2010 Question 1 (i):

1. i) Determine the dimensions of the following quantities in the form $M^{\alpha}L^{\beta}T^{\gamma}$: [50%]
   a) total pressure,
   b) stream function,
   c) wall shear stress,
   d) dynamic viscosity,
   e) pressure gradient and
   f) drag coefficient.

(a) Total Pressure: P_o

$$\frac{M^{\alpha}\frac{L^{\beta}}{T^{2\gamma}}}{L^{2\beta}} = M^{\alpha}L^{-\beta}T^{-2\gamma}$$

(b) Stream function (Not yet covered)

(c) Wall shear stress: $\tau = \mu \frac{\partial u}{\partial y}$

$M^{\alpha}L^{-\beta}T^{-2\gamma}$ $ML^{-1}T^{-2}$

(d) Dynamics Viscosity $\mu = \frac{\tau \partial y}{\partial u}$

$$\frac{M^{\alpha}L^{-\beta}T^{-2\gamma} \cdot L^{\beta}}{\frac{L^{\beta}}{T^{\gamma}}} = M^{\alpha}L^{-\beta}T^{-\gamma}$$ $ML^{-1}T^{-1}$

(e) Pressure Gradient $\frac{dp}{dx}$

$$\frac{M^{\alpha}L^{-\beta}T^{-2\gamma}}{L^{\beta}} = M^{\alpha}L^{-2\beta}T^{-2\gamma}$$ $ML^{-2}T^{-2}$

(f) Drag Coefficient

dimentionless $\rightarrow$ $M^{0}L^{0}T^{0}$ 0

4\. (a) State, in words, the general control volume form for the equation of momentum. [10%]

(b) Using a rectangular control volume, derive the partial differential equation describing the unsteady two-dimensional, inviscid y-component of the momentum equation. What form does the equation reduce to if one imposes the principle of two-dimensional mass conservation? [35%]

(c) How is wall shear stress related to fluid deformation? Considering parallel flow above a flat plate, derive the wall shear stress relationship

$$\tau_w = \mu \frac{\partial u}{\partial n}$$

where n is the surface normal direction to the wall. [35%]

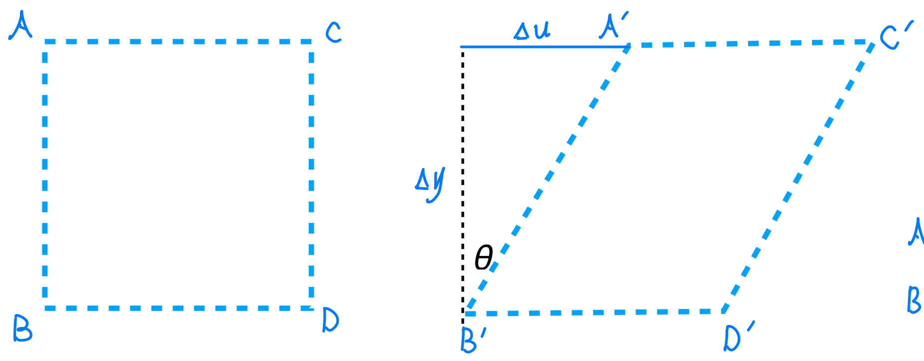
(d) By using control volume analysis for steady, fully developed, laminar flow in a pipe of diameter D, derive an expression relating the pressure drop Δp over a length l of the pipe to the wall shear stress τ_w. [20%]

Consider section 1-2 and the definition of wall shear stress

Wall shear stress is related to the rate of deformation in time

This relation is the dynamic viscosity

Draw a figure of problem identifying rate of deformation:



$AA' = (u+\Delta u)\Delta t$

$BB' = u\Delta t$

Determine the angle θ in terms of Δu and Δy

$\therefore \tan\theta = \frac{\Delta u}{\Delta y}\Delta t \quad \therefore \theta = \arctan\left(\frac{\Delta u}{\Delta y}\Delta t\right) \xrightarrow{\text{small angle approximation}} \frac{\Delta u}{\Delta y}\Delta t$

2014 Question 4 (c):

Time rate of deformation is therefore

$$\frac{\Delta\theta}{\Delta t} = \frac{\Delta u \Delta t}{\Delta y \Delta t} = \frac{\Delta u}{\Delta y}$$

$$\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{\partial u}{\partial y}$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_w$$

2014 Question 3 (a)

3. (a) In order to estimate the drag and overturning moment due to wind experienced by a rocket on its launch pad, it is decided to carry out wind tunnel tests. The loading on the rocket is required at a wind speed of 40 m/s. A $1/15^{th}$ scale model of the rocket is placed in a wind tunnel that operates at a pressure of 15 atmospheres and at atmospheric temperature.

i) If the flow about the model is dynamically similar to that about the full-scale rocket, calculate the air speed required in the wind tunnel. State any assumptions you make. [25%]

ii) The drag and overturning moment measured on the model at this speed are 5.2 kN and 4.6 kNm respectively. Calculate the drag force and the overturning moment acting on the full scale rocket at a wind speed of 40 m/s. [30%]

(i) *For dynamic similarity we expect*

$$\left(\frac{\rho U l}{\mu}\right)_{Full} = \left(\frac{\rho U l}{\mu}\right)_{Model} = Re$$

$$\rho = pRT \quad \therefore \quad \rho_{model} = 15\,pRT = 15\,\rho_{Full}$$

$$U_{air} = \frac{\rho_F U_F l_F \cdot \mu_M}{\mu_F \cdot \rho_M \cdot l_M}$$

$$= \frac{\rho_F U_F l_F \cdot \mu_F}{\mu_F \cdot 15\rho_F \cdot \frac{1}{15} l_F}$$

$$= U_{Full\ model} = 40\ m/s$$

(ii) In dynamically similar model drag and moment coefficients will be the same

$$C_{DF} = C_{DM} = \frac{D}{\frac{1}{2}\rho U^2 cA}$$

$$C_{MF} = C_{MM} = \frac{M}{\frac{1}{2}\rho U^2 cA}$$

2014 Question 3 (b)

(b) Spheres of various diameters D and densities σ are allowed to fall freely under gravity g through various fluids which have density ρ and kinematic viscosity ν. The terminal velocity of the falling spheres V for each experiment is then measured. Find an expression connecting V with the other variables and hence suggest a suitable form of graph in which the results could be presented. [45%]

$f(D, 6, g, \rho, \nu)$

6 variables - 3 dimensions = 3 dimensionless quatities

$Re = \frac{\rho U D}{\mu}$, $\nu = \frac{\mu}{\rho}$

$\therefore Re = \frac{UD}{\nu} = \frac{VD}{\nu}$ in this case

$$\overset{D}{(L^{a})}\overset{6}{\left(\frac{M}{L^3}\right)^{\beta}}\overset{g}{\left(\frac{L}{T^2}\right)^{\gamma}}\overset{\rho}{\left(\frac{M}{L^3}\right)^{\delta}}\overset{\nu}{\left(\frac{L^2}{T}\right)^{\epsilon}} = M^0 L^0 T^0$$

$$\begin{cases} \beta + \delta = 0 \to M \\ a - 3\beta + \gamma - 3\delta + 2\epsilon = 0 \to L \\ -2\gamma - \epsilon = 0 \to T \end{cases} \quad \therefore \begin{cases} \delta = -\beta \\ \epsilon = -2\gamma \\ a = 3\delta \end{cases}$$

$$\frac{VD}{\nu} \propto D^{3\delta} 6^{\beta} g^{\gamma} \rho^{-\beta} \nu^{-2\epsilon}$$

$$\therefore Re \propto \left(\frac{6}{\rho}\right)^{\beta} \left(\frac{D^3 g}{\nu^2}\right)^{\epsilon}$$