Static Stability - Tutorial Sheet:

The Piper Cherokee PA-28-180 aircraft of Figures 1 and 2 is a single engine propeller driven four seat aircraft which was originally introduced in 1961. There have been over 30,000 PA-28 Cherokee family aircraft built of which only 10 were of the 180 series. Data for this aircraft is given in Table 2.

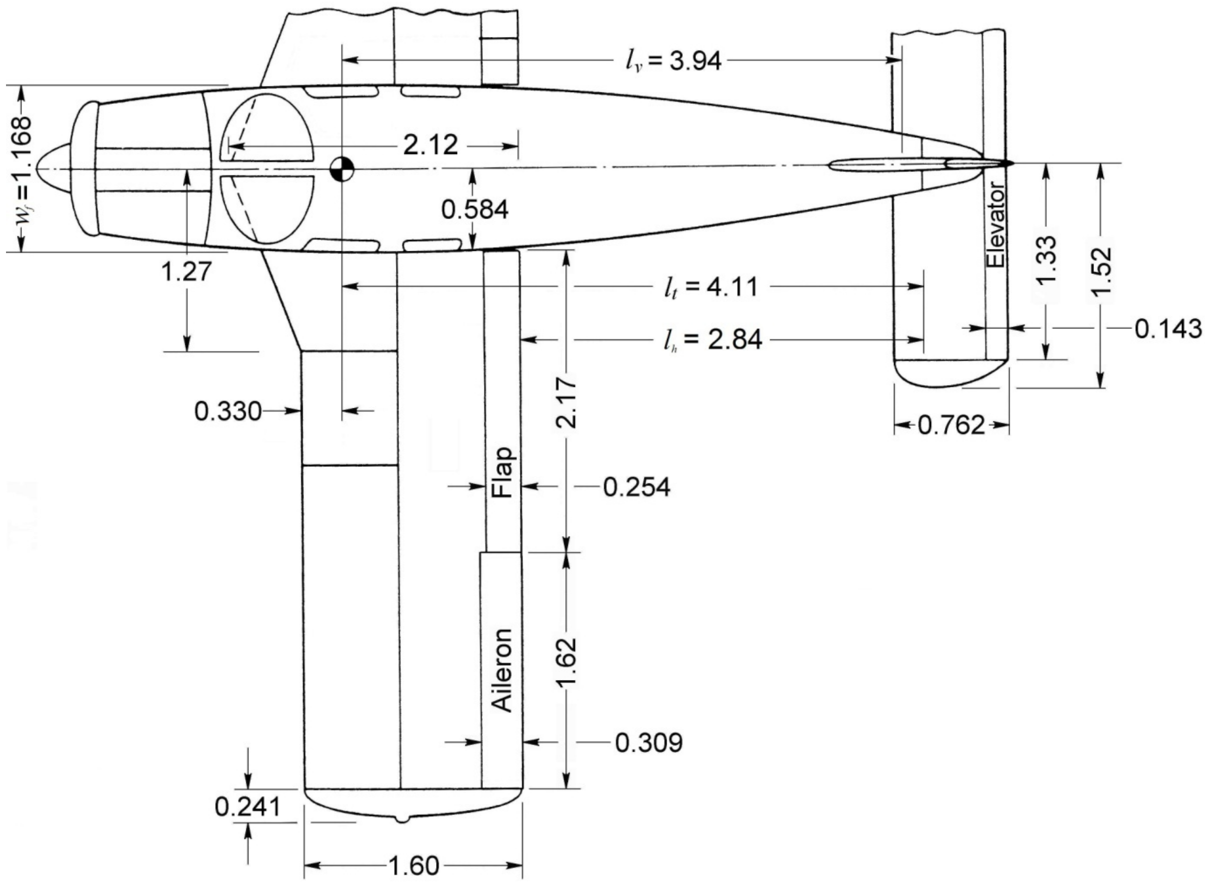

Figure 1: Top view of the Piper Cherokee PA-28-180 aircraft
(all dimensions are in metres)

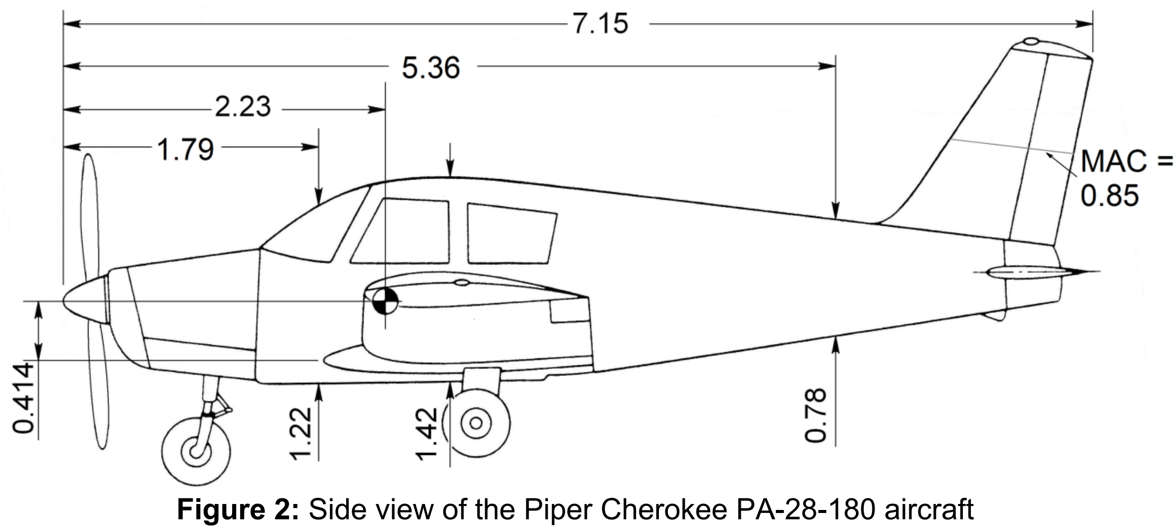

Figure 2: Side view of the Piper Cherokee PA-28-180 aircraft
(all dimensions are in metres)

Table 1: General Aircraft data for the Piper Cherokee PA-28-180

Wing		**Vertical Tail**	
Area S	= 14.9 m^2	Area S_v	= 1.059 m^2
Span b	= 9.23 m	Rudder Area S_R	= 0.334 m^2
Mean aerodynamic chord $\bar{c}$	= 1.60 m	a_v	= 3.04/rad
Wing Lift Curve Slope a_W	= 4.50/rad	η_V	= 1.1
i_w	= -1.01°	MAC $\bar{c}_V$	= 0.85 m
Oswald's efficiency factor e	= 0.6	**Horizontal Tail**	
Dihedral angle Γ_W	= 7°	Area S_H	= 2.27 m^2
Swept back angle $\Lambda_{c/4W}$	= 0°	Aspect Ratio AR_H	= 4.10
Taper ratio λ_W	= 0.985	a_H	= 3.68/rad
Aircraft		η_H	= 1.1
Mass at take-off	= 1105 kg	Elevator Area S_e	= 0.380 m^2
I_x	= 1450.3 kgm^2	**Fuselage**	
I_y	= 1693.0 kgm^2	$(dC_M/d\alpha)_F$	= 0.321/rad
I_z	= 3133.8 kgm^2		
I_{xz}	= 0 kgm^2	**Steady Level Flight Conditions**	
		Design Cruise speed	= 45 m/s
Drag Polar	$C_D = 0.03363 + 0.0943 C_L^2$	Cruise altitude	= 3000 m
		Air density ρ	= 0.9098kg/m^3

Assume the aircraft is flying at the cruise altitude with the steady and level flight conditions of Table 1.

1. Calculate the Horizontal Tail Volume Coefficient $\bar{V}_H$

Let us assume that this aircraft has a fixed tailplane and pitch control and trim is achieved through elevator deflections.

The elevator hinge moment can be assumed to be $C_H = -0.05\alpha_H - 0.59\delta_E - 1.146\delta_T$
The elevator contribution to tailplane lift is $a_{\delta_E} = 3.12$ 1/rad and the trim-tab's is negligible.
The downwash gradient at the tailplane ac is $d\varepsilon/d\alpha = 0.55$.

2. Calculate the horizontal tail setting angle (i_H) required for trimmed flight at the design cruise condition. To minimize drag in this condition $\delta_E = \delta_T = 0$.
3. Calculate the stick-fixed static margin. You may assume the fuselage has no effect. Is the aircraft stable in pitch?
4. Will the aircraft be speed stable?
5. How would an inclusion of the fuselage's aerodynamics into the analysis affect the stick fixed static margin and tail setting angle required for trim? You may assume that the fuselage generates no pitching moment at an angle of attack of zero.

The pilot adjusts the throttle and now flies at a cruise speed of 35 m/s at the same altitude.

6. Will the aircraft be speed stable?
7. Calculate the elevator deflection required to trim the aircraft in this flight condition
8. Calculate the trim-tab deflection required to trim the aircraft in the stick-free case.
9. Calculate the stick-free static margin.

<u>If you have found the above of interest:</u>

The actual PA-28-180 has an all moving tailplane, designed to rotate about its aerodynamic centre. The yoke is connected directly to the tailplane. As a result, control forces felt by the pilot would be very low. The elevator therefore functions as an anti-servo device, accentuating the control loads.

The pitching moment (about the tailplane aerodynamic center) generated by an elevator deflection can be given by $dC_{MH}/d\delta_E = -0.42$ 1/rad.
Stick control forces (F_c) can be related to the relevant hinge moment (H) by the control system's gearing ratio (G) using the equation

$$F_c = G\ H.$$

10. Derive an expression for the elevator to tailplane deflection ratio $(d\delta_E/di_H)$ required to give the pilot a minimum stick control force per tailplane deflection dF_c/di_H, as defined by airworthiness standards. What flight condition do you think this should be designed for?
11. In this case, how would the stick-free static margin differ from that found in Q9?

1. $\bar{V}_H = (\bar{x}_H - \bar{x}_W)\dfrac{S_H}{S_{ref}}$

$x_w = \frac{1}{4}\bar{c} = 0.4m$

$\therefore \bar{V}_H = \dfrac{(2.23m + 4.11m) - (2.23m - 0.33m + 0.4m)}{1.6m}\dfrac{2.27m^2}{14.9m^2} = 0.38468$

2. To achieve trimmed flight, $L = W$, $C_M = 0$

$C_L = \dfrac{L}{\frac{1}{2}\rho V^2 S_{ref}} = \dfrac{W}{\frac{1}{2}\rho V^2 S_{ref}}$ $\quad\therefore$ At take-off mass, $C_L = 0.7898$

$C_M = C_{M0W} + (\bar{x}_{CG} - \bar{x}_W)C_L - \eta_H \bar{V}_H C_{LH}$ ($C_{M0H} = 0$ for symmetric airfoil)

$= (\bar{x}_{CG} - \bar{x}_W)C_L - \eta_H \bar{V}_H C_{LH} = 0$

$\therefore \eta_H \bar{V}_H C_{LH} = (\bar{x}_{CG} - \bar{x}_W)C_L$

$\therefore C_{LH} = \dfrac{(\bar{x}_{CG} - \bar{x}_W)}{\eta_H \bar{V}_H}C_L = -0.0817$

$\because C_{LH} = a_H(\alpha_H + i_H - \varepsilon - \alpha_{0H}) + a_{\delta E}\delta_E + a_{\delta T}\delta_T = a_H[(1-\frac{d\varepsilon}{d\alpha})\alpha + i_H - \varepsilon_0 - \alpha_{0H}] + a_{\delta E}\delta_E + a_{\delta T}\delta_T$

$\therefore i_H = \dfrac{C_{LH}}{a_H} + (\dfrac{d\varepsilon}{d\alpha} - 1)\alpha + \varepsilon_0 + \alpha_{0H} - a_{\delta E}\delta_E - a_{\delta T}\delta_T = \dfrac{C_{LH}}{a_H} + (\dfrac{d\varepsilon}{d\alpha} - 1)\alpha + \varepsilon_0 + \alpha_{0H}$

for this plane $a_{0W} = a_{0H} = 0$

$\therefore \varepsilon = \dfrac{d\varepsilon}{d\alpha}(a + i_W - a_0 W) = \dfrac{d\varepsilon}{d\alpha}a + \varepsilon_0$

$\therefore \varepsilon_0 = \dfrac{d\varepsilon}{d\alpha}(i_W - a_{0W}) = -0.0097 rad$

$\because C_L = a(\alpha - \alpha_0) = C_{L0} + a\alpha = a_W(\alpha + i_W - a_{0W}) + \eta_H \dfrac{S_H}{S_{ref}} a_H[(1-\frac{d\varepsilon}{d\alpha})\alpha + i_H - \varepsilon_0 - \alpha_{0H}]$

$\therefore a = \dfrac{dC_L}{d\alpha} = a_W + \eta_H a_H \dfrac{S_H}{S_{ref}}(1 - \dfrac{d\varepsilon}{d\alpha}) = 4.775 /rad$

$\therefore C_{L0} = C_L - a\alpha = a_W(i_W - \alpha_{0W}) + \eta_H \dfrac{S_H}{S_{ref}} a_H(i_H - \varepsilon_0 - \alpha_{0H})$

$= -0.0733 + 0.6167 i_H$

$\therefore i_H = \dfrac{C_{LH}}{a_H} + (\dfrac{d\varepsilon}{d\alpha} - 1)(\dfrac{C_L - C_{L0}}{a}) + \varepsilon_0 + \alpha_{0H} = -0.12012 rad$

3. $K_n = \bar{x}_{np} - \bar{x}_{CG} = -\dfrac{dC_m}{dC_L}$

$\therefore K_n = -\bar{x}_{CG} + \bar{x}_W + \eta_H \bar{V}_H \dfrac{dC_{LH}}{dC_L}$

For $\dfrac{dC_{LH}}{dC_L}$, $\dfrac{dC_{LH}}{dC_L} = \dfrac{\frac{dC_{LH}}{d\alpha}}{\frac{dC_L}{d\alpha}} = \dfrac{a_H(1 - \frac{d\varepsilon}{d\alpha})}{a}$

$\therefore K_n = -\bar{x}_{cg} + \bar{x}_w + \eta_H \bar{V}_H \frac{a_t}{a}(1-\frac{d\varepsilon}{d\alpha}) = 0.1904$

4. $V_{imD} = (\frac{B}{A})^{\frac{1}{4}}$ for $L=W$ $\therefore V_{imD} = 44.59 m/s$ $\therefore V_{mD} = 51.759 m/s > 45 m/s$

$\therefore$ speed unstable

5. $C_M \uparrow$

$\therefore K_n \downarrow$ $\therefore$ destabilising

6. 7. 8. ...