Circuit Model Components

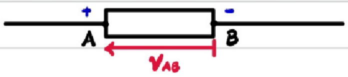
Revision

Charge q coulomb (C) · $e^- = -1.6 \times 10^{-19}$ C · proton $= 1.6 \times 10^{-19}$ C

Current $i \triangleq \frac{dq}{dt}$ amps (A)

1C positive charge in 1s = 1A

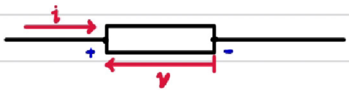
Voltage Potential $v \triangleq \frac{dw}{dq}$ volts (V) amount of work to make 1C of charge from - to +



· $V_{AB} \triangleq V_A - V_B$ · $V_{BA} \triangleq V_B - V_A$ · $V_{AB} = -V_{BA}$

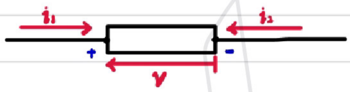
Power $p \triangleq \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = vi$ watt (W)

· power absorbed



$p = vi$

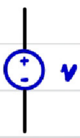
· power absorbed = - power supplied



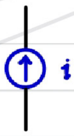
$p = vi_1 = -vi_2$

Independent Ideal Sources

· voltage source



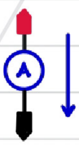
· current source



Voltmeter and Ammeters

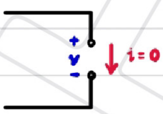
· voltmeter

· ammeter

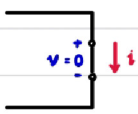


Circuit

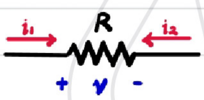
· open-circuit



· short-circuit



Ohm's Law for linear resistors $R = \frac{v}{i}$ ohm (Ω)



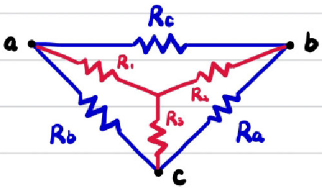
$v = Ri_1 = -Ri_2$

* conductance $G = \frac{1}{R}$ simens (S)

· series resistors $R_{total} = \Sigma R_i$

· parallel resistors $\frac{1}{R_{total}} = \Sigma \frac{1}{R_i}$

· Delta-wye transformation *

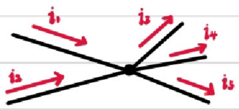


Delta configuration $R_a = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$

Wye configuration $R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$

KCL (Kirchhoff's Current Law)

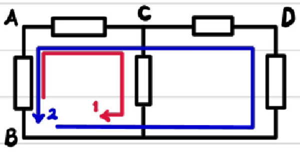
conservation of charge total current in = total current out



$i_1 + i_2 = i_3 + i_4 + i_5$

KVL (Kirchoff's Voltage Law)

Conservation of energy Σ voltage around any closed loop = 0



loop 1 $V_{AB} + V_{CA} + V_{BC} = 0$

loop 2 $V_{BD} + V_{DC} + V_{CA} + V_{AB} = 0$

Independent and Dependent Sources

Independent battery

· voltage source  v $R \longrightarrow 0$ · current source  i $R \longrightarrow \infty$

Dependent amplifier

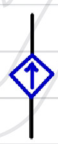
· voltage source

v

VCVS (Voltage Controlled Voltage Source) $V_{AB} = kV_{CD}$

CCVS (Current Controlled Voltage Source) $V_{AB} = ki_2$

· current source



i

VCCS (Voltage Controlled Current Source) $i_1 = kV_{CD}$

CCCS (Current Controlled Current Source) $i_1 = ki_2$

Capacitors

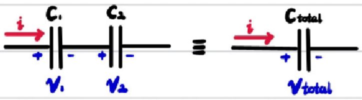
Capacitor $C = \frac{q}{v} \longrightarrow i = C\frac{dv}{dt} \longrightarrow v = \frac{1}{C}\int i\,dt$ farad (F)

continuous — $i = C\frac{dv}{dt}$

can be discontinuous — i

steady-state when $\frac{dv}{dt} = 0 \longrightarrow i = 0 \longrightarrow$ equivalent to open-circuit

Series



$$v_{total} = \sum v_i \longrightarrow \frac{dv_{total}}{dt} = \sum \frac{dv_i}{dt} \longrightarrow \frac{i}{C_{total}} = \sum \frac{i}{C_i} \longrightarrow \frac{1}{C_{total}} = \sum \frac{1}{C_i}$$

Parallel $i_{total} = \sum i_i \longrightarrow \int i_{total}\,dt = \sum \int i_i\,dt \longrightarrow VC_{total} = \sum VC_i \longrightarrow C_{total} = \sum C_i$

Inductors

Inductor $v = L\frac{di}{dt} \longrightarrow i = \frac{1}{L}\int v\,dt$ henry (H)

continuous — $v = L\frac{di}{dt}$

can be discontinuous — v

steady-state when $\frac{di}{dt} = 0 \longrightarrow v = 0 \longrightarrow$ equivalent to short-circuit

Series $v_{total} = \sum v_i \longrightarrow \int v_{total}\,dt = \sum \int v_i\,dt \longrightarrow iL_{total} = \sum iL_i \longrightarrow L_{total} = \sum L_i$

Parallel $i_{total} = \sum i_i \longrightarrow \frac{di_{total}}{dt} = \sum \frac{di_i}{dt} \longrightarrow \frac{V}{L_{total}} = \sum \frac{V}{L_i} \longrightarrow \frac{1}{L_{total}} = \sum \frac{1}{L_i}$

DC Circuit Analysis

Nodal Analysis

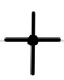
Recipe

1. mark reference node / ground and label nodes
2. apply KCL at each node and label currents through branches / omit earth
3. write currents as function of node voltages
4. plug in knowns and solve for unknowns linear algebra

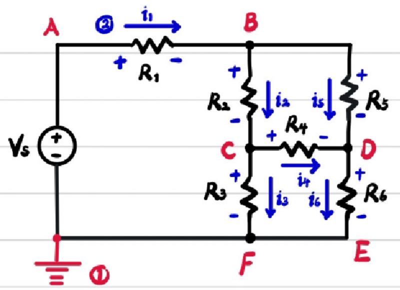
no connection

or

connection



Example



① show in diagram above

② At B, $i_1 = i_2 + i_5$; At C, $i_2 = i_3 + i_4$; At D, $i_4 + i_5 = i_6$

③ At B, $\frac{V_{AB}}{R_1} = \frac{V_{BC}}{R_2} + \frac{V_{BD}}{R_5}$; At C, $\frac{V_{BC}}{R_2} = \frac{V_{CF}}{R_3} + \frac{V_{CD}}{R_4}$; At D, $\frac{V_{CD}}{R_4} + \frac{V_{BD}}{R_5} = \frac{V_{DE}}{R_6}$

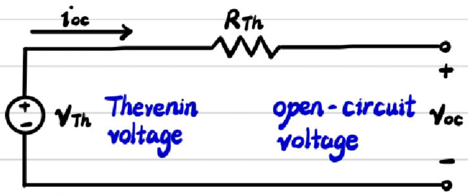
$\therefore \frac{V_A - V_B}{R_1} = \frac{V_B - V_C}{R_2} + \frac{V_B - V_D}{R_5}$; $\frac{V_B - V_C}{R_2} = \frac{V_C - V_F}{R_3} + \frac{V_C - V_D}{R_4}$; $\frac{V_C - V_D}{R_4} = \frac{V_B - V_D}{R_5} + \frac{V_D - V_E}{R_6}$

* $V_A = V_S = V_A - V_E$ $\quad \therefore V_E = 0V$

④ ……

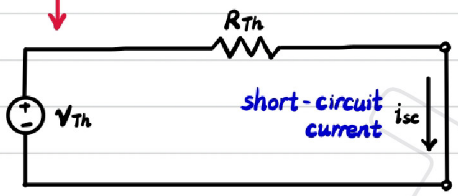
Equivalent Circuits

Thevenin equilibrium circuit



open-circuit $\longrightarrow i_{oc} = 0$

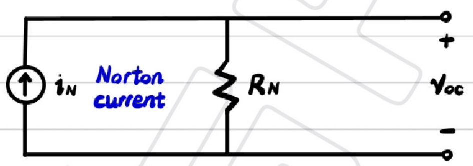
KVL $\rightarrow V_{Th} - i_{oc}R_{Th} = V_{oc} \longrightarrow V_{Th} = V_{oc}$



short-circuit $\longrightarrow i_{sc} = \frac{V_{Th}}{R_{Th}}$

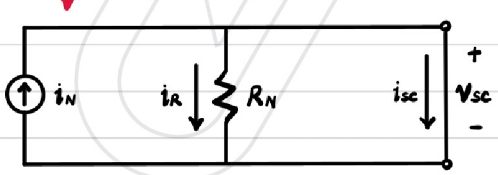
$\longrightarrow R_{Th} = \frac{V_{Th}}{i_{sc}}$

Norton equivalent circuit



open-circuit

KVL $\rightarrow V_{oc} = i_N R_N \longrightarrow R_N = \frac{V_{oc}}{i_N}$



short-circuit $\longrightarrow V_{sc} = 0 \longrightarrow i_R = \frac{V_{sc}}{R_N} = 0$

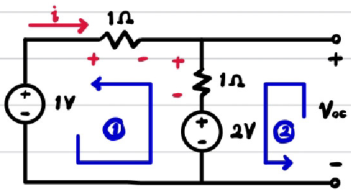
KCL $\rightarrow i_N = i_R + i_{sc} \longrightarrow i_N = i_{sc} \longrightarrow R_N = R_{Th}$

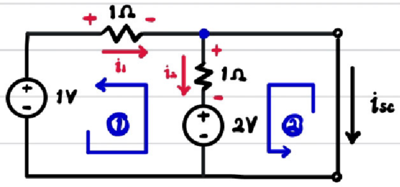
Recipe ① compute / measure $V_{oc} = V_{Th}$ in open-circuit

② compute / measure $i_{sc} = i_N$ in short-circuit

③ find $R_{Th} = R_N = \frac{V_{oc}}{i_{sc}}$

Example





① find V_{oc} KVL $\xrightarrow{①}$ $1 - 1 \cdot i - 1 \cdot i - 2 = 0\,(V) \longrightarrow i = -\frac{1}{2}\,(A)$

$\xrightarrow{②}$ $1 \cdot i + 2 - V_{oc} = 0\,(V) \longrightarrow V_{oc} = 1.5\,(V) = V_{Th}$

② find i_{sc} KCL $\longrightarrow i_1 = i_2 + i_{sc} \longrightarrow i_{sc} = 3\,(A)$

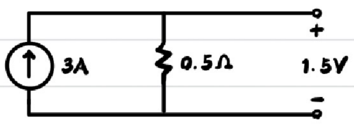
KVL $\xrightarrow{①}$ $1 - 1 \cdot i_1 - 1 \cdot i_2 - 2 = 0\,(V) \longrightarrow i_1 = 1\,(A)$

$\xrightarrow{②}$ $1 \cdot i_2 + 2 = 0 \longrightarrow i_2 = -2\,(A)$

③ find R_{Th} $R_{Th} = R_N = \frac{V_{oc}}{i_{sc}} = 0.5\,(\Omega)$

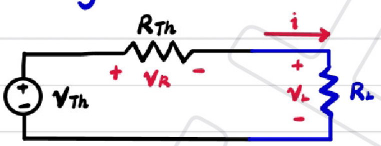
for Thevenin equivalent circuit

for Norton equivalent circuit



Maximum Power Transfer

Voltage divider rule



← represent other circuits

Power absorbed by load $P_L = V_L \cdot i$

KVL $\longrightarrow V_{Th} - V_R - V_L = 0 \longrightarrow V_{Th} - iR_{Th} - iR_L = 0 \longrightarrow i = \frac{V_{Th}}{R_{Th} + R_L}$

$\longrightarrow V_{Th} - iR_{Th} - V_L = 0 \longrightarrow V_L = V_{Th} - iR_{Th} = \frac{R_L}{R_{Th} + R_L} V_{Th}$

$$\rightarrow P_L = \frac{R_L}{(R_{Th} + R_L)^2}(V_{Th})^2$$

Maximised P_L

$$\frac{dP_L}{dR_L} = 0 \rightarrow \frac{(V_{Th})^2(R_{Th} + R_L)^2 - R_L(V_{Th})^2 \cdot 2 \cdot (R_{Th} + R_L)}{(R_{Th} + R_L)^4} = \frac{(V_{Th})^2(R_{Th} - R_L)}{(R_{Th} + R_L)^3} = 0 \rightarrow R_{Th} = R_L$$

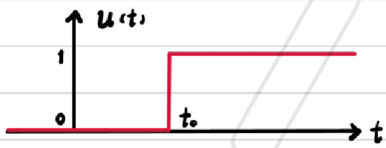
Reactive Circuit Transient Response

Reactive Circuit Transient Response

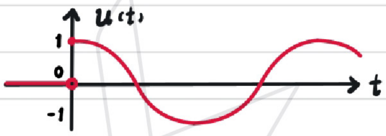
Aim interested in computing the time response of RLC circuits to time-varying sources

Time-varying sources

unit step $u(t-t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t \geq t_0 \end{cases}$



Heaveside function $u(t)\cos(\omega t)$



Recipe ① identify the instances $t_1, t_2, t_3, \ldots$ where discrete changes / switches

capacitors $v_c(t_i^-) = v_c(t_i^+)$ for $\forall i$

inductors $i_L(t_i^-) = i_L(t_i^+)$ for $\forall i$

② if initial condition is not given, assume steady state at $t = t_1^-$

capacitors $\frac{dv}{dt}(t_1) = 0$ inductors $\frac{di}{dt}(t_1) = 0$

sketch circuit for $t < t_1$.

replace capacitors as open-circuit $\longrightarrow$ compute $v_c(t_1^-)$

replace inductors as short-circuit $\longrightarrow$ compute $i_L(t_1^-)$

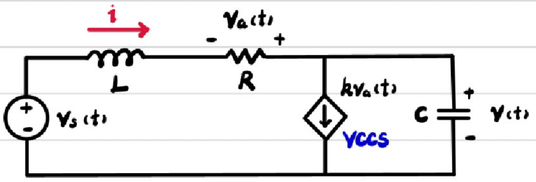
③ sketch circuit for all intervals, $t_1 < t < t_2$, $t_2 < t < t_3$, ...

④ derive differential equations for circuits in each interval

$N_{\text{differential equations}} = N_C + N_L = N_{\text{initial conditions}}$

⑤ solve differential equations

Example

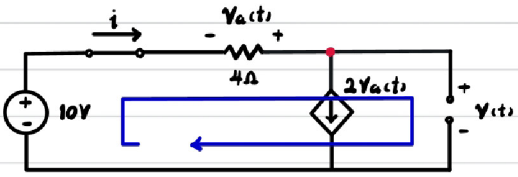


Suppose $v_s = 6u(t) + 10\,(V)$, $k = 2$, $L = 0.1\,(H)$,

$R = 4\,(\Omega)$, $C = 0.625\,(F)$, find $v(t)$

① $v(0^-) = v(0^+)$ and $i(0^-) = i(0^+)$

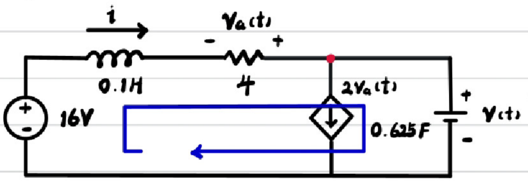
② for $t < 0$,



KCL $\longrightarrow i(0^-) = kv_a(0^-) = 2\cdot[4\cdot -i(0^-)] \longrightarrow i(0^-) = 0\,(A) \longrightarrow v_a(0^-) = 0\,(V)$

KVL $\longrightarrow 10 + v_a(0^-) - v(0^-) = 0\,(V) \longrightarrow v(0^-) = 10\,(V)$

for $t > 0$, $i(0^+) = i(0^-) = 0\,(A)$ and $v(0^+) = v(0^-) = 10\,(V)$



KCL $\longrightarrow i(t) = kv_a(t) + C\frac{dv}{dt} = kR[-i(t)] + C\frac{dv}{dt} \longrightarrow \frac{dv}{dt} = \frac{1+kR}{C}\,i(t) = \frac{72}{5}\,i(t)$

KVL $\longrightarrow 16 - L\frac{di}{dt} + v_a(t) - v(t) = 0 \longrightarrow 16 - L\frac{di}{dt} + R[-i(t)] - v(t) = 0$

$\longrightarrow 16 - 0.1\frac{di}{dt} - 4i - v = 0$

$\longrightarrow \frac{di}{dt} + 40i + 10v = 160 \longrightarrow \frac{5}{72}\frac{d^2v}{dt^2} + \frac{25}{9}\frac{dv}{dt} + 10v = 160$

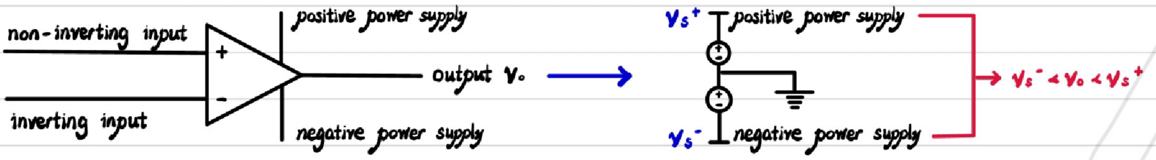
$\longrightarrow \frac{d^2v}{dt^2} + 40\frac{dv}{dt} + 144v = 2304$

$\longrightarrow v(t) = 0.75e^{-4t} - 6.75e^{-36t} + 16\,(V)$ for $t > 0$

Operational Amplifier

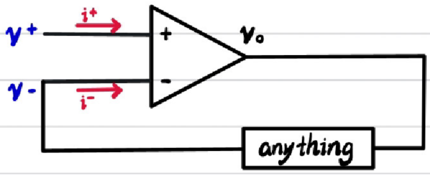
Operation Amplifier (Op-amp)

Circuit diagram

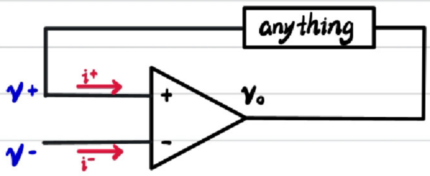


Feedback configuration

negative feedback



positive feedback



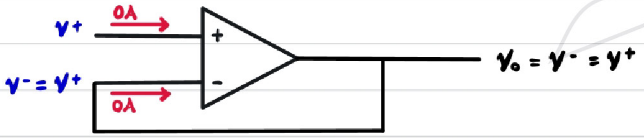
- $V^- < V^+ \longrightarrow V_o \uparrow$
- $V^+ > V^- \longrightarrow V_o \downarrow$

balanced quickly

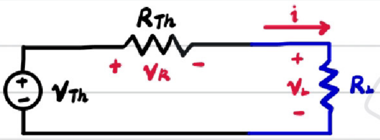
① $i^- = i^+ \xrightarrow{\text{infinity } R} 0A$

② $V^- = V^+$

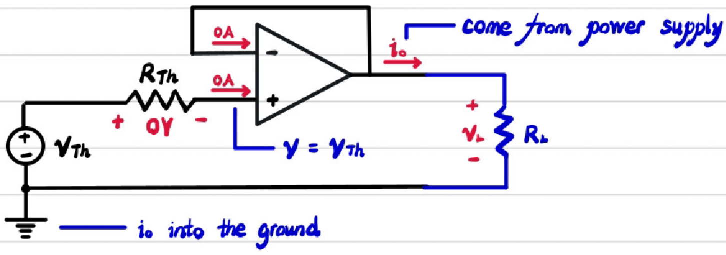
Voltage follower / buffer



example



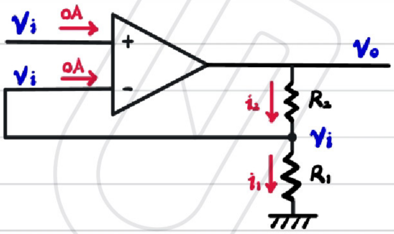
$\xrightarrow{\text{find } V_{Th}}$



$\longrightarrow V_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$

$\longrightarrow V_L = V_{Th}$ regardless R_{Th}

Non-inverting amplifier



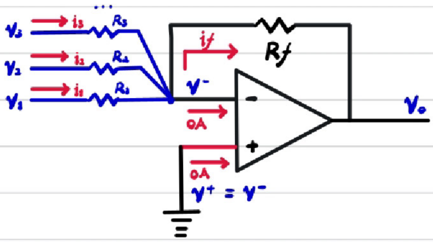
KCL $\longrightarrow i_2 = i_1 + 0A \longrightarrow i_2 = i_1$

$\longrightarrow \frac{V_o - V_i}{R_2} = \frac{V_i - 0}{R_1} \longrightarrow V_o = (1 + \frac{R_2}{R_1}) V_i$

$\downarrow$

> 1

Inverting summing amplifier



KCL $\longrightarrow$ $i_f + 0A = i_1 + i_2 + i_3$

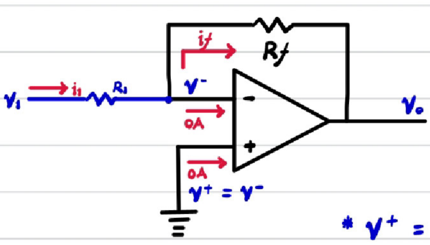
$$\longrightarrow \frac{v^- - v_o}{R_f} = \frac{v_1 - v^-}{R_1} + \frac{v_2 - v^-}{R_2} + \frac{v_3 - v^-}{R_3}$$

$$\longrightarrow -\frac{v_o}{R_f} = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}$$

* $v^+ = v^- = 0V$

$$\longrightarrow v_o = -R_f\left(\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}\right)$$

Inverting amplifier



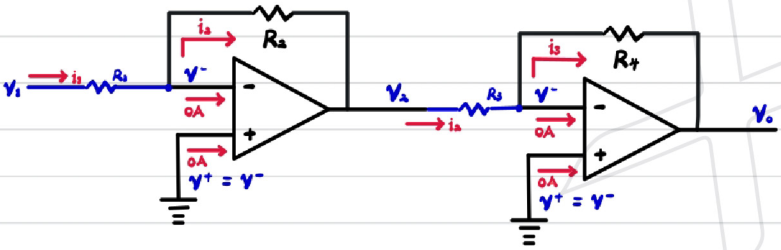
* $v^+ = v^- = 0V$

$$v_o = -\frac{R_f}{R_1} v_1$$

> 1 amplify

< 1 attenuates 减弱

in series $\longrightarrow$ non-inverting amplifier

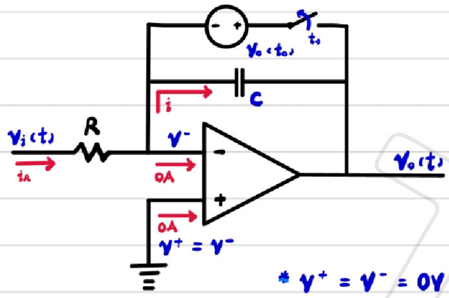


$$v_3 = -\frac{R_2}{R_1} v_1$$

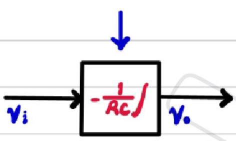
$$v_o = -\frac{R_4}{R_3} v_3$$

$$v_o = \frac{R_4 R_2}{R_1 R_3} v_1$$

Inverting integrator



* $v^+ = v^- = 0V$



KCL $\longrightarrow$ $i_R = i + 0A$

$$t \geq t_o \longrightarrow i_R = \frac{v_i - v^-}{R} = \frac{v_i}{R} = i = C\frac{d}{dt}[v^- - v_o(t)] = -C\frac{dv_o}{dt}$$

$$\longrightarrow \frac{dv_o}{dt} = -\frac{1}{RC} v_i(t)$$

$$\longrightarrow v_o(t) - v_o(t_o) = -\frac{1}{RC}\int_{t_o}^{t} v_i(\tau)\,d\tau$$

$$\longrightarrow v_o(t) = v_o(t_o) - \frac{1}{RC}\int_{t_o}^{t} v_i(\tau)\,d\tau$$

Differential and Algebaric Equations

Design and solution of differential and algebraic equations

example

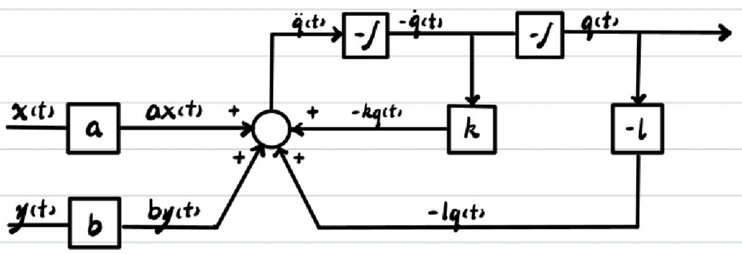
implement a circuit that solves the equation

$\ddot{q}(t) + k\dot{q}(t) + l q(t) = a x(t) + b y(t)$, $q(0) = q_0$, $\dot{q}(0) = r_0$

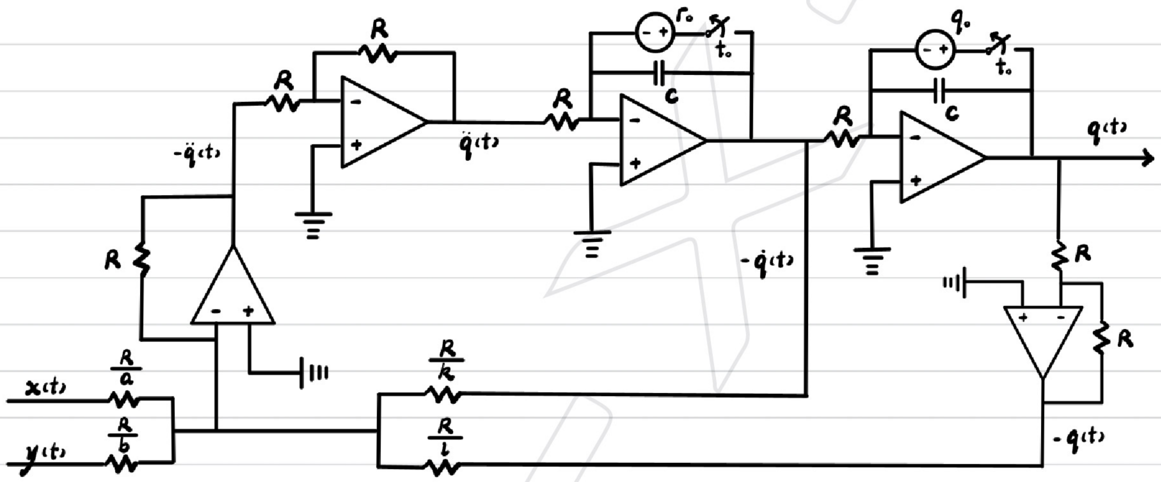
where x and y are the inputs and q is output of the circuit

① $\ddot{q}(t) = a x(t) + b y(t) - k\dot{q}(t) - l q(t)$

② draw a block diagram



③ substitude Op-amp choose $RC = 1$



* typical range R $100\,\Omega \sim 100\,k\Omega$ C $1\,\mu F \sim 100\,mF$

i $1\,\mu A \sim 1\,mA$ V $1\,mV \sim 10\,mV$

* why not differentiators ?

example

measure $x(t) = s(t) + n(t) = s(t) + \sum_n a_n \sin(\omega_n t)$

signal noise not interested in

$\longrightarrow \frac{dx}{dt} = \frac{ds}{dt} + \sum_n a_n \omega_n \cos(\omega_n t)$

$> 1 \longrightarrow$ amplify noise

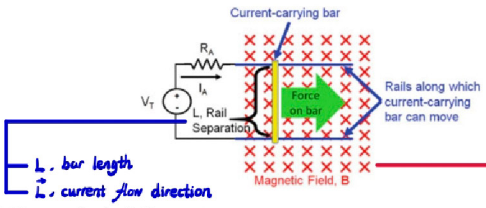
DC Motors and Generators *

DC Motors

Linear Motors

horizontal (Lorentz) force

$\vec{f} = I_A \vec{L} \times \vec{B}$



into × • out of

Fig. 5.2 Diagram of a simple linear motor

In this case

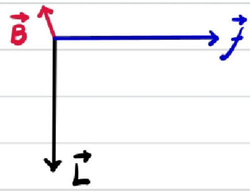
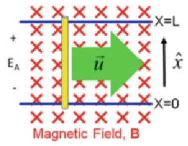


Fig. 5.3 Right-hand rule to find the direction of force for linear motor



conductor cuts across $\vec{B}$ ⟶ voltage induced

$E_A = uLB$

Fig. 5.5 Electromotive force (emf) induced due to motion of conducting bar in the presence of a magnetic field

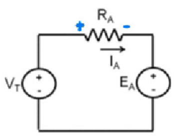


Fig. 5.6 Equivalent circuit for a linear machine

KVL $\Sigma V = 0$ in close loop $V_T - R_A I_A - E_A = 0$

$\longrightarrow I_A = \frac{V_T - E_A}{R_A} = \frac{V_T - uLB}{R_A} \propto -u$ also $f \propto I_A$

⟶ balance to have constant u

Rotating DC Motors

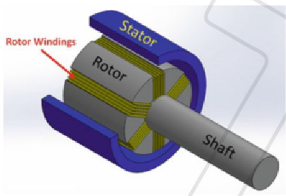


Fig. 5.8 Basic structure of DC motor

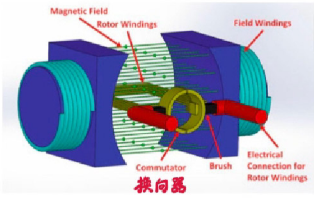


Fig. 5.9 Simplified diagram for DC motor focusing on a single rotor winding to illustrate the motor operation

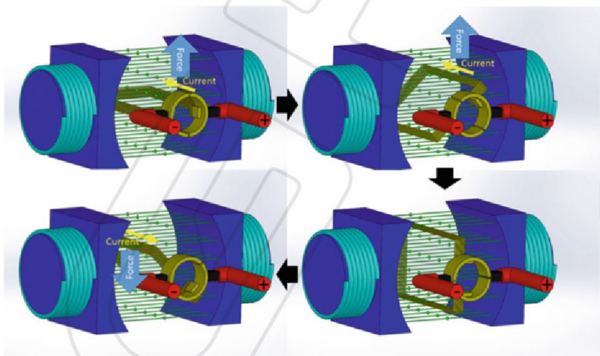
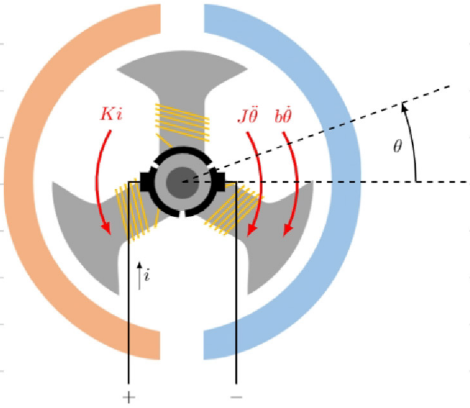


Fig. 5.10 Diagram illustrating basic motor operation emphasizing the importance of the commutator to change current direction so that the Lorentz force, and hence the torque, can always be in the same direction as the motor turns



$J\ddot{\theta} = T_{out} = T_{dev} - T_{loss}$

* different source of losses

- $T_{fluid} \propto \omega^2 = \dot{\theta}^2$ —— ignore unless very high speed
- $T_{viscous} \propto \omega = \dot{\theta}$ —— dominate

$\longrightarrow T_{loss} = b\dot{\theta} \longrightarrow J\ddot{\theta} = T_{dev} - b\dot{\theta}$

Torque on the rotor (due to Lorentz force)

$T_{dev} = K\phi I_A = \tilde{K}_t I_A$

- K: constant
- ϕ: magnetic flux produced by each stator pole 定子极
- $\tilde{K}_t$: torque constant

Back emf $E_A = \tilde{K}_a \omega = \tilde{K}_a \dot{\theta}$

Power generated $P_{dev} = T_{dev}\,\omega = \tilde{K}_t I_A \omega$ = absorbed $P_A = E_A I_A = \tilde{K}_a \omega I_A \longrightarrow K = \tilde{K}_t = \tilde{K}_a$

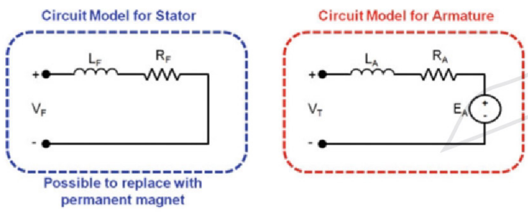


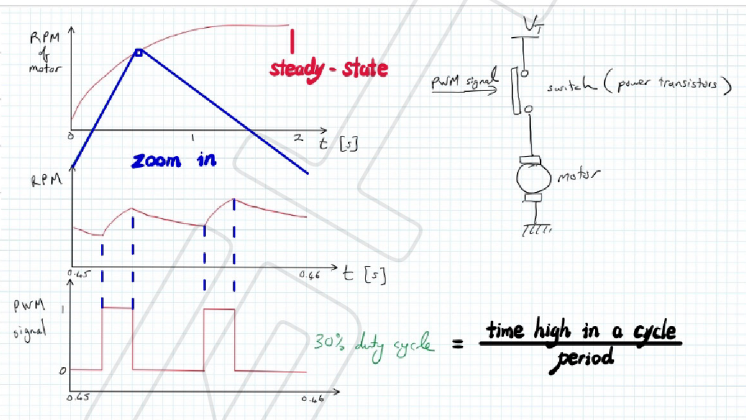
Fig. 5.11 Circuit model for DC motor

— KVL $V_T - V_L - V_R - E_A = 0$

$\longrightarrow V_T = L_A \frac{dI_A}{dt} + I_A R_A + \tilde{K}_a \omega$

$$\longrightarrow \begin{cases} V_T = L_A \dot{I}_A + R_A I_A + K\dot{\theta} \\ J\ddot{\theta} = K I_A - b\dot{\theta} \end{cases} \longrightarrow \begin{bmatrix} \dot{I}_A \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{1}{L_A}(V_T - R_A I_A - K\dot{\theta}) \\ \frac{1}{J}(K I_A - b\dot{\theta}) \end{bmatrix}$$

PWM (Pulse Width Moderation)



period logical signal

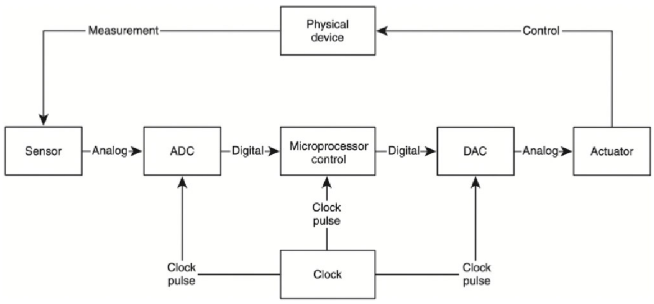
switching between 0 to 1

0: low

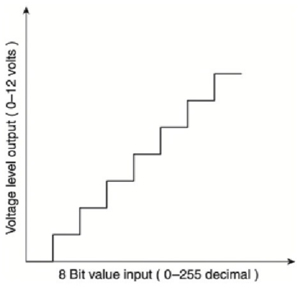
1: high

A/D (Analogue 模拟的 to digital) and D/A Conversion

Microprocessor Control System



DAC (Digital-to-Analogue Conversion)



Binary	Decimal	Voltage
0000 0000	0	0.0
0000 0001	1	0.00390625
1000 0000	128	6.0
1111 1111	255	11.953125

$(a_{n-1}a_{n-2}\dots a_0)_2 = \sum_{k=0}^{n-1} a_k 2^k$

1 below highest

n bit = 2^n different values ⟶ resolution $= \dfrac{\text{Range}}{2^n} = \dfrac{\text{Max Analogue} - \text{Min Analogue}}{2^n}$

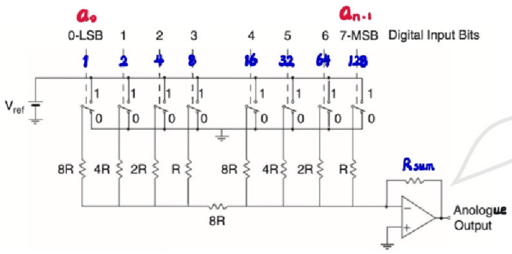
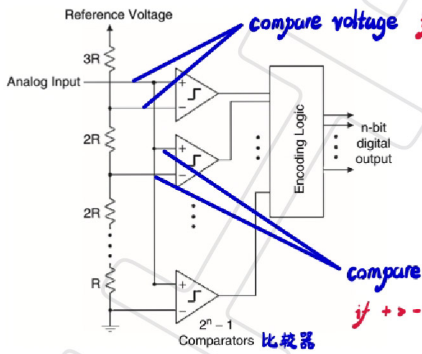


FIGURE 31.5 DAC architecture—most digital-to-analog converters (DACs) follow a standard architecture of a switch network, a resistive network, and an amplifier.

ADC (Analogue-to-Digital Conversion)

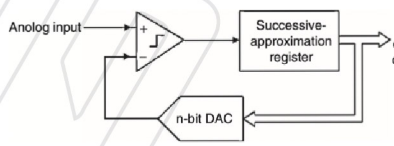


compare voltage if + < -

compare

if + > - ⟶ everything above off & everything below on

FIGURE 31.1 Flash ADC—a flash converter has $2n - 1$ comparators operating in parallel. It relies on the uniformity of the resistors for linearity.

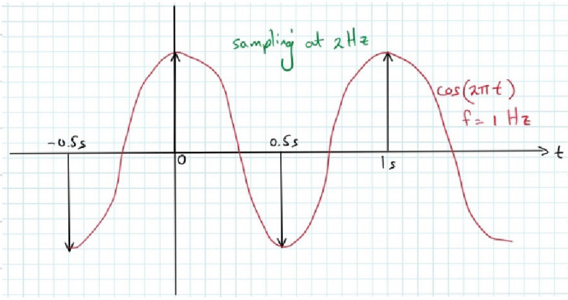


iteration smaller but slower

FIGURE 31.2 SAR ADC—a successive-approximation (SAR) converter has one comparator that iterates through a series of "guesses" to determine a digital representation of the signal.

Nyquist Sampling Theorem

sample twice as fast as highest frequency component to reconstruct signal



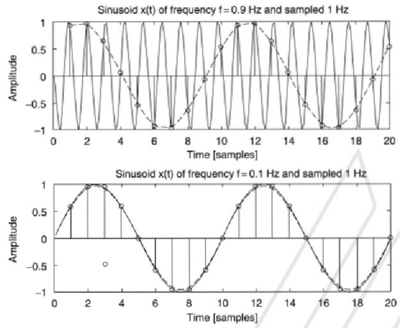


FIGURE 3.17 Illustration of aliasing appearing during sampling of a sinusoid $x(t) = \sin 2\pi \cdot 0.9t$ at the insufficient sampling frequency 1 Hz (sampling period $T = 1$) (*upper graph*). The sampled signal exhibits aliasing with its major component similar to a signal $x(t) = \sin 2\pi \cdot 0.1t$ sampled with the same rate (*lower graph*).