Introduction to Aerospace - Tutorial Sheet

Fixed-Wing Aircraft Performance

1. A small jet aircraft is in steady level flight at an altitude of 1000 m, and a true air speed of 400 kn. If it now flies at the same weight and L/D ratio at 5000 m, what must its TAS be in: m/s, km/h, kn.

Table 1: Characteristics of different airplanes and gliders

Aircraft	Span b (m)	Wing Area S (m^2)	C_{D_0}	k	Max. Take-off Mass (kg)	Empty Mass	Max Fuel Mass (kg)	Max Payload Mass (kg)
BAe 146 Series 100	26.34	77.3	0.018	1.15	33,500	20,200	9,200	8,100
BAe Hawk T Mk1	9.39	16.69	0.020	1.20	4,411			
Nimbus 3 Glider	22.9	16.2	0.008	1.20	750			
BAe Jet Provost	11.3	19.9	0.025	1.25	3,360		910	420

2. Characteristics of three aeroplanes and a glider are given in Table 1. For each aircraft, in level flight, find the minimum drag speed in kn (EAS), the maximum lift/drag ratio $(L/D)_{max}$ and the lift coefficient at minimum drag speed. You can first compute the coefficient A and B.

3. The characteristics of the BAe Jet Provost are given in Table 1.

   (a) Plot curves of profile drag, induced drag and total drag against EAS. You may assume the aircraft took off at its Maximum Takeoff Weight (MTOW) and maximum fuel load, and has consumed 50% of its fuel.

   (b) The engine of the Jet Provost is a Rolls-Royce Viper 202, which has a maximum continuous rate of 9.6 kN thrust at static sea-level conditions. You may assume the thrust to be independent of forward speed, but follow the classical decay of thrust with altitude discussed in lectures. Calculate the aircraft's maximum level speed at sea-level and altitudes of 10, 20, 30 and 40 thousand feet.

   (c) What is the absolute ceiling of the aircraft?

   (d) What is the maximum climb rate achievable by the aircraft at an altitude of 20,000 ft and full throttle? What velocity will it occur at?

1. $\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{\frac{k}{\pi AR} C_L^2 + C_{D0}} \longrightarrow C_L$ constant

$L = W = \frac{1}{2}\rho V^2 S C_L \longrightarrow \rho V^2$ constant

$\therefore V_{5000} = V_{1000}\sqrt{\frac{\rho_{1000}}{\rho_{5000}}} = 491.67 kn = 252.94 m/s = 910.59 km/h$

2. $AR = \frac{b^2}{S_{ref}}$

$A = \frac{1}{2}\rho_0 S_{ref} C_{D0}$

$B = \frac{2kW^2}{\rho_0 S_{ref} \pi AR}$

$(L/D)_{max} = \frac{1}{2}\sqrt{\frac{\pi AR}{kC_{D0}}}$

$V_{imD} = \sqrt{n}\,(\frac{B}{A})^{\frac{1}{4}} = (\frac{B}{A})^{\frac{1}{4}}$

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AR =

    8.9754    5.2829   32.3710    6.4166

A =

    0.8522    0.2045    0.0794    0.3047

B =

   1.0e+07 *

    9.3033    1.3244    0.0064    0.5527

LDm =

   18.4538   13.1485   51.4621   12.6991

Vimd =

  198.6880  174.3834   58.3314  126.8543
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3. (a) ...

(b) $T = D = \frac{1}{2}\rho V^2 S C_D$

$\therefore V_i = (\frac{T}{2A} + ((\frac{T}{2A})^2 - \frac{B}{A})^{\frac{1}{2}})^{\frac{1}{2}}$

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T =

   1.0e+03 *

    9.6000    7.7918    6.2261    4.8838    3.5075

Vim =

  341.7871  306.3574  271.4514  236.4880  190.7124
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(c) Abs ceiling when $L = W_{half}$ and $T = D_{min}$

$\therefore T = D_{min} = \frac{W_{half}}{(L/D)_{max}} = 1.4396 T_0$

$\therefore 6 = \frac{(3360 - \frac{1}{2}\times 910)\,9.81}{1.439\, T_0 (L/D)_{max}} = 0.1624 \longrightarrow h = 14838.47 m$

(d) $(L/D)_{max}\,\bar{V}_c = \tau\bar{V} - \frac{1}{2}(\bar{V}^3 + \frac{1}{\bar{V}})$

$\therefore$ To maximum climb rate, $\frac{d\bar{V}_c}{d\bar{V}} = \tau - \frac{3}{2}\bar{V}^2 + \frac{1}{2\bar{V}^2} = 0 = \frac{T}{D_{min,1}} - \frac{3}{2}\bar{V}^2 + \frac{1}{2\bar{V}^2}$

$\because T = 6^{0.7} T_0 = 9600 \times (0.3241)^{0.7} = 6107.47\,(N)$

$D_{min,1} = \ldots$ $\quad\therefore \bar{V} = 1.2149$ $\quad\therefore \ldots$

4. The engines of the BAe Series 100 (characteristics given in Table 1) are four Avco Lycoming ALF 502R-3 turbofans whose specific fuel consumption when cruising is 0.73 1/hr. Find the maximum distance that the aircraft is capable of cruising at constant C_L and velocity.

   You may assume that:

   - 2000 kg of fuel are consumed during the takeoff and climb to 22,000 ft,
   - 1200 kg of fuel must be available at the beginning of the descent to deal with the remainder of the flight, and
   - during the level flight phase, the airspeed always has its optimum value.

   Consider the following three cases:

   (a) The aircraft took-off at maximum takeoff weight and had the maximum payload onboard.

   (b) The aircraft took-off at maximum takeoff weight and had the maximum fuel load onboard.

   (c) The aircraft took-off with no payload and maximum fuel load onboard.

   Hence draw the aircraft's Payload-Range diagram. What is the initial and final EAS and TAS required to achieve maximum range in each of these conditions?

5. A glider is flown so that its flight path is truly horizontal. Show that the equation of motion along the flight-path becomes:

$$-\left(AV_i^2 + BV_i^{-2}\right) = \frac{m}{2\sigma}\frac{dV_i^2}{dx}$$

   Hence show that the distance covered in slowing down from Vi1 to Vi2 is

$$x_2 - x_1 = \frac{m}{4A\sigma}\ln\left(\frac{V_{i1}^4 + V_{imD}^4}{V_{i2}^4 + V_{imD}^4}\right)$$

   What is the horizontal distance covered by the Nimbus 3 Glider, the characteristics of which are given in Table 1, when the pilot is performing a stall test at 10,000 ft. You may assume the staring indicated airspeed is $V_{i1} = 120$ kn and the aircraft $C_{L_{max}} = 1.75$.

6. An aeroplane performs a loop, starting from level flight at a true airspeed V_1. The thrust vector lies along the flight path and the pilot adjusts the thrust so that it equals the drag at all times. Show that in order to maintain a constant radius loop the pilot must adjust the load factor n so that

$$n = N + cos(\gamma)$$

   where N is a constant and γ is the instantaneous flight path angle of the aircraft in the loop.

   By considering the equations of motion along and perpendicular to the flight path, show that the aircraft's speed at any instant is given by

$$V = V_1 e^{(\cos\gamma - 1)/N}$$

   You may assume that the mass of the airplane stays constant during the loop.

4. (a) $W_{ini} = 31500\,kg$, $W_{fin} = 29500\,kg$

(b) $W_{ini} = 31500\,kg$, $W_{fin} = 25500\,kg$

(c) $W_{ini} = 27400\,kg$, $W_{fin} = 21400\,kg$

According to the $R = \frac{V_{mD,1}(L/D)_{max}}{c}\left[\frac{4}{\bar{v}+\bar{v}^{-3}}\right]\left(1-\sqrt{\frac{W_{ini}}{W_{fin}}}\right)$

$\therefore$ To maximise R, minimise $\left(\frac{4}{\bar{v}+\bar{v}^{-3}}\right)$ $\therefore$ maximise $\bar{v}+\bar{v}^{-3}$

$\therefore \frac{d(\bar{v}+\bar{v}^{-3})}{d\bar{v}} = 1-3\bar{v}^{-4} = 0$ $\quad\therefore \bar{v}^{4}-3=0$ $\quad\therefore \bar{v} = \sqrt[4]{3} = \frac{V_i}{V_{imD,1}}$

$\therefore V_i = \sqrt[4]{3}\,V_{imD,1} = \sqrt[4]{3}\left(\frac{B}{A}\right)^{\frac{1}{4}}$

$\therefore$ Vii =

130.4467   130.4467   121.6615

Vif =

126.2376   117.3675   107.5189

Vti =

182.7717   182.7717   170.4625

Vtf =

176.8743   164.4462   150.6470

5. m is constant

$$\therefore \frac{dmV}{dt} = m\frac{dV}{dt} = m\frac{dV}{dx}\cdot\frac{dx}{dt} = m\frac{dV}{dx}V = m\frac{dV^2}{dx}\frac{1}{2} = \frac{m}{2}\frac{dV^2}{dx} = \frac{m}{2\sigma}\frac{dV_i^2}{dx} = -D = -(AV_i^2+BV_i^{-2})$$

$$\therefore \Delta x = \int_{V_{i1}}^{V_{i2}}\frac{m}{2\sigma}\frac{-1}{(AV_i^2+BV_i^{-2})}\,dV_i^2 = \frac{m}{2A\sigma}\int_{V_{i1}}^{V_{i2}} -\frac{1}{V_i^2+V_{imo}^4V_i^{-2}}\,dV_i^2 = \frac{m}{2A\sigma}\left[-\frac{1}{2}\ln(V_i^4-V_{imo}^4)\right]_{V_{i1}}^{V_{i2}}$$

$$= \frac{m}{4A\sigma}\ln\left(\frac{V_{i,1}^4+V_{imo}^4}{V_{i,2}^4+V_{imo}^4}\right)$$

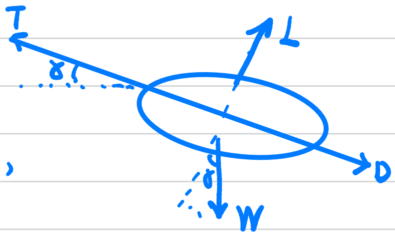
$V_{i,1} = 120\,kn$

$V_{i,2} = \left(\frac{2W}{\rho_0 S C_{L,max}}\right)^{\frac{1}{2}} = V_S$

$V_{imo} = 174.3834\,kn$

$\therefore \Delta x = 8762\,m$

6. m is constant



$m\frac{dV}{dt} = T - D - W\sin\gamma \quad (\rightarrow)$

$F_{centripetal} = \frac{mV^2}{R} = L - W\cos\gamma \quad (\uparrow)$

$\therefore \frac{mV^2}{WR} = n - \cos\gamma$

$\therefore n = \frac{mV^2}{WR} + \cos\gamma = N + \cos\gamma$

For constant loop. $T = D$

$\therefore m\frac{dV}{dt} = -W\sin\gamma$

$\because mV\frac{d\gamma}{dt} = L - W\cos\gamma$

When $\frac{d\gamma}{dt} = \Omega$, $V = \Omega R$

$mV\frac{d\gamma}{dt} = NW$

$\therefore \frac{m\frac{dV}{dt}}{mV\frac{d\gamma}{dt}} = \frac{-W\sin\gamma}{L - W\cos\gamma} = \frac{-W\sin\gamma}{NW} = \frac{-\sin\gamma}{N}$

$\therefore \frac{dV}{d\gamma} = -\frac{V\sin\gamma}{N}$

$\therefore \int \frac{1}{V} dV = \int -\frac{\sin\gamma}{N} d\gamma$

$\therefore V = V_1 e^{(\cos\gamma - 1)/N}$

7. Electric aircraft rely on batteries to provide the electrical energy required to power their motors. Show that for an electric powered fixed-wing aircraft, its range is given by

$$R = \eta_p \eta_e E_{wB} (L/D)_{max} \frac{W_B}{W} \frac{2}{\overline{V}^2 + \overline{V}^{-2}}$$

and endurance by

$$\Delta t = \frac{\eta_p \eta_e E_{wB} (L/D)_{max}}{V_{mD}} \frac{W_B}{W} \frac{2}{\overline{V}^3 + \overline{V}^{-1}}$$

where E_{wB} is the battery specific energy (energy per unit weight), W_B/W is the aircraft's battery mass fraction, η_P is the propeller efficiency and η_e is the total efficiency of the electrical propulsive system. What velocity will the range and endurance or an all-electric fixed-wing aircraft be maximised at?

8. Hybrid-Electric aircraft rely on a combination of fuel and electricity to power their propellers. Consider a parallel system layout where, through a gearbox, the sum of the power generated by a turboshaft engine and an electric motor drives a propeller. The power-split factor $\Psi > 0$ is defined as the ratio of the power generated by the turboshaft engine to the total power driving the propeller. Show that the weight fraction required for a hybrid-electric aircraft to travel a range R is given by

$$\ln\left(\frac{W_{fin}}{W_{init}}\right) = -\frac{c\,\Psi\,R}{2\eta_p (L/D)_{max}} \left(\overline{V}^2 + \frac{1}{\overline{V}^2}\right)$$

and the battery mass fraction required is given by

$$\frac{W_B}{W_{init}} = \frac{1-\Psi}{c\,\eta_e\,E_{wB}\,\Psi}\left(1 - \frac{W_{fin}}{W_{init}}\right).$$

9. To minimise flight time and fuel consumption, eastward travelling long-haul flights aim to take advantage of the polar or subtropical jet streams. In the presence of head or tail winds, the standard form of the Breguet range equation is no longer valid.

   (a) Show that in the presence of a tailwind of speed V_{tw}, the range R of a jet aircraft, operating at constant velocity and L/D, is given by:

   $$R = -\frac{2V_{mD}(L/D)_{\max}}{c}\left(\frac{\overline{V} + \overline{V}_{tw}}{\overline{V}^2 + \overline{V}^{-2}}\right) \ln\left(\frac{W_{fin}}{W_{init}}\right),$$

   where $\overline{V}_{tw} = V_{tw}/V_{mD}$ and all other symbols have their usual meaning.

   (b) Using equation above or otherwise, prove that the range is maximised when

   $$\overline{V}_{tw} = -\frac{\overline{V}}{2}\left(\frac{\overline{V}^4 - 3}{\overline{V}^4 - 1}\right).$$

   (c) In January 2015, a British Airways Boeing 777-200 operating from New York's JFK to London Heathrow encountered tailwind speeds of the order of 320 km/h, arriving 1.5 hours ahead of schedule. How much fuel did the 777-200 above save due to the tailwinds encountered? You may assume that at the start of the cruise segment the aircraft weighs 490×10^3 lbs, that $V_{mD} = 373$ kn and that the cruise segment, with no wind, would require a minimum 84×10^3 lbs of fuel.

7. $P = \frac{dE}{dt} = \frac{TV}{\eta_p}$

$\because T = D$

$\therefore \frac{dE}{dt} = \frac{DV}{\eta_p} = \frac{W\bar{V}V_{mD}}{\eta_p (L/D)_{max}}\bar{D} = \frac{WV_{mD}}{\eta_p (L/D)_{max}} \frac{1}{2}(\bar{V}^3 + \frac{1}{\bar{V}})$

$\therefore E = E_{wb} W_b \eta_e = \frac{WV_{mD}}{\eta_p (L/D)_{max}} \frac{1}{2}(\bar{V}^3 + \frac{1}{\bar{V}})\Delta t$

$\therefore \Delta t = \frac{\eta_p \eta_e E_{wb} (L/D)_{max}}{V_{mD}} \frac{W_B}{W} \frac{2}{\bar{V}^3 + \bar{V}^{-1}}$

$\therefore R = V\Delta t = V_{mD} \bar{V} t = \eta_p \eta_e E_{wb} (L/D)_{max} \frac{W_B}{W} \frac{2}{\bar{V}^2 + \bar{V}^{-2}}$

To maximise R, minimise $\bar{V}^2 + \bar{V}^{-2}$ $\quad \therefore 2\bar{V} - 2\frac{1}{\bar{V}^3} = 0 \quad \therefore \bar{V} = 1$

To maximise t, minimise $\bar{V}^3 + V^{-1}$ $\quad \therefore 3\bar{V}^2 - \frac{1}{\bar{V}^2} = 0 \quad \therefore \bar{V} = \sqrt[4]{\frac{1}{3}}$

8. $\psi = \frac{P_T}{P_t}$

$\therefore P_T = P_t \psi = \frac{TV}{\eta_p}\psi \quad \because T = D \quad \therefore P_T = \frac{DV}{\eta_p}\psi = \frac{WV\bar{D}}{\eta_p (L/D)_{max}}\psi$

$\therefore \frac{ds}{dW} = \frac{\frac{ds}{dt}}{\frac{dW}{dt}} = -\frac{V}{\dot{m}g} = -\frac{V}{cP_E} = -\frac{\eta_p (L/D)_{max}}{cW\psi} 2(\bar{V}^2 + \frac{1}{\bar{V}^2})$

$\therefore R = -\frac{2\eta_p (L/D)_{max}}{c\psi}(\bar{V}^2 + \frac{1}{\bar{V}^2}) \ln(\frac{W_{fin}}{W_{ini}})$

$P_E = P_t - P_T = (1-\psi)P_t$

$\therefore \frac{dE}{dx} = \frac{\frac{dE}{dt}}{\frac{dx}{dt}} = \frac{\frac{P_E}{\eta_e}}{V} = \frac{(1-\psi)P_t}{\eta_e V} = \frac{(1-\psi)W}{\eta_e \eta_p (L/D)_{max}}\bar{D}$

$\because W = W_{ini} e^{-\frac{\psi c \bar{D}}{\eta_p (L/D)_{max}}x}$

$\therefore E = \int \frac{(1-\psi)W_{ini}}{\eta_e \eta_p (L/D)_{max}}\bar{D} e^{-\frac{\psi c\bar{D}}{\eta_p (L/D)_{max}}x} dx = E_{wb} \cdot W_B$

9. (a) $\frac{ds}{dW} = \frac{\frac{ds}{dt}}{\frac{dW}{dt}} = \frac{V + V_{tw}}{-\dot{m}g} = \frac{V + V_{tw}}{-cT}$

$\because T = D$

$\therefore \frac{ds}{dW} = \frac{V + V_{tw}}{-cD} = \frac{(V + V_{tw})}{-cW\bar{D}}(L/D)_{max}$

$= -\frac{2V_{mD}(L/D)_{max}}{c}\frac{1}{W}(\frac{\bar{V} + \bar{V}_{tw}}{\bar{V}^2 + \bar{V}^{-2}})$

$\therefore R = -\frac{2V_{mD}(L/D)_{max}}{c}(\frac{\bar{V} + \bar{V}_{tw}}{\bar{V}^2 + \bar{V}^{-2}}) \ln(\frac{W_{fin}}{W_{ini}})$

(b) R is maximum when $\frac{\bar{V} + \bar{V}_{tw}}{\bar{V}^2 + \bar{V}^{-2}}$ is maximum

$$\therefore \left(\frac{\bar{V}+\bar{V}_{tw}}{\bar{V}^2+\bar{V}^{-2}}\right)' = 0 = \frac{\bar{V}_{tw}(\bar{V}^2+\bar{V}^{-2}) + (\bar{V}+\bar{V}_{tw})(2\bar{V}-2\bar{V}^{-3})}{(\bar{V}^2+\bar{V}^{-2})^2}$$

$$\therefore \bar{V}_{tw}(\bar{V}^2+\bar{V}^{-2}) = (\bar{V}+\bar{V}_{tw})(2\bar{V}-2\bar{V}^{-3})$$

$$\therefore \bar{V}_{tw}(2\bar{V}^3+\bar{V}^2-2\bar{V}+\bar{V}^{-2}) = 2\bar{V}^2-2\bar{V}^{-2}$$

$$\therefore \bar{V}_{tw} = -\frac{\bar{V}}{2}\left(\frac{\bar{V}^4-3}{\bar{V}^4-1}\right)$$

(C) ...