# Tutorial 2

## Check your knowledge

d 1. An airspeed indicator shows the _________ based on pressure measurements taken from a ______________.

   (a) "ground speed" - "static port"
   (b) "equivalent airspeed" - "static port"
   (c) "true airspeed" - "pitot tube"
   (d) "equivalent airspeed" - "pitot-static tube"

a 2. The altimeter shows _________ based on pressure measurements taken from a ______________.

   (a) "geopotential height" - "static port"
   (b) "geometric height" - "pitot tube"
   (c) "geometric height" - "static port"
   (d) "density altitude" - "pitot tube"

3. Are the following statements *true* or *false*?

   Unless stated otherwise, you may assume a standard day and that gravitational acceleration is a function of altitude alone.

   T (a) At sea level, the geometric and geopotential heights are both zero.
   F (b) At any given pressure level, above sea level, the geopotential height will be higher less than the geometric height. $\int_0^{h_t} g\,dz = g_0 h_g$
   F (c) The Standard Atmosphere model given as Table 1, indicates that at a geometric geopotential height of 20 km, the atmospheric pressure will be 5,475 Pa.
   T (d) The reference sea-level temperature $T_0$ for the ISA model is 15°C.
   F (e) Equivalent Airspeed readings taken by a pilot on a non-standard day ($T \neq T_0$ or $P_0 \neq 101325$ Pa) will not be correct.
   T (f) Using ISA tables to find the True Airspeed of an aircraft from Equivalent Airspeed readings taken on a non-standard day will yield incorrect results.
   F (g) The ground speed of an aircraft can be calculated by adding a known tailwind component to its equivalent true airspeed.
   F (h) Thrust specific fuel consumption is defined as the mass of fuel consumed per unit thrust per unit time produced.
   F (i) The fuel consumption of a turboshaft engine is typically expressed in terms of thrust power specific fuel consumption.
   F (j) The thrust produced at a given altitude is a function of the ~~local pressure only~~ $\rho$.
   T (k) All other things being equal, an increase in temperature would result in a reduction of lift being generated. $P = \rho RT$
   F (l) Wave drag is encountered only in the supersonic transonic speed regime and arises due to the formation of shock waves over the aircraft.
   T (m) Wind axes are a type of body-fixed axis system, where the $x$-axis is aligned to the freestream flow.

## Application Exercise

The Perlan II, seen in figure 1, is a pressurized sailplane designed to fly up to an altitude of 90,000 ft to conduct atmospheric research. The drag curve of the aircraft can be assumed to be parabolic and is known to be approximately

$$C_D = 0.012 + 0.013C_L^2.$$

The aircraft has a fully loaded mass of 816 kg, a wing area of 24.4 $m^2$ and a maximum lift coefficient $C_{L_{max}} = 1.4$.

Figure 1: Rendering of Perlan II in high altitude flight (Airbus)

1. Derive the 2D axial equations of motion for a glider in wings level flight. Hence show that for a steady, constant slope glide

$$\tan\gamma = -\frac{C_D}{C_L}$$

   For the purposes of this question you may assume small angles. [30%]

2. What is the minimum possible flight speed for the aircraft at 90,000 ft? Express your answer in EAS, TAS and Mach number. How would these quantities change with altitude? [20%]

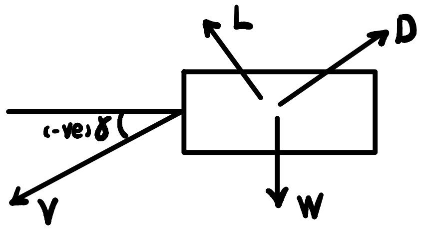
3. Starting from an altitude of 50,000 ft and maintaining a constant EAS of 54 kts, what distance over ground will the aircraft be able to cover? [20%]

4. What lift coefficient should the pilot be operating the aircraft at to maximise the glide distance possible? [30%]

### Extra Credit

5. How might your result above differ from the lift coefficient required to achieve the maximum flight time? Is this flight condition achievable?

1.




$$L = W\cos(-\gamma) = W\cos\gamma = \frac{1}{2}\rho V^2 S C_L$$

$$D = W\sin(-\gamma) = -W\sin\gamma = \frac{1}{2}\rho V^2 S C_D$$

$$\therefore \tan\gamma = -\frac{D}{L} = -\frac{C_D}{C_L}$$

2. $W = L = \frac{1}{2}\rho_0 V_s^2 S C_{L\max} = mg = 816\,kg \times 9.81\,m/s^2 = 8004.96\,N$

$$\therefore V_{s,EAS} = \left(\frac{2W}{\rho_0 S C_{L\max}}\right)^{\frac{1}{2}} = 19.560\,m/s$$

$$\because \frac{1}{2}\rho V_{s,TAS}^2 = \frac{1}{2}\rho_0 V_{s,EAS}^2$$

At 90000 ft, $h = 27432.33\,m$

$$\therefore V_{s,TAS} = \left(\frac{\rho_0 V_{s,EAS}^2}{\rho}\right)^{\frac{1}{2}} = \left(\frac{V_{s,EAS}^2}{6}\right)^{\frac{1}{2}} = 138.033\,m/s$$

$$\therefore M = \frac{V_{s,TAS}}{c} = \frac{138.033\,m/s}{300.5\,m/s} = 0.459$$

3. When $V_{EAS} = 54\,kts = 27.78\,m/s$, $h = 50000\,ft = 15240.19\,m$

$$\tan\gamma = -\frac{C_D}{C_L} = -\frac{0.012 + 0.013C_L^2}{C_L}$$

For $C_L$, $C_L = \frac{W}{\frac{1}{2}\rho_0 V_{EAS}^2 S} = 0.694$

$$\therefore \tan\gamma = -8.268 \times 10^{-3}$$

$$\because \tan\gamma = \frac{\Delta h}{\Delta d}$$

$$\therefore d = \frac{-h}{\tan\gamma} = 579195\,m$$

4. $\because d = \dfrac{-h}{\tan\gamma}$

$\therefore$ to maximise $d$, minimise $-\tan\gamma$

$$\because -\tan\gamma = \frac{0.012 + 0.013C_L^2}{C_L} = \frac{0.012}{C_L} + 0.013C_L$$

$$\therefore \frac{d(-\tan\gamma)}{dC_L} = -0.012C_L^{-2} + 0.013 = 0$$

$$\therefore C_L = \frac{2}{13}\sqrt{39} = 0.961$$

5. For maximum flight time, $V_i$ constant

$$t = \frac{d}{V_{EAS}\cos\gamma} = \frac{-h\tan\gamma}{\left(\frac{2W}{\rho_0 C_L S}\right)^{\frac{1}{2}}\cos\gamma} = \frac{h(\rho_0 C_L S)^{\frac{1}{2}}(0.012 + 0.013C_L^2)}{(2W)^{\frac{1}{2}} C_L \cos\gamma}$$

$$= \frac{h(\rho_0 C_L S)^{\frac{1}{2}}(0.012 + 0.013C_L^2)}{(2W)^{\frac{1}{2}} C_L} \frac{2W}{\rho_0 V_i^2 S C_L} = \frac{h(2W)^{\frac{1}{2}}(0.012 + 0.013C_L^2)}{(\rho_0 S)^{\frac{1}{2}} V_i^2 C_L^{\frac{3}{2}}}$$

$\therefore t \propto \dfrac{0.012 + 0.023C_L^2}{C_L^{\frac{3}{2}}}$ $\quad\therefore$ let $t_{C_L} = 0.012C_L^{-\frac{3}{2}} + 0.023C_L^{\frac{1}{2}}$

$\therefore \dfrac{dt_{C_L}}{dC_L} = -\dfrac{9}{500}C_L^{-\frac{5}{2}} + \dfrac{23}{2000}C_L^{-\frac{1}{2}} = 0$ $\quad\therefore C_L = \dfrac{6}{23}\sqrt{23}$

$\therefore \dfrac{d^2 t_{C_L}}{dC_L^2} = \dfrac{9}{200}C_L^{-\frac{7}{2}} - \dfrac{23}{4000}C_L^{-\frac{3}{2}}$ when $C_L = \dfrac{6}{23}\sqrt{23}$ $(C_L > 0)$

$= 0.016 > 0$ $\quad\therefore$ minimum point exists

$\therefore$ The flight condition is not achievable

Table 1: Atmospheric properties in the lower atmosphere (1976 COESA model)

| Altitude $h$ (m) | Temperature $T$ (K) | Pressure $P$ (Pa) | Density ratio $\sigma$ | Speed of Sound $a_s$ (m/s) |
|---|---|---|---|---|
| 0 | 288.1 | 101325 | 1.00000 | 340.3 |
| 500 | 284.9 | 95461 | 0.95287 | 338.4 |
| 1000 | 281.6 | 89875 | 0.90746 | 336.4 |
| 1500 | 278.4 | 84556 | 0.86373 | 334.5 |
| 2000 | 275.1 | 79495 | 0.82162 | 332.5 |
| 2500 | 271.9 | 74683 | 0.78111 | 330.6 |
| 3000 | 268.6 | 70109 | 0.74214 | 328.6 |
| 3500 | 265.4 | 65764 | 0.70468 | 326.6 |
| 4000 | 262.1 | 61640 | 0.66868 | 324.6 |
| 4500 | 258.9 | 57728 | 0.63410 | 322.6 |
| 5000 | 255.6 | 54020 | 0.60091 | 320.5 |
| 5500 | 252.4 | 50507 | 0.56907 | 318.5 |
| 6000 | 249.1 | 47181 | 0.53853 | 316.4 |
| 6500 | 245.9 | 44035 | 0.50926 | 314.4 |
| 7000 | 242.6 | 41061 | 0.48123 | 312.3 |
| 7500 | 239.4 | 38251 | 0.45439 | 310.2 |
| 8000 | 236.1 | 35600 | 0.42871 | 308.1 |
| 8500 | 232.9 | 33099 | 0.40415 | 305.9 |
| 9000 | 229.6 | 30742 | 0.38069 | 303.8 |
| 9500 | 226.4 | 28524 | 0.35829 | 301.6 |
| 10000 | 223.1 | 26436 | 0.33690 | 299.5 |
| 10500 | 219.9 | 24474 | 0.31651 | 297.3 |
| 11000 | 216.6 | 22632 | 0.29708 | 295.1 |
| 11500 | 216.6 | 20916 | 0.27455 | 295.1 |
| 12000 | 216.6 | 19330 | 0.25374 | 295.1 |
| 12500 | 216.6 | 17865 | 0.23450 | 295.1 |
| 13000 | 216.6 | 16510 | 0.21672 | 295.1 |
| 13500 | 216.6 | 15259 | 0.20029 | 295.1 |
| 14000 | 216.6 | 14102 | 0.18510 | 295.1 |
| 14500 | 216.6 | 13033 | 0.17107 | 295.1 |
| 15000 | 216.6 | 12045 | 0.15810 | 295.1 |
| 15500 | 216.6 | 11131 | 0.14611 | 295.1 |
| 16000 | 216.6 | 10287 | 0.13504 | 295.1 |
| 16500 | 216.6 | 9508 | 0.12480 | 295.1 |
| 17000 | 216.6 | 8787 | 0.11534 | 295.1 |
| 17500 | 216.6 | 8121 | 0.10659 | 295.1 |
| 18000 | 216.6 | 7505 | 0.09851 | 295.1 |
| 18500 | 216.6 | 6936 | 0.09104 | 295.1 |
| 19000 | 216.6 | 6410 | 0.08414 | 295.1 |
| 19500 | 216.6 | 5924 | 0.07776 | 295.1 |
| 20000 | 216.6 | 5475 | 0.07187 | 295.1 |
| 22000 | 218.6 | 4000 | 0.05202 | 296.4 |
| 24000 | 220.6 | 2930 | 0.03777 | 297.8 |
| 26000 | 222.6 | 2153 | 0.02750 | 299.1 |
| 28000 | 224.6 | 1586 | 0.02008 | 300.5 |
| 30000 | 226.6 | 1172 | 0.01470 | 301.8 |