1. Write down the dimensionally-correct form for the pitching moment coefficient of an aerofoil of chord c in fluid stream of speed u, the pitching moment <u>per unit span</u> being denoted by m. Justify the form by using a dimensional argument. *(see section 1-6)*

$$C_M = \frac{m}{\frac{1}{2}\rho u^2 c^2}$$

$$m \rightarrow \frac{Nm}{m} \rightarrow \frac{ML}{T^2}$$

$$\rho \rightarrow \frac{kg}{m^3} \rightarrow \frac{M}{L^3}$$

$$u^2 \rightarrow \frac{m^2}{s^2} \rightarrow \frac{L^2}{T^2}$$

$$c^2 \rightarrow m^2 \rightarrow L^2$$

$\frac{1}{2}\rho u^2 c^2$ hure unit of $\frac{M}{L^3} \cdot \frac{L^2}{T^2} \cdot L^2 = \frac{ML}{T^2}$

There for C_M is a dimensionless number

2. Which of the following quantities are dimensionless? Notation is standard; e.g. L=lift, A=area, γ=ratio of specific heats, υ=kinematic viscosity, c=chord, p=pressure, ρ=density

(i) L/pA

(ii) $\dfrac{L}{\frac{1}{2}\gamma p M^2 A}$ (where M is the Mach No.)

(iii) $\Delta p/\rho u$ (where Δp is a pressure difference)

(iv) $gh/(u\sqrt{\gamma p/\rho})$

(v) $\sigma/(\rho u^2 c)$ (where σ is surface tension with units of force per unit length)

(vi) $\Delta p\, d^2/(\rho \upsilon^2)$ (where d is length)

(vii) $(\tau_w/\rho)^{1/2} y/\upsilon$ (where τ_w is a shear stress and y is a length)

(viii) $\dfrac{\Delta\rho\, gh}{\rho u^2}$ (where Δρ is a density difference and h is a length)

(i) $\dfrac{N}{N/m^2 \cdot m^2} = \dfrac{N}{N} \rightarrow \checkmark$

(ii) $\dfrac{N}{J/kg{\cdot}K \cdot N/m^2 \cdot m^2} = kg{\cdot}K/J \quad \times$ $\qquad \dfrac{N}{N/m^2 \cdot m^2} = \dfrac{N}{N} \checkmark$

(iii) $\dfrac{Pa}{kg/m^3 \cdot m/s} = \dfrac{N/m^2}{kg/m^2 s} = \dfrac{kg/s}{kg/m^2 s} = m^2 \quad \times$ $\qquad \dfrac{N/m^2}{kg/m^3 \cdot m/s} = \dfrac{kgm/s^2}{kg\, s/m^2} = \dfrac{m}{s}$

(iv) $\dfrac{m/s^2 \cdot m}{m/s \cdot \sqrt{J/kg{\cdot}K/kg/m^3}} = \dfrac{m/s^3}{\sqrt{Jm^3/K}} \quad \times$ $\qquad \dfrac{m/s^2 \cdot m}{m/s\ (\frac{kgm/s^2}{kg/m^3})^{\frac{1}{2}}} = \dfrac{m^2/s^2}{m/s \cdot m/s} \checkmark$

(v) $\dfrac{N/m}{kg/m^3 \times m^2/s^2 \times m} = \dfrac{kgm/s^2}{kg/s^2} = m \quad \times$ $\qquad \dfrac{kg/s^2}{kg/m^3 \cdot m^2/s^2 \cdot m} = \dfrac{kg/s^2}{kg/s^2} \checkmark$

(vi) $\dfrac{N/m^2 \times m^2}{kg/m^3 \times (\frac{kg/ms}{kg/m^3})^2} = \dfrac{kgm^2/s^2}{kg/m^3 \times m^4/s^2} = \dfrac{kgm^2/s^2}{kg\,m/s^2} = m \quad \times$ $\qquad \dfrac{kg/ms^2 \cdot m^2}{kg/m^3 \cdot m^4/s^2} = \dfrac{kgm/s^2}{kgm/s^2} \checkmark$

(vii) $\left(\dfrac{(\frac{kgm/s^2}{m^2})}{kg/m^3}\right)^{\frac{1}{2}} \times \dfrac{m}{(\frac{kg/ms}{kg/m^3})} = \left(\dfrac{kg/ms^2}{kg/m^3}\right)^{\frac{1}{2}} \times \dfrac{m}{m^2/s} = \dfrac{m}{s} \times \dfrac{s}{m} \checkmark$

(viii) $\dfrac{\frac{kg}{m^3} \times \frac{m}{s} \times m}{\frac{kg}{m^3} \times (\frac{m}{s})^2} = \dfrac{\frac{kg}{ms}}{\frac{kg}{ms^2}} = \dfrac{kg}{ms} \cdot \dfrac{ms^2}{kg} = s \quad \times$ $\qquad \dfrac{kg/m^3 \cdot m/s^2 \cdot m}{kg/m^3 \cdot m^2/s^2} = \dfrac{kg/ms^2}{kg/ms^2} \checkmark$

3. Compare the percentage change in car aerodynamic drag that can be achieved by
   (i) Reduction of drag coefficient C_D from 0.35 to 0.3
   (ii) Reduction of linear dimensions of the car by 10 percent
   (iii) Reduction of speed from 70 to 65 mph
   (Ans -14.3%, -19%, -13.8%)

$$D = \frac{1}{2} C_D \rho U^2 A^2$$

(i) $$\frac{0.3 - 0.35}{0.35} \times 100\% = -14.286\% = -14.3\%$$

(ii) $$\frac{(1-10\%)^2 - 1}{1} \times 100\% = -19\%$$

(iii) $$\frac{65^2 - 70^2}{70^2} \times 100\% = -13.78\% = -13.8\%$$

4. A syringe plunger has a diameter of 50mm and moves in a cylinder of the same internal diameter. The other end of the syringe has an opening of 5mm diameter. If the syringe contains water and the plunger is forced into the syringe at a steady rate of 20mm/s, what will be the average velocity at which the water escapes? **(Ans 2000 mm/s)** (Hint: see section 1-3)

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

$$\therefore u_2 = \frac{\rho_1 u_1 A_1}{\rho_2 A_2} = \frac{20\,mm/s \times (50\,mm)^2}{(5\,mm)^2} = 2000\,mm/s$$

5. Read through Buckingham's rule in lecture 1-7. The drag on the hull of a ship depends in part on the height of water waves produced by the hull. The potential energy associated with these waves therefore depends on the acceleration due to gravity g. We can show that the wave drag on the hull D is a function of ρ_0, V_0, c and g, i.e.

$$D = f(\rho_0, V_0, c, g)$$

where c is a length associated with the hull (say its maximum width), ρ_0, is the density of the water and V_0 is the ships speed. If we define the non-dimensional drag coefficient as

$$C_D = \frac{D}{\frac{1}{2}\rho_0 V^2 c^2}$$

use dimensional analysis to shown that $C_D = f_1(\rho_0, V_0, c, g)$ is a function of the similarity parameter Fr where Fr = $V_0/\sqrt{gc}$ and is known as the Froude number. (see lecture 1-7)

$$C_D = f_1(\rho_0, V_0, c, g)$$

$$= (\rho_0)^{\alpha} (V_0)^{\beta} (c)^{\gamma} (g)^{\delta}$$

$$= \left[\frac{M}{L^3}\right]^{\alpha} \left[\frac{L}{T}\right]^{\beta} [L]^{\gamma} \left[\frac{L}{T^2}\right]^{\delta}$$

$$M \rightarrow \alpha = 0$$

$$L \rightarrow -3\alpha + \beta + \gamma + \delta = 0$$

$$T \rightarrow -\beta - 2\delta = 0$$

$$\therefore \delta = -\frac{1}{2}\beta$$

$$\therefore \gamma = -\frac{1}{2}\beta$$

$$\therefore C_D \propto (V_0)^{\beta} (c)^{-\frac{1}{2}\beta} (g)^{-\frac{1}{2}\beta}$$

$$\therefore C_D \propto \frac{V_0}{\sqrt{gc}}$$

6. The power required to propel an aircraft in steady level flight, at a height where the density is 1/3 of that sea level, is 20,000 kW. A 1/20 scale model of the aircraft is mounted in a wind tunnel, operating at sea-level temperature and a pressure of 5 atmospheres (1 atmosphere = sea-level pressure), for test at the same true air speed as the full scale aircraft.

   (i) Define a non-dimensional power coefficient (recall power=force x velocity)
   (ii) Assuming the same power coefficient for the full scale and model conditions estimate the power required to drive the wind tunnel when the model is inserted.
   (iii) If the full scale and model temperatures are 232K and 288K and presuming that viscosity is proportional to the square root of temperature, what is the ratio of the two Reynolds number of the two flows? Comment on how this might effect the prediction of the drag coefficient. **(Ans 750 Kw, 0.673)**

(i) $$F \times u = \frac{ML}{T^2} \times \frac{L}{T} = \frac{ML^2}{T^3}$$

$$p = \frac{M}{T^2L}, \quad \rho = \frac{M}{L^3}, \quad u^2 = \frac{L^2}{T^2} \qquad \therefore \frac{1}{2}\rho u^2 = \frac{M}{LT^2}$$

$$\therefore \frac{Fu}{\frac{1}{2}\rho u^2 x} = \frac{ML^2}{T^3} \times \frac{LT^2}{M} \times \frac{1}{x} = \frac{L^3}{T} \times \frac{1}{x} = 1$$

$$\therefore x = \frac{L^3}{T} = uA \qquad \therefore C_p = \frac{Fu}{(\frac{1}{2}\rho u^2)uA}$$

(ii) $$p \propto \rho u^2, \quad P = \rho RT$$

$$\therefore C_p = \frac{P}{\frac{1}{2} \times \frac{1}{3} \times 1} = \frac{P_m}{\frac{1}{2} \times 1 \times (\frac{1}{20})^2 \times 5}$$

$$\therefore P_m = 750\ \text{kW}$$

(iii) $$\frac{Re}{Re'} = \frac{\frac{\rho u c}{\mu}}{\frac{\rho' u' c'}{\mu'}} = \frac{\rho u c}{\rho' u' c'} \frac{\mu'}{\mu} = \frac{\rho u c}{\rho' u' c'} \sqrt{\frac{T'}{T}}$$

$$= \frac{\frac{1}{3} \times 1 \times 1}{5 \times 1 \times 1 \times (\frac{1}{20})} \sqrt{\frac{288}{232}} = 1.486$$

$$\therefore \frac{Re'}{Re} = 0.673$$