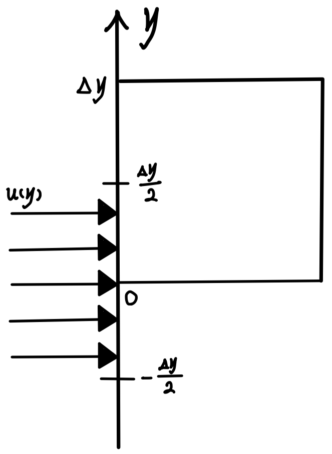
1. The x-component velocity, u(y), normal to the face of a 2D control volume which has a centre at y=0 and whose height is Δy, so that $-\Delta y/2 < y < \Delta y/2$, varies as

$$u(y) = u_0 + a\,y + b\,y^2$$

(i) Draw a diagram of the problem.
(ii) Show that the average velocity, $\overline{U}$, over the face is $\overline{U} = u_0 + b\,(\Delta y)^2/12$

(i)



(ii) $$\bar{U} = \frac{\int_{-\frac{\Delta y}{2}}^{\frac{\Delta y}{2}} u(y)\,dy}{\Delta y}$$

$$= \frac{1}{\Delta y}\left[u_0 y + \frac{1}{2}ay^2 + \frac{1}{3}by^3\right]_{-\frac{\Delta y}{2}}^{\frac{\Delta y}{2}}$$

$$= \frac{1}{\Delta y}\left[u_0(\Delta y) + \frac{1}{12}b(\Delta y)^3\right]$$

$$= u_0 + \frac{1}{12}b(\Delta y)^2$$

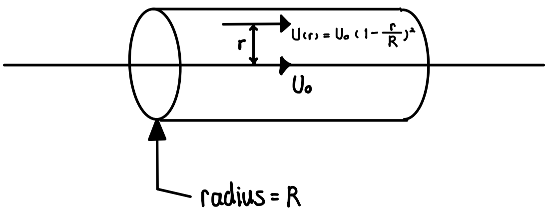
2. The velocity profile across a circular pipe of radius R is given by $U(r)=U_0(1-(r/R)^2)$, where U_0 is the velocity at the centre and r is radial distance. This is a similar problem to question 3 but requires integration over a circular cross section.

   To perform this integral we can use the method of integration of a *solid of revolution* (this will be covered in more detail in your mathematics lectures). In this approach we consider the volume under a surface $U(r)$ as the summation (or integral) or a series of circular areas (πr^2) of radius $r(U)$ multiplied by a thickness du and so the volume is given by $\int U(r)\,dA = \int_0^{U_0} \pi r^2(U)\,du$

   (i) Derive an expression for the mass flow rate in the pipe in terms of U_0, r, R and ρ assuming steady flow. Recall that mass flow $Q = \rho \overline{U} A = \rho \int U\,dA$ where $\overline{U}$ is the average velocity, and A is the cross-sectional area.

   (ii) If the diameter of the pipe is 4cm and the velocity at the centre is 1m/s, calculate the Reynolds number Re, assuming the fluid in the pipe is air, defining Re as $\rho \overline{U} D/\mu$, where $\overline{U}$ is the average velocity of the air in the pipe.

   (Ans $\rho\pi U_0 R^2/2$, 1308 using ρ=1.19 kg/m^3 and μ=1.82x10^{-5} Kg m/s)



(i) $Q = \rho \bar{U} A = \rho \dfrac{U(R) + U(R)}{2} (\pi R^2) = \dfrac{1}{2} (\rho U_0 \pi R^2)$

(ii) $R = \frac{1}{2} \times 4cm = 2cm$

$\bar{U} = \frac{U_0}{2} = \frac{1}{2} m/s$

$\therefore Re = \dfrac{\rho \bar{U} D}{\mu} = \dfrac{1.19 kg/m^3 \times 0.5 m/s \times 4cm}{1.82 \times 10^{-5} kg\, m/s}$

$= 1307.69$

$= 1310$

3. What is the vorticity of the following flows?

u = x, v = y

u = 1-y^2, v = 0

(Ans 0, *2y*)

For $u = x,\ v = y$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial y}{\partial x} - \frac{\partial x}{\partial y} = 1 - 1 = 0$$

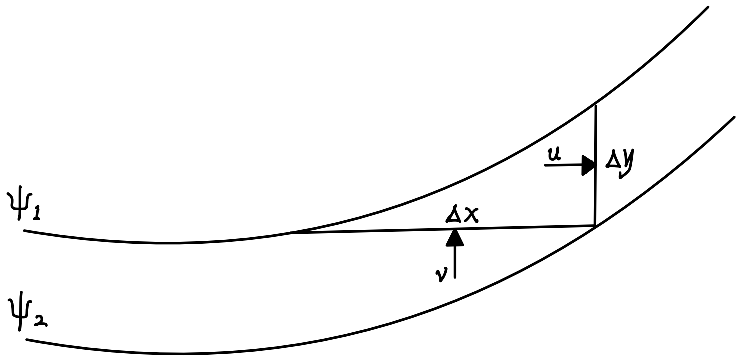
For $u = 1 - y^2,\ v = 0$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 - (1 - y^2)\,dy = 0 - (-2y) = 2y$$

4. By considering the flow rate between two streamlines and recalling that the streamfunction is constant along a streamline and is related to the flow rate between the streamlines derive the following differential expressions:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

(Hint: *Start with a diagram and also review lecture 2-4*)



$$\psi_1 - \psi_2 = \int u\, dy = u \Delta y \qquad \therefore \Delta \psi = u \Delta y$$

$$\psi_2 - \psi_1 = \int v\, dx = v \Delta x \qquad \therefore -\Delta \psi = v \Delta x$$

$\therefore$ Let $\psi, x, y \longrightarrow 0$

$$\delta \psi = u \delta y \qquad \therefore u = \frac{\delta \psi}{\delta y}$$

$$-\delta \psi = v \delta x \qquad \therefore v = -\frac{\delta \psi}{\delta x}$$