AERO40005 PROPERTIES OF MATERIALS

Tutorial questions

By Dr. Q Li

Note: Questions 1-4 are for "Elasticity"

Questions 5-7 are for "Plasticity"

Question 1

(a) What are the three most common crystal structures for metals? Name the densely packed planes and directions for the two cubic systems (for "Plasticity).

(b) Gold has an fcc structure and a lattice constant of 0.40788 nm. How many atoms are in the fcc unit cell? Calculate the value for the atomic radius of gold based on this information. Indicate the following planes and direction (100), (111), [110], [210] in the fcc lattice.

(c) Pure iron is polymorphic and changes from bcc to fcc at 912°C. Calculate the density change associated with this structural change. Lattice parameter of α-iron (bcc) is a=0.293 and of γ-iron (fcc) a=0.363 nm.

(a) HCP (Hexagonal closely Packed crystals)

FCC (Face Central Cubic)

· density packed planes {1 1 1}

· density packed directions <1 1 0>

BCC (Body Central Cubic)

· no density packed planes

· density packed directions <1 1 1>

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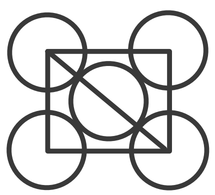
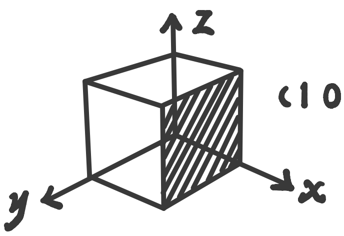
(b) $N = \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$



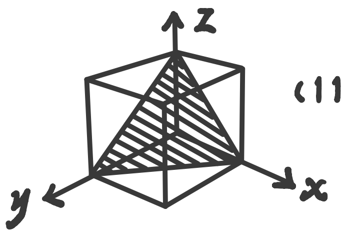
According to the diagram.

$a^2 + a^2 = [(1+2+1)r]^2 = 16r^2$

$\therefore r = \frac{\sqrt{2}}{4} a = 0.14421\,nm$



(100)



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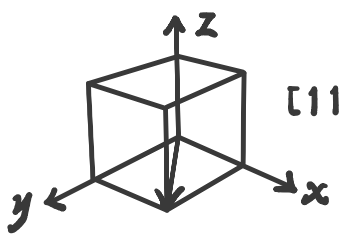
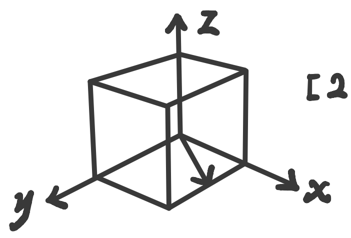


[110]



[210]

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(c) $N_{bcc} = \frac{1}{8} \times 8 + 1 = 2$

$N_{fcc} = \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$

$$\rho_{bcc} = \frac{m}{V} = \frac{2M}{(0.293\,nm)^3}$$

$$\rho_{fcc} = \frac{m}{V} = \frac{4M}{(0.363\,nm)^3}$$

$$\therefore \Delta\rho = \frac{\rho_{fcc} - \rho_{bcc}}{\rho_{bcc}} \times 100\% = 5.175\%$$

Question 2

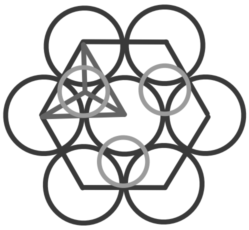
Magnesium has a hexagonal close-packed structure.

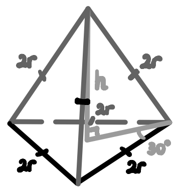
(a) Assuming that the atoms can be represented as hard spheres, calculate the percentage of the available volume occupied by atoms in each material, and comment on your answer.

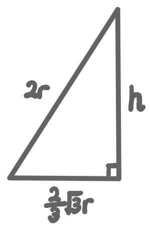
(b) The density of magnesium crystal is 1740 kg m^{-3} respectively. Calculate (i) the dimensions of the unit cells, and hence (ii) the radii of magnesium atoms.

Hint: For magnesium of HCP structure, 1 atom weighs 24.31 amu (this information can be taken from the periodic table, 1 amu =$1.661*10^{-27}$ kg).

(a)







$\therefore h = \frac{2}{3}\sqrt{6}r$

$$N = 3 + 12 \times \frac{1}{6} + 2 \times \frac{1}{2} = 6$$

$$V = A(2h) = 6\left(\frac{1}{2} \times 2r \times \sqrt{3}r\right) \times \left(2 \times \frac{2}{3}\sqrt{6}r\right) = 24\sqrt{2}r^3$$

$$V_{atom} = 6\left(\frac{4}{3}\pi r^3\right) = 8\pi r^3$$

$$\therefore \%AV = \frac{V_{atom}}{V} \times 100\% = 74.0\%$$

(b) (i) $$V = \frac{m}{\rho} = \frac{6 \times 24.31 \times 1.661 \times 10^{-27}\,kg}{1740\,kg/m^3} = 1.3924 \times 10^{-28}\,m^3$$

(ii) $V = 24\sqrt{2}r^3$ $\quad \therefore r = \sqrt[3]{\frac{V}{24\sqrt{2}}} = 1.6008 \times 10^{-10}\,m$

$$\therefore h = \frac{2}{3}\sqrt{6}r = 2.6141 \times 10^{-10}\,m$$

Question 3

The potential energy of a pair of adjacent atoms in a solid is

$$U = -\frac{A}{r^m} + \frac{B}{r^n}$$

where r is the distance between atom centres.

(a) Express this relationship in terms of the equilibrium spacing, r_0, of the atoms and the potential energy at this spacing, U_0.

(b) For n=6 and m=2 and a bond energy of -3.2 eV and r_0= 0.4 nm calculate A and B (N.B. 1eV = 1.602×10^{-19} Joules).

(a) $U_0 = -\frac{A}{(r_0)^m} + \frac{B}{(r_0)^n}$

(b) $\frac{dU}{dr} = mAr^{-m-1} - nBr^{-n-1} = 0$ when $U = U_0$

$\therefore\ 2A(0.4\times10^{-9}m)^{-3} - 6B(0.4\times10^{-9}m)^{-7} = 0$ $\quad\therefore\ B = \frac{1}{3}A(0.4\times10^{-9}m)^4$

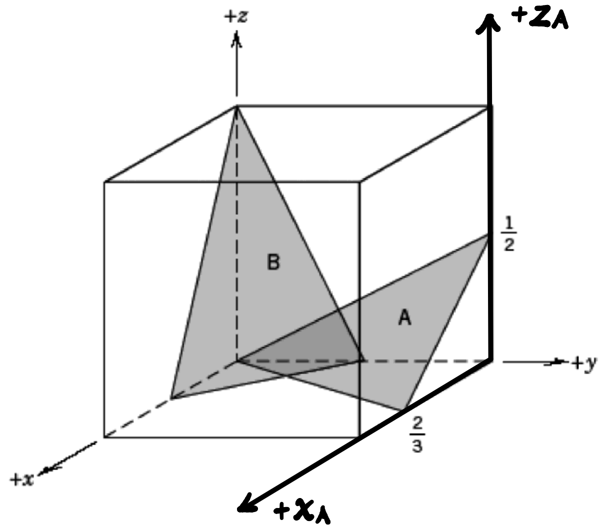
$\because\ -3.2\times1.602\times10^{-19}J = -\frac{A}{(0.4\times10^{-9}m)^2} + \frac{B}{(0.4\times10^{-9}m)^6} = -\frac{2}{3}\frac{A}{(0.4\times10^{-9}m)^2}$

$\therefore\ A = 1.230\times10^{-37}$

$B = 1.050\times10^{-75}$

Questions 4

Determine the Miller indices for the planes shown in the following unit cell:



For A,

intercept $[\frac{2}{3}\ \bar{1}\ \frac{1}{2}]$

reciprocals $(\frac{3}{2}\ \bar{1}\ 2)$

$\therefore$ plane $(3\ \bar{2}\ 4)$

For B,

intercept $[\frac{1}{2}\ \frac{1}{2}\ 1]$

reciprocals $(2\ 2\ 1)$

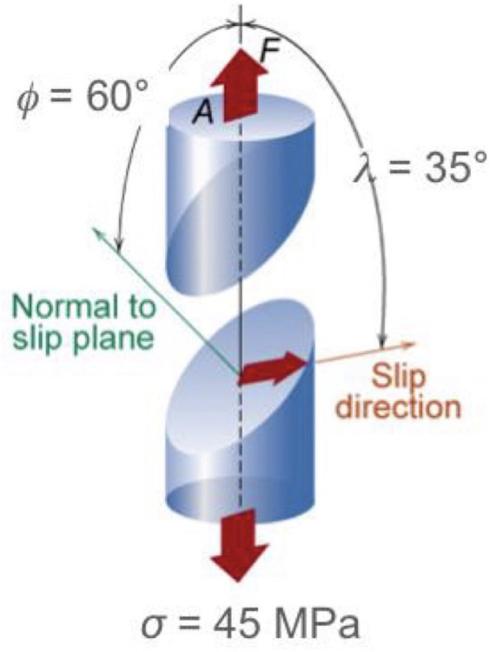
$\therefore$ plane $(2\ 2\ 1)$

Question 5

A bar of cross-section A is made from a single-crystal metallic material, with one active slip system identified as shown in the figure below. Critical resolved shear stress Tcrss = 20.7MPa

(a) Will this single crystal yield?

(b) If not, what stress is needed?



(a) $\tau = \frac{F_s}{A_s}$

$= \frac{F\cos\lambda}{\frac{A}{\cos\phi}}$

$= \frac{F}{A}\cos\lambda\cos\phi$

$= \sigma\cos\lambda\cos\phi$

$= 18.43\,MPa$

$< \tau_{crss}$

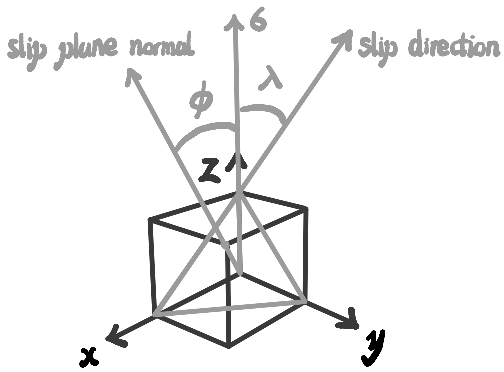
(b) $\sigma_n \cos\lambda\cos\phi > \tau_{crss}$

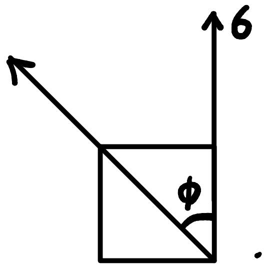
$\sigma_n > \frac{\tau_{crss}}{\cos\lambda\cos\phi}$

$\therefore \sigma_n > 50.54\,MPa$

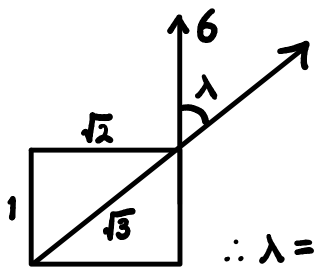
Question 6

Consider a single crystal of silver oriented such that a tensile stress is applied along a [001] direction. If slip occurs on a (111) plane and in a $[\bar{1}01]$ direction, and is initiated at an applied tensile stress of 1.4 MPa, calculate the critical resolved shear stress.





$\therefore \phi = 45°$



$\therefore \lambda = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.736°$

$$\therefore \tau = \sigma \cos\lambda \cos\phi = 0.5715 \text{ MPa}$$

Question 7

(a) Describe in your own words the four strengthening mechanisms.

(b) The lower yield point for iron with an average grain diameter of $5*10^{-2}$ mm is 135 MPa. At a grain diameter of $8*10^{-3}$ mm, the yield point increases to 260 MPa. At what grain diameter will the yield point be 205 MPa?

(a) ...

(b) $\sigma_Y = a + kd^{-\frac{1}{2}}$

$\therefore\ 135 \times 10^6\,Pa = a + k(5 \times 10^{-5}\,m)^{-\frac{1}{2}}$

$260 \times 10^6\,Pa = a + k(8 \times 10^{-6}\,m)^{-\frac{1}{2}}$

$\therefore\ a = 51666666.\dot{6}$

$k = 589255.651$

$\therefore$ When $\sigma_Y = 205\,MPa$

$d = 1.4768431 \times 10^{-5}\,m$

$= 1.48 \times 10^{-2}\,mm$