Introduction to Aerospace

Tutorial 4

1. Specific Excess Power (SEP) represents:
   (a) an aircraft's flight envelope;
   (b) an aircraft's specific energy level throughout the aircraft's envelope;
   (c) the effect of altitude and airspeed on an aircraft's drag curve.
   (d) the level of excess thrust available throughout the aircraft's envelope;
2. Specific energy or energy height represents
   (a) the aircraft's total energy per unit weight;
   (b) the rate of change of an aircraft's energy with height;
   (c) the aircraft's total energy per unit mass;
   (d) the aircraft's kinetic energy per unit mass at any given height;
3. The $P_S = 0$ contour indicates:
   (a) maximum flightspeed and minimum flightspeed
   (b) maximum flightspeed and absolute ceiling
   (c) maximum flightspeed and service ceiling
   (d) maximum flightspeed, minimum flightspeed and absolute ceiling
4. Which of the following do SEP contours take into consideration:
   (a) all wing design characteristics;
   (b) pilot limits;
   (c) thrust lapse;
   (d) structural design limits.
5. Which of the following do SEP curves not depend on?
   (a) Aircraft weight.
   (b) Pilot technique.
   (c) Load factor.
   (d) Throttle setting.

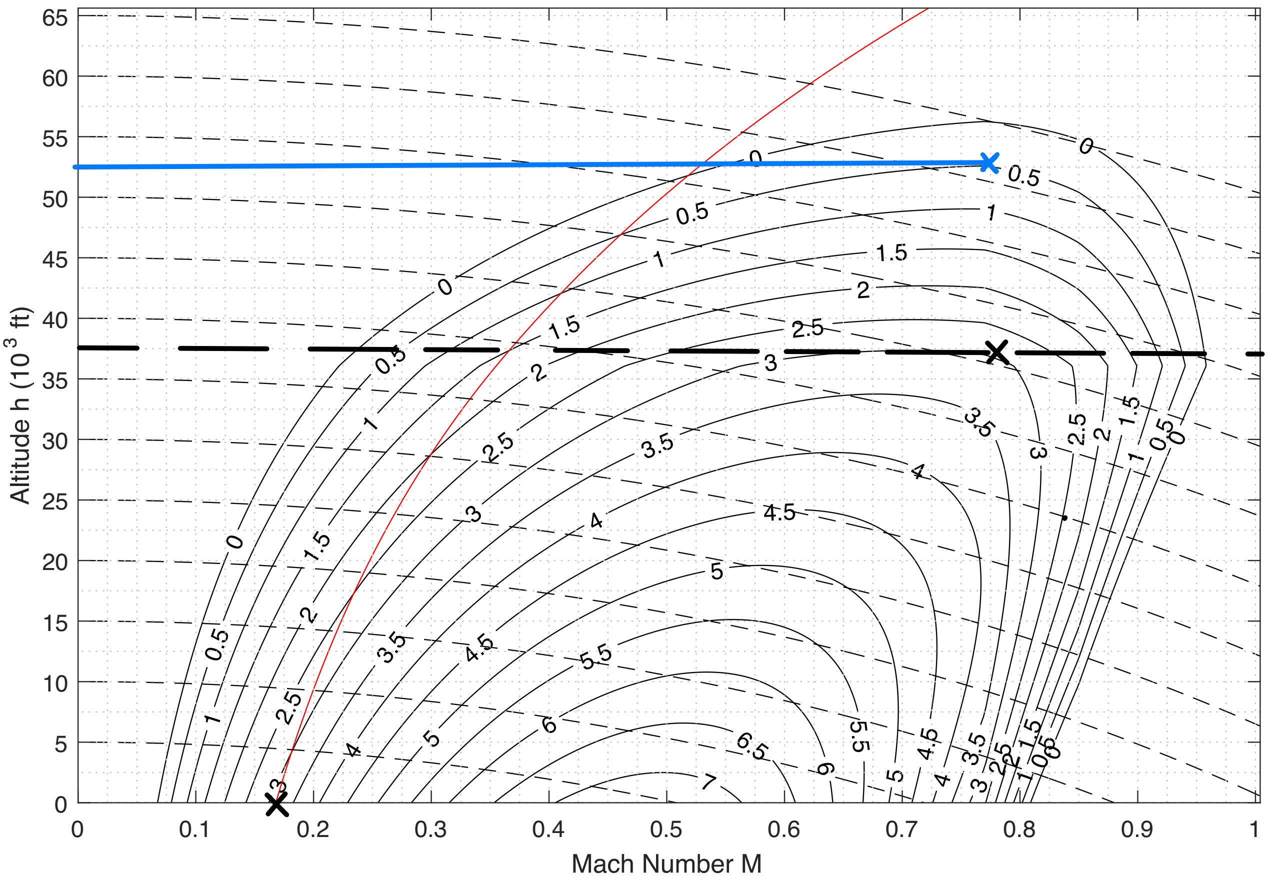

Figure 1: Plot of Specific Excess Power contours P_S (1000 ft/min) for a jet airliner, with Energy Heights (- -) and Stall curve (in red) superimposed

6. Which of the following pieces of information cannot be extracted from the SEP plot given in figure 1?

   (a) The stall speed is approximately 111 kts EAS. $0.17 \times 340.7\, m/s$

   (b) The maximum rate of climb possible at 37,500 ft is approximately 3,000 ft/min.

   (c) The maximum aircraft (L/D) is approximately 17.7. $\frac{V}{L/D} = \frac{VT_x}{W}$ when $P_s = 0$

   (d) The best cruise speed is approximately 200 kts EAS.

7. The service ceiling of the aircraft whose SEP plot is given in figure 1 will be:

   (a) 56,000 ft;

   (b) 37,000 ft;

   (c) 52,500 ft;

   (d) 65,000 ft.

8. Which velocity "schedule" should a pilot follow to climb in the minimum time possible?

   (a) The velocities where SEP contours are tangent to energy height lines for each altitude.

   (b) The velocities where SEP contours are normal to energy height lines for each altitude.

   (c) The velocities where rate of climb is maximised for each altitude.

   (d) The maximum velocity achievable at each altitude.

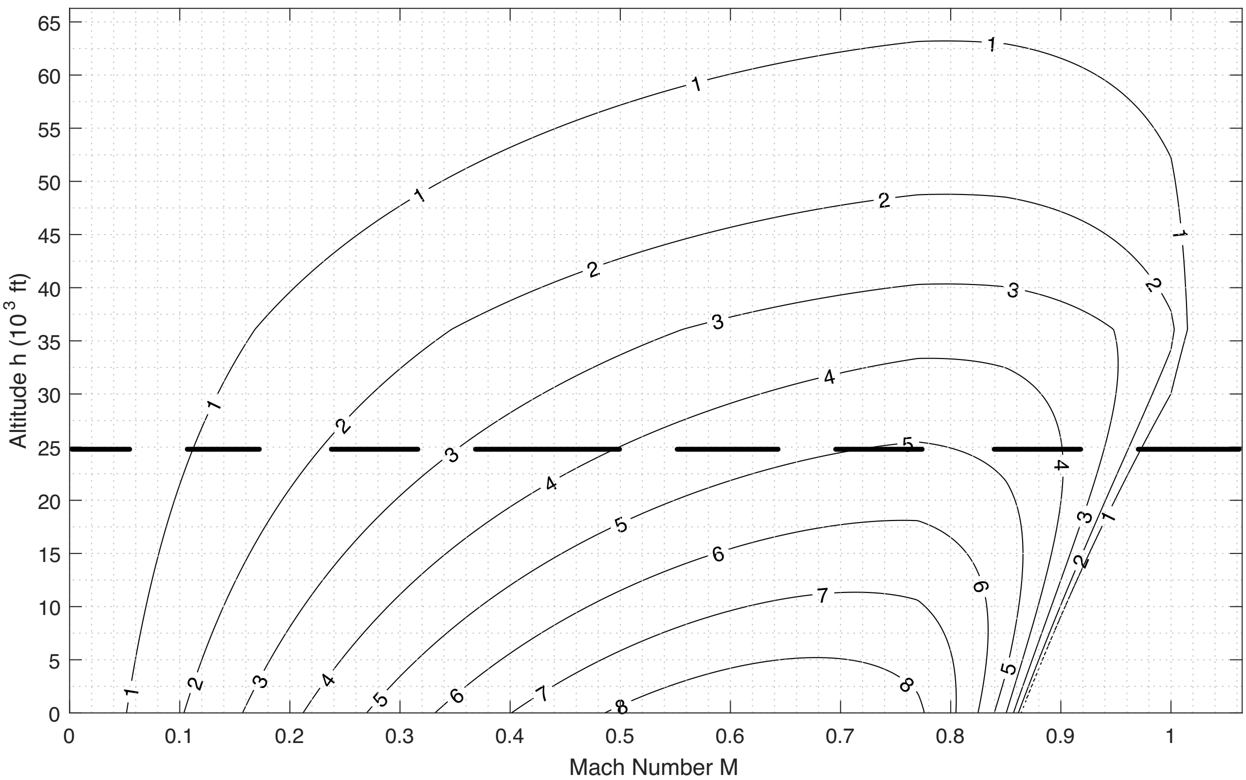

Figure 2: $P_S = 0$ contours at varying load factor, n, for a low-powered fighter.

9. The fighter whose SEP contours are given in Figure 2 is engaged in aerial combat. What would the radius of the tightest level turn achievable by the pilot with a 70° bank angle at an altitude of 25 000 ft?

$$n = \frac{L}{W} = \frac{L}{L\cos\phi} = \frac{1}{\cos\phi} = 2.92$$

$$R = \frac{V^2_{mo}}{g} \quad \frac{\bar{V}^2}{\sqrt{n^2-1}}$$

   (a) 460 m *If $M_{stall} < 0.3$
   (b) 3,150 m
   (c) 4,533 m
   (d) There is insufficient info to approximate.

10. What is the maximum load factor that the pilot of the fighter whose SEP contours are given in Figure 2 can achieve when flying at an altitude of 25 000 ft?

   (a) There is insufficient info to approximate.
   (b) Approximately 1 g;
   (c) Approximately 3 g;
   (d) Approximately 5 g;

	SI Units ⟹	The Standard Atmosphere							A.101/102
Alt. Feet	Alt. m.	T/K	a	ρ	σ	$\sqrt{\sigma}$	p/p0	p (Pa)	μ
0	0	288.2	340.7	1.225	1.0000	1.0000	1.0000	101325	1.795E-05
1640	500	284.9	338.8	1.167	0.9529	0.9762	0.9421	95463	1.779E-05
3281	1000	281.7	336.9	1.112	0.9075	0.9526	0.8870	89878	1.763E-05
4921	1500	278.4	334.9	1.058	0.8638	0.9294	0.8346	84561	1.747E-05
6562	2000	275.2	332.9	1.007	0.8217	0.9065	0.7846	79502	1.731E-05
8202	2500	271.9	331.0	0.957	0.7812	0.8838	0.7371	74690	1.715E-05
9843	3000	268.7	329.0	0.909	0.7422	0.8615	0.6920	70117	1.699E-05
11483	3500	265.4	327.0	0.863	0.7048	0.8395	0.6491	65774	1.683E-05
13123	4000	262.2	325.0	0.819	0.6688	0.8178	0.6084	61651	1.666E-05
14764	4500	258.9	323.0	0.777	0.6342	0.7964	0.5698	57739	1.650E-05
16404	5000	255.7	320.9	0.736	0.6010	0.7753	0.5332	54031	1.633E-05
18045	5500	252.4	318.9	0.697	0.5692	0.7545	0.4986	50519	1.617E-05
19685	6000	249.2	316.8	0.660	0.5387	0.7339	0.4658	47193	1.600E-05
21325	6500	245.9	314.7	0.624	0.5094	0.7137	0.4347	44047	1.583E-05
22966	7000	242.7	312.7	0.590	0.4814	0.6938	0.4054	41073	1.566E-05
24606	7500	239.4	310.6	0.557	0.4545	0.6742	0.3776	38264	1.549E-05
26247	8000	236.2	308.4	0.525	0.4289	0.6549	0.3515	35612	1.532E-05
27887	8500	232.9	306.3	0.495	0.4043	0.6358	0.3268	33112	1.515E-05
29528	9000	229.7	304.2	0.467	0.3808	0.6171	0.3035	30755	1.498E-05
31168	9500	226.4	302.0	0.439	0.3584	0.5987	0.2816	28536	1.480E-05
32808	10000	223.2	299.8	0.413	0.3371	0.5806	0.2610	26448	1.463E-05
34449	10500	219.9	297.6	0.388	0.3167	0.5627	0.2417	24486	1.445E-05
36089	**11000**	**216.66**	**295.4**	**0.364**	**0.2972**	**0.5452**	**0.2235**	**22644**	**1.427E-05**
37730	11500	216.7	295.5	0.336	0.2747	0.5241	0.2066	20931	1.428E-05
39370	12000	216.7	295.5	0.311	0.2539	0.5039	0.1909	19344	1.428E-05
41010	12500	216.7	295.5	0.287	0.2346	0.4844	0.1764	17877	1.428E-05
42651	13000	216.7	295.5	0.266	0.2168	0.4657	0.1631	16522	1.428E-05
44291	13500	216.7	295.5	0.245	0.2004	0.4477	0.1507	15270	1.428E-05
45932	14000	216.7	295.5	0.227	0.1852	0.4303	0.1393	14112	1.428E-05
47572	14500	216.7	295.5	0.210	0.1712	0.4137	0.1287	13042	1.428E-05
49213	15000	216.7	295.5	0.194	0.1582	0.3977	0.1190	12053	1.428E-05
50853	15500	216.7	295.5	0.179	0.1462	0.3824	0.1099	11139	1.428E-05
52493	16000	216.7	295.5	0.166	0.1351	0.3676	0.1016	10295	1.428E-05
54134	16500	216.7	295.5	0.153	0.1249	0.3534	0.0939	9514	1.428E-05
55774	17000	216.7	295.5	0.141	0.1154	0.3397	0.0868	8793	1.428E-05
57415	17500	216.7	295.5	0.131	0.1067	0.3266	0.0802	8127	1.428E-05
59055	18000	216.7	295.5	0.121	0.0986	0.3140	0.0741	7510	1.428E-05
60696	18500	216.7	295.5	0.112	0.0911	0.3018	0.0685	6941	1.428E-05
62336	19000	216.7	295.5	0.103	0.0842	0.2901	0.0633	6415	1.428E-05
63976	19500	216.7	295.5	0.095	0.0778	0.2789	0.0585	5928	1.428E-05
65617	20000	216.7	295.5	0.088	0.0719	0.2682	0.0541	5479	1.428E-05

NB: Conversion factors - 1 knot = 1.689 ft/s = 0.5148 m/s

Long Question

Figure 3 shows the specific excess power (P_S) plot obtained when flight testing a jet trainer at a wing loading $W/S = 4937$ N/m^2. The aircraft has the following characteristics:

- wing reference area $S = 16.7$ m^2,
- wing aspect ratio $\mathcal{R} = 7$,
- loading efficiency $k = 1.25$,
- specific fuel consumption $c = 0.45$ 1/hour
- maximum sea-level thrust to weight ratio $T_0/W = 0.352$.

The aircraft's thrust may be assumed to be independent of Mach number and dependent on altitude only. In the troposphere you may use $T/T_0 = \sigma^{0.7}$ and $T/T_0 = 1.439\sigma$ in the stratosphere.

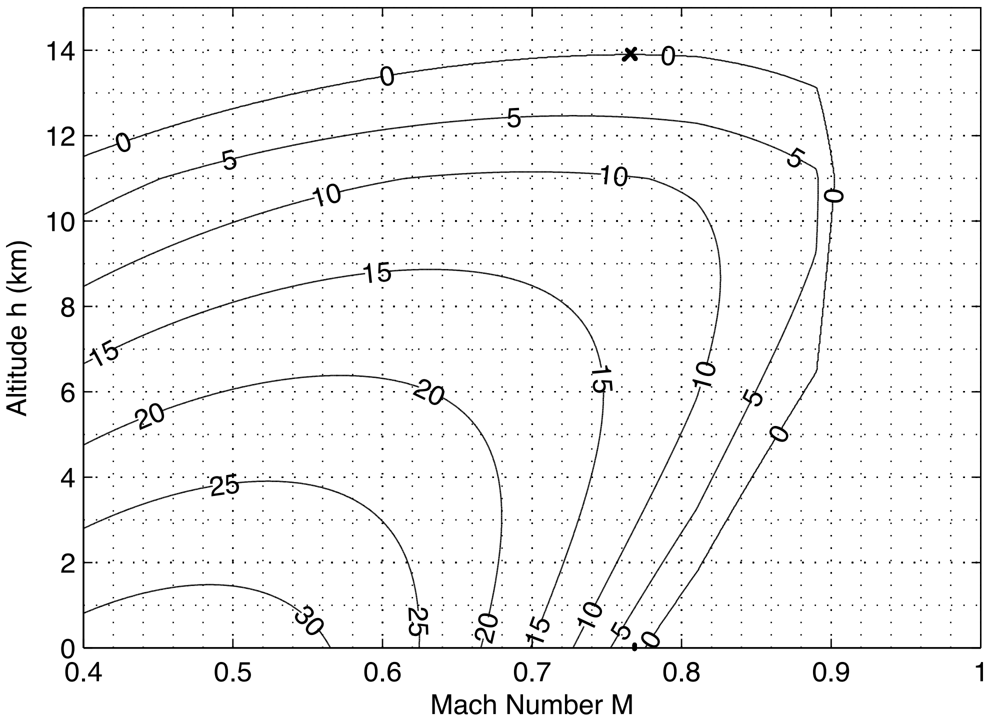

Figure 3: Variation of specific excess power (in m/s) with altitude and Mach number

Based on Figure 3:

1. approximately calculate the aircraft's minimum drag Equivalent Airspeed V_{imD}; [20%]
2. approximately calculate the aircraft zero-lift drag coefficient C_{D_0}; [35%]
3. what is the maximum endurance of this aircraft if 1400 kg of fuel is available? State any assumptions made. [25%]
4. what is the maximum climb gradient the aircraft can achieve at sea-level at the end of its loiter? [20%]

1. $M_{mD} = 0.76$

V_{mD} ($13.8\times10^3 m \approx 14000m$) $= 0.76 \times 295.5 = 224.58$ (m/s)

$\therefore V_{imD} = V_{mD}\sqrt{6} = 0.4303 V_{mD} = 96.6$ (m/s)

2. At absolute ceiling,

$$L/D = (L/D)_{max} = \frac{1}{2}\sqrt{\frac{\pi A\!\!R}{kC_{D_0}}}$$

When $P_s = 0$, $V\left(\frac{T_x}{W} - \frac{1}{L/D}\right) = 0$

$$\therefore (L/D)_{max} = \frac{W}{T} = \frac{W}{T_0 \cdot 1.4396} \text{ at } 13800m = 10.660$$

$$\therefore C_{D_0} = \frac{\pi A\!\!R}{[2(L/D)_{max}]^2 k} = 0.0387$$

3. $E = -\frac{(L/D)_{max}}{C}\left[\frac{2}{\bar{V}^2 + \bar{V}^{-2}}\right]\ln\left(\frac{W_f}{W_i}\right)$

while $\bar{V} = 1$, $E_{max} = -\frac{(L/D)_{max}}{C}\ln\left(\frac{W_f}{W_i}\right) = 4.316\,h$

4. $P_s = V\left(\frac{T_x}{W} - \frac{1}{L/D}\right)$ while V is constant

$\sin\gamma = \frac{T-D}{W_f} = \frac{T_0}{W_f} - \frac{D}{W_f} = \frac{T_0}{W_f} - \frac{1}{(L/D)_{max}}$ ($L \approx W$ for small γ)

$\therefore \gamma_{max} = 19.2°$