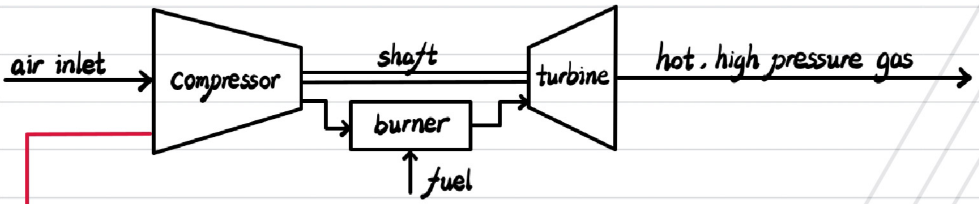
The Basic

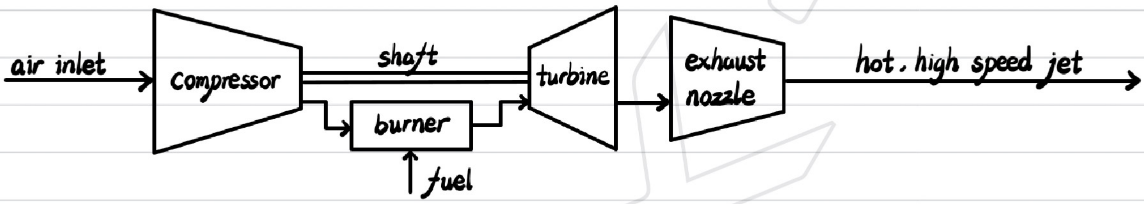
Schemes of Air-breathing Turbo-engines

Basic Gas Generator

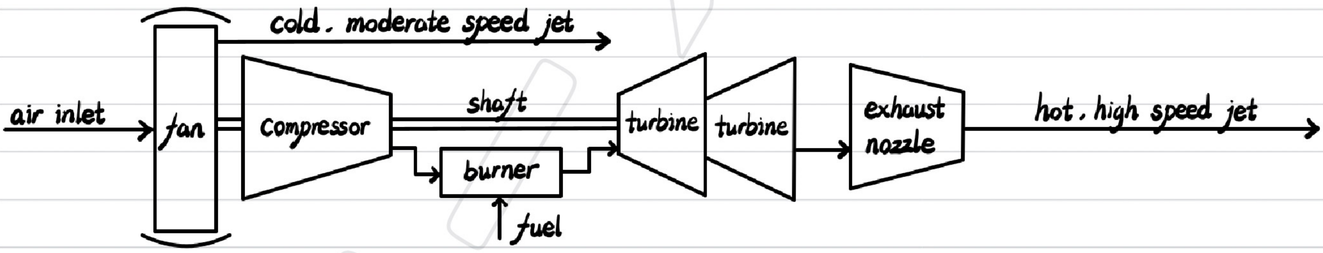


- has more stage than turbine $\longleftarrow$ $\frac{\partial p}{\partial x} > 0$ adverse pressure gradient prone to seperation
- $\frac{dA}{dx} < 0$ for annulus 环 $\longrightarrow$ $\frac{dV_{ax}}{dx} = 0$ $\longrightarrow$ repeated design reduce design cost

Turbojet

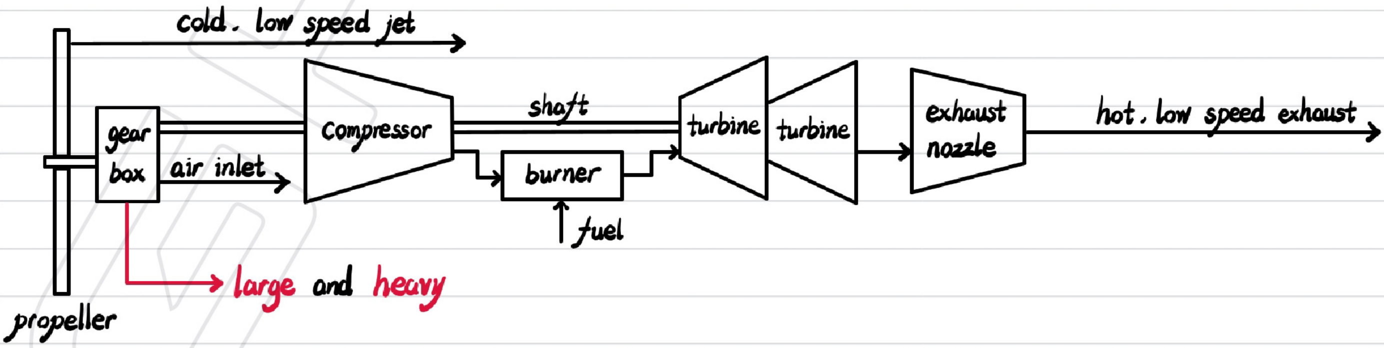


Turbofan



* might use several shafts to reduce the speed mismatch between fan / compressor and turbine

Turboprop

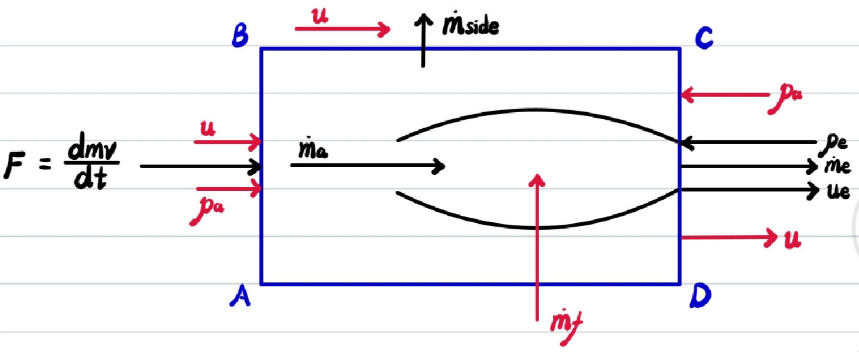


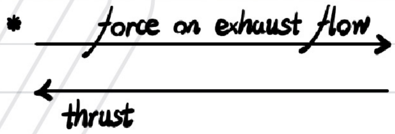
Propfan*

Thrust Equation

Single Jet

① apply control volume





② apply conservation of mass

· $\dot m_e = \dot m_a + \dot m_f = (1+f)\,\dot m_a$ · $\dot m_{CD} + \dot m_{side} = \dot m_{AB} + \dot m_f$

③ apply conservation of momentum

$$(F + p_a A_{AB}) - [p_e A_e + p_a(A_{CD} - A_e)] \xrightarrow{A_{AB} = A_{CD}} F - (p_e - p_a)A_e$$

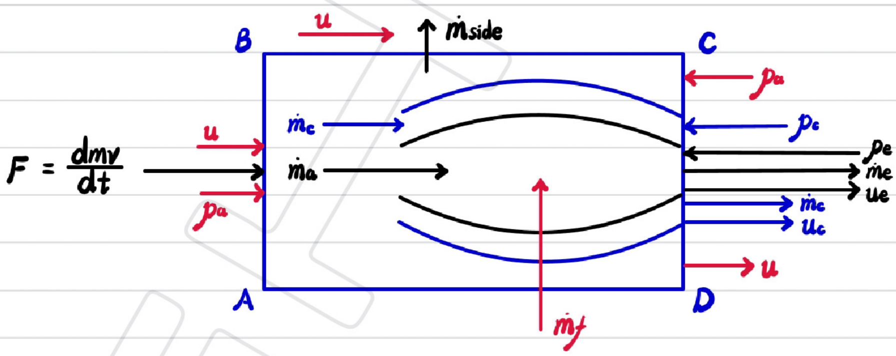
$$= [\dot m_e u_e + (\dot m_{CD} - \dot m_e)u + \dot m_{side}u] - \dot m_{AB}u = (1+f)\dot m_a u_e + (\dot m_{CD} - \dot m_e + \dot m_{side} - \dot m_{AB})u$$

$$= \dot m_a(1+f)u_e + (\dot m_f - \dot m_e)u = \dot m_a(1+f)u_e - \dot m_a u$$

$$\longrightarrow F = \dot m_a[(1+f)u_e - u] + (p_e - p_a)A_e$$

* $p_e = p_a$ in the design regime

Two Exhaust Jet



$$F = \dot m_a[(1+f)u_e - u] + \dot m_c(u_c - u) + (p_e - p_a)A_e + (p_c - p_a)A_c$$

Turbofan

$$F = \dot m_a[(1+f)u_e + \beta u_{ef} - (1+\beta)u] + (p_e - p_a)A_e + (p_{ef} - p_a)A_{ef}$$

* $\dot m_{fan} = \beta \dot m_a$

Engine Peformance

Overall Efficiency $\eta_o = \dfrac{\text{thrust power}}{\text{rate of energy consumption}} = \dfrac{Fu}{\dot{m}_f Q_R}$ *ignore KE_{fuel} * $\eta_o = \eta_{prop}\eta_{th}$

Propulsion Efficiency $\eta_{prop} = \dfrac{\text{thrust power}}{\text{production of propeller kinetic energy}}$

$$= \frac{Fu}{\dot{m}_a[(1+f)\frac{1}{2}u_e^2 - \frac{1}{2}u^2]}$$

$$\xrightarrow[p_e \approx p_a]{f \ll 1} \frac{\dot{m}_a(u_e - u)u}{\frac{1}{2}\dot{m}_a(u_e^2 - u^2)} = \frac{2u}{u_e + u}$$

Thermal Efficiency $\eta_{th} = \dfrac{\text{production of propeller kinetic energy}}{\text{rate of energy consumption}} = \dfrac{\dot{m}_a[(1+f)\frac{1}{2}u_e^2 - \frac{1}{2}u^2]}{\dot{m}_f Q_R}$

Takeoff Thrust $F|_{u=0} = \dot{m}_a(1+f)u_e + (p_e - p_a)A_e$

Specific Thrust $\dfrac{F}{\dot{m}_a} = (1+f)u_e - u + \dfrac{1}{\dot{m}_a}(p_e - p_a)A_e \xrightarrow[p_e \approx p_a]{f \ll 1} u_e - u$

TSFC (Thrust Specific Fuel Consumption) $\dfrac{\dot{m}_f}{F} = \dfrac{f}{F/\dot{m}_a}$

Breguet's Range Formula for level flight

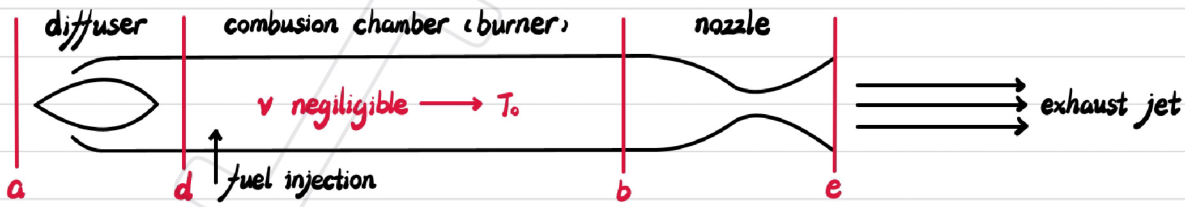
$$F = D = \frac{1}{L/D}L = \frac{1}{L/D}mg \longrightarrow Fu = \frac{mgu}{L/D} = \eta_o(\dot{m}_f Q_R) \longrightarrow u = \frac{(L/D)\eta_o \dot{m}_f Q_R}{mg}$$

$$\frac{dm}{dt} = \frac{dm}{dl}\frac{dl}{dt} = \frac{dm}{dl}u \xrightarrow{\frac{dm}{dt} = -\dot{m}_f} \frac{dm}{dl}u = -\dot{m}_f \longrightarrow \int dl = \int \frac{u}{-\dot{m}_f}dm = \int -\frac{(L/D)\eta_o Q_R}{mg}dm$$

$$\longrightarrow l = \eta_o (L/D)\frac{Q_R}{g}\int_{m_1}^{m_2} -\frac{1}{m}dm \longrightarrow l = \eta_o (L/D)\frac{Q_R}{g}\ln\left(\frac{m_1}{m_2}\right)$$

Ideal Ramjet

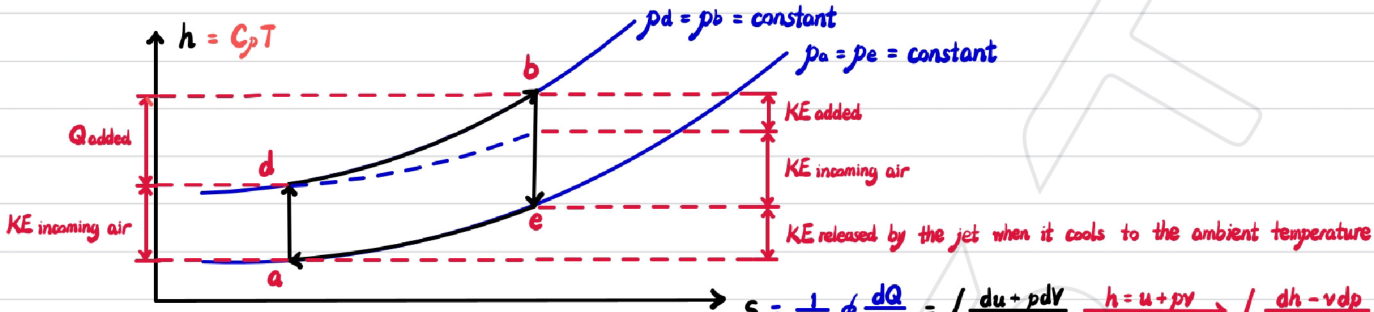
The Ramjet can be practical only at supersonic speed



Ideal Ramjet

- compression and expansion process are reversible and adiabatic
- combustion occurs at constant pressure
- specific heat ratio γ and gas constant R are constant throughout engine
- nozzle in design regime $\longrightarrow$ $p_e = p_a$

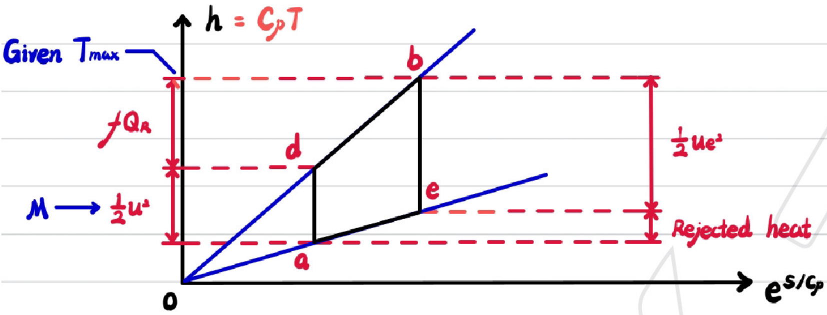
h-s (Enthalpy - entropy) Diagram



$$s = \frac{1}{m}\oint \frac{dQ}{T} = \int \frac{du + pdV}{T} \xrightarrow{h = u + pv} \int \frac{dh - vdp}{T}$$

$$= \int \frac{C_p}{T} dT - \int \frac{\left(\frac{RT}{p}\right)}{T} dp = C_p \ln T - R \ln p$$

h - e^{s/C_p} Diagram



$$e^{s/C_p} = e^{(C_p \ln T - R \ln p)/C_p} = e^{\ln\left(\frac{T}{p^{R/C_p}}\right)} = \frac{T}{p^{R/C_p}}$$

① find f $\quad \dot{Q} + \dot{m}_{in} h_{o,in} = \dot{W} + \dot{m}_{out} h_{o,out}$ $\quad$ * energy conservation law

$$\longrightarrow m_f Q_R + m_a h_{oa} = m_b h_{ob} \longrightarrow f Q_R + h_{od} = (1+f) h_{ob}$$

$$\longrightarrow f = \frac{h_{ob} - h_{od}}{Q_R - h_{ob}} = \frac{T_{ob} - T_{od}}{Q_R / C_p - T_{ob}}$$

· $T_{od} = T_a \left(1 + \frac{\gamma - 1}{2} M^2\right) \longrightarrow p_{od} = p_{ob} = p_a \left(\frac{T_{od}}{T_a}\right)^{\frac{\gamma}{\gamma - 1}}$

· $T_{ob} = T_{max}$

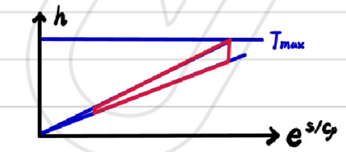
② find u_e $\quad \dot{Q} + \dot{m}_{in} h_{o,in} = \dot{W} + \dot{m}_{out} h_{o,out}$ $\quad$ * energy conservation law

$$\longrightarrow h_{ob} = h_{oe} \longrightarrow h_{ob} = h_e + \tfrac{1}{2} u_e^2 \longrightarrow \tfrac{1}{2} u_e^2 = h_{ob} - h_e = C_p (T_{ob} - T_e)$$

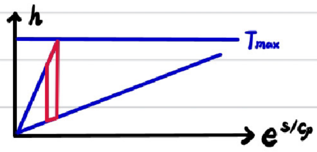
· $T_e = T_{ob} \left(\frac{p_e}{p_{ob}}\right)^{\frac{\gamma - 1}{\gamma}} = T_{ob} \left(\frac{p_a}{p_{ob}}\right)^{\frac{\gamma - 1}{\gamma}}$

③ find efficiencies

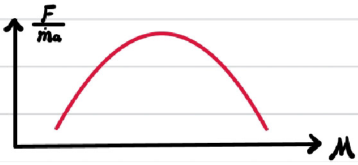
* at small M.



at large M.

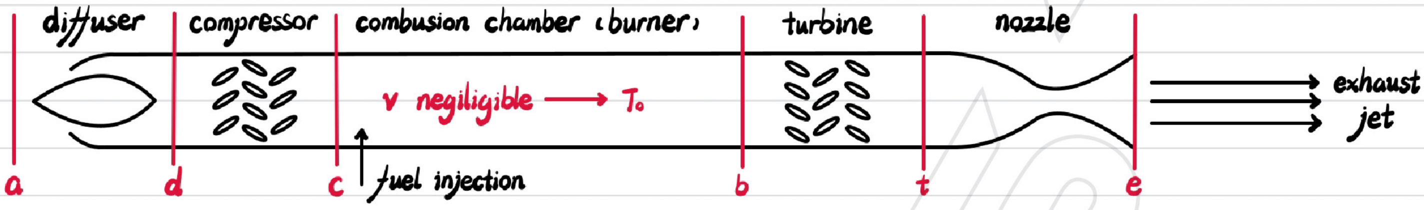


specific thrust $= \frac{F}{m_a} \xrightarrow[p_e - p_a]{f \ll 1} u_e - u$

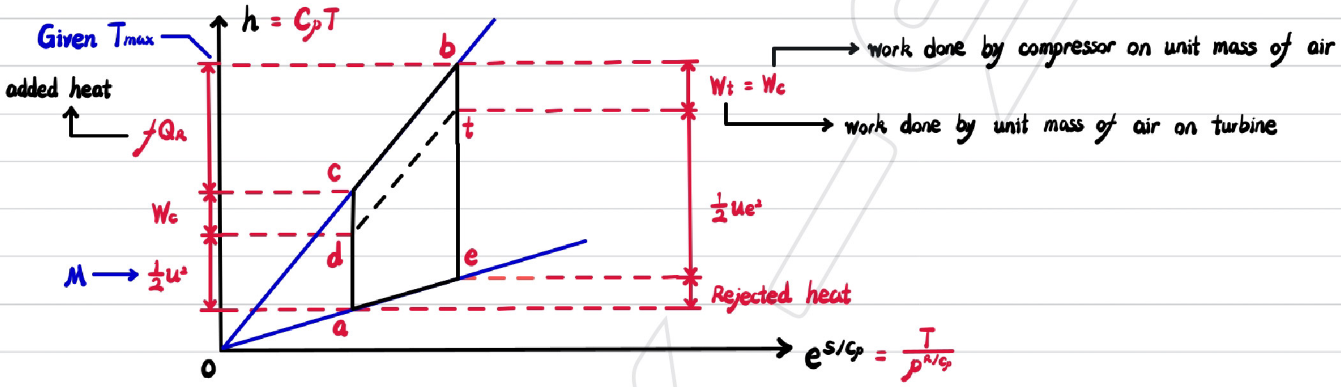


Ideal Turbojet

The Turbojet



$h - e^{s/C_p}$ Diagram



① find f $\quad \dot{Q} + \dot{m}_{in} h_{o,in} = \dot{W} + \dot{m}_{out} h_{o,out}$ energy conservation law

$$\longrightarrow fQ_R + h_{oc} = (1+f)h_{ob} \longrightarrow f = \frac{h_{ob} - h_{oc}}{Q_R - h_{ob}} = \frac{T_{ob} - T_{oc}}{Q_R / C_p - T_{ob}}$$

· $T_{od} = T_a\left(1 + \frac{\gamma - 1}{2} M^2\right) \longrightarrow p_{od} = p_a\left(\frac{T_{od}}{T_a}\right)^{\frac{\gamma}{\gamma - 1}}$

· $p_{oc} = p_{ob} = p_{rc}\, p_{od} \longrightarrow T_{oc} = T_{od}\, p_{rc}^{\frac{\gamma - 1}{\gamma}}$

* $M \uparrow$ ⟶ optimal $p_{rc} \downarrow$ ⟶ at $M \approx 3$ a turbojet become ramjet

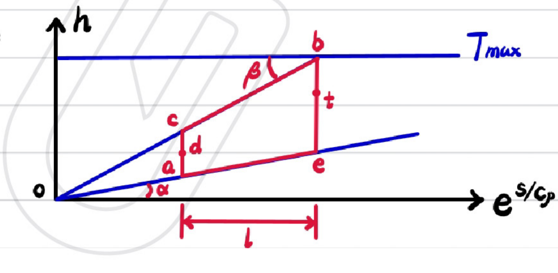
② find u_e $\quad h_{ot} = h_e + \frac{1}{2}u_e^2$

· $h_{ob} - h_{ot} = h_{oc} - h_{od} \longrightarrow T_{ot} = T_{ob} - T_{oc} + T_{od}$

· $T_e = T_{ob}\left(\frac{p_e}{p_{ob}}\right)^{\frac{\gamma}{\gamma - 1}} = T_{ob}\left(\frac{p_a}{p_{ob}}\right)^{\frac{\gamma}{\gamma - 1}}$

③ find efficiencies

*



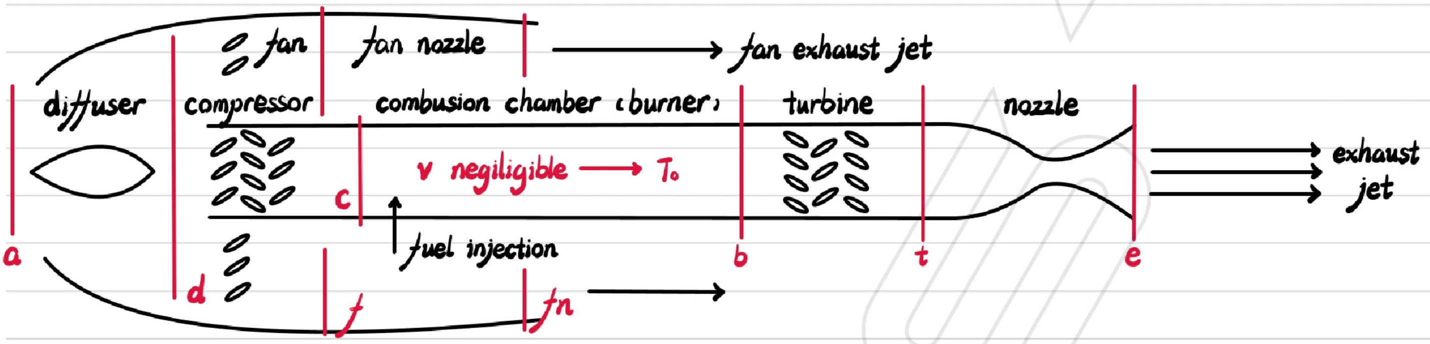
$$\eta_{th} = \frac{\dot{m}_a\left[(1+f)\frac{1}{2}u_e^2 - \frac{1}{2}u^2\right]}{\dot{m}_a f Q_R} \xrightarrow{f \ll 1} \frac{\frac{1}{2}u_e^2 - \frac{1}{2}u^2}{fQ_R}$$

$$= \frac{fQ_R - \text{Rejected heat}}{fQ_R} = 1 - \frac{\text{Rejected heat}}{fQ_R}$$

$$= 1 - \frac{l \tan\alpha}{l \tan\beta} = 1 - \frac{\tan\alpha}{\tan\beta} = 1 - \frac{T_a}{T_c}$$

Ideal Turbofan

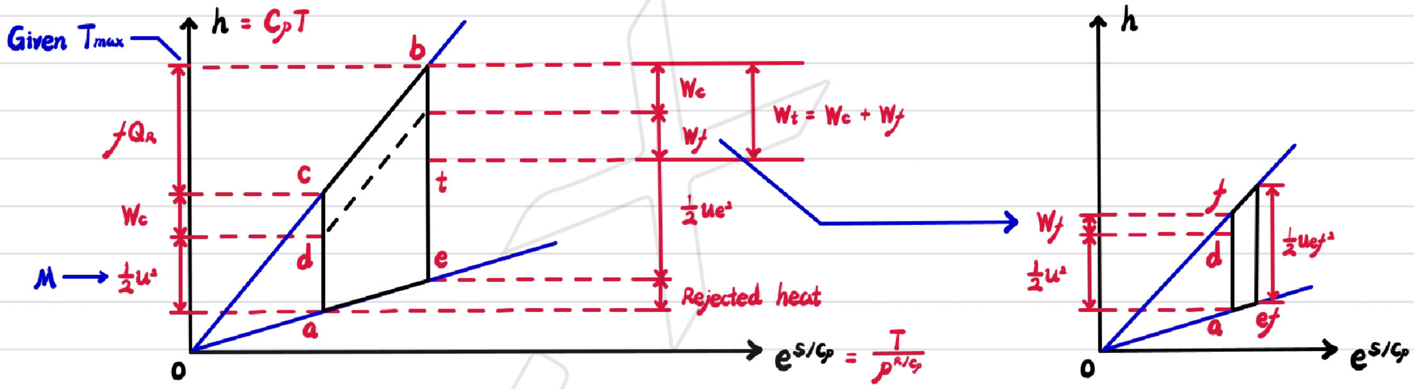
The Turbofan



to get momentum $\Delta I = mu$ for a mass of air, must gain $KE = \frac{1}{2}mu^2$ as cost

$\longrightarrow KE = \frac{1}{2}mu^2 = \frac{1}{2}(mu)u = \frac{1}{2}\Delta I u = \frac{1}{2m}\Delta I^2 \propto u$ or $\propto \frac{1}{m}$ $\longrightarrow$ use more propellant 推进物

$h - e^{s/c_p}$ Diagram



① find f $\quad \dot{Q} + \dot{m}_{in} h_{o,in} = \dot{W} + \dot{m}_{out} h_{o,out}$ energy conservation law

$$\longrightarrow fQ_R + h_{oc} = (1+f)h_{ob} \longrightarrow f = \frac{h_{ob} - h_{oc}}{Q_R - h_{ob}} = \frac{T_{ob} - T_{oc}}{Q_R/C_p - T_{ob}}$$

· $T_{od} = T_a(1 + \frac{1+\gamma}{2}M^2) \longrightarrow p_{od} = p_a(\frac{T_{od}}{T_a})^{\frac{\gamma-1}{\gamma}}$

· $p_{oc} = p_{ob} = p_{rc}\,p_{od} \longrightarrow T_{oc} = T_{od}\,p_{rc}^{\frac{\gamma}{\gamma-1}}$

② find u_{ef} $\quad h_{of} = h_{ef} + \frac{1}{2}u_{ef}^2 \longrightarrow \frac{1}{2}u_{ef}^2 = h_{of} - h_{ef} = C_pT_{of} - C_pT_{ef}$

· $p_{of} = p_{rf}\,p_{od} \longrightarrow T_{of} = T_{od}\,p_{rf}^{\frac{\gamma}{\gamma-1}}$

· $T_{ef} = T_{of}(\frac{p_{ef}}{p_{of}})^{\frac{\gamma-1}{\gamma}} = T_{of}(\frac{p_a}{p_{of}})^{\frac{\gamma-1}{\gamma}}$

③ find u_e $\quad h_{ot} = h_e + \frac{1}{2}u_e^2 \longrightarrow \frac{1}{2}u_e^2 = h_{ot} - h_e = C_p(T_{ot} - T_e)$

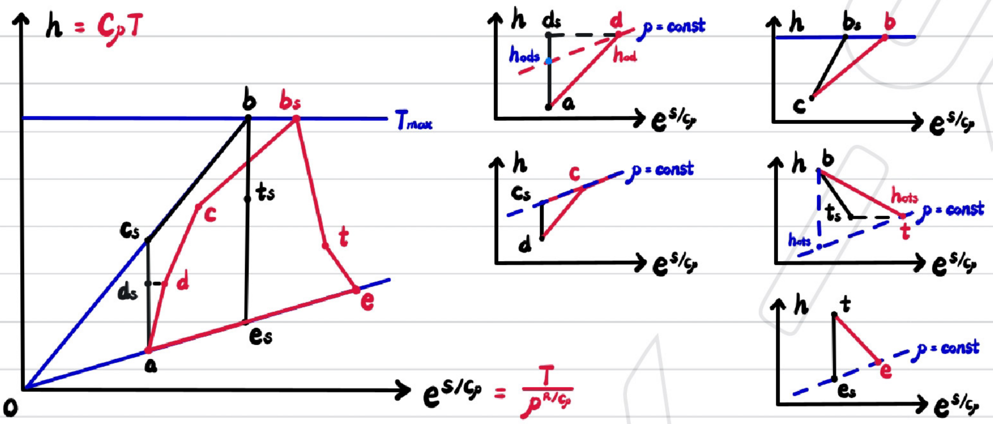
$\cdot\ h_{ob} - h_{ot} = (h_{oc} - h_{od}) + \beta(h_{of} - h_{od}) \longrightarrow T_{ot} = T_{ob} - T_{oc} - \beta T_{of} + (1+\beta) T_{od}$

$\cdot\ T_e = T_{ob}\left(\frac{p_e}{p_{ob}}\right)^{\frac{\gamma}{\gamma-1}} = T_{ob}\left(\frac{p_a}{p_{ob}}\right)^{\frac{\gamma}{\gamma-1}}$

④ find efficiencies * $F = \dot{m}_a[(1+f)u_e + \beta u_{ef} - (1+\beta)u]$

The Effect of Losses

$h - e^{s/c_p}$ Diagram



* $r_d = \frac{p_{od}}{p_{oa}}$

$r_b = \frac{p_{ob}}{p_{oc}}$

$r_n = \frac{p_{oe}}{p_{ot}}$

* $\frac{p_A}{p_B} = \left(\frac{T_{As}}{T_B}\right)^{\frac{\gamma_A}{\gamma_A - 1}}$

① find f $\quad \dot{Q} + \dot{m}_{in} h_{o,in} = \dot{W} + \dot{m}_{out} h_{o,out}$ energy conservation law * $c_p = \frac{\gamma}{\gamma-1}R$

$\longrightarrow fQ_{Rs} + h_{oc} = f\eta_b Q_R + h_{oc} = (1+f)h_{ob} \longrightarrow f = \frac{h_{ob} - h_{oc}}{\eta_b Q_R - h_{ob}} = \frac{T_{ob} - T_{oc}}{\eta_b Q_R / c_p - T_{ob}}$

· diffuser $\quad T_{od} = T_a\left(1 + \frac{\gamma-1}{2}M^2\right) \xrightarrow{\eta_d = \frac{h_{ods} - h_a}{h_{od} - h_a}} T_{ods} \longrightarrow p_{od} = p_a\left(\frac{T_{ods}}{T_a}\right)^{\frac{\gamma}{\gamma-1}}$

· compressor $\quad p_{oc} = p_{rc} p_{od} \longrightarrow T_{ocs} = T_{od}\, p_{rc}^{\frac{\gamma-1}{\gamma}} \xrightarrow{\eta_c = \frac{h_{ocs} - h_{od}}{h_{oc} - h_{od}}} T_{oc}$

· burner $\quad p_{ob} = p_{oc} r_b$

② find u_{ef} $\quad h_{of} = h_{ef} + \frac{1}{2}u_{ef}^2 \longrightarrow \frac{1}{2}u_{ef}^2 = h_{of} - h_{ef} = c_{pf}(T_{of} - T_{ef})$ * $c_{pf} = \frac{\gamma_f}{\gamma_f - 1}R_f$

· fan $\quad p_{of} = p_{rf} p_{od} \longrightarrow T_{ofs} = T_{od}\, p_{rf}^{\frac{\gamma_f - 1}{\gamma_f}} \xrightarrow{\eta_f = \frac{h_{ofs} - h_{od}}{h_{of} - h_{od}}} T_{of}$

· fan nozzle $\quad T_{efs} = T_{of}\left(\frac{p_{ef}}{p_{of}}\right)^{\frac{\gamma_f - 1}{\gamma_f}} = T_{of}\left(\frac{p_a}{p_{of}}\right)^{\frac{\gamma_f - 1}{\gamma_f}} \xrightarrow{\eta_{fn} = \frac{h_{of} - h_{ef}}{h_{of} - h_{efs}}} T_{ef}$

③ find u_e $\quad h_{ot} = h_e + \frac{1}{2}u_e^2 \longrightarrow \frac{1}{2}u_e^2 = h_{ot} - h_e = c_{pn}(T_{ot} - T_e)$ * $c_{pn} = \frac{\gamma_n}{\gamma_n - 1}R_n$

· turbine $\quad h_{ob} - h_{ot} = (h_{oc} - h_{od}) + \beta(h_{of} - h_{od}) \longrightarrow T_{ot} = T_{ob} - T_{oc} - \beta T_{of} + (1+\beta)T_{od}$

$\xrightarrow{\eta_t = \frac{T_{ob} - T_{ot}}{T_{ob} - T_{ots}}} T_{ots} \longrightarrow p_{ot} = p_{ob}\left(\frac{T_{ots}}{T_{ob}}\right)^{\frac{\gamma_n}{\gamma_n - 1}}$

· nozzle $\quad T_{es} = T_{ob}\left(\frac{p_e}{p_{ob}}\right)^{\frac{\gamma_n - 1}{\gamma_n}} = T_{ob}\left(\frac{p_a}{p_{ob}}\right)^{\frac{\gamma_n - 1}{\gamma_n}} \xrightarrow{\eta_n = \frac{h_{ot} - h_{oe}}{h_{ot} - h_{oes}}} T_e$

④ find efficiencies

Power Supply

Turbofan Engine $P_{fan} = \dot{m}_a \beta (h_{of} - h_{od}) = \dot{m}_a \beta C_p (T_{of} - T_{od})$

Turboshaft Engine $h_{ob} - h_{ot} = (h_{oc} - h_{od}) + \frac{P_{shaft}}{\dot{m}_a}$

Turboprop Engine $h_{ob} - h_{ot} = (h_{oc} - h_{od}) + \frac{P_{pr}}{\dot{m}_a}$

$\xrightarrow[\text{propeller efficiency } \eta_{pr}]{\text{gear efficiency } \eta_g} F_{pr} u = \eta_g \eta_{pr} P$

$\longrightarrow \frac{F}{\dot{m}_a} = (1+f) u_e - u + \frac{F_{pr}}{\dot{m}_a}$

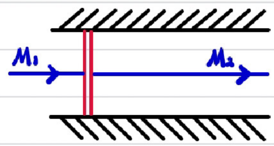
Engine Components

Intake and Nozzles

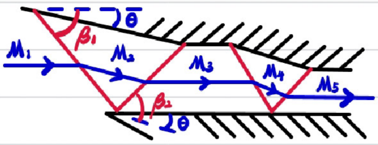
Normal Shocks Aerodynamics 2

Oblique Shock Waves Aerodynamics 2

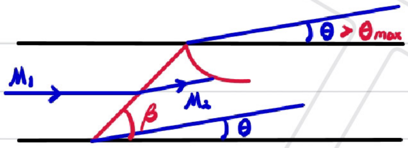
Supersonic Intake Idea losses $= \frac{p_{02}}{p_{01}} \xrightarrow{\text{Taylor expansion}} 1 - \frac{16\gamma}{3(\gamma+1)^2}(M_1 - 1)^3 \xrightarrow{M_1 \to 1} 0$



$\xrightarrow{\text{more efficient}}$

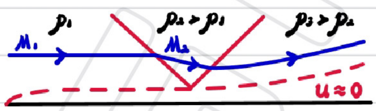


Intake Failure Due to Mach Reflection Aerodynamic 2



normal shock / strong shock near the wall $\longrightarrow$ large losses

Supersonic Intake Failure Due to Boundary Layer Seperation



* might re-attach downstream

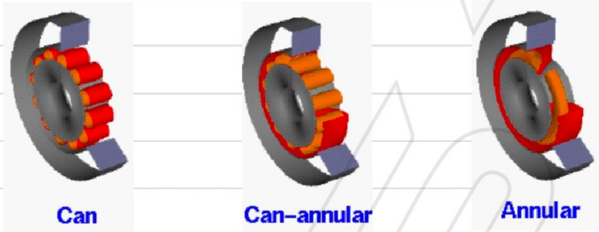
Petrov's formulae $\frac{p_{downstream}}{p_{upstream}} = 0.287 + (1 - 0.287) M_{upstr}$ or $1 + \frac{0.2\gamma M_{upstr}^2}{(M_{upstr}^2 - 1)^{1/4}}$ $(\gamma = 1.4)$

* vaild only for a turbulent boundary layer developed under an action of zero pressure gradient

at least over certain distance upstream from the seperation point and only for smooth wall

Combustors

Combustor Type



easy to design / test ⟷ smaller volume

Combustor Requirements

① high η_b and small pressure loss ($r_b \to 1$)   * solved by morden design

② small space and low weight

③ stable operation over a very wide range of conditions

④ outlet temperature T_{ob} < 1100 K (uncooled turbine blades) - 1850K (cooled turbine blades)

⑤ T_b should be evenly distributed

⑥ combustion maintain in the u_{air} = 30 ~ 60 m/s > $u_{flame\ propagation}$

⑦ avoid coking (formation of carbon deposits)

⑧ engine should not emit too much smoke

⑨ minimise emissions (NO_x, CO, unburned hydrocarbons)

Zonal Combustion

fuel ← fuel-rich — stoichiometric ratio — lean → air

less efficient ↓ ← T_{max} & stable → ↓ efficient

fuel injection | 30% added gradually avoid over-cooling | remaining T↓

15% ~ 20% → primary zone → secondary zone → tertiary / dilution 稀释 zone →

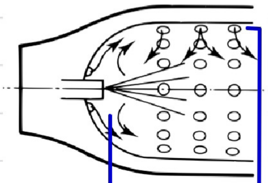
stoichiometric ratio* | complete combustion | cooling

* not completely burn due to dissociation ⟶ high T, u molecules break down when collide

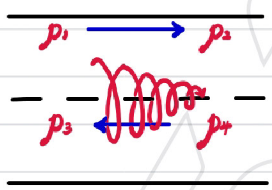
Primary Combustion Zone

since $U_{air} > U_{flame\ propagation}$ ⟶ reverse flow

① swirl 旋涡



swirl vane   tangential blowing



* $p_1 - p_3 > p_2 - p_4$

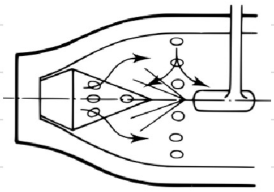
↓

$p_1 - p_2 > p_3 - p_4$

can be negative

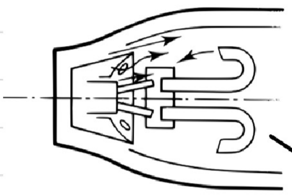
creating swirl ⟶ consuming energy ⟶ losses ↑

② upstream blowing



⟶ difficult to prevent overheat ⟶ afterburner

③ vaporiser



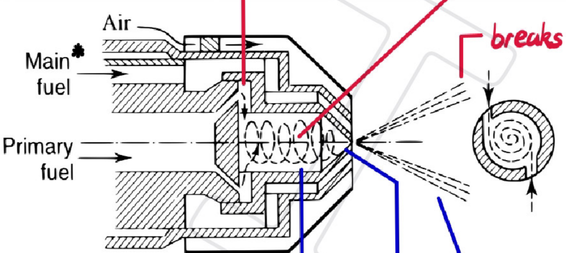
⟶ fuel evaporates ⟶ fuel-rich mixture ⟶ flow back

Fuel Injection

① duplex atomiser 喷雾剂

fuel supply at large p tangentially

form vapore / air core near the cone axis



breaks up ⟶ spray of droplets ⟵$^{\text{size}\downarrow}$ pressure drop ↑

near fixed

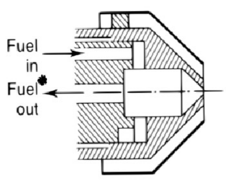
keep optimal

conicial sheet

swirl   conical chamber

* only switch at high load

② spill atomiser

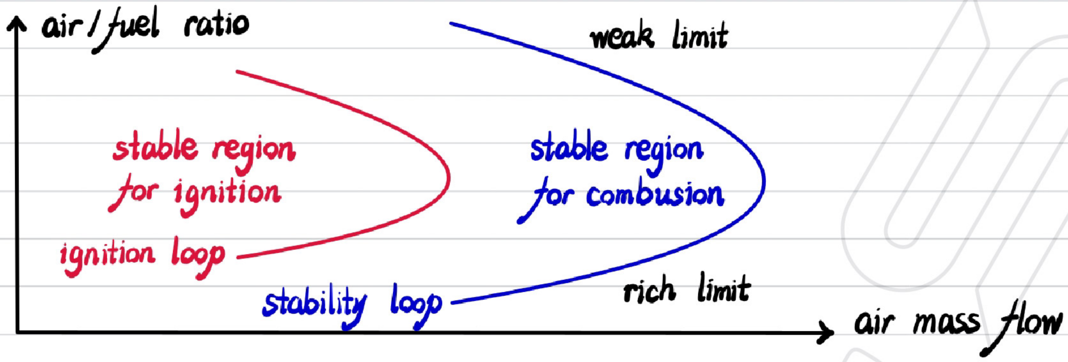


* spilled off extra fuel

③ vaporiser ⟶ small atomiser is needed to ignite the fuel when engine is started

Stability Loop

* flame - out ⟶ quenching of the flame



stabilility region ∝ reaction rate ∝ number of molecule collision ∝ T or ρ or p

Pressure Losses and Combustion Intensity

① skin friction ② large - scale turbulence ⟶ create intentially for good mixing

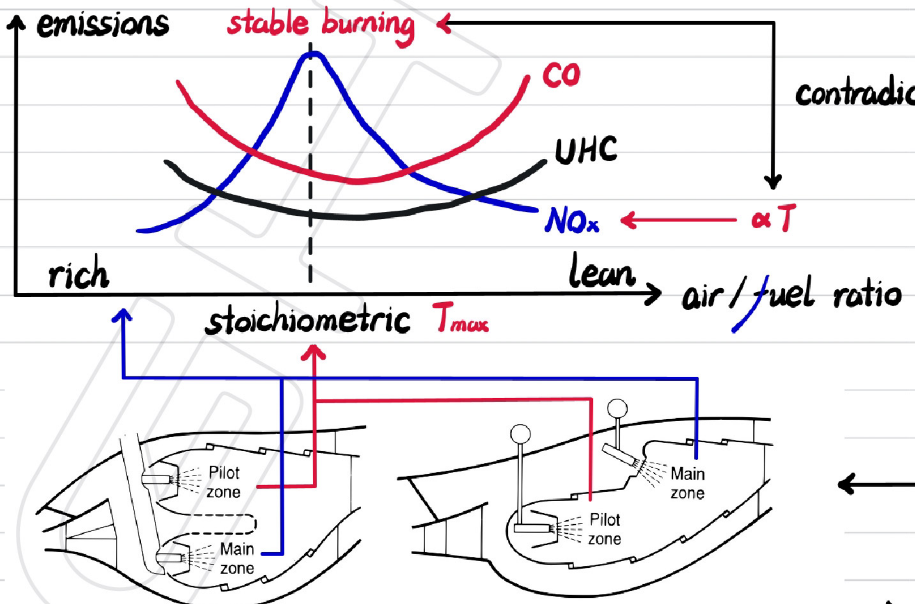
③ heat addition $\longrightarrow T\uparrow \xrightarrow{p=\rho RT} \rho\downarrow \xrightarrow{\dot{m}=\frac{d(\rho V)}{dt}=const} u_{downstr}\uparrow$ ⟶ fundemental loss

PLF (pressure loss factor) $= \dfrac{p_{02}-p_{01}}{\dot{m}_a^2/(2\rho_c A_{max}^2)} = K_1 + K_2\left(\dfrac{T_{ob}}{T_{oc}}-1\right)$ * constant

combustion intensity factor $= \dfrac{\text{heat release rate}}{\text{combustor volume} \times \text{pressure}}$ ⟶ design quality / difficulty

Emissions

NO_x, CO, unburned hydrocarbons (UHC) ⟶ smog 雾霾 & acid rain & ozone depletion



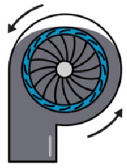
contradict ⟶ fuel staging

at primary zone

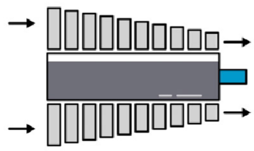
⟶ allow using more lean mixtures

Ideal Compressor Stage

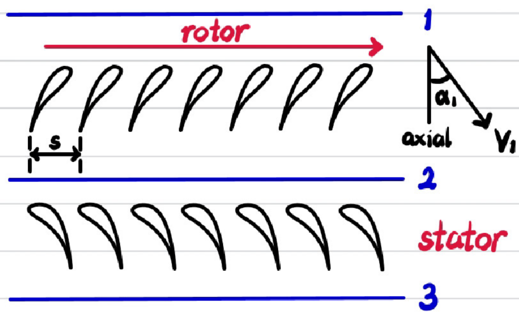
Basic Cascade 层叠 Theory

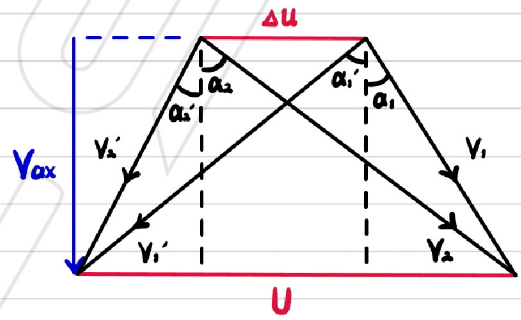


radial compressor



axial compressor ⟶ considered

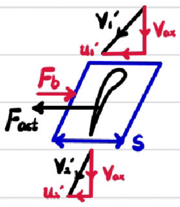




① $\Delta u = V_2 \cos\alpha_2 - V_1 \cos\alpha_1 = V_1' \cos\alpha_1' - V_2' \cos\alpha_2' = U - (V_1 \cos\alpha_1 + V_2' \cos\alpha_2')$

$\longrightarrow \Delta u = V_{ax}(\tan\alpha_2 - \tan\alpha_1) = V_{ax}(\tan\alpha_1' - \tan\alpha_2') = U - V_{ax}(\tan\alpha_1 + \tan\alpha_2')$

② apply control volume on a single blade



$F_b = \dot{m}_b(u_2' - u_1') = \dot{m}_b V_{ax}(\tan\alpha_1' - \tan\alpha_2')$

$= \dot{m}_b \Delta u = \dot{m}_b V_{ax}(\tan\alpha_2 - \tan\alpha_1) = \dot{m}_b[U - V_{ax}(\tan\alpha_1 + \tan\alpha_2')]$

③ $\dot{W} = P = Fu = \sum F_b U = \dot{m} U \Delta u = \dot{m} U V_{ax}(\tan\alpha_1' - \tan\alpha_2') = \dot{m} U V_{ax}(\tan\alpha_2 - \tan\alpha_1)$

$= \dot{m}[U^2 - UV_{ax}(\tan\alpha_2' + \tan\alpha_1')]$

$\xrightarrow{\dot{W} = \dot{m} C_p \Delta T} T_{02} - T_{01} = T_{03} - T_{01} = \frac{U\Delta u}{C_p} = \frac{UV_{ax}}{C_p}(\tan\alpha_1' - \tan\alpha_2') = \frac{UV_{ax}}{C_p}(\tan\alpha_2 - \tan\alpha_1)$

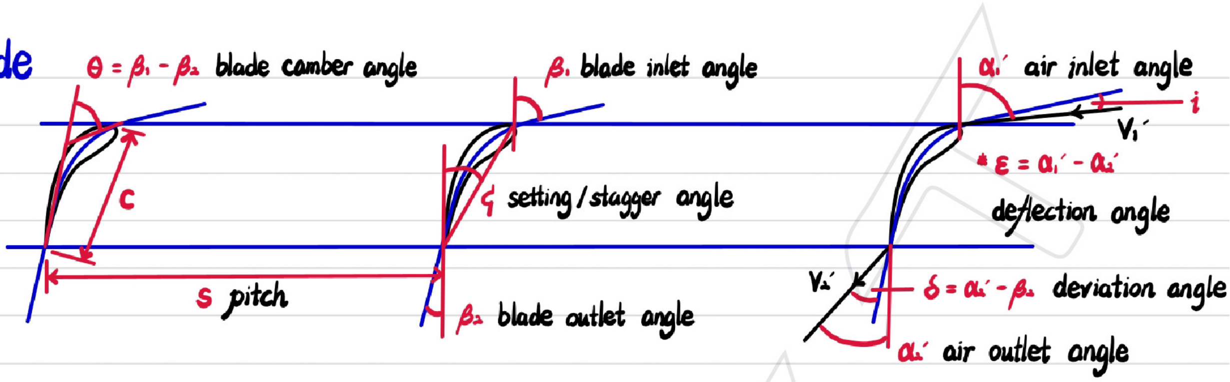
$= \frac{1}{C_p}[U^2 - UV_{ax}(\tan\alpha_1 + \tan\alpha_2')] \propto -V_{ax}$

④ $\frac{p_{03}}{p_{01}} = \left(\frac{T_{03s}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \xrightarrow{\eta_s = \frac{h_{03s} - h_{01}}{h_{03} - h_{01}}} \frac{p_{03}}{p_{01}} = \left(1 + \eta_s \frac{T_{03} - T_{01}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$

Ideal Cascade Calculation

· $\eta_s = 1$ · no deviation angle $\delta = \alpha_2' - \beta_2$ · flow velocity is constant with radius

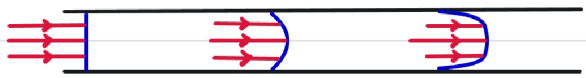
Rotor Blade



Real Flow Effects in Compressors

Radial Flow * due to centrifugal force * not discussed but important

Blockage



$u \rightarrow 0$ at the wall

$\longrightarrow T_{03} - T_{01} = \lambda \dfrac{U \Delta u}{c_p}$ * λ work-done factor < 1

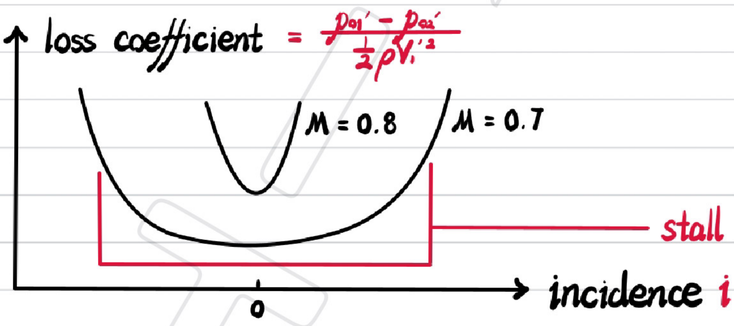
Deviation $s \uparrow \longrightarrow \dot{m}_b \uparrow \xrightarrow{F_b = \text{const}} V_{ax}(\tan\alpha_1' - \tan\alpha_2') \downarrow \longrightarrow \alpha_1' - \alpha_2' = \varepsilon \downarrow$

$\xrightarrow{\dot{W} = \text{const}} \alpha_2' \uparrow \longrightarrow \text{AoA} \uparrow \longrightarrow$ stall & possible losses

* for soildity $\sigma = \frac{c}{s} < 0.5 \longrightarrow$ approximate as single airfoil flow

for soildity $\sigma = \frac{c}{s} > 1.5 \longrightarrow \delta \downarrow$ & friction loss $\uparrow$

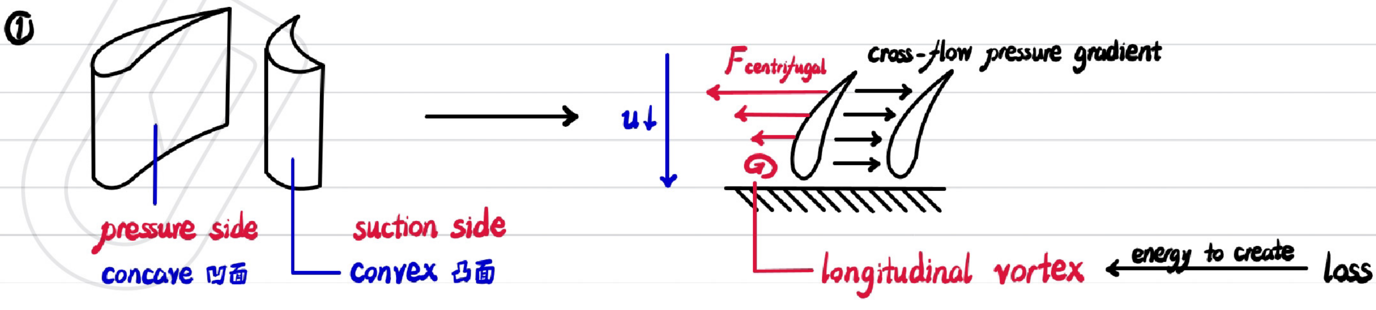
Losses



Friction Losses boundary layer theory & solidity

Secondary Losses

①

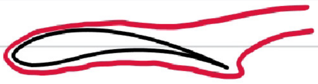


①

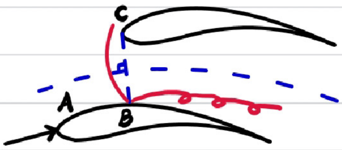
secondary losses ∝ pressure difference on the side of the blade ∝ $C_{L,\text{blade}}$

Stall Losses

stall losses ∝ pressure ratio across the row of blades ⟶ de Haller number $\frac{V_2'}{V_1'} > 0.72$

*  first occurs on the suction side ∝ incidence i

Shock Losses



$s\downarrow \longrightarrow$ distance between A and B $\downarrow \xrightarrow{M_1\downarrow}$ shock loss $\downarrow$

Repeating Stage and Degree of Reaction

for repeating stage $\longrightarrow \alpha_1 = \alpha_3 \longrightarrow V_1 = V_3 \xrightarrow{\text{design to provide } \Delta p_{max}} \Delta p_{rotor} = \Delta p_{stator}$

⟶ significant reduction in development costs

degree of reaction = $\dfrac{\text{static ethalpy rise in the rotor}}{\text{static ethalpy rise in the stage}}$

$$\xrightarrow{C_p = \text{const}} \Lambda = \frac{T_2 - T_1}{T_3 - T_1} = \frac{h_2 - h_1}{h_3 - h_1} = \frac{(h_{02} - \frac{1}{2}V_2^2) - (h_{01} - \frac{1}{2}V_1^2)}{(h_{03} - \frac{1}{2}V_3^2) - (h_{01} - \frac{1}{2}V_1^2)} = \frac{(h_{02} - h_{01}) - \frac{1}{2}(V_2^2 - V_1^2)}{(h_{03} - h_{01}) - \frac{1}{2}(V_3^2 - V_1^2)}$$

$$\xrightarrow{\text{repeating stage}} \frac{(h_{02} - h_{01}) - \frac{1}{2}(V_{2}^2 - V_{1}^2)}{h_{03} - h_{01}} = 1 - \frac{V_2^2 - V_1^2}{2C_p(T_{03} - T_{01})}$$

$$= 1 - \frac{[V_{ax}^2 + (U - V_{ax}\tan\alpha_2')^2] - [V_{ax}^2 + (U - V_{ax}\tan\alpha_1')^2]}{2UV_{ax}(\tan\alpha_1' - \tan\alpha_2')}$$

$$= 1 - \frac{2U(\tan\alpha_1' - \tan\alpha_2') + V_{ax}(\tan^2\alpha_2' - \tan^2\alpha_1')}{2U(\tan\alpha_1' - \tan\alpha_2')}$$

$$= 1 - 1 - \frac{V_{ax}}{2U}(\tan\alpha_1' + \tan\alpha_2') = -\frac{V_{ax}}{2U}(\tan\alpha_1' + \tan\alpha_2')$$

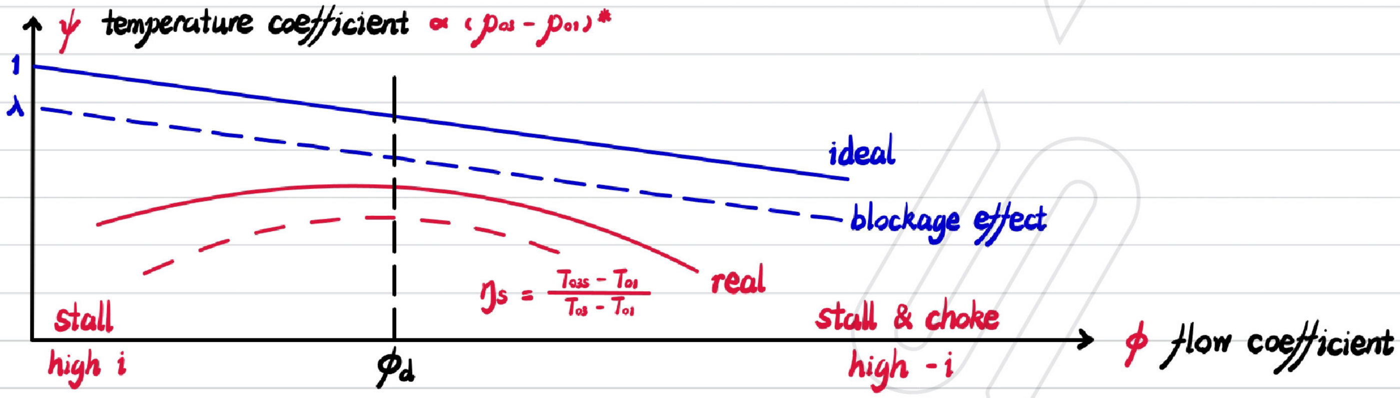
$= 0.5$ if pressure rise is equally distributed $(\alpha_1 = \alpha_2')$

* first stage and last stage unlikely to be repeating

Off-design Performance*

Temperature and Flow Coefficient

$$\psi = \frac{c_p(T_{03} - T_{01})}{U^2} = \lambda\left[1 - \frac{V_{ax}}{U}(\tan\alpha_1 + \tan\alpha_2')\right] = \lambda\left[1 - \phi(\tan\alpha_1 + \tan\alpha_2')\right]$$

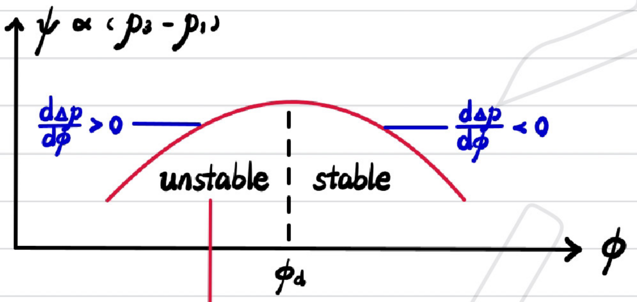


$$* \quad \frac{p_{03}}{p_{01}} = \left(\frac{T_{03s}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \eta_s \frac{T_{03} - T_{01}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \left(1 + \eta_s \psi \frac{U^2}{c_p T_{01}}\right)^{\frac{\gamma}{\gamma-1}} \xrightarrow{c_0 = \sqrt{\gamma R T} = [\gamma(\frac{\gamma-1}{\gamma}c_p)T]^{\frac{1}{2}}} \left[1 + \eta_s \psi (\gamma - 1)\frac{U^2}{c_0^2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\xrightarrow{\text{Taylor expansion}} \frac{p_{03} - p_{01}}{p_{01} U^2} = \eta_s \psi$$

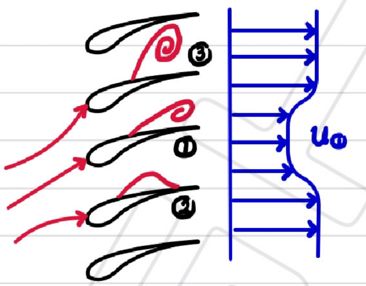
Surge



$\dot m \uparrow \longrightarrow \Delta p \uparrow \longrightarrow V_{ax} \uparrow \longrightarrow \dot m \uparrow \longrightarrow$ surge $\longrightarrow$ resonance

Rotating Stall

when $\frac{d\Delta p}{d\phi}$ * often observed with surge



① stall $\longrightarrow \Delta p \downarrow \longrightarrow u_0 \downarrow \longrightarrow$ streamline deviation

→ ② reduction of i $\longrightarrow$ flow reattach

→ ③ increases of i $\longrightarrow$ flow seperate at one more blade

$\longrightarrow$ shift seperation region from ① to ③

Stage Stacking

assume all the stage are repeating

$$\dot m = \rho V_{ax} A \longrightarrow \rho_1 V_{ax1} h_1 = \rho_2 V_{ax2} h_2 \longrightarrow \frac{\rho_1 V_{ax1} h_1}{\rho_{1d}} = \frac{\rho_2 V_{ax2} h_2}{\rho_{2d}}$$

$\longrightarrow \frac{\rho_1}{\rho_{1d}} V_{ax1} h_1 = \frac{\rho_2}{\rho_{2d}} V_{ax2} h_2 \longrightarrow \frac{\rho_{2d}}{\rho_{1d}} V_{ax1} h_1 = \frac{\rho_2}{\rho_1} V_{ax2} h_2$

if $\phi < \phi_d \longrightarrow \psi > \psi_d \longrightarrow \Delta p > \Delta p_d \longrightarrow \Delta \rho > \Delta \rho_d \longrightarrow V_{ax2} < V_{ax1}$

$\longrightarrow$ last stage stalled

* if $\phi > \phi_d \longrightarrow$ last stage chocked

Non-dimensional Parameter of a Compressor

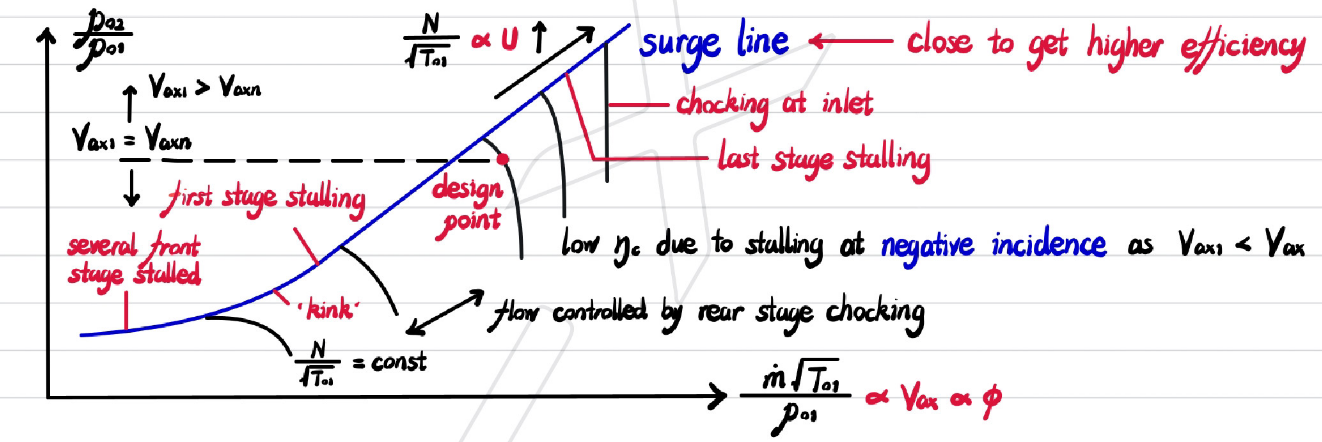
· size D · rotational speed N · mass flow rate $\dot{m}$ · gas constant R

· stagnation pressure at the inlet p_{01} · stagnation temperature at the inlet T_{01}

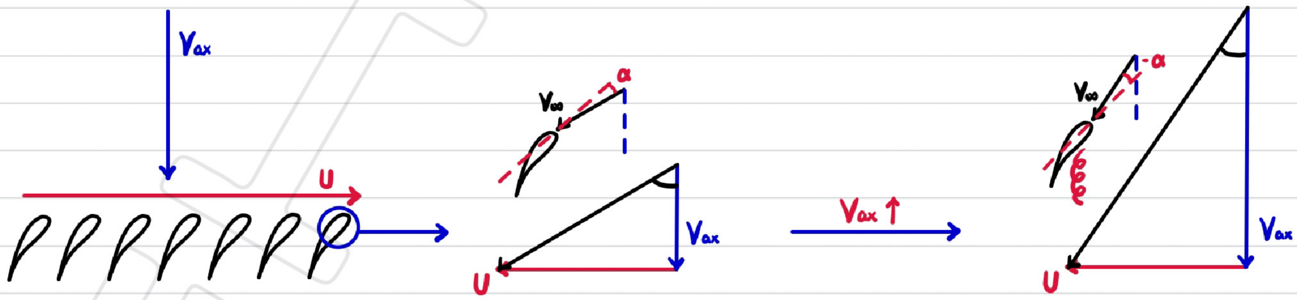
non-dimensional
- → "mass flow rate" $\frac{\dot{m}\sqrt{RT_{01}}}{D^2 p_{01}} \longrightarrow \frac{\dot{m}\sqrt{T_{01}}}{p_{01}}$
- → rotational "speed" $\frac{ND}{\sqrt{RT_{01}}} \longrightarrow \frac{N}{\sqrt{T_{01}}}$

Off-design Regimes

average characteristics of all stages

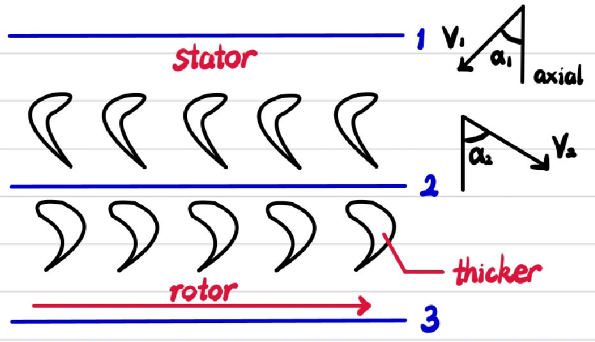


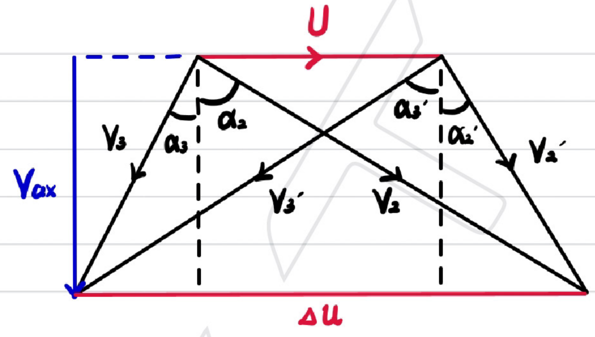
* negative incidence



Axial Turbines

Basic Theory





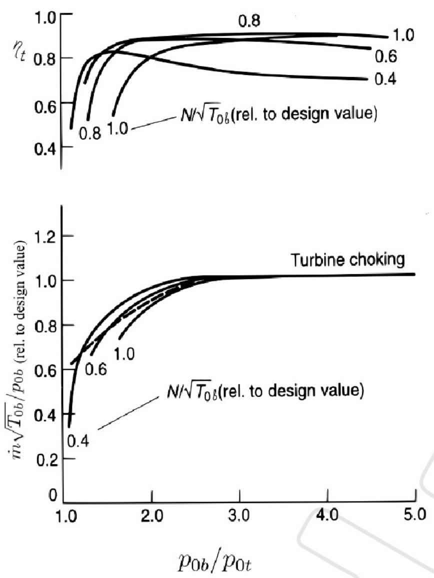
$$\Delta T_0 = \frac{U\Delta u}{c_p} = \frac{UV_{ax}}{c_p}(\tan\alpha_2 + \tan\alpha_3) = \frac{UV_{ax}}{c_p}(\tan\alpha_2' + \tan\alpha_3')$$

limit of performance by · stresses in the blades

· losses due to compressibility effect

Equilibrium Running line

Typical Turbine Stage Characteristic



· compressor and turbine are connected by a shaft

⟶ same rotational speed N

⟶ limit possible regine of compressor / turbine

· $W_t \approx W_c$ for turbojet

$W_t \approx W_c + W_{fan} / W_{prop}$ for turbofan / turboprop

· $\frac{p_{0b}}{p_{0t}} = \frac{p_{0b}}{p_{0c}} \frac{p_{0c}}{p_{0d}} \frac{p_{0d}}{p_a} \frac{p_a}{p_{0t}}$

⟶ $p_{0d} \approx p_a \approx p_{0t}$ for turboprop

Equilibrium Running Line