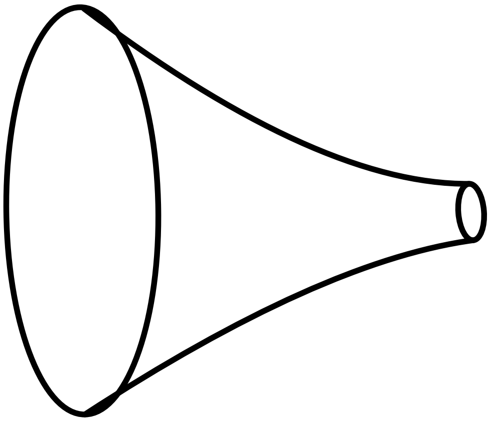
1. Consider a low-speed subsonic wind tunnel with a 12 to 1 contraction ratio (change in area) for the nozzle. If the flow in the test section is at standard sea-level conditions with a velocity of 50m/s, calculate the height difference in a U-tube mercury manometer with one side connected to the nozzle inlet and the other to the test section. Use the general handout for any required data.

   (Ans 0.011m assuming ρ_{air}= 1.19kg/m^3, $\rho_{mercury}$= 13 600kg/m^3)



$$\rho_a u_a A_a = \rho_m u_m A_m$$

$$\therefore u_a = \frac{1}{12}\frac{1.19 \times 50}{1.19} = \frac{25}{6}\ (m/s)$$

At inlet side,

$$P = \frac{1}{2}\rho u_a^2 = \frac{1}{2} \times 1.19 \times \left(\frac{25}{6}\right)^2 = 10.32986\ (Pa)$$

$$\therefore h_1 = \frac{P}{\rho g} = \frac{10.32986}{9.81 \times 13600} = 7.7426 \times 10^{-5}\ (m)$$

At test side,

$$P = \frac{1}{2}\rho u_a^2 = \frac{1}{2} \times 1.19 \times (50)^2 = 1487.5\ (Pa)$$

$$\therefore h_2 = \frac{P}{\rho g} = \frac{1487.5}{9.81 \times 13600} = 0.011149\ (m)$$

$$\therefore h_2 - h_1 = 0.0110716\ (m) = 0.0111\ (m)$$

2. An airplane is flying at standard sea level where the pressure is 101.3KPa. The measurement of the Pitot tube mounted on the wing tip reads 15.21 PSI (104.8 KPa). What is the velocity of the airplane?

(Ans 76.7m/s assuming ρ_{air}= 1.19kg/m^3)

$$P = P_0 + \frac{1}{2}\rho V^2$$

$$\therefore 104.8\,kPa = 101.3\,kPa + \frac{1}{2} \times 1.19\,kg/m^3 \times V^2$$

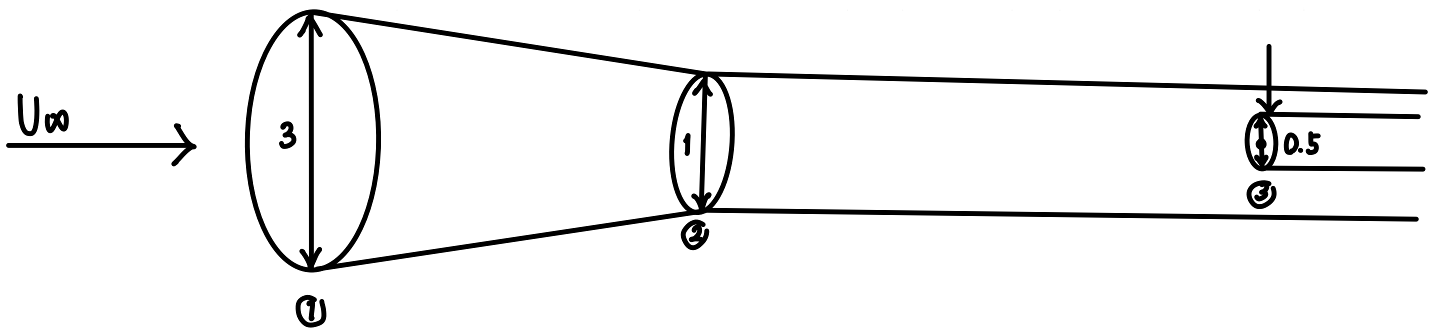
$$\therefore V = 76.696\,m/s$$

$$= 76.7\,m/s$$

3. cylindrical wind tunnel contraction (nozzle) has a diameter of 3m at the upstream end and 1m at the downstream end where it leads into the working section, also of 1m diameter. In the front part of the working section the airflow speed is 30ms^{-1} and the pressure is atmospheric, while the rear part contains a long model rocket of diameter 0.5m. Draw a figure of the configuration and calculate the change in pressure relative to atmospheric pressure (neglecting all effects of viscosity)

   a. upstream of the contraction
   b. at the front stagnation point of the model
   c. on the side of the model well behind the nose

   (Ans 528.9Pa, 535.5Pa,-416.5Pa)



a. $U_1 = (\frac{1}{3})^2 U_2 = \frac{10}{3}$ (m/s)

$P_1 + \frac{1}{2}\rho(U_1)^2 = P_2 + \frac{1}{2}\rho(U_2)^2 \quad \because P_2 = P_a$

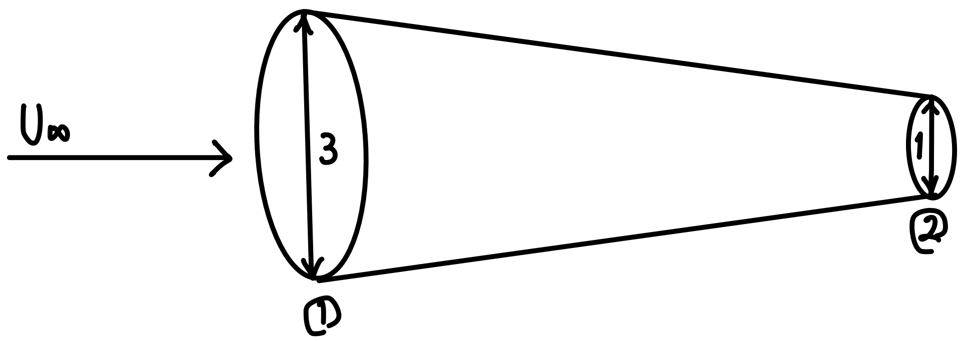
$\Delta P = \frac{1}{2}\rho(U_2)^2 - \frac{1}{2}\rho(U_1)^2 = 528.9$ (Pa)

b. $\Delta P = \frac{1}{2}\rho_a(U_2)^2 = \frac{1}{2} \times 1.19 \times (30)^2 = 535.5$ (Pa)

c. $U_3 = \frac{(0.5)^2\pi}{(0.5)^2\pi - (0.25)^2\pi} U_2 = 40$ (m/s)

$\therefore \Delta P = \frac{1}{2}\rho_a(U_2)^2 - \frac{1}{2}\rho_a(U_3)^2 = -416.5$ (Pa)

4. A nozzle has a diameter of 3m at the upstream end and 1m at the downstream end. The flow speed at the downstream end, where the pressure and temperature are at atmospheric conditions, is 30ms^{-1}. Determine
   a. the flux of momentum out of the nozzle
   b. the axial force on the nozzle walls (*Hint see lecture 2.7*)
   c. Justify the assumption of constant density
   (Ans (i) 848 N (ii) 3016 N Downstream, taking ρ= 1.2kg/m³)



a. $\rho(U_2)^2A_2 = 1.2\times(30)^2\times\pi(\frac{1}{2})^2 = 848.23\ (N)$

b. $U_1 = (\frac{1}{3})^2\times 30 = \frac{10}{3}\ (m/s)$

$$\rho(U_2)^2A_2 - \rho(U_1)^2A_1 = F + P_1A_1 - P_2A_2 - P_a(A_1-A_2)$$

$$F_1 = \rho(U_2)^2A_2 - \rho(U_1)^2A_1 + (P_1-P_2)A_1$$

$$= \rho(U_2)^2A_2 - \rho(U_1)^2A_1 + \frac{1}{2}\rho[(U_2)^2-(U_1)^2]A_1$$

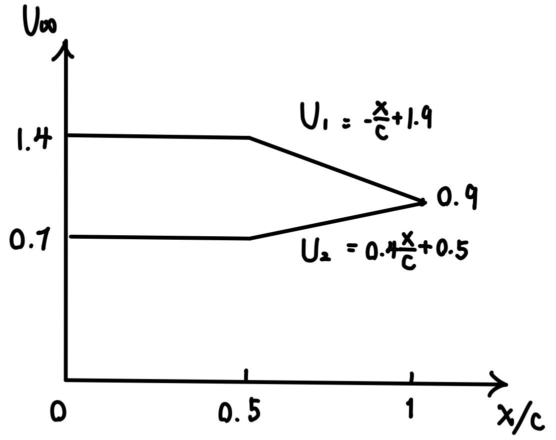
$$= \rho\left[A_2(U_2)^2 - \frac{A_1(U_2)^2}{2} - \frac{A_1(U_1)^2}{2}\right]$$

$$= 1.2(225\pi - 1025\pi) = -3015.93\text{ N}$$

$$\therefore F_2 = 3016\text{ N}$$

c. $U_{max} = 30m/s$, $M = \frac{30m/s}{330m/s} = 0.091 < 0.3$ ∴ incompressible

5. An aerofoil is situated in an airstream with free stream velocity U_∞ and density ρ_∞. The aerofoil section of chord c is designed to that the upper-surface velocity is 1.4 U_∞ up to x/c=0.5 (50% chord) and then decreases linearly to 0.9 U_∞ at x/c=1 (the trailing edge) while the lower-surface velocity is 0.7 U_∞ up to x/c=0.5 and then increases linearly to 0.9 U_∞ at the trailing edge. Plot a graph of velocity against x/c. Plot also a graph of C_p against x/c assuming inviscid flow. From the area of your second graph find the lift coefficient due to pressure. **(Ans 1.085)**



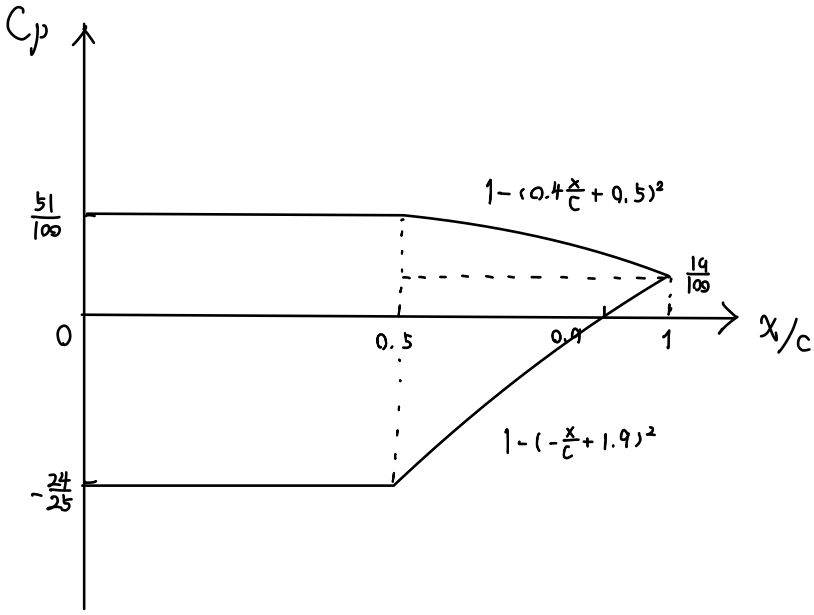
For inviscid flow, $P + \frac{1}{2}\rho V^2 = P_0$, ρ do not change

$$\therefore P_\infty + \frac{1}{2}\rho U_\infty^2 = P + \frac{1}{2}\rho U^2 \qquad \therefore P = P_\infty + \frac{1}{2}\rho(U_\infty^2 - U^2)$$

$$\therefore C_p = \frac{P - P_\infty}{\frac{1}{2}\rho U_\infty^2} = \frac{\frac{1}{2}\rho(U_\infty^2 - U^2)}{\frac{1}{2}\rho U_\infty^2} = 1 - \left(\frac{U}{U_\infty}\right)^2$$

$$\because C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 A} = \frac{(P - P_\infty)A}{\frac{1}{2}\rho U_\infty^2 A}$$

$$= C_p$$



$$\therefore \text{Area} = 0.5\times\left(\frac{51}{100}+\frac{24}{25}\right)$$

$$+\left(\int_{0.5}^{1} 1-(0.4\frac{x}{c}+0.5)^2\,d\frac{x}{c}\right)$$

$$+\left(-\int_{0.5}^{0.9} 1-(-\frac{x}{c}+1.9)^2\,d\frac{x}{c}-\int_{0.9}^{1} 1-(-\frac{x}{c}+1.9)^2\,d\frac{x}{c}\right)$$

$$=\frac{147}{200}+\frac{107}{600}+\frac{68}{375}-\frac{29}{3000}$$

$$=1.085$$