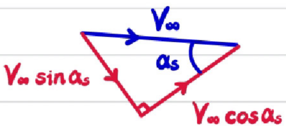
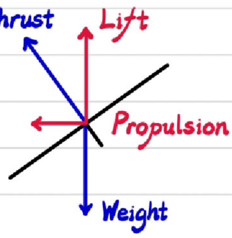
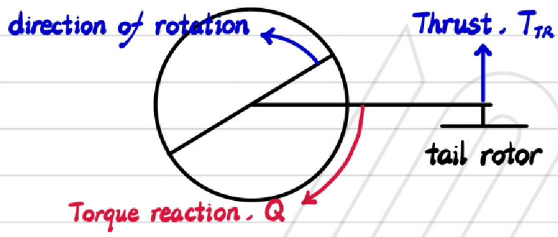
Introductions

Axis Conventions

Rotors



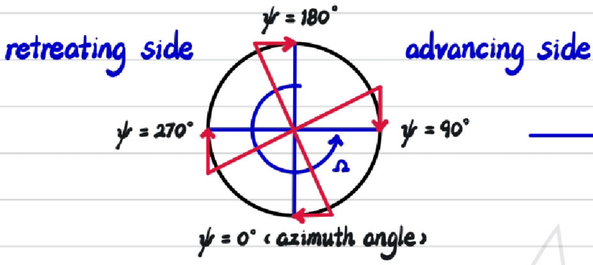




$\longrightarrow$ $Q = T_{TR}\, l_{TR}$

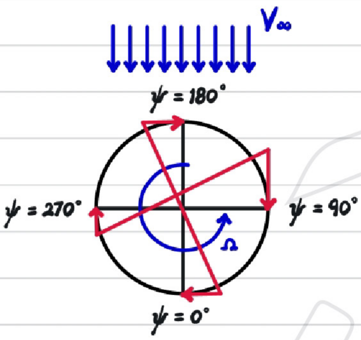
Dissimetry of lift

hover



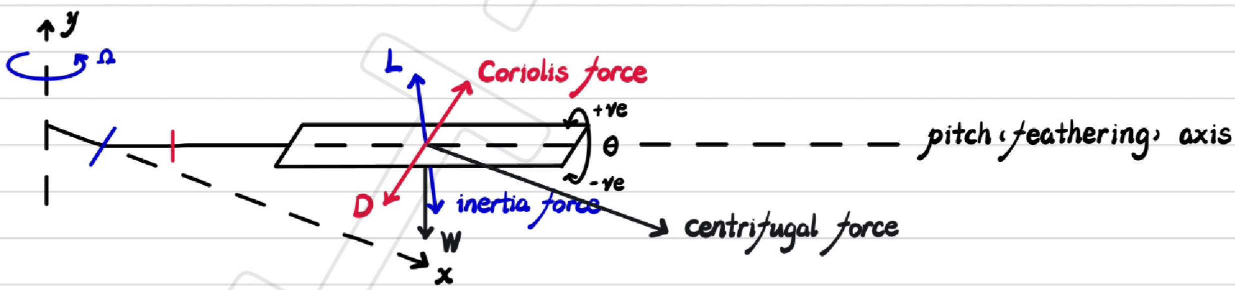
$\longrightarrow$ $V(r, \psi) = \Omega r$

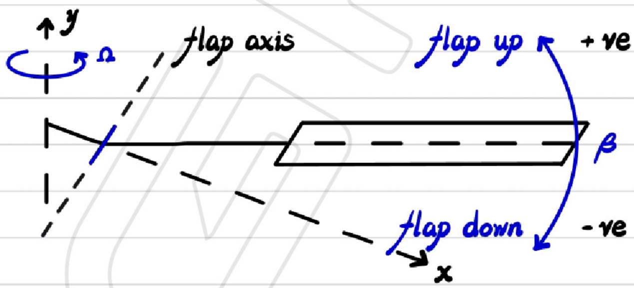
forward flight

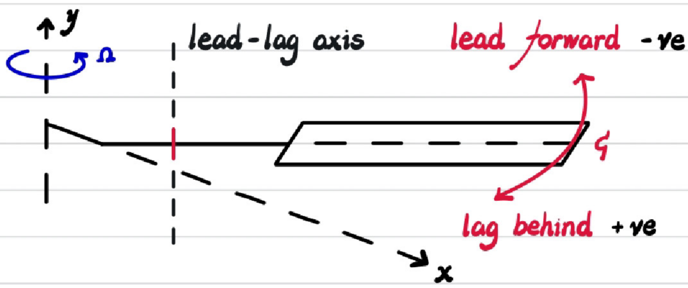


$\longrightarrow$ $V(r, \psi) = \Omega r + V_\infty \sin\psi$

Blade motion



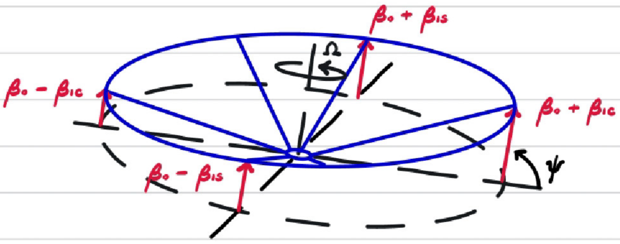




a blade is a single - degree - of - freedom dynamic system in flap and lag

→ mass-spring-damper system → $m\ddot{x} + c\dot{x} + kx = F$ → often decoupled flap and lag

Rotor tip path plane (TPP)



$\xrightarrow{\text{Fourier series}}$ $\beta = \beta_0$ coning angle

$+ (\beta_{1c} + \ldots + \beta_{nc})\cos\psi$ longitudinal

$+ (\beta_{1s} + \ldots + \beta_{ns})\sin\psi$ lateral

→ $\beta = \beta_0 + \beta_{1c}\cos\psi + \beta_{1s}\sin\psi$

a dynamic system forced near natural frequency, ω_n, reponds with 90° phase lag

$\Delta F \xRightarrow{90°} \Delta\beta \xRightarrow{90°} \Delta\theta$

forward flight → lift disymmetry $\xrightarrow{90°}$ TPP tilt aft

$\xrightarrow{\text{coning}}$ AoA lower on rear and higher on forward

→ lift disymmetry $\xrightarrow{90°}$ TPP tilt toward advancing side

pitch input $\theta = \theta_0 - \theta_{1s}\sin\theta + \theta_{1c}\cos\theta$ $\xrightarrow{90°}$ $\beta = \beta_0 + \beta_{1c}\cos\psi + \beta_{1s}\sin\psi$

Configurations

Autogiro / Gyroplane

Single rotor

· conventional · NOTAR (No-Tail-Rotor) using Coandă effect · fenestron / fan-in-fin

· tip jets no anti-torque device necessary

Dual rotors

· tandem two longitudinal counter-rotating rotors · coaxial · side-by-side

· synchropter two equal counter-rotating intermeshing rotors on different axes

Compound helicopters

· thrust compound additional forward thrust · lift compound small wings

Tilt - rotors

· tilt - rotor · tilt - wing

Multi - rotors

Control Mechanism

Variable pitch conventional helicopters

· uniformly collective feathering Swashplate plate up/down $\longrightarrow$ altitude control

· cyclically longitudinal / lateral cyclic feathering Swashplate plate tilting $\longrightarrow$ direction control

Variable speed multi-rotor systems

Types of Rotors

Teetering 半刚性（跷跷板） · both blades flapping together · no lead-lag hinges

Articulated 全铰接式 · flapping hinges · lead-lag hinges · feathering hinges

Hingeless 无铰接式（刚性）

Bearingless 无轴承 · further evolution of hingeless rotor

Momentum Theory

Introduction

Methodology

· rotor $\xrightarrow{\text{idealisation}}$ infinitesimally thin actuator disk

· conservation laws apply over the control volume

mass $\oiint_s \rho \vec{V} \cdot d\vec{S} = 0 \longrightarrow \dot{m} = \rho A \sqrt{(V_\infty \cos\alpha_s)^2 + (V_\infty \sin\alpha_s + v_i)^2}$

momentum $\vec{F} = \oiint_s p d\vec{S} + \oiint_s (\rho \vec{V} \cdot d\vec{S}) \vec{V} \longrightarrow T = \dot{m} w$

energy $W = \oiint_S \frac{1}{2}(\rho\vec{V}\cdot d\vec{S})|\vec{V}|^2 \longrightarrow P = T(V_\infty \sin\alpha_s + v_i) = \frac{1}{2}\dot{m}w(2V_\infty \sin\alpha_s + w)$

Assumptions

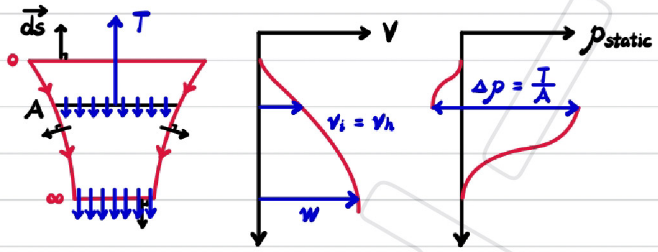
- flow through rotor / being distributed in a well defined streamtube
- far above the disk air is at the rest
- 1-dimensional
- quasi-steady $\frac{\partial}{\partial t} = 0$
- ideal fluid $\longrightarrow$ incompressible $\rho = const$ and inviscid $\mu = \nu = 0$

Non-dimensional terms US convention

- thrust coefficient $C_T = \frac{T}{\rho A V_{tip}^2} = \frac{T}{\rho A \Omega^2 R^2}$
- power coefficient $C_P = \frac{P}{\rho A V_{tip}^3} = \frac{P}{\rho A \Omega^3 R^3}$
- torque coefficient $C_Q = \frac{Q}{\rho A V_{tip}^2 R} = \frac{Q}{\rho A \Omega^2 R^3} = \frac{P/\Omega}{\rho A \Omega^2 R^3} = C_P$
- induced flow ratio $\lambda = \frac{V_c + v_i}{V_{tip}} = \frac{V_c + v_i}{\Omega R} = \lambda_c + \lambda_i$
- rotor solidity $\sigma = \frac{\text{blade area}}{\text{rotor area}} = \frac{N_b c R}{\pi R^2} = \frac{N_b c}{\pi R}$ typically $\sigma = 0.07 \sim 0.12$

Axial Flight

Hover $V_c = 0$



- conservation of mass $\dot{m} = \rho v_h A = \rho w A_\infty$
- — momentum $T = \dot{m} w$
- — energy $P = T v_h = \frac{1}{2}\dot{m} w^2$

$\longrightarrow \dot{m} w v_h = \frac{1}{2}\dot{m} w^2 \longrightarrow w = 2v_h \longrightarrow A = 2A_\infty \longrightarrow v_h = \sqrt{\frac{T}{2\rho A}} \longrightarrow P = \frac{T^{3/2}}{\sqrt{2\rho A}}$

Disk loading $DL = \frac{T}{A} = 2\rho v_h^2$

Power loading $PL = \frac{T}{P} = \frac{1}{v_h} \longrightarrow$ power requirements $\propto \frac{1}{PL} = v_h \propto \sqrt{DL}$

Non-ideal effects for power to hover

- ideal power $C_P = \frac{P}{\rho A \Omega^3 R^3} = \frac{T v_h}{\rho A \Omega^3 R^3} = \frac{T^{3/2}}{\sqrt{2}\rho^{1/2} A^{1/2} \Omega^3 R^3} = \frac{C_T^{3/2}}{\sqrt{2}}$
- induced power $C_{P_i} = \frac{\kappa C_T^{3/2}}{\sqrt{2}}$ typically $\kappa \approx 1.1 \sim 1.15$

· profile power $C_{P_0} = \dfrac{P_0}{\rho A \Omega^3 R^3}$ where $P_0 = N_b Q_0 = \Omega N_b \int_0^R dD\, y\, dy = \Omega N_b \int_0^R \frac{1}{2}\rho (\Omega y)^2 c\, C_{d_0}\, y\, dy$

$$= \frac{\frac{1}{8}\rho N_b \Omega^3 R^4 C_{d_0}}{\rho A \Omega^3 R^3} = \frac{1}{8}\left(\frac{N_b c}{\pi R}\right) C_{d_0} = \frac{1}{8}\sigma C_{d_0}$$

$\longrightarrow$ $C_P = C_{P_i} + C_{P_0} = \dfrac{\kappa C_T^{\frac{3}{2}}}{\sqrt{2}} + \dfrac{1}{8}\sigma C_{d_0}$

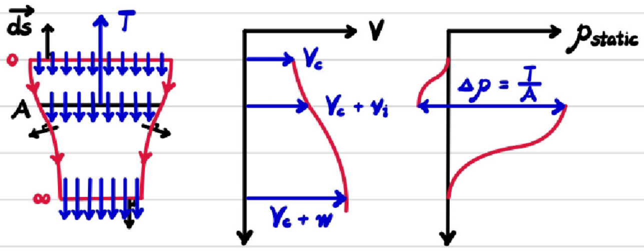
FM (Figure of Merit)

$$FM = \frac{\text{ideal power to hover}}{\text{actual power to hover}} = \frac{C_T^{\frac{3}{2}}/\sqrt{2}}{\kappa C_T^{\frac{3}{2}}/\sqrt{2} + \frac{1}{8}\sigma C_{d_0}} < 1$$

$FM \propto \frac{1}{\sigma}$ but low σ $\longrightarrow$ higher α needed to produce same lift $\longrightarrow$ blade stall

$\longrightarrow$ better to plot FM vs $\dfrac{C_T}{\sigma} = \dfrac{T}{\rho N_b c R \Omega^2 R^2}$ = blade loading coefficient

Axial climb $V_c > 0$



· conservation of mass $\dot{m} = \rho(V_c + v_i)A = \rho(V_c + w)A_\infty$

· — momentum $T = \dot{m}(V_c + w) - \dot{m}V_c = \dot{m}w$

· — energy $P = T(V_c + v_i) = \frac{1}{2}\dot{m}w(2V_c + w)$

$\longrightarrow$ $\dot{m}w(V_c + v_i) = \frac{1}{2}\dot{m}(w^2 + 2V_c w)$ $\longrightarrow$ $w = 2v_i$

$\longrightarrow$ $T = \dot{m}w = \rho A(V_c + v_i)(2v_i)$ $\longrightarrow$ $v_h^2 = \dfrac{T}{2\rho A} = (V_c + v_i)v_i$ $\longrightarrow$ $\left(\dfrac{v_i}{v_h}\right)^2 + \dfrac{V_c}{v_h}\dfrac{v_i}{v_h} - 1 = 0$

$\xrightarrow{\frac{v_i}{v_h} > 0}$ $\dfrac{v_i}{v_h} = -\dfrac{V_c}{2v_h} + \sqrt{\left(\dfrac{V_c}{2v_h}\right)^2 + 1}$

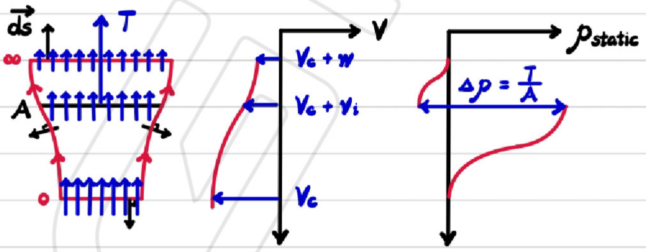
- $\xrightarrow{\frac{V_c}{2v_h} \ll 1}$ $\dfrac{v_i}{v_h} \approx -\dfrac{V_c}{2v_h} + 1$
- $\xrightarrow{\frac{V_c}{2v_h} \gg 1}$ $\dfrac{v_i}{v_h} \approx 0$

Axial descent $V_c < 0$

· $-2v_i < V_c < 0$ $\longrightarrow$ rotor still produce +ve thrust $\longrightarrow$ induced velocity point downwards

$\longrightarrow$ no well-defined streamtube $\longrightarrow$ cannot apply momentum theory

· $V_c < -2v_i$



· conservation of mass $\dot{m} = \rho(V_c + w)A_\infty = \rho(V_c + v_i)A$

· — momentum $T = \dot{m}V_c - \dot{m}(V_c + w) = -\dot{m}w$

· — energy $P = T(V_c + v_i) = -\frac{1}{2}\dot{m}w(2V_c + w)$

$\longrightarrow$ $-\dot{m}w(V_c + v_i) = -\frac{1}{2}\dot{m}(w^2 + 2V_c w)$ $\longrightarrow$ $w = 2v_i$

$\longrightarrow\ T = -\dot{m}w = -\rho A (V_c + v_i)(2v_i) \longrightarrow v_h^2 = \frac{T}{2\rho A} = -(V_c + v_i)v_i$

$\longrightarrow\ \left(\frac{v_i}{v_h}\right)^2 + \frac{V_c}{v_h}\frac{v_i}{v_h} + 1 = 0 \xrightarrow{\frac{V_c}{v_h} \le -2} \frac{v_i}{v_h} = -\frac{V_c}{2v_h} - \sqrt{\left(\frac{V_c}{2v_h}\right)^2 - 1}$

Power requirement

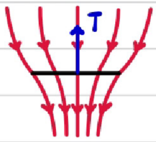
· axial climb $\frac{P}{P_h} = \frac{V_c + v_i}{v_h} = \frac{V_c}{2v_h} + \sqrt{\left(\frac{V_c}{2v_h}\right)^2 + 1}$

- $\xrightarrow{\frac{V_c}{2v_h} \ll 1} \frac{P}{P_h} = 1 + \frac{\Delta P}{P_h} = \frac{V_c}{2v_h} + 1$
- $\xrightarrow{\frac{V_c}{2v_h} \gg 1} \frac{P}{P_h} \approx \frac{V_c}{v_h} \longrightarrow P_c \approx TV_c$

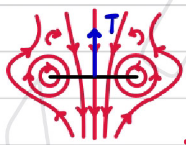
· axial descent $\frac{P}{P_h} = \frac{V_c + v_i}{v_h} = \frac{V_c}{2v_h} - \sqrt{\left(\frac{V_c}{2v_h}\right)^2 - 1}$

Working states

· normal working state $V_c > 0$

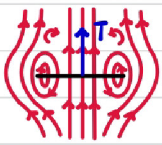


· vortex ring state $-2v_h < V_c < 0$

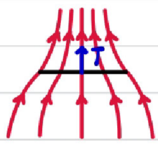


20-50% load flucturation

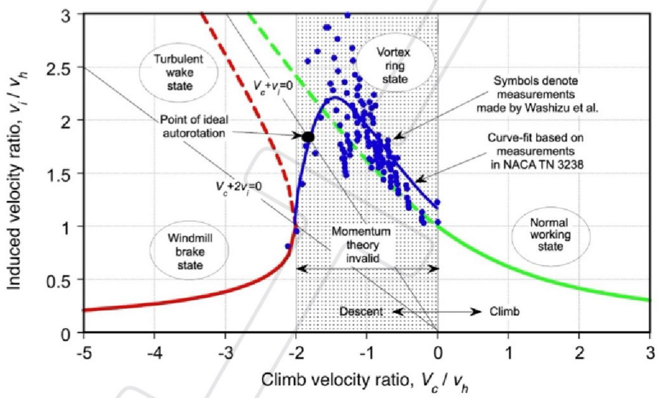
· turbulent wake state $-2v_i < V_c < -2v_h$

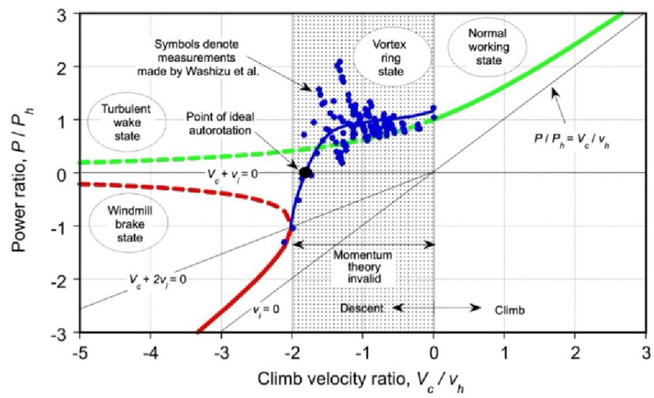


· windmill 风车 brake state $V_c < -2v_i$



extract energy from the flow





Credit: J.G Leishman

Autorotation recover safe flight in the event of mechanical failure

· ideal $\frac{P}{P_h} = 0 \longrightarrow P = T(V_c + v_i) = 0 \longrightarrow \frac{V_c}{v_h} = -1.75$

· actual profile losses $\longrightarrow P = T(V_c + v_i) + P_0 = 0 \longrightarrow \frac{V_c}{v_h} \approx -1.85$

Coaxial rotors

· ideal assume $T = \frac{W}{2}$ and infinitesimal seperation $\longrightarrow$ $T_{total} = 2T$

$\longrightarrow$ $v_h = \sqrt{\frac{T}{\rho A}}$ and $P_{i,total} = \frac{(2T)^{\frac{3}{2}}}{\sqrt{2\rho A}} = \sqrt{2}\left(2\frac{T^{\frac{3}{2}}}{\sqrt{2\rho A}}\right)$ $\longrightarrow$ $K_{int} = \sqrt{2}$

· experimental $K_{int} \approx 1.16$ $\longrightarrow$ momentum theory overprediction

$\longrightarrow$ assume lower motor operates at the vena contracta 收缩流束 of the upper rotor, not affecting upper rotor and have constant induced velocity

· simple approach $T_l = T_u = \frac{W}{2}$ $\longrightarrow$ $K_{int} \approx 1.281$

· accurate approach $Q_l = Q_u$ $\longrightarrow$ $K_{int} \approx 1.219$

Tip Losses

Accounting for tip losses

reduction in effective disk area $A_e = \pi R_e^2 = \pi (BR)^2 = B^2 A$

· Prantl's tip loss factor $B = 1 - \frac{1.386\lambda}{N_b}$

· Gessow & Myer's tip loss factor $B = 1 - \frac{c}{2R}$

Forward Flight

Non-dimensional terms US convention

· advance ratio $\mu = \frac{V_\infty \cos\alpha_s}{\Omega R}$

· inflow ratio $\lambda = \frac{V_\infty \sin\alpha_s + v_i}{\Omega R} = \mu \tan\alpha_s + \lambda_i$

Momentum theory

$T = \dot{m}w = 2\dot{m}v_i = 2\rho A v_i \sqrt{(V_\infty \cos\alpha_s)^2 + (V_\infty \sin\alpha_s + v_i)^2} \xrightarrow{\text{high speed, } V_\infty \gg v_i} 2\rho A v_i V_\infty$

$\longrightarrow$ $v_h^2 = \frac{T}{2\rho A} = v_i \sqrt{(V_\infty \cos\alpha_s)^2 + (V_\infty \sin\alpha_s + v_i)^2}$

$\longrightarrow$ $\lambda_h^2 = \lambda_i^2\sqrt{\mu^2 + \lambda^2}$ $\longrightarrow$ $\lambda_i = \frac{\lambda_h^2}{\sqrt{\mu^2 + \lambda^2}} = \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}}$ and $\lambda = \mu \tan\alpha_s + \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}}$

· at $\alpha_s = 0$, $\lambda = \lambda_i$ $\longrightarrow$ $\lambda_i = \frac{\lambda_h^2}{\sqrt{\mu^2 + \lambda_i^2}}$ $\longrightarrow$ $\lambda_i^4 + \lambda_i^2\mu^2 = \lambda_h^4$ $\longrightarrow$ $\left(\frac{\lambda_i}{\lambda_h}\right)^4 + \left(\frac{\mu}{\lambda_h}\right)^2\left(\frac{\lambda_i}{\lambda_h}\right)^2 - 1 = 0$

$\longrightarrow$ $\frac{\lambda_i}{\lambda_h} = \sqrt{-\frac{1}{2}\left(\frac{\mu}{\lambda_h}\right)^2 + \sqrt{\frac{1}{4}\left(\frac{\mu}{\lambda_h}\right)^4 + 1}}$

· at $\alpha_s = 0$, highspeed $\longrightarrow$ $\lambda_i \approx \frac{\lambda_h^2}{\mu} = \frac{C_T}{2\mu}$

· fixed point method $\lambda_0 = \lambda_h = \sqrt{\frac{C_T}{2}}$ $\longrightarrow$ $\lambda_{n+1} = \mu \tan\alpha_s + \frac{C_T}{2\sqrt{\mu^2 + \lambda_n^2}}$ where $\varepsilon = \left\| \frac{\lambda_{n+1} - \lambda_n}{\lambda_{n+1}} \right\|$

· Newton-Rapson method $\lambda_0 = \lambda_h = \sqrt{\frac{C_T}{2}}$ $\longrightarrow$ $\lambda_{n+1} = \lambda_n - \frac{f(\lambda_n)}{f'(\lambda_n)}$ where $\varepsilon = \left\| \frac{\lambda_{n+1} - \lambda_n}{\lambda_{n+1}} \right\|$

$\longrightarrow$ $\lambda_{n+1} = \lambda_n - \frac{\lambda - \mu\tan\alpha_s - C_T(2\sqrt{\mu^2 + \lambda_n^2})^{-1}}{1 + C_T[2\lambda_n(\mu^2 + \lambda_n^2)^{\frac{3}{2}}]^{-1}}$

Rotor-wing analogy

assume mean downwash were uniform over the disk and wake lay in plane with rotor disk $\longrightarrow$ $\alpha_s = 0$

$\longrightarrow$ liken the rotor to an ellipitical wing $\longrightarrow$ $\tan\alpha_i = \frac{v_i}{V_\infty} = \frac{\lambda_i}{\mu} \approx \alpha_i$

$\longrightarrow$ $AR = \frac{b^2}{A} = \frac{(2R)^2}{\pi R^2} = \frac{4}{\pi}$ and $C_L = \frac{T}{\frac{1}{2}\rho(\mu^2\Omega^2R^2)^2 A} = \frac{2C_T}{\mu^2}$

$\longrightarrow$ $\alpha_i = \frac{C_L}{\pi AR} = \frac{C_T}{2\mu^2}$ $\longrightarrow$ $\frac{\lambda_i}{\mu} = \frac{C_T}{2\mu^2}$ $\longrightarrow$ $\lambda_i = \frac{C_T}{2\mu}$

Rotor power

$P = T(V_\infty \sin\alpha_s + v_i)$ $\longrightarrow$ $\frac{P}{P_h} = \frac{T(V_\infty \sin\alpha_s + v_i)}{T v_h}$

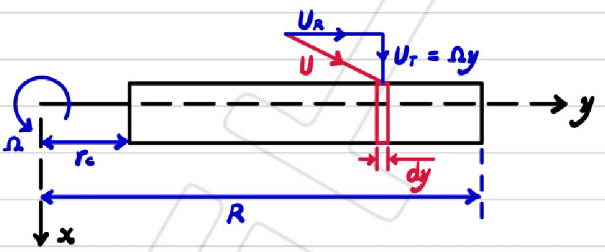
$\longrightarrow$ $\frac{P}{P_h} = \frac{\lambda}{\lambda_h} = \frac{\mu\tan\alpha_s}{\lambda_h} + \frac{\lambda_h}{\sqrt{\mu^2 + \lambda^2}}$

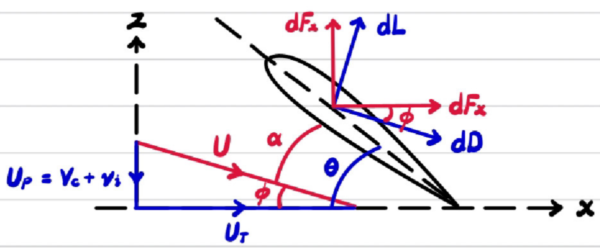
$\xrightarrow{\tan\alpha_s = \frac{D}{W} \approx \frac{D}{T}}$ $\frac{P}{P_h} \approx \frac{\mu}{\lambda_h}\frac{D}{T} + \frac{\lambda_h}{\sqrt{\mu^2 + \lambda^2}} = P_{propulsive} + P_{induced} = P_{parasitic} + P_{induced}$

BEMT (Blade Element Momentum Theory)

BET (Blade Element Theory)

Axis conventions





Assumptions

· $U_P \ll U_T$ and neglect U_R $\longrightarrow$ $U \approx U_T$

· $\phi \ll 1$ $\longrightarrow$ $\phi = \tan^{-1}(\frac{U_P}{U_T}) \approx \frac{U_P}{U_T}$ $\longrightarrow$ $\alpha = \theta - \frac{U_P}{U_T}$

Loads

- $dL = \frac{1}{2}\rho(\Omega y)^2 c C_l \, dy$ • $dD = \frac{1}{2}\rho(\Omega y)^2 c C_d \, dy$

- $dF_x = dL\sin\phi + dD\cos\phi \approx dL\phi + dD$ • $dF_z = dL\cos\phi - dD\sin\phi \approx dL$

- $dT = N_b dF_z \approx N_b dL$

- $dQ = N_b (dF_x y) \approx N_b (dL\phi + dD) y$

- $dP = \Omega dQ \approx N_b (dL\phi + dD)\Omega y$

Non-dimensional terms

- distance from root $\quad r = \frac{y}{R}$

- inflow ratio $\quad \lambda = \frac{V_c + v_i}{V_{tip}} = \frac{V_c + v_i}{\Omega R} = \frac{V_c + v_i}{\Omega y} r = \frac{U_P}{U_T} r = \phi r$

- thrust coefficient $\quad dC_T = \frac{N_b \, dL}{\rho A \Omega^2 R^2} = \frac{\frac{1}{2} N_b \rho (\Omega y)^2 c C_l \, dy}{\rho \pi R^2 \Omega^2 R^2} = \frac{1}{2}\frac{N_b c}{\pi R}\left(\frac{y}{R}\right)^2 C_l \frac{dy}{R} = \frac{1}{2}\sigma C_l r^2 dr$

$$\longrightarrow \quad C_T = \frac{1}{2}\sigma \int_0^1 C_l r^2 \, dr$$

- power coefficient $\quad dC_P = \frac{N_b (dL\phi + dD)\Omega y}{\rho A \Omega^3 R^3} = \frac{N_b (dL\phi + dD)}{\rho A \Omega^2 R^2}\frac{y}{R} = \frac{1}{2}\sigma (C_l \phi r^3 + C_d r^3) dr$

$$\longrightarrow \quad C_P = \frac{1}{2}\sigma \int_0^1 (\lambda C_l r^2 + C_d r^3) \, dr$$

- torque coefficient $\quad C_Q = C_P$

Thrust approximations

linearised aerodynamics $\quad C_l = C_{l\alpha}(\alpha - \alpha_0) = C_{l\alpha}(\theta - \phi - \alpha_0) \xrightarrow{\text{symmetric airfoil}} C_l = C_{l\alpha}(\theta - \phi)$

$$\longrightarrow \quad C_T = \frac{1}{2}\sigma \int_0^1 C_{l\alpha}(\theta - \phi) r^2 \, dr = \frac{1}{2}\sigma C_{l\alpha} \int_0^1 (\theta r^2 - \lambda r) \, dr$$

- untwisted blade $\theta = \theta_0$ in uniform inflow $\lambda = \text{constant}$

$$C_T = \frac{1}{2}\sigma C_{l\alpha} \int_0^1 (\theta_0 r^2 - \lambda r) \, dr = \frac{1}{2}\sigma C_{l\alpha}\left[\frac{1}{3}\theta_0 r^3 - \frac{1}{2}\lambda r^2\right]_0^1 dr = \frac{1}{2}\sigma C_{l\alpha}\left(\frac{1}{3}\theta_0 - \frac{1}{2}\lambda\right)$$

$$\xrightarrow{\text{at hover, } \lambda = \frac{v_h}{\Omega R} = \sqrt{\frac{T}{2\rho A}}\frac{1}{\Omega R} = \sqrt{\frac{C_T \Omega^2 R^2}{2}}\frac{1}{\Omega R} = \sqrt{\frac{C_T}{2}}} \quad C_T = \frac{1}{2}\sigma C_{l\alpha}\left(\frac{1}{3}\theta_0 - \frac{1}{2}\sqrt{\frac{C_T}{2}}\right) \quad \text{solve iteratively}$$

$$\longrightarrow \quad \theta_0 = \frac{6C_T}{\sigma C_{l\alpha}} + \frac{3}{2}\sqrt{\frac{C_T}{2}}$$

- linear twisted blade $\theta(r) = \theta_0 + r\theta_{tw}$ in uniform inflow $\lambda = \text{constant}$

$$C_T = \frac{1}{2}\sigma C_{l\alpha}\int_0^1 [(\theta_0 + r\theta_{tw})r^2 - \lambda r]\,dr = \frac{1}{2}\sigma C_{l\alpha}\left(\frac{1}{3}\theta_0 + \frac{1}{4}\theta_{tw} - \frac{1}{2}\lambda\right)$$

· linear twisted blade $\theta(r) = \theta_{0.75} + (r - 0.75)\theta_{tw}$ in uniform inflow λ = constant

$$C_T = \frac{1}{2}\sigma C_{l\alpha}\int_0^1 \{[\theta_{0.75} + (r-0.75)\theta_{tw}]\,r^2 - \lambda r\}\,dr = \frac{1}{2}\sigma C_{l\alpha}\left(\frac{1}{3}\theta_{0.75} - \frac{1}{2}\lambda\right)$$

Power approximations

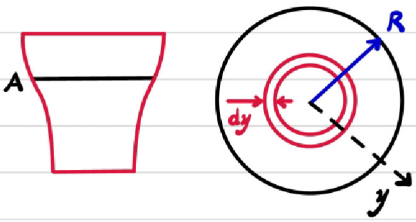
$$C_Q = C_P = \frac{1}{2}\sigma\int_0^1 (\lambda C_l r^2 + C_d r^3)\,dr$$

· constant sectional drag coefficient C_d = constant in uniform inflow λ = constant

$$C_Q = C_P = \lambda C_T + \frac{1}{8}\sigma C_d \xrightarrow{\text{at hover, } \lambda = \sqrt{\frac{C_T}{2}}} \frac{C_T^{\frac{3}{2}}}{\sqrt{2}} + \frac{1}{8}\sigma C_d$$

Axial Flight

Annular momentum theory (Froude)



· conservation of mass $d\dot{m} = \rho(V_c + v_i)\,dA = \rho(V_c + v_i)(2\pi y\,dy)$

· — momentum $dT = d\dot{m}w = \rho(V_c + v_i)(2\pi y\,dy)(2v_i)$

· — energy $dP = dT(V_c + v_i) = \rho(V_c + v_i)^2(2\pi y\,dy)(2v_i)$

$$\xrightarrow{\lambda_c = \frac{V_c}{\Omega R} \text{ and } \lambda_i = \frac{v_i}{\Omega R}} dC_T = 4\lambda\lambda_i r\,dr = 4\lambda(\lambda - \lambda_c)r\,dr$$

$$\longrightarrow dC_P = dC_Q = 4\lambda^2\lambda_i r\,dr = 4\lambda^2(\lambda - \lambda_c)r\,dr$$

· hover $\lambda_c = 0$ in uniform inflow λ = constant

$$C_T = 4\lambda^2\int_0^1 r\,dr = 2\lambda^2 \quad \text{and} \quad C_P = 4\lambda^3\int_0^1 r\,dr = 2\lambda^3$$

· hover $\lambda_c = 0$ in non-uniform inflow $\lambda(r) = \lambda_{tip} r^n$ for $n \geq 0$

$$C_T = 4\lambda_{tip}^2\int_0^1 r^{2n+1}\,dr = \frac{2\lambda_{tip}^2}{n+1} \longrightarrow \lambda_{tip} = \sqrt{n+1}\sqrt{\frac{C_T}{2}}$$

$$C_P = 4\lambda_{tip}^3\int_0^1 r^{3n+1}\,dr = \frac{4\lambda_{tip}^3}{3n+2} = \frac{2(n+1)^{\frac{3}{2}}}{3n+2}\frac{C_T^{\frac{3}{2}}}{\sqrt{2}} \longrightarrow \kappa = \frac{2(n+1)^{\frac{3}{2}}}{3n+2} \geq 1$$

$\longrightarrow \kappa = 1$ when $n = 0$ $\longrightarrow$ minimum induced power when uniform inflow λ = constant

BEMT

$$dC_T = \frac{1}{2}\sigma C_{l\alpha}(\theta r^2 - \lambda r)\,dr = 4\lambda(\lambda - \lambda_c)r\,dr \longrightarrow \lambda^2 + \left(\frac{\sigma C_{l\alpha}}{8} - \lambda_c\right)\lambda - \frac{\sigma C_{l\alpha}}{8}\theta r$$

$$\longrightarrow \lambda(r) = \sqrt{\left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)^2 + \left(\frac{\sigma C_{l\alpha}}{8}\right)\theta r} - \left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)$$

· hover $\lambda_c = 0$

$$\lambda(r) = \sqrt{\left(\frac{\sigma C_{l\alpha}}{16}\right)^2 + \left(\frac{\sigma C_{l\alpha}}{8}\right)\theta r} - \frac{\sigma C_{l\alpha}}{16} = \frac{\sigma C_{l\alpha}}{16}\left[\sqrt{1 + \left(\frac{32}{\sigma C_{l\alpha}}\right)\theta r} - 1\right]$$

· hover $\lambda_c = 0$ for minimum power λ = constant

θr = constant $\longrightarrow$ $\theta(r) = \frac{\theta_{tip}}{r}$ ideal twist distribution*

* helicopter always use linear twist $\theta(r) = \theta_0 + r\theta_{tw}$ for forward flight consideration

Ideal twist performance in hover

$$C_T = \frac{1}{2}\sigma C_{l\alpha}\int_0^1 (\theta_{tip} - \lambda) r\,dr = \frac{1}{4}\sigma C_{l\alpha}(\theta_{tip} - \lambda) \longrightarrow \theta_{tip} = \frac{4C_T}{\sigma C_{l\alpha}} + \lambda = \frac{4C_T}{\sigma C_{l\alpha}} + \sqrt{\frac{C_T}{2}}$$

$$\xrightarrow{\lambda = \text{constant} = \phi r = \phi_{tip}} C_T = \frac{1}{4}\sigma C_{l\alpha}(\theta_{tip} - \phi_{tip}) = \frac{1}{4}\sigma C_{l\,tip}$$

The optimum hovering motor

· minimise induce power λ = constant and $\theta = \frac{\theta_{tip}}{r}$ $\longrightarrow$ $C_{P_i} = \frac{C_T^{\frac{3}{2}}}{\sqrt{2}}$

· minimise profile power α_1 = constant for $\left(\frac{C_l}{C_d}\right)_{max}$

$$\longrightarrow dC_T = \frac{1}{2}\sigma C_{l\alpha}(\theta_{tip} - \lambda) r\,dr = \frac{1}{2}\sigma C_{l\alpha}\alpha_1 r^2\,dr \longrightarrow \frac{1}{2}\sigma C_{l\alpha}\alpha_1 r^2\,dr = 4\lambda^2 r\,dr$$

$$\longrightarrow \lambda = \sqrt{\frac{1}{8}\sigma r C_{l\alpha}\alpha_1} = \text{constant} \longrightarrow \sigma r = \frac{N_b c}{\pi R} r = \text{constant} \longrightarrow \sigma(r) = \frac{\sigma_{tip}}{r} \text{ and } c(r) = \frac{c_{tip}}{r}$$

$$\longrightarrow \theta = \alpha + \phi = \alpha_1 + \frac{\lambda}{r} = \alpha_1 + \frac{1}{r}\sqrt{\frac{1}{8}\sigma r C_{l\alpha}\alpha_1} = \alpha_1 + \frac{1}{r}\sqrt{\frac{1}{8}\sigma_{tip} C_{l\alpha}\alpha_1}$$

$$\longrightarrow C_T = \frac{1}{2}C_{l\alpha}\alpha_1\int_0^1 \frac{\sigma_{tip}}{r} r^2\,dr = \frac{1}{4}\sigma_{tip} C_{l\alpha}\alpha_1$$

Power estimation

$$C_P = C_{P_i} + C_{P_0} = \int_0^1 \lambda\,dC_T\,dr + \frac{1}{2}\sigma\int_0^1 C_d r^3\,dr \quad \text{where} \quad C_d = C_{d_0} + d_1\alpha + d_2\alpha^2$$

Equivalent weight solidity

for non-rectangular blade, local solidity varies along the span

$\longrightarrow$ weight solidity $\sigma = \int_0^1 \sigma(r)\,dr$ = solidity of equivalent rectangular blade

· thrust weighted solidity most common

$$C_T = \frac{1}{2}\int_0^1 \sigma(r) C_l r^2\,dr = \frac{1}{2}\sigma_e\int_0^1 C_l r^2\,dr \longrightarrow \sigma_e = 3\int_0^1 \sigma(r) r^2\,dr$$

$\xrightarrow{\text{the optimum hovering motor}}$ $\sigma_e = 3\int_0^1 \frac{\sigma_{tip}}{r} r^2 dr = \frac{3}{2}\sigma_{tip} \longrightarrow C_{P_0} = \frac{1}{6}\sigma_{tip} C_{d_0} = \frac{1}{9}\sigma_e C_{d_0} < \frac{1}{8}\sigma_{rect} C_{d_0}$

$\xrightarrow{\text{linear taper}}$ $\sigma_e = 3\int [\sigma_{root} + (\sigma_{tip} - \sigma_{root}) r] r^2 dr = \sigma_{root} + \frac{3}{4}(\sigma_{tip} - \sigma_{root}) = \sigma_{0.75}$

· power weighted solidity

$$C_P = \int_0^1 \lambda dC_T + \frac{1}{2}\int_0^1 \sigma(r) C_d r^3 dr = \int_0^1 \lambda dC_T + \frac{1}{2}\sigma_e \int_0^1 C_d r^3 dr \longrightarrow \sigma_e = 4\int_0^1 \sigma(r) r^2 dr$$

Tip Losses

BET

· reduce effective radius $R_e = BR \longrightarrow C_T = \frac{1}{2}\sigma C_{l\alpha} \int_0^B (\theta r^2 - \lambda r) dr$

· increase effective inflow $\lambda_e = \frac{\lambda}{B} \longrightarrow C_T = \frac{1}{2}\sigma C_{l\alpha} \int_0^1 (\theta r^2 - \frac{\lambda}{B} r) dr$

BEMT

Prandtl's tip loss function $F = \frac{2}{\pi}\cos^{-1}(e^{-f})$ where $f = \frac{N_b}{2}\left(\frac{1-r}{r\phi}\right)$

$\longrightarrow dC_T = 4F\lambda(\lambda - \lambda_c) r\, dr$

Forward Flight

Complications

· compressibility effects · unsteady effects · non-linear aerodynamics · possibility of stall

· reverse flow · complex induced velocity from the rotor wake

Flapping motion

$$\beta(r, \psi) = \beta_0 + \sum_{n=1}^{N} (\beta_{nc}\cos n\psi + \beta_{ns}\sin n\psi)$$

$\xrightarrow{\text{ignoring higher harmonics}}$ $\beta(r,\psi) = \beta_0 + \beta_{1c}\cos\psi + \beta_{1s}\sin\psi \longrightarrow \dot{\beta} = \Omega \mathring{\beta} = \Omega(-\beta_{1c}\sin\psi + \beta_{1s}\cos\psi)$

Periodic pitch variation

$$\theta(r,\psi) = \theta_{in\text{-}built} + \theta_{elastic} + \theta_{collective} + \theta_{cyclic}$$

$\longrightarrow \theta(r,\psi) = \theta_{tw}(r) + \theta_{el}(r,\psi) + \theta_0 + \theta_{1s}\sin\psi + \theta_{1c}\cos\psi$

BET

- $U_T(y, \psi) = \Omega y + V_\infty \sin\psi = \Omega y + \mu\Omega R \sin\psi$

- $U_P(y, \psi) = V_c + v_i + y\dot{\beta} + V_\infty \beta\cos\psi = (\lambda_c + \lambda_i)\Omega R + y\dot{\beta} + \mu\Omega R\beta\cos\psi$

- $U_R(y, \psi) = V_\infty \cos\psi = \mu\Omega R \cos\psi$

- $dF_z \approx dL = \frac{1}{2}\rho U^2 c C_l dy \approx \frac{1}{2}\rho U_T^2 c C_{l\alpha}(\theta - \phi) dy \approx \frac{1}{2}\rho U_T^2 c C_{l\alpha}(\theta - \frac{U_P}{U_T}) dy = \frac{1}{2}\rho c C_{l\alpha}(\theta U_T^2 - U_P U_T) dy$

- $dF_x \approx dL\phi + dD \approx \frac{1}{2}\rho c C_{l\alpha}(\theta U_P U_T - U_P^2) dy + \frac{1}{2}\rho U_T^2 c C_d dy = \frac{1}{2}\rho c C_{l\alpha}(\theta U_P U_T - U_P^2 + \frac{C_d}{C_{l\alpha}} U_T^2) dy$

- $dF_r = -\beta dF_z + dD_{radial}$ act outward

$\longrightarrow$ $T = \frac{N_b}{2\pi}\int_0^{2\pi}\int_0^R dF_z \, d\psi = \frac{N_b R}{4\pi}\rho c C_{l\alpha}\int_0^{2\pi}\int_0^1 \theta U_T^2 - U_P U_T \, dr \, d\psi$

$\longrightarrow$ $H = \frac{N_b}{2\pi}\int_0^{2\pi}\int_0^R dF_x \sin\psi + dF_r \cos\psi \, d\psi$

$\longrightarrow$ $Y = \frac{N_b}{2\pi}\int_0^{2\pi}\int_0^R -dF_x \cos\psi + dF_r \sin\psi \, d\psi$ to advancing side

$\longrightarrow$ $Q = \frac{N_b}{2\pi}\int_0^{2\pi}\int_0^R dF_x r \, d\psi = \frac{N_b R}{4\pi}\rho c C_{l\alpha}\int_0^{2\pi}\int_0^1 (\theta U_P U_T - U_P^2 + \frac{C_d}{C_{l\alpha}} U_T^2) r \, dr \, d\psi$

$\longrightarrow$ $M_x = \frac{N_b}{2\pi}\int_0^{2\pi}\int_0^R dF_z r \sin\psi \, d\psi$

$\longrightarrow$ $M_y = \frac{N_b}{2\pi}\int_0^{2\pi}\int_0^R dF_z r \cos\psi \, d\psi$

Simplify expression for C_T

- small angle · rigid blade $\longrightarrow$ $\theta_{el} = 0$

- simplified air characteristic $\longrightarrow$ ignore stall, compressibility effect, U_R, ...

- neglect reverse flow · no elastic β, ζ or θ DoF · no higher harmonic term

- constant inflow $\longrightarrow$ satisfactory for high μ but introduce errors in flapping motion calculation

- no tip loss · no root cutout

$$C_T = \frac{T}{\rho A (\Omega R)^2} = \frac{1}{\rho A (\Omega R)^2}\frac{N_b R}{4\pi}\rho c C_{l\alpha}\int_0^{2\pi}\int_0^1 (\Omega R)^2 [\theta (r + \mu\sin\psi)^2 - (\lambda_c + \lambda_i + \frac{r}{\Omega}\dot{\beta} + \mu\beta\cos\psi \times r + \mu\sin\psi)] \, dr \, d\psi$$

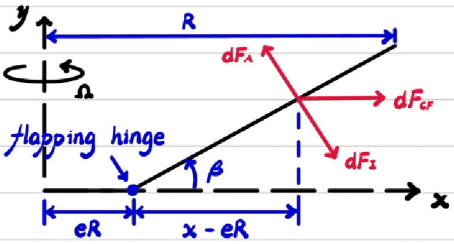
$$\longrightarrow C_T = \frac{6C_{l\alpha}}{4\pi}\int_0^{2\pi}\int_0^1 (\theta_0 + \theta_{tw} r + \theta_{1c}\cos\psi + \theta_{1s}\sin\psi \times r + \mu\sin\psi)^2 - (\lambda_0 + r\mathring{\beta} + \mu\beta\cos\psi \times r + \mu\sin\psi) \, dr \, d\psi$$

$$\longrightarrow C_T = \frac{6C_{l\alpha}}{2}\left[\frac{\theta_0}{3}(1 + \frac{3}{2}\mu^2) + \frac{\theta_{tw}}{4}(1 + \mu^2) + \frac{\mu}{2}\theta_{1s} - \frac{\lambda_0}{2}\right]$$

Rotor Dynamics

Flapping Dynamics

Fundamental flap mode

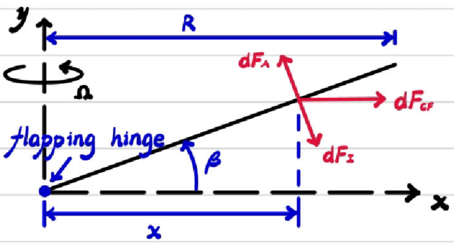


balance of moment about flapping hinge

$\longrightarrow$ external moments = inertial moments

$\longrightarrow$ $\Sigma M_{ext} = I_b \ddot{\beta}$

Assumptions



- hinge locate at origin $e = 0$
- rigid blade undergoes pure flapping
- Ω = constant $\longrightarrow$ F_{CF} = constant
- uniform mass distribution
- gravity forces are negligible
- $F_{CF} \gg F_A$ $\longrightarrow$ β is small

Blade element formulation

$\Sigma M_{ext} = I_b \ddot{\beta} \longrightarrow M_A + M_{CF} + M_I = 0$ in clockwise about hinge

- $dM_A = -dF_A \cdot x = -dL\,x \longrightarrow M_A = -\int_0^R Lx\,dx$
- $dM_{CF} = dF_{CF} \cdot \beta \cdot x = (m\,dx \cdot \Omega^2 x)\,\beta x = m\Omega^2 x^2 \beta\,dx \longrightarrow M_{CF} = \int_0^R \Omega^2 \beta m x^2\,dx = \frac{1}{3} m \Omega^2 \beta R^3$
- $dM_I = dF_I \cdot x = (m\,dx \cdot \ddot{\beta} x)\,x = m x^2 \ddot{\beta}\,dx \longrightarrow M_I = \int_0^R \ddot{\beta} m x^2\,dx = \frac{1}{3} m \ddot{\beta} R^3$

Coning angle

blade reach equilibrium when $\Sigma M_{ext} = 0 \longrightarrow M_A + M_{CF} = 0$

$$\longrightarrow -\int_0^R Lx\,dx + \frac{1}{3} m \Omega^2 \beta_0 R^3 = 0 \longrightarrow \beta_0 = \frac{3\int_0^R Lx\,dx}{m\Omega^2 R^3} \propto \frac{F_A}{F_{CF}} \xrightarrow{F_{CF} = \text{constant}} \beta_0 \propto F_A$$

large β_0 · increase stress on the structure · reduce A_{eff} $\longrightarrow$ reduce lift production

Flapping equation of motion in hover

$$-\int_0^R Lx\,dx + \Omega^2 \beta \int_0^R m x^2\,dx + \ddot{\beta} \int_0^R m x^2\,dx = 0 \xrightarrow{I_b = \int_0^R m x^2\,dx} (\Omega^2 \beta + \ddot{\beta})\, I_b = \int_0^R Lx\,dx$$

$$\xrightarrow{\mathring{\beta} = \frac{\partial \beta}{\partial \psi} = \frac{\partial \beta}{\partial t}\frac{\partial t}{\partial \psi} = \frac{\dot{\beta}}{\Omega}} (\Omega^2 \beta + \Omega^2 \overset{\circ\circ}{\beta})\, I_b = \int_0^R Lx\,dx \longrightarrow \beta + \overset{\circ\circ}{\beta} = \frac{1}{I_b \Omega^2} \int_0^R Lx\,dx$$

$\cdot \int_0^R Lx\,dx = \int_0^R (\frac12\rho U_T^2 c C_l) x\,dx \xrightarrow{\text{linearised aerodynamics}} \int_0^R [\frac12 \rho U_T^2 c C_{l\alpha}(\theta-\phi)]\, x\,dx$

$\xrightarrow{\text{rectangular blade}} \frac12\rho c C_{l\alpha}\int_0^R U_T^2(\theta - \frac{U_P}{U_T})\, x\,dx = \frac12 \rho c C_{l\alpha}\int_0^R (U_T^2\theta - U_P U_T)\, x\,dx$

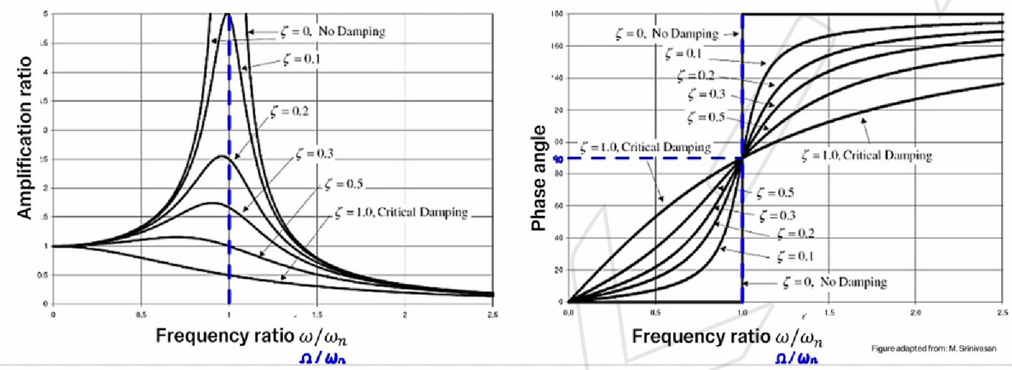
$\longrightarrow \frac12\rho c C_{l\alpha}\int_0^R[\Omega^2x^2\theta - (\dot\beta x + v_i)(\Omega x)]\,x\,dx = \frac12\rho c\Omega^2 C_{l\alpha}\int_0^R \theta\Omega^2 x^3 - \Omega\dot\beta x^3 - \Omega v_i x^2\,dx$

$\longrightarrow \frac18\rho c C_{l\alpha}\Omega^2R^4(\theta - \frac{\dot\beta}{\Omega} - \frac{4v_i}{3\Omega R}) \longrightarrow \frac18 \rho c C_{l\alpha}\Omega^2R^4(\theta-\frac{\dot\beta}{\Omega}-\frac43\lambda_i)$

$\longrightarrow \beta + \overset{**}{\beta} = \gamma\bar M_\beta$ where lock number $\gamma = \frac{\rho c C_{l\alpha} R^4}{I_b} \propto \frac{F_A}{F_I}$ (typically 5 ~ 12) and $\bar M_\beta = \frac18(\theta - \overset{*}{\beta} - \frac43\lambda_i)$

$\longrightarrow$ mass-spring-damper system with aerodynamic damping $\frac{c}{m} = 2\omega_{n,\beta}\xi = \frac18\gamma$

$\longrightarrow \omega_{n,\beta} = 1(\psi) = \Omega(t)$ and $\xi = \frac{\gamma}{16}(\psi)$ (typically 0.5 ~ 0.7 in hover) $= \frac{\gamma}{16\Omega}(t) \longrightarrow$ very well damped



$\Delta F \xrightarrow{90^\circ} \Delta\beta$

$\longrightarrow$ stabilised the flapping motion $\longrightarrow$ allow rotor operate at or near resonance, $\omega_{n,\beta} = \Omega$

$\xrightarrow{\text{harmonic matching}} \beta_0 + \frac{\gamma}{8}(-\beta_{1c}\sin\psi + \beta_{1s}\cos\psi) = \frac18\gamma(\theta_0+\theta_{1c}\cos\psi+\theta_{1s}\sin\psi - \frac43\lambda_i)$

$\longrightarrow$ periodic solution does not cover transients $\longrightarrow$

- $\beta_0 = \frac18\gamma(\theta_0 - \frac43\lambda_i)$
- $\beta_{1c} = -\theta_{1s}$
- $\beta_{1s} = \theta_{1c}$

$\longrightarrow \Delta\beta \xrightarrow{90^\circ} \Delta\theta$

Flapping equation of motion in forward flight

$U_T = \Omega x + V_\infty\sin\psi = \Omega x + \mu\Omega R\sin\psi$ and $U_P = (\lambda_c+\lambda_i)\Omega R + \dot\beta x + \mu\Omega R\beta\cos\psi$

$\xrightarrow{\text{linear twisted blades, } \theta = \theta_0 + \frac{x}{R}\theta_{tw}} \bar M_\beta = \theta(\frac18 + \frac{\mu}{3}\sin\psi + \frac{\mu^2}{4}\sin^2\psi) + \theta_{tw}(\frac1{10} + \frac{\mu}{4}\sin\psi + \frac{\mu^2}{6}\sin^2\psi)$

$- \lambda(\frac16+\frac{\mu}{4}\sin\psi) - \overset{*}{\beta}(\frac18+\frac{\mu}{6}\sin\psi) - \beta\mu\cos\psi(\frac16+\frac{\mu}{4}\sin\psi)$

$\xrightarrow{\text{harmonic matching}} \beta_0 = \gamma\{(\theta_0+\theta_{1c}\cos\psi+\theta_{1s}\sin\psi)(\frac18+\frac{\mu}{3}\sin\psi+\frac{\mu^2}{4}\sin^2\psi) + \theta_{tw}(\frac1{10}+\frac{\mu}{4}\sin\psi+\frac{\mu^2}{6}\sin^2\psi)$

$- \lambda(\frac16+\frac{\mu}{4}\sin\psi) - (-\beta_{1c}\sin\psi + \beta_{1s}\cos\psi)(\frac18+\frac{\mu}{6}\sin\psi)$

$$- (\beta_0 + \beta_{1c}\cos\psi + \beta_{1s}\sin\psi)\mu\cos\psi(\tfrac{1}{6} + \tfrac{\mu}{4}\sin\psi)\}$$

$$\longrightarrow \beta_0 = \gamma\{[\theta_0(\tfrac18 + \tfrac{\mu}{3}\sin\psi + \tfrac{\mu^2}{8}) + \theta_{1c}(\tfrac18\cos\psi + \tfrac{\mu^2}{16}\cos\psi) + \theta_{1s}(\tfrac18\sin\psi + \tfrac{\mu}{6} + \tfrac{3\mu^2}{16}\sin\psi)$$

$$+ \theta_{tw}(\tfrac{1}{10} + \tfrac{\mu}{4}\sin\psi + \tfrac{\mu^2}{12}) - \lambda(\tfrac16 + \tfrac{\mu}{4}\sin\psi) - [-\beta_{1c}(\tfrac18\sin\psi + \tfrac{\mu}{12}) + \beta_{1s}(\tfrac18\cos\psi)]$$

$$- [\beta_0\mu(\tfrac16\cos\psi) + \beta_{1c}\mu(\tfrac{1}{12} + \tfrac{\mu}{16}\sin\psi) + \beta_{1s}\mu(\tfrac{\mu}{16}\cos\psi)]\}$$

where $\sin\psi\cos\psi = \frac12\cancelto{\text{dropped}}{\sin 2\psi} = 0$, $\cos^2\psi = \frac12(1+\cancelto{\text{dropped}}{\cos 2\psi}) = \frac12$, $\sin^2\psi = \frac12(1-\cancelto{\text{dropped}}{\cos 2\psi}) = \frac12$.

$$\cos\psi\sin^2\psi = \tfrac12\sin 2\psi\sin\psi = \tfrac14(\cos\psi - \cancelto{\text{dropped}}{\cos 3\psi}) = \tfrac14\cos\psi.$$

$$\cos^2\psi\sin\psi = \tfrac12\sin 2\psi\cos\psi = \tfrac14(\sin\psi + \cancelto{\text{dropped}}{\sin 3\psi}) = \tfrac14\sin\psi.$$

$$\sin^3\psi = \tfrac12(\sin\psi - \sin\psi\cos 2\psi) = \tfrac12[\sin\psi - \tfrac12(-\sin\psi + \cancelto{\text{dropped}}{\sin 3\psi})] = \tfrac34\sin\psi$$

$\longrightarrow$ periodic solution

$\longrightarrow \beta_0 = \gamma[\frac{\theta_0}{8}(1+\mu^2) + \frac{\theta_{tw}}{10}(1+\frac65\mu^2) - \frac{\lambda}{6} + \frac{\mu}{6}\theta_{1s}] \propto \gamma$ and $\frac{C_T}{6}$

$\longrightarrow \beta_{1s} - \theta_{1c} = -\frac{4\beta_0\mu}{3(1+\frac12\mu^2)}$

$\longrightarrow \beta_{1c} + \theta_{1s} = -\frac{8\mu(\theta_0 - \frac34\lambda + \frac34\mu\theta_{1s} + \frac34\theta_{tw})}{3(1-\frac12\mu^2)}$

$\longrightarrow \propto \mu$ and $\frac{C_T}{6}$

Flapping equation of motion include hinge offset

· $dM_A = -dF_A(x-eR) = -dL(x-eR) \longrightarrow M_A = -\int_{eR}^{R} L(x-eR)\,dx$

· $dM_{cf} = dF_{cf}\beta(x-eR) = (m\,dx)(\Omega^2 x)(x-eR) \longrightarrow M_{cF} = \int_{eR}^{R}\Omega^2\beta m x(x-eR)\,dx \approx \frac13 m\Omega^2\beta R^3(1-\frac32 e)$

· $dM_I = dF_I(x-eR) = (m\,dx)[\ddot\beta(x-eR)](x-eR) \longrightarrow M_I = \int_{eR}^{R}\ddot\beta m(x-eR)^2\,dx \approx \frac13 m\ddot\beta R^3(1-3e)$

$$\longrightarrow \int_{eR}^{R}\ddot\beta m(x-eR)^2\,dx + \int_{eR}^{R}\Omega^2\beta m x(x-eR)\,dx = \int_{eR}^{R} L(x-eR)\,dx$$

$$\xrightarrow{I_b = \int_{eR}^{R} m(x-eR)^2 dx} I_b\ddot\beta + \Omega^2 I_b\left(\frac{\int_{eR}^{R} m(x-eR)^2 + meR(x-eR)\,dx}{I_b}\right)\beta = \int_{eR}^{R} L(x-eR)\,dx$$

$\longrightarrow \overset{**}{\beta} + \nu_\beta^2\beta = \gamma\overline{M_\beta}$ where $\nu_\beta^2 = 1 + \frac{eR\int_{eR}^{R} m(x-eR)\,dx}{I_b} = 1 + \frac{3e}{2(1-3e)} \approx 1 + \frac{3e}{2} = (\frac{\omega_{n,\beta}(t)}{\Omega})^2 = \omega_{n,\beta}(\psi)^2 < 1.1$

$\longrightarrow$ rotor operate near resonance, $\omega_{n,\beta} > \Omega \longrightarrow$ aerodynamic damping $\frac{c}{m} = 2\omega_{n,\beta}\xi = \gamma(\frac18 - \frac{e}{6})$

at hovering, $\overset{**}{\beta} + \frac18\gamma\overset{*}{\beta} + \nu_\beta^2\beta = \frac18\gamma(\theta + \frac45\theta_{tw} - \frac43\lambda_i)$

$\xrightarrow{\text{harmonic matching}}$ periodic solution

$\longrightarrow \beta_0 = \frac{\gamma}{8\nu_\beta^2}(\theta_0 + \frac45\theta_{tw} - \frac43\lambda_i)$

$\longrightarrow \beta_{1c}(\nu_\beta^2 - 1) + \frac{\gamma}{8}\beta_{1s} = \frac{\gamma}{8}\theta_{1c}$

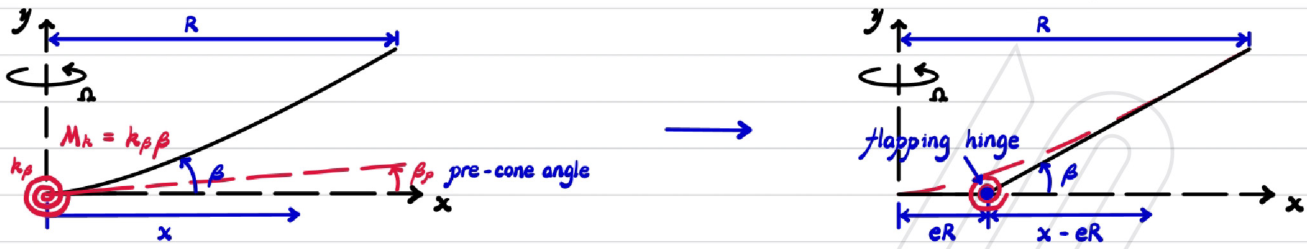
$\longrightarrow \beta_{1s}(\nu_\beta^2 - 1) - \frac{\gamma}{8}\beta_{1c} = \frac{\gamma}{8}\theta_{1s}$

$\longrightarrow \Delta\beta \xrightarrow{<90^\circ} \Delta\theta$

· hinge offset enable part of F_{CF} act on hub $\longrightarrow$ $M_{hub} = -F_{CF}\,eR$

$\longrightarrow$ improve control by increase M_A (increase β)

Flapping equation of motion for hingless or semi-rigid rotors



equivalent hinge offset $R - eR = y_{tip}\dfrac{0.75R - eR}{y_{0.75R}}$ with resisting spring k_β

$\longrightarrow$ $\int_{eR}^{R} \ddot{\beta} m (x-eR)^2\,dx + \int_{eR}^{R} \Omega^2 \beta m x (x-eR)\,dx + k_\beta(\beta - \beta_p) = \int_{eR}^{R} L(x-eR)\,dx$

$\longrightarrow$ $\overset{**}{\beta} + \nu_\beta^2 \beta = \gamma \overline{M_\beta} + \dfrac{\omega_{n\beta}^2}{\Omega^2}\beta_p$ where $\nu_\beta^2 = 1 + \dfrac{eR\int_{eR}^{R} m(x-eR)\,dx}{I_b} + \dfrac{k_\beta}{I_b\Omega^2} \approx 1 + \dfrac{3e}{2} + \dfrac{\omega_{n\beta}^2}{\Omega^2} = \left(\dfrac{\omega_{n,\beta}(t)}{\Omega}\right)^2 = \omega_{n,\beta}(\psi)^2$

$\longrightarrow$ smaller offset, e, needed to obtain same ν_β value

$\longrightarrow$ rotor operate near resonance (off resonance), $\omega_{n,\beta} > \Omega$

· large ν_β $\longrightarrow$ higher $\omega_{n,\beta}$

$\longrightarrow$ higher manoeurvring capabilities and easiler to triming but higher vibration load from rotor

at hovering $\xrightarrow{\text{harmonic matching}}$ periodic solution $\longrightarrow$

$$\beta_0 = \frac{\gamma}{8\nu_\beta^2}\left(\theta_0 - \frac{4}{3}\lambda_i\right) + \frac{\omega_{n,\beta}^2}{\Omega^2}\beta_p$$

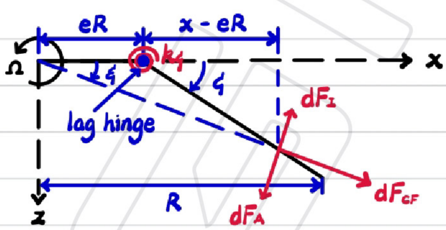
$$\beta_{1s} = \frac{\theta_{1c} + (\nu_\beta^2 - 1)\frac{\gamma}{8}\theta_{1s}}{1 + \left[(\nu_\beta^2 - 1)\frac{\gamma}{8}\right]^2}$$

$$\beta_{1c} = \frac{-\theta_{1s} + (\nu_\beta^2 - 1)\frac{\gamma}{8}\theta_{1c}}{1 + \left[(\nu_\beta^2 - 1)\frac{\gamma}{8}\right]^2}$$

$\beta_{1s}, \beta_{1c} \longrightarrow \Delta\beta \xRightarrow{<90^\circ} \Delta\theta$

Lead-lag Dynamics

Fundamental lag mode



balance of moment about lag hinge

$\longrightarrow$ external moments = inertial moments

$\longrightarrow$ $\Sigma M_{ext} = I_b \ddot{\zeta}$

Assumptions

· small hinge offset $\longrightarrow$ $I_\zeta \approx I_b$ · rigid blade undergoes pure lead-lag

· Ω = constant $\longrightarrow$ F_{CF} = constant · uniform mass distribution

Blade element formulation

$\Sigma M_{ext} = I_b \ddot{\zeta} \longrightarrow M_A + M_{CF} + M_I + M_k = 0$ in anticlockwise about hinge

· $dM_A = -dF_A(x-eR) \approx -dD(x-eR) \longrightarrow M_A = -\int_{eR}^{R} D(x-eR)\,dx$

· $dM_{CF} = dF_{CF}(\zeta-\xi)(x-eR) = (m\,dx)(\Omega^2 x)(\zeta - \zeta\frac{x-eR}{x})(x-eR) = m\Omega^2 \zeta eR(x-eR)\,dx$ where $\zeta - \xi \propto \frac{1}{x}$

$\longrightarrow M_{CF} = \int_{eR}^{R} \Omega^2 \zeta eRm(x-eR)\,dx = \frac{1}{2}m\Omega^2\zeta eR^3(1-e)^2 \approx \frac{1}{2}m\Omega^2\zeta eR^3(1-2e) = 0$ if $e=0$

· $dM_I = dF_I(x-eR) = (m\,dx)[\ddot{\zeta}(x-eR)](x-eR) \longrightarrow M_I = \int_{eR}^{R}\ddot{\zeta}m(x-eR)^2\,dx \approx \frac{1}{3}m\ddot{\zeta}R^3(1-3e)$

· mechanical spring moment $M_k = k_\zeta \zeta$

Lagging equation of motion

$$\int_{eR}^{R}\ddot{\zeta}m(x-eR)^2\,dx + \int_{eR}^{R}\Omega^2\zeta eRm(x-eR)\,dx + k_\beta\beta = \int_{eR}^{R}D(x-eR)\,dx$$

$\longrightarrow \overset{**}{\zeta} + \nu_\zeta^2\zeta = \gamma\overline{M}_\zeta$ where $\nu_\zeta^2 = \frac{eR\int_{eR}^{R}m(x-eR)\,dx}{I_\zeta} + \frac{k_\zeta}{I_\zeta\Omega^2} = \frac{3e}{2(1-3e)} + \frac{\omega_{n\zeta}^2}{\Omega^2} \approx \frac{3e}{2} + \frac{\omega_{n\zeta}^2}{\Omega^2} = \left(\frac{\omega_{n,\zeta}(t)}{\Omega}\right)^2 = \omega_{n,\zeta}(\psi)^2$

$\longrightarrow \omega_{n,\beta}$ typically $0.2 \sim 0.4\Omega$ without $k_\beta \ll \Omega$ $\longrightarrow \omega_{n,\beta}$ typically $0.5 \sim 1.5\Omega$ with k_β

$\xrightarrow{e_\beta = e_\zeta,\ I_b = I_\beta = I_\zeta,\ k_\beta = k_\zeta} \nu_\beta^2 = 1 + \nu_\zeta^2$

Ground resonance

$D \ll L \longrightarrow$ very lightly damped $\longrightarrow$ aeroelastic and aerothermal instabilities e.g. ground resonance

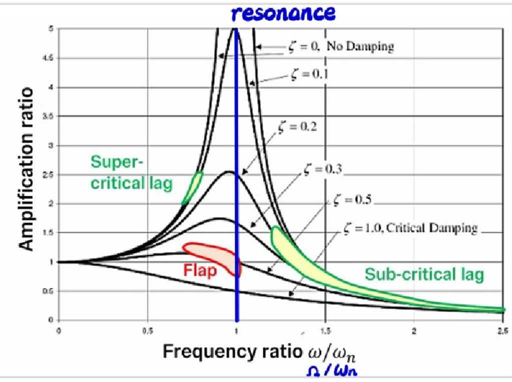
ground resonance occurs when
- $\longrightarrow$ regressive lag mode frequency close to fuselage mode frequency
- $\longrightarrow$ regressive lag mode frequency < rotor speed, Ω

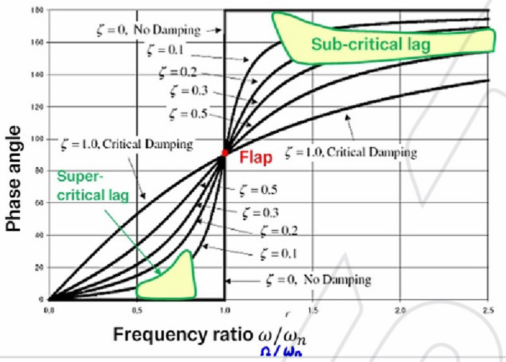
avoid ground resonance (teetering and articulated rotor) and air resonance (hingeless and bearingless rotor)

$\longrightarrow$ non-linear mechanical lag dampers

$\longrightarrow$ artifical damping + supression of aeromechanical phenomena

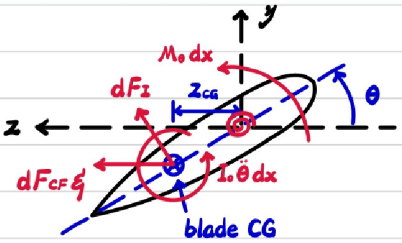
- $\longrightarrow$ sub-critical rotors $\omega < \Omega$ low lag response by high $\frac{\omega}{\omega_n}$ $\longleftrightarrow$ require dampers at resonance
- $\longrightarrow$ super-critical rotors $\omega > \Omega$ high lag response by low $\frac{\omega}{\omega_n}$ $\longleftrightarrow$ do not need dampers





Torsional Dynamics

Fundamental torsion (pitching) mode



balance of moment about pitching axis

$\longrightarrow$ external moments = inertial moments

$\longrightarrow$ $\Sigma M_{ext} = I_b \ddot{q}$

Assumptions

· rigid blade undergoes pure pitching · Ω = constant $\longrightarrow$ F_{CF} = constant · torsional spring at root

Blade element formulation

$\Sigma M_{ext} = I_b \ddot{\theta} \longrightarrow M_A + M_{CF} + M_I + M_K = 0$ in clockwise about root

· $dM_A = -M_\theta \, dx \longrightarrow M_A = -\int_0^R M_\theta \, dx$

· $dM_{CF} = I_\theta \Omega^2 \theta \, dx \longrightarrow M_{CF} = \int_0^R I_\theta \Omega^2 \theta \, dx$

· $dM_I = (m\,dx \times \ddot{\theta} z_{CG}) z_{CG} + I_0 \ddot{\theta} \, dx \longrightarrow M_I = \int_0^R m z_{CG}^2 \ddot{\theta} \, dx + \int_0^R I_0 \ddot{\theta} \, dx$

· mechanical spring moment $M_K = k_\theta (\theta - \theta_{con})$ where θ_{con} is pilot input

Torsion equation of motion

$$\int_0^R I_\theta \Omega^2 \theta \, dx + \int_0^R m z_{CG}^2 \ddot{\theta} \, dx + \int_0^R I_0 \ddot{\theta} \, dx + k_\theta (\theta - \theta_{con}) = \int_0^R M_\theta \, dx$$

$$\xrightarrow{I_f = \int_0^R I_0 \, dx = \int_0^R m z_{CG}^2 + I_0 \, dx} I_f (\ddot{\theta} + \Omega^2 \theta) + k_\theta (\theta - \theta_{con}) = I_f (\ddot{\theta} + \Omega^2 \theta) + \omega_{n\theta}^2 I_f (\theta - \theta_{con}) = \int_0^R M_\theta \, dx$$

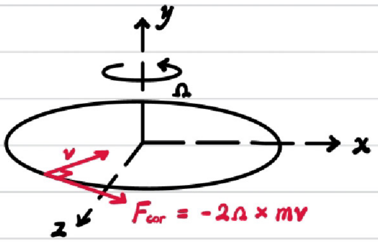
$$\longrightarrow \frac{I_f}{I_b} (\overset{**}{\theta} + \nu_\theta^2 \theta) = \gamma \overline{M_\theta} + \frac{I_f}{I_b} \frac{\omega_{n\theta}^2}{\Omega^2} \theta_{con} \quad \text{where} \quad \nu_\theta^2 = 1 + \frac{\omega_{n\theta}^2}{\Omega^2} = \left(\frac{\omega_{n,\theta}(t)}{\Omega} \right)^2 = \omega_{n,\theta}(\psi)^2$$

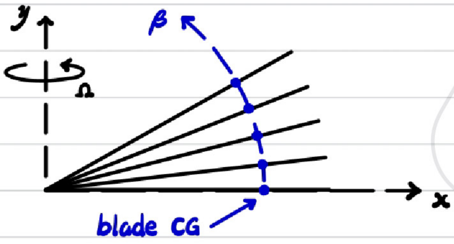
$\longrightarrow$ $\omega_{n,\theta}$ without $k_\theta = \Omega$ $\longrightarrow$ $\omega_{n,\theta}$ typically $5 \sim 10\Omega$ with $k_\theta \gg \Omega$ and $\zeta \approx 0.1$

Coupling Effects

Coupled flap-lag motion

assume $e_\beta \approx e_\zeta$ and high torsional stiffness, k_θ $\longrightarrow$ blade only goes flapping and lead-lag motion

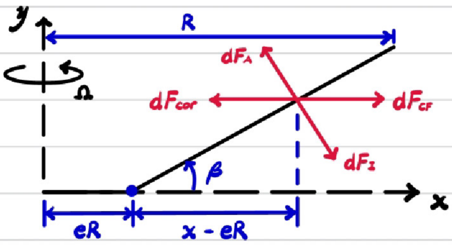




$\beta \uparrow \longrightarrow x_{CG} \downarrow \longrightarrow \Omega \uparrow$

$\longrightarrow F_{cor} \uparrow \longrightarrow \zeta \downarrow$

· flapping equation of motion



$M_A + M_{CF} + M_{cor} + M_I = 0$

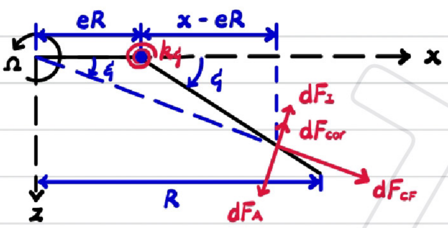
where $dM_{cor} = -dF_{cor}\beta(x - eR)$

and $dF_{cor} = 2\Omega \times mv = -2\Omega(mdx)[\dot{\zeta}(x - eR)]$

$$\longrightarrow \int_{eR}^{R} m\beta\Omega^2 x(x-eR)\,dx + \int_{eR}^{R} m\ddot{\beta}(x-eR)^2\,dx - \int_{eR}^{R} 2m\Omega\dot{\zeta}\beta(x-eR)^2\,dx = \int_{eR}^{R} L(x-eR)\,dx$$

$$\longrightarrow I_b(\ddot{\beta} + \nu_\beta^2\Omega^2\beta - 2\Omega\dot{\zeta}\beta) = \gamma\overline{M}_\beta \longrightarrow \overset{**}{\beta} + \nu_\beta^2\beta - 2\beta\overset{*}{\zeta} = \gamma\overline{M}_\beta$$

· lagging equation of motion



$M_A + M_{CF} + M_{cor} + M_I = 0$

where $dM_{cor} = -dF_{cor}(x - eR)$

and $dF_{cor} = 2\Omega \times mv = -2\Omega(mdx)[\beta\dot{\beta}(x - eR)]$

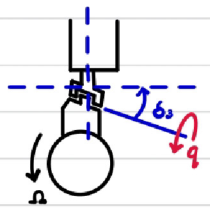
$$\longrightarrow \int_{eR}^{R} m\zeta\Omega^2 eR(x-eR)\,dx + \int_{eR}^{R} m\ddot{\zeta}(x-eR)^2\,dx + \int_{eR}^{R} 2m\Omega\beta\dot{\beta}(x-eR)^2\,dx = \int_{eR}^{R} D(x-eR)\,dx$$

$$\longrightarrow I_b(\ddot{\zeta} + \nu_\zeta^2\Omega^2\zeta - 2\Omega\beta\dot{\beta}) = \gamma\overline{M}_\zeta \longrightarrow \overset{**}{\zeta} + \nu_\zeta^2\zeta + 2\overset{*}{\beta}\beta = \gamma\overline{M}_\zeta$$

$\xrightarrow{M_\zeta \ll M_\beta}$ non-linear Coriolis force in the lagging equation of motion is significant

$\xrightarrow{\text{linear perturbation, } \beta = \beta_0 + \dot{\beta}\Delta t}$ $\overset{**}{\beta} + \nu_\beta^2\beta + 2\beta_0\overset{*}{\zeta} = \gamma\overline{M}_\beta$ and $\overset{**}{\zeta} + \nu_\zeta^2\zeta + 2\overset{*}{\beta}\beta_0 = \gamma\overline{M}_\zeta$

Pitch-flap (δ_3) coupling



δ_3 hinge $\longrightarrow$ $\beta = q\cos\delta_3$ and $\theta = -q\sin\delta_3$ $\longrightarrow$ $\theta = -\tan\delta_3\,\beta$ stabilised helicopter

$\longrightarrow$ $\ddot{\beta} + \nu_\beta^2\beta = \frac{\gamma}{8}(\theta + \Delta\theta - \dot{\beta} - \frac{4}{3}\lambda_i) = \frac{\gamma}{8}(\theta - k_{p\beta}\beta - \dot{\beta} - \frac{4}{3}\lambda_i)$

where $k_{p\beta} = \tan\delta_3$, typically approximately 1 ($\delta_3 \approx 45°$)

$\longrightarrow$ $\ddot{\beta} + \frac{\gamma}{8}\dot{\beta} + (\nu_\beta^2 + \frac{\gamma}{8}\tan\delta_3)\beta = \frac{\gamma}{8}(\theta - \frac{4}{3}\lambda_i)$

$\longrightarrow$ $k_\beta \uparrow$ $\longrightarrow$ $\omega_{n,\beta}\uparrow = (\nu_\beta^2 + \frac{\gamma}{8}\tan\delta_3)^{\frac{1}{2}}\Omega$ and $\beta_0 \downarrow \xlongequal{\nu_\beta = 1} \frac{\gamma(\theta - \frac{4}{3}\lambda_i)}{8 + \gamma\tan\delta_3}$ $\longrightarrow$ less vibration and noise

$\longrightarrow$ often used in tail rotor to reduce flap response and save weight by avoid need for lead-lag hinge

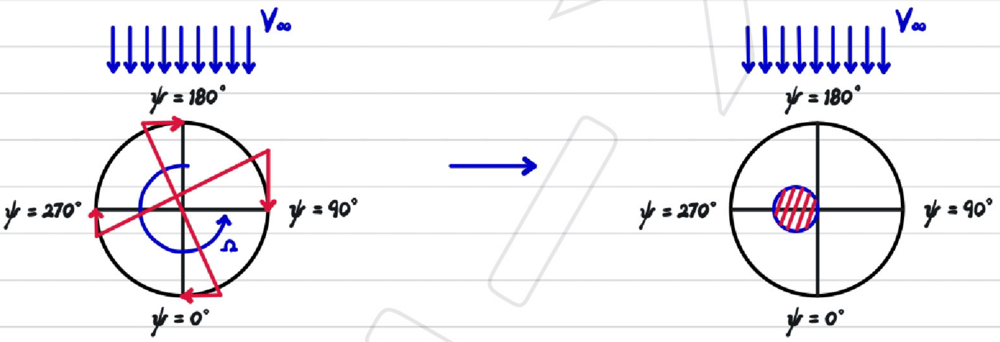
Pitch-Lag (δ_1) coupling

$\Delta\theta = -k_{p\zeta}\zeta$ $\longrightarrow$ strongly influence lag damping

The Rotor Wake

Reverse Flow

Forward flight



reverse flow region margin at $U_T = 0$ $\longrightarrow$ $\Omega y + V_\infty\sin\psi = \Omega y + \mu\Omega R\sin\psi = 0$

$\longrightarrow$ $\Omega R(r + \mu\sin\psi) = 0$ $\longrightarrow$ $r + \mu\sin\psi = 0$ $\longrightarrow$ $r = -\mu\sin\psi$

$\longrightarrow$ $x^2 + y^2 = r^2 = \mu^2\sin^2\psi = -\mu y$ where $y = r\sin\psi = -\mu\sin^2\psi$

$\longrightarrow$ $x^2 + (y + \frac{\mu}{2})^2 = (\frac{\mu}{2})^2$ $\longrightarrow$ circle with diameter μ and centre located at $(\frac{\mu}{2}, \pi)$

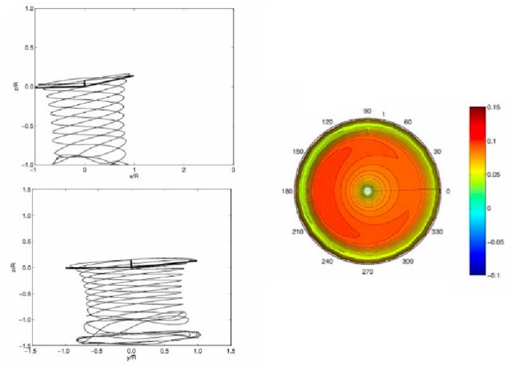
Rotor Wake

Introduction

constant induced velocity only valid in hover, simplified cases (e.g. momentum theory) and preliminary analysis

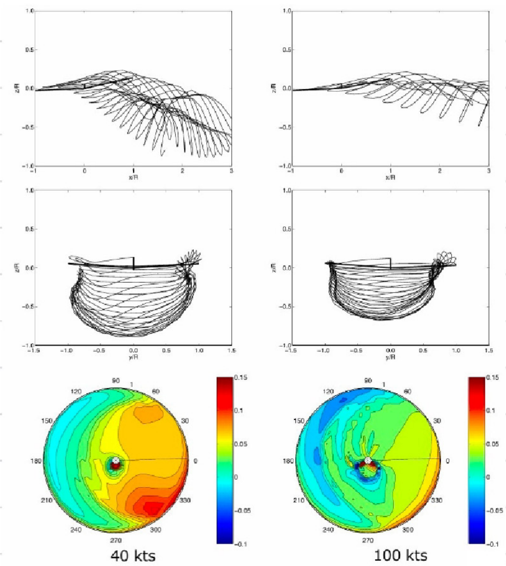
$\longrightarrow$ more accurate predictions include rotor wake is needed for forward flight and rotor loads

Hover



- wake is radially axisymmetric
- strong tip vortices following helical pattern
- vortex sheet related to gradient of circulation

Forward flight



- wake no longer axisymmetric
- wake convected behind in epicycloidal 外摆线 pattern
- highly sensitive to ΔT, μ, α_s and interference

Inflow method

low fidelity $\longrightarrow$ cheap · constant inflow · linear inflow models · dynamic inflow

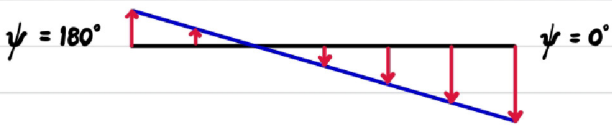
mid fidelity $\longrightarrow$ expensive · prescribed wake · free wake models $\longrightarrow$ Lagrangian

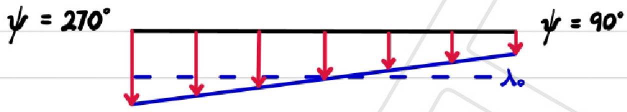
high fidelity $\longrightarrow$ very expensive · CFD · CFD and CSD $\longrightarrow$ Eulerian

Linear inflow models

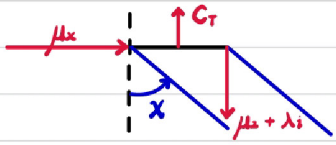
$$\lambda_i(r,\psi) = \lambda_0(1 + k_x r\cos\psi + k_y r\sin\psi)$$

with mean induced velocity $\lambda_0 = \lambda_i = \dfrac{C_T}{2\sqrt{\mu^2+\lambda^2}}$ from momentum theory





· skew angle



$\longrightarrow$ $\chi = \tan^{-1}\left(\frac{\mu_x}{\mu_z + \lambda_i}\right)$

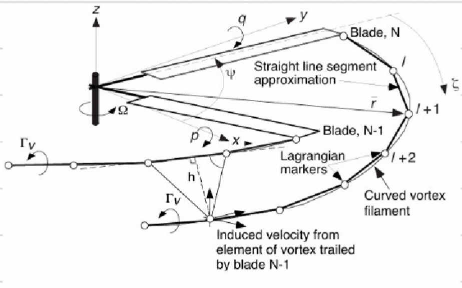
→ Coleman $k_x = \tan\left(\frac{\chi}{2}\right)$ and $k_y = 0$

→ Drees $k_x = \frac{4}{3}\left(\frac{1 - \cos\chi - 1.8\mu^2}{\sin\chi}\right)$ and $k_y = -2\mu$

Dynamic inflow

downwash $\lambda(r, \psi) = \lambda_0 + \lambda_{1c} r\cos\psi + \lambda_{1s} r\sin\psi$ $\longrightarrow$ $\frac{1}{\Omega}[\tau]\begin{Bmatrix}\dot{\lambda}_0 \\ \dot{\lambda}_{1s} \\ \dot{\lambda}_{1c}\end{Bmatrix} + \begin{Bmatrix}\lambda_0 \\ \lambda_{1s} \\ \lambda_{1c}\end{Bmatrix} = L_{nl}\begin{Bmatrix}C_T \\ -C_{My} \\ C_{Mx}\end{Bmatrix}_{aero}$

Vortex wake models



Credit: Bhagwat & Leishman

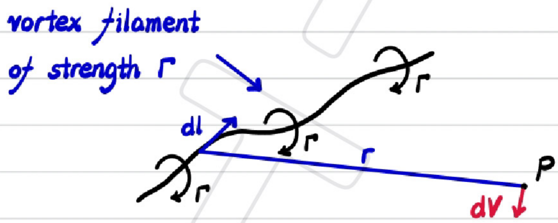
· wake filament discretized into a number of vortex filament

· wake discretized in time into a number of equal ψ steps

→ solve iteratively due to mutual interactions or prescribed

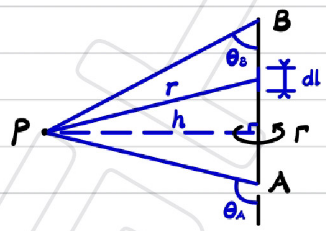
→ position and strength of the wake vortex filament

→



Biot-Savart law $dV = \frac{\Gamma}{4\pi}\frac{dl \times r}{|r|^3}$

→



induced velocity $v_i = \frac{\Gamma}{4\pi h}(\cos\theta_B - \cos\theta_A)$

→ calculate circulation strength, Γ by lifting-line blade model → solve iteratively

Free wake models

· relaxation or iterative $\longrightarrow$ enforce periodicity · time marching

Rotorcraft Performance

Power

Hover

assume $T \approx W \longrightarrow P = P_{induced} + P_{profile} = K\dfrac{W^{\frac{3}{2}}}{\sqrt{2\rho A}} + \dfrac{1}{8}N_b\rho c R(\Omega R)^3 C_{d_0}$

$\longrightarrow C_P = K\dfrac{C_W^{\frac{3}{2}}}{\sqrt{2}} + \dfrac{1}{8}\sigma C_{d_0}$

$\longrightarrow P_{induced} \propto \dfrac{1}{\sqrt{\rho}}$ and $P_{profile} \propto \rho$

Climb

$\dfrac{P}{P_h} = \dfrac{V_c + v_i}{v_h} = \dfrac{V_c}{2v_h} + \sqrt{\left(\dfrac{V_c}{2v_h}\right)^2 + 1} \xrightarrow{\text{low rate of climb}} \dfrac{P}{P_h} = \dfrac{V_c}{2v_h} + 1$

$\longrightarrow \dfrac{P_h + \Delta P \text{ excess power}}{P_h} = \dfrac{V_c}{2v_h} + 1 \longrightarrow \dfrac{\Delta P}{P_h} = \dfrac{V_c}{2v_h}$

$\xrightarrow{P_h = Tv_h} \Delta P = \dfrac{TV_c}{2}$

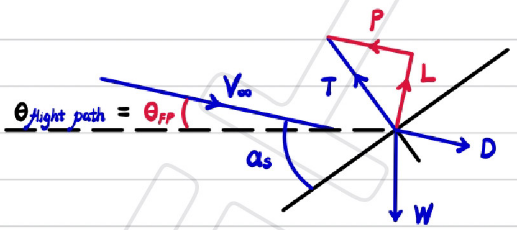
$\xrightarrow{T = W \text{ at maximum climb rate}} V_{c,max} = \dfrac{2\Delta P}{W}$

Forward flight

$P = P_{induced} + P_{profile} + P_{parasitic} + P_{climb} + P_{\text{anti-torque (tail rotor)}} + P_{\text{auxiliary systems}} + P_{\text{transmisson and rotor losses}}$

$= K_{trans}(P_{i_{MR}} + P_{o_{MR}} + P_p + P_c + P_{i_{TR}} + P_{o_{TR}} + P_{AUX})$

· level flight force equilibrium



$T\cos(\alpha_s - \theta_{FP}) = W \xrightarrow{\alpha_s - \theta_{FP} \ll 1} T \approx W$

$T\sin(\alpha_s - \theta_{FP}) = D\cos\theta_{FP} \xrightarrow{\alpha_s - \theta_{FP} \ll 1} T(\alpha_s - \theta_{FP}) \approx D$

$\longrightarrow \alpha_s = \theta_{FP} + \dfrac{D}{W}$

· induced power $P_i = \kappa T v_i \longrightarrow C_{P_i} = \kappa\dfrac{C_T^2}{2\sqrt{\mu^2 + \lambda^2}} \xrightarrow{\text{high speed}} C_{P_i} = \kappa\dfrac{C_T^2}{2\mu}$

where κ induced power factor $= \kappa_{hover}\cosh(7.5\mu^2)$

and download factor = 1.04 ~ 1.08 (large fuselage or compact main rotor) adds to lift

· profile power $P_o = \frac{1}{8} N_b \rho c R (\Omega R)^3 C_{do} (1 + K\mu^2) \longrightarrow C_{P_o} = \frac{1}{8} 6 C_{do} (1 + K\mu^2)$

where $K = 4.65$ for $\mu < 0.5$

$\xrightarrow{\text{compressibility effect}} \frac{\Delta C_{P_o}}{6} = 0.007 \Delta M_{dd} + 0.052 \Delta M_{dd}$ for $M \geq M_{dd}$

$\xrightarrow{\text{reverse flow}} C_{P_o} = \frac{1}{8} 6 C_{do} (1 + K\mu^2 + \frac{3}{8}\mu^4)$

· parasitic power $P_p = \frac{1}{2} \rho V_\infty^2 S_{ref} C_{Df} V_\infty \longrightarrow C_{P_p} = \frac{1}{2} (\frac{S_{ref} C_{Df}}{A}) \mu^3 = \frac{1}{2} (\frac{f}{A}) \mu^3$

where f equivalent wetted area $= 0.9 \sim 5\, m^2$

· climb power $P = TV_\infty \sin\alpha_s \approx TV_\infty (\theta_{FP} + \frac{D}{W}) \approx TV_c + DV_\infty = P_c + P_p \longrightarrow C_{P_c} = C_T \lambda_c$

· tail rotor power $Q_{MR} = \frac{P_{MR}}{\Omega_{MR}} = l_{TR} T_{TR} \longrightarrow T_{TR} = \frac{(P_i + P_o + P_p + P_c)_{MR}}{\Omega_{MR} l_{TR}}$

$\xrightarrow{\text{interference effect}} T_{TR} = K_{TRB} \frac{P_{MR}}{\Omega_{MR} l_{TR}}$

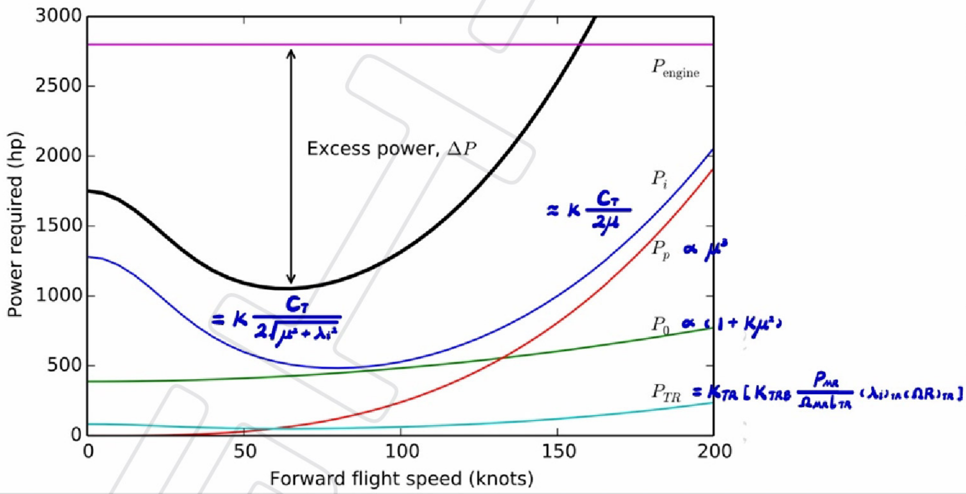
where K_{TRB} tail rotor blockage factor ≈ 1.09 (pusher) < (tractor)

$\longrightarrow P_{i_{TR}} = K_{TR} K_{TRB} \frac{P_{MR}}{\Omega_{MR} l_{TR}} v_{i_{TR}}$ where K_{TR} tail rotor induced power factor ≈ 1.2

$P_{o_{TR}} = \frac{1}{8} N_{b_{TR}} \rho c_{TR} R_{TR} (\Omega R)^3_{TR} C_{do_{TR}} (1 + K\mu_{TR}^2)$

· auxilary power $P_{AUX} = P_{\text{hydraulic systems}} + P_{\text{electrical systems}} + P_{\text{windage loss from high drag rotating component (e.g. hub)}}$

· transmission loss 2% per gear reduction stage



Performance indicators

Lift - to - drag ratio

$$\frac{L}{D} = \frac{T\cos(\alpha_s - \theta_{FP})}{P/V_\infty} \approx \frac{W}{P/V_\infty} = \frac{WV_\infty}{P}$$

Speed for minimum power

$V_{mp} \Longleftrightarrow$ maximum rate of climb $\quad V_{c,max} = \dfrac{P_{avail} - P}{T}$

$\Longleftrightarrow$ minimum autorotative rate of descent

$\Longleftrightarrow$ maximum endurance

$$C_P = C_Q \approx K\frac{C_T^2}{2\mu} + \frac{1}{8}\sigma C_{d0}(1+K\mu^2) + \frac{1}{2}\left(\frac{f}{A}\right)\mu^3 + C_T\lambda_c \xrightarrow{\text{low speed},\ \alpha_s \ll 1} C_P = K\frac{C_T^2}{2\mu} + \frac{1}{2}\left(\frac{f}{A}\right)\mu^3$$

$$\longrightarrow \text{at } V_{mp} = \frac{\mu_{mp}\Omega R}{\cos\alpha_s} \approx \mu_{mp}\Omega R,\quad \frac{dC_P}{d\mu} = -K\frac{C_T^2}{2\mu^2} + \frac{3}{2}\left(\frac{f}{A}\right)\mu^2 = 0$$

$$\longrightarrow \mu_{mp} = \left(\frac{KC_T^2}{3f/A}\right)^{\frac{1}{4}} \xrightarrow{\lambda_h = \sqrt{\frac{C_T}{2}}} \lambda_h\left(\frac{4K}{3f/A}\right)^{\frac{1}{4}}$$

$$\longrightarrow V_{mp} = v_h\left(\frac{4K}{3f/A}\right)^{\frac{1}{4}}$$

Speed for maximum range

$V_R \Longleftrightarrow (L/D)_{max} \Longleftrightarrow \left(\dfrac{P}{V_\infty}\right)_{min} \Longleftrightarrow$ minimum P_0

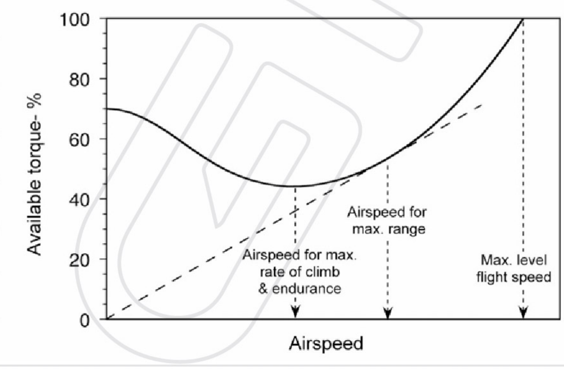
$$C_P = C_Q \approx K\frac{C_T^2}{2\mu} + \frac{1}{8}\sigma C_{d0}(1+K\mu^2) + \frac{1}{2}\left(\frac{f}{A}\right)\mu^3 + C_T\lambda_c \xrightarrow{\text{low speed},\ \alpha_s \ll 1} C_P = K\frac{C_T^2}{2\mu} + \frac{1}{2}\left(\frac{f}{A}\right)\mu^3$$

$$\longrightarrow \text{at } V_R = \frac{\mu_R\Omega R}{\cos\alpha_s} \approx \mu_R\Omega R,\quad \frac{d(C_P/\mu)}{d\mu} = -K\frac{C_T^2}{\mu^3} + \left(\frac{f}{A}\right)\mu = 0$$

$$\longrightarrow \mu_R = \left(\frac{KC_T^2}{f/A}\right)^{\frac{1}{4}} \xrightarrow{\lambda_h = \sqrt{\frac{C_T}{2}}} \lambda_h\left(\frac{4K}{f/A}\right)^{\frac{1}{4}} \longrightarrow V_R = v_h\left(\frac{4K}{f/A}\right)^{\frac{1}{4}}$$

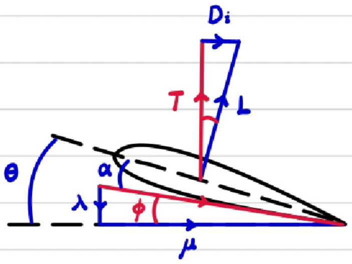
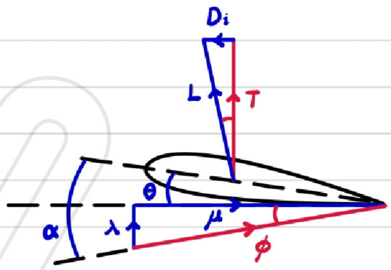
Maximum forward speed

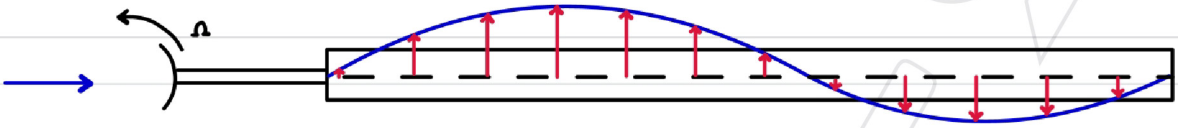
limit by
- P_{avail} and Q_{avail} $\longrightarrow$ most helicopter is torque-limited
- parasitic drag
- $P_{avail\,TR}$ and $Q_{avail\,TR}$
- rotor aerodynamic $\longrightarrow$ compressibility effect and stall
- aeroelastic vibratory loads
- structural constraints



Autorotation

BET

$V_c + v_i > 0$  $\Longleftrightarrow$ $V_c + v_i < 0$ 



driving region $V_c < -v_i$ driven region $V_c > -v_i$

$\longrightarrow$ $Q = \int_0^R (dD - dL\phi)\, y\, dy = 0$ for autorotation $\longrightarrow$ Ω adjusted for stable equilibrium

$\xrightarrow{\text{forward flight}}$ $C_Q \approx K\frac{C_T^2}{2\mu} + \frac{1}{8}\sigma C_{d0}(1+K\mu^2) + \frac{1}{2}\left(\frac{f}{A}\right)\mu^3 + C_T\lambda_c + K_{trans}\frac{1}{8}\sigma_{TR}C_{d0,TR}(1+K\mu_{TR}^2) = 0$

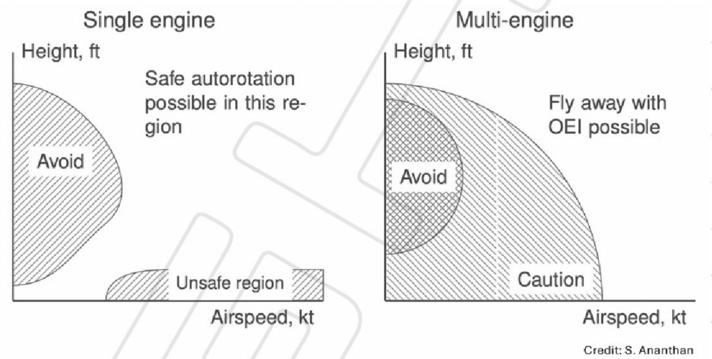
$\longrightarrow$ $\lambda_c \equiv \lambda_d = -K\frac{C_T}{2\mu} - \frac{1}{8C_T}\sigma C_{d0}(1+K\mu^2) - \frac{1}{2C_T}\left(\frac{f}{A}\right)\mu^3 - \frac{K_{trans}}{8C_T}\sigma_{TR}C_{d0,TR}(1+K\mu_{TR}^2)$

Autorotation index

stored kinetic energy in the rotor system

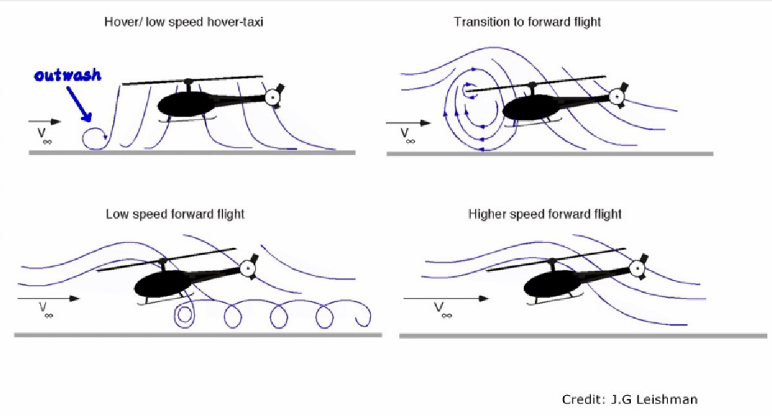
- $\longrightarrow$ $AI = \frac{I_R\Omega^2}{2W}$
- $\longrightarrow$ Sikorsky $AI = \frac{I_R\Omega^2}{2W\cdot DL} = \frac{I_R\Omega^2}{2W(T/A)}$

Height-Velocity (HV) curve



Ground Effect

Ground effect



→ increase T for given P

Outwash

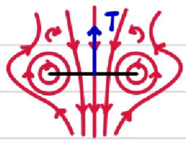
· strength related to DL · may uplift debris, damage ground facilities, ...

· brownout when close to dust or sand

The Vortex Ring State (VRS)

The vortex ring state

Occurs at low forward speeds and moderate descent rates →



20~50% load flucturation

Avoidance diagram

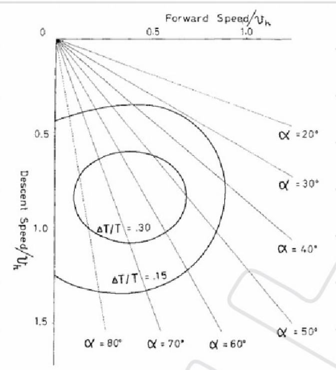


Fig. 8 $\Delta T/T$.

Washizu & Azuma (1966)

← thrust flucturation

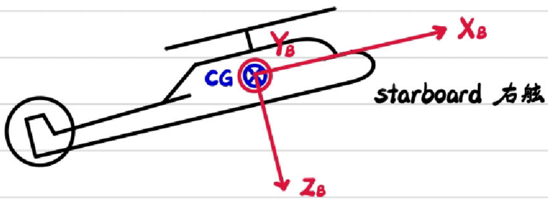
Boundary defination

$\bar{\mu}_{WTV_{CRIT}}$ critical wake transparent velocity $= \sqrt{k^2 \bar{\mu}_x^2 + (\bar{\mu}_z^2 + \bar{\lambda}_i^2)} = 0.74$

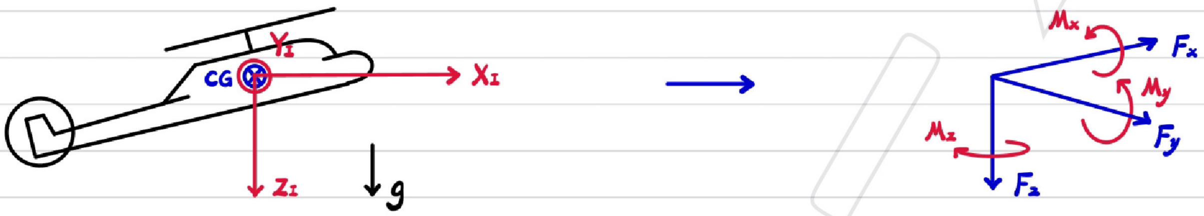
Flight Dynamics

Coordinate System

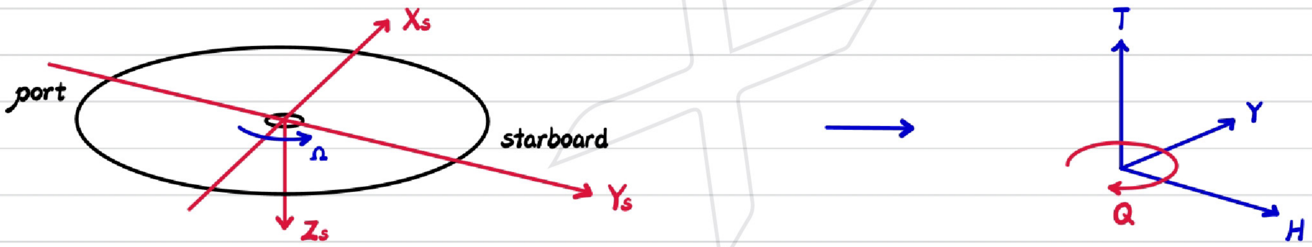
Body axes



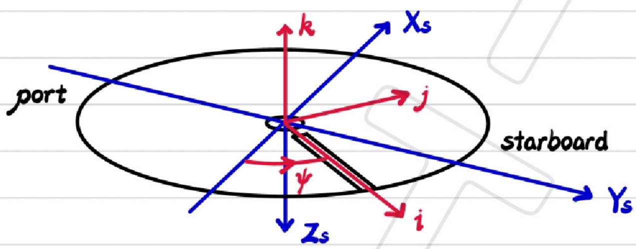
Inertia axes



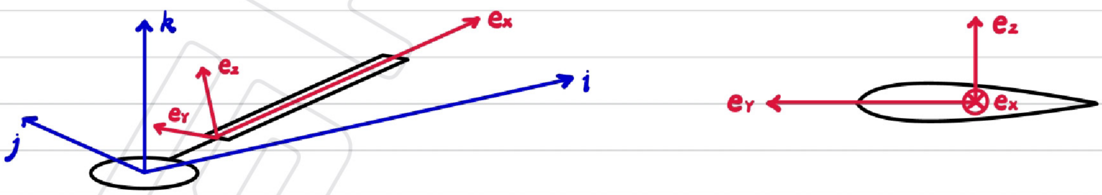
Shaft axes



Hub rotating axes



Blade axes



Equation of Motion

First-order ODEs

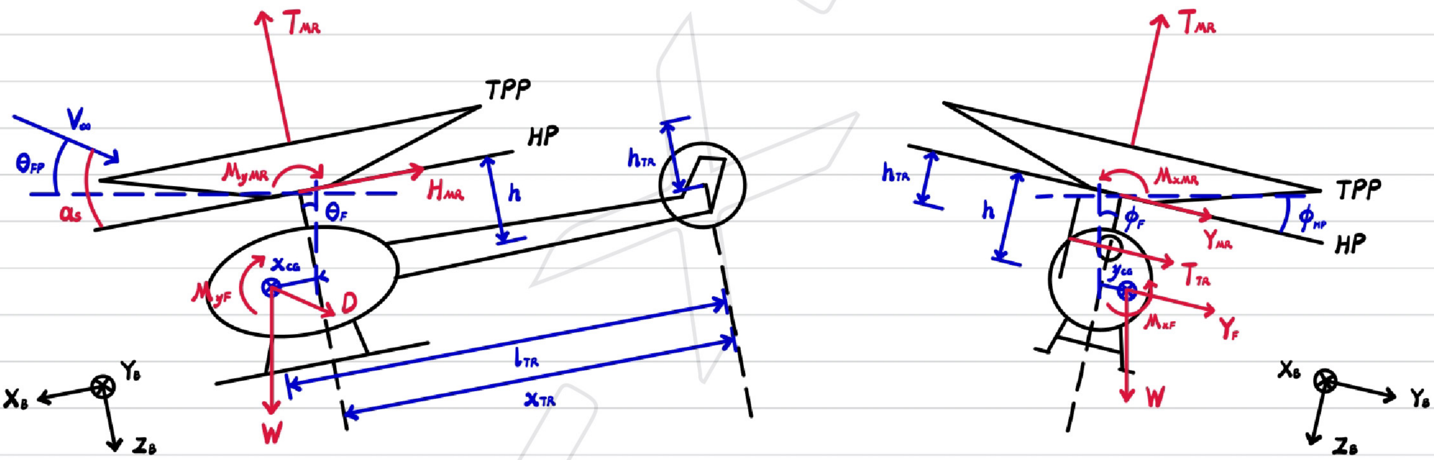
$\dot{y} = g(\dot{y}, y, u; t)$

- state $\rightarrow$ $y = [y_F, y_R, y_I]$
  - rigid body $\rightarrow$ $y_F = [u_F, v_F, w_F, p, q, r, \phi_F, \theta_F, \psi_F]$
  - rotor $\rightarrow$ $y_R = [\beta_0, \beta_{1c}, \beta_{2c}, \dots, \beta_{kc}, \beta_{1s}, \beta_{2s}, \dots, \beta_{ks}]$
  - inflow $\rightarrow$ $y_I = [\lambda_0, \lambda_{1c}, \lambda_{1s}, \lambda_{TR}]$
- control
  - conventional helicopter $\rightarrow$ $u = [\theta_0, \theta_{1c}, \theta_{1s}, \theta_{TR}]$
  - variable RPM drone $\rightarrow$ $u = [U_1, U_2, \dots, U_N]$

Trim

coupled helicopter trim

- fuselage not present $\rightarrow$ wind-tunnel or isolated rotor trim
- fuselage present $\rightarrow$ free-flight or propulsive trim



vertical forces $\quad W - T_{MR}\cos\theta_F\cos\phi_F + D\sin\theta_{FP} - H_{MR}\sin\theta_F + (Y_{MR} + Y_F + T_{TR})\sin\phi_F = 0$

small angle approximation $\rightarrow$ $W - T_{MR} = 0$

longitudinal forces $\quad T_{MR}\sin\theta_F\cos\phi_F - D\cos\theta_{FP} - H_{MR}\cos\theta_F = 0$

small angle approximation $\rightarrow$ $T_{MR}\theta_F - D - H_{MR} = 0$

lateral forces $\quad T_{MR}\cos\theta_F\sin\phi_F + (Y_{MR} + Y_F + T_{TR})\cos\phi_F = 0$

small angle approximation $\rightarrow$ $T_{MR}\phi_F + (Y_{MR} + Y_F + T_{TR}) = 0$

rolling moment about hub $\quad M_{xMR} + M_{xF} + W(-y_{cg}\cos\phi_F + h\sin\phi_F) + Y_F h + T_{TR} h_{TR} = 0$

small angle approximation $\rightarrow$ $M_{xMR} + M_{xF} + W(-y_{cg} + h\phi_F) + Y_F h + T_{TR} h_{TR} = 0$

pitching moment about hub $M_{yMR} + M_{yF} + W(h\sin\theta_F - x_{cg}\cos\theta_F) + D(-h\cos\alpha_s - x_{cg}\sin\alpha_s) = 0$

$\xrightarrow{\text{small angle approximation}}$ $M_{yMR} + M_{yF} + W(h\theta_F - x_{cg}) - Dh = 0$

torque $Q_{MR} = l_{TR} T_{TR}$

inflow $\lambda_{MR} = \mu_{MR}\tan\alpha_s + \dfrac{C_{T,MR}}{2\sqrt{\mu_{MR}^2 + \lambda_{MR}^2}}$ and $\lambda_{TR} = \mu_{TR}\tan\alpha_{s,TR} + \dfrac{C_{T,TR}}{2\sqrt{\mu_{TR}^2 + \lambda_{TR}^2}}$

flapping $\ddot{\beta} + \nu_\beta^2\beta = \gamma\overline{M}_\beta$

$\longrightarrow$ find $X = [\theta_0, \theta_{1c}, \theta_{1s}, \theta_{0TR}, \lambda_{MR}, \lambda_{TR}, \theta_F, \phi_F]^T$ for trim