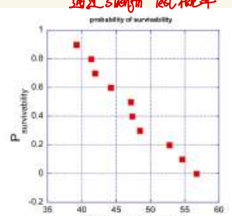
leading crack.

Survivability

通过strength test概率

measured strength, MPa	P_survive
39.200	0.90000
41.300	0.80000
41.900	0.70000
44.200	0.60000
47.100	0.50000
47.300	0.40000
48.400	0.30000
52.700	0.20000
54.600	0.10000
56.700	0.0000



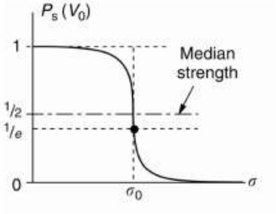


根据所需的survivability选material

- repeat stress tess
- plot graph.

Weibull's probability of survivability (for uniform tensile test)

先plot σ — survivability. 然后看 fit 的 σ_0, m (拟合)



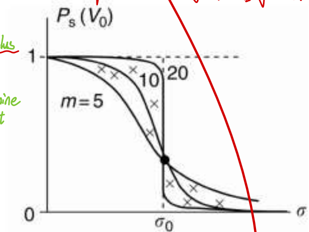
applied stress

Weibull's modulus

$$P_s(V_0) = \exp\left\{-\left(\frac{\sigma}{\sigma_0}\right)^m\right\}$$

where σ_0 and m are constants to be determined by fitting to data

need to determine in experiment



Note: for $\sigma = \sigma_0$, $P_s(V_0) = 1/e = 0.37$. σ_0 is the stress that allows 37% of survival.

- Chalk, brick, pottery and cement: $m \sim$ 1-10; engineering ceramics (SiC, Al_2O_3): $m \sim$ 10-20
- $m >$ 40-50: effectively means a well-defined strength (curve looks like a step function)

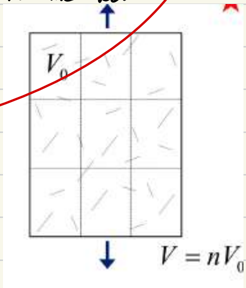
For large size of materials, easier to defect. why? need makesure every part survive in the test

n个小的同时 survive

$$P_s(V) = \{P_s(V_0)\}^n = [P_s(V_0)]^{V/V_0} \qquad n=\frac{V}{V_0}$$

同时取 ln

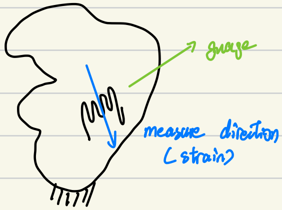
$$\ln P_s(V) = \frac{V}{V_0}\ln P_s(V_0) \Rightarrow P_s(V_0) = \exp\left\{-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right\}$$



if not uniform, $\longrightarrow$ stress varies with position

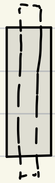
$$P_s(V) = \exp\left\{-\frac{1}{\sigma_0^m V_0}\int \sigma^m dV\right\}$$

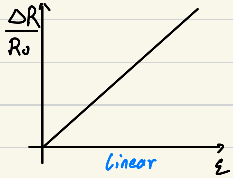
Materials Lab



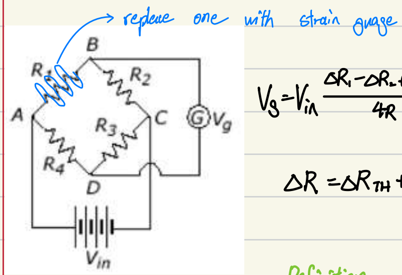
How it works?

$R = \rho \frac{L}{A}$



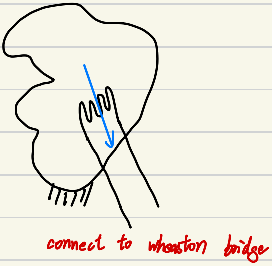


$\frac{\Delta R}{R} = S\varepsilon$



$V_g = V_{in} \frac{\Delta R_1 - \Delta R_2 + \Delta R_3 - \Delta R_4}{4R} = V_{in} \frac{\Delta R}{4R} = V_{in} \frac{1}{4} S\varepsilon$

$\Delta R = \Delta R_{TH} + \Delta R_{actual}$



Definition

Hardness $= 3\sigma_Y$, $H = \frac{F}{A}$

Hardness test :

- Vickers → square pyramid
- Rockwell → cone
- Brinel → sphere
- Berkovic → triangle pyramid
- Knoop → pyramid with rhombidal base

Summary - Plasticity

① Crystalline
- Nucleation ⟶ Growth ⟶ Irregular ⟶ Grain Boundary.
- Same atom packing. Different orientations

② Plastic deformation and dislocation

Carrier of plastic deformation

dislocations
- edge: dislocation line ⊥ Burger's vector (extra half-plane)
- screw: //

when critical shear stress reach → dislocations

Plastic, will not move back after unload

- Metal: easy ← non-directional, close-packed
  - Cu, Al, Ag → Ductile, soft (properties)
- Covelent: hard ← directional, almost no dislocation
  - Si, Diamond → Hard, brittle
- Ionic ceramic: hard ← non-directional but avoid "++" "--" neighbours
  - → Hard, brittle

Slip systems
- Slip plane: crystallographic plane ..... provide easiest distance to move / greatest atomic density
- slip direction: crystallographic direction .....
- slip system: plane + system + condition
- slip step: displacement ......

→ FCC
- {111} closed packed plane
- <110> ------ direction
- 12 slip systems

BCC
- Only closed packed direction
- hard to active

HCP { Closed packed plane & direction
Very few slip system (only in elevated Temperature)
↓ Ductility fcc > bcc > hcp

Schmid factor (single crystals) { Decide slip system active under certain stress

$\sigma \cos\lambda \cos\phi$

for $\tau \geq \tau_{cri}$ active

The Biggest τ will active first

Strengthening Solid solution

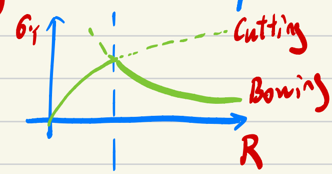
{ Substitutional Similar size
Interstitial Smaller size

substitute

fit gap

Precipitate

{ Cutting Small, closely-spaced
Bowing Fewer, wider-spaced



Grain size → Reduction Grain size
Only ↓ Increasing Grain Boundary area

{ Ductility ↑
Strength ↑

$\sigma_Y \propto \frac{1}{\text{grain size}^{\frac{1}{2}}}$

Work hardening cold work → dislocation density ↑
{ forging
rolling
drawing

generate stress ←

↓ hinders of dislocation

$\% CW = \frac{A_0 - A_d}{A_0} \times 100$

↓ yield strength ↑
tensile strength ↑
ductility

heat treatment
ductility ↑
strength ↓

4 different strengthening (Details)

Precipitation
Particles Introduce + thermal treatment
By solving
closely-space precipitate form
precipitate act as obstacles to dislocation
overcome
Bowing
Cutting
Higher stress needed
strength ↑

Grain
why ductility ↑
more choice of sliping system
Grain size ↓
Grain Boundary density ↑
Obstacle to dislocation motion
Higher stress needed
strength ↑

Solid solution
solute atoms in host lattice
atom size dismatch
Local stress-strain field Interact
dislocation mobility ↓
Higher stress needed
strength ↑

Cold work
plastic deformation Occur
dislocation multiplication ↑
dislocation density ↑
Interaction of dislocations
dislocation mobility ↓
Higher stress needed
strength ↑

Why strength much lower than estimate ?

Model assumes all bonds in plane need to break

only one bond break at a time will leading much lower stress