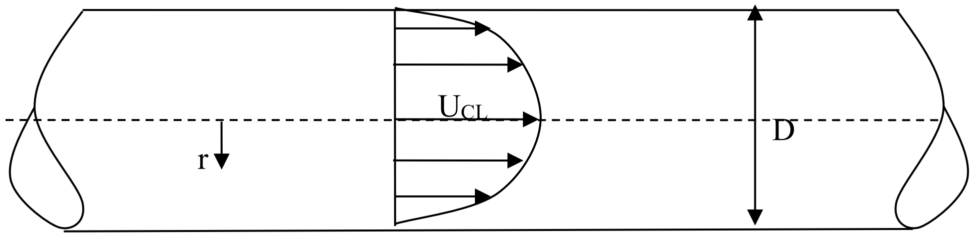


1. The figures shows the velocity profile of laminar flow in a circular pipe. The shape is a parabola so that at the wall of the pipe, $du/dr = -2U_{CL}/(D/2)$. Work out the "friction coefficient" $C_f = \tau_\varpi/(\frac{1}{2}\,\rho\, u_{CL}^2)$ as a function of U_{CL}, D, ρ, μ and demonstrate that $C_f=8/Re_D$ where Re_D is based on the centreline velocity U_{CL}. Evaluate C_f for airflow in a pipe of 8mm diameter with a centre-line speed of $3ms^{-1}$ assuming $\rho = 1.19$ kg m^{-3}, $\mu = 1.82\text{x}10^{-5}$ kg $m^{-1}s^{-1}$. If the centre-line velocity is raised to $15ms^{-1}$ would you expect the flow to be laminar or turbulent? **(Ans: $C_f = 5.1 \times 10^{-3}$)**

Note: the friction coefficient is usually based on the average velocity over the cross section, which for a paraboloid profile in a circular pipe is $u_{CL}/2$.

$$\left.\frac{du}{dr}\right|_w = \frac{-2U_{CL}}{\frac{D}{2}} = \frac{-4U_{CL}}{D}, \quad U_\infty = U_{CL}$$

$$\tau_w = \mu \left.\frac{du}{dr}\right|_w = \mu \frac{4U_{CL}}{D}$$

$$\therefore C_f = \frac{\tau_w}{\frac{1}{2}\rho (U_\infty)^2} = \frac{\mu \frac{4U_{CL}}{D}}{\frac{1}{2}\rho (U_{CL})^2} = \frac{8\mu}{\rho U_{CL} D}$$

$$= \frac{8}{Re_D}$$

When $U_{CL} = 3m/s$

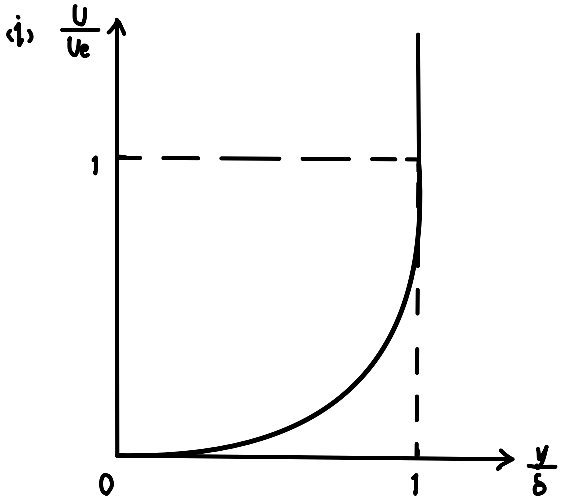
$$\therefore C_f = \frac{8 \times 1.82 \times 10^{-5}\, kg/ms}{1.19\, kg/m^3 \times 3\, m/s \times 8\, mm} = 5.098 \times 10^{-3}$$

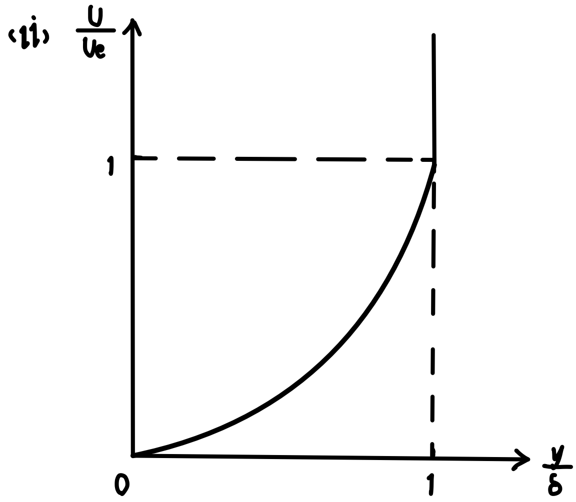
When $U_{CL} = 15m/s$

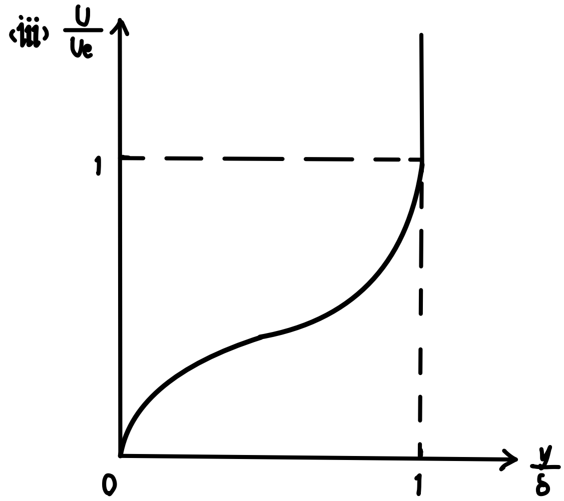
$$\therefore C_{f2} = \frac{3m/s}{15m/s} C_f = 1.0196 \times 10^{-3}$$

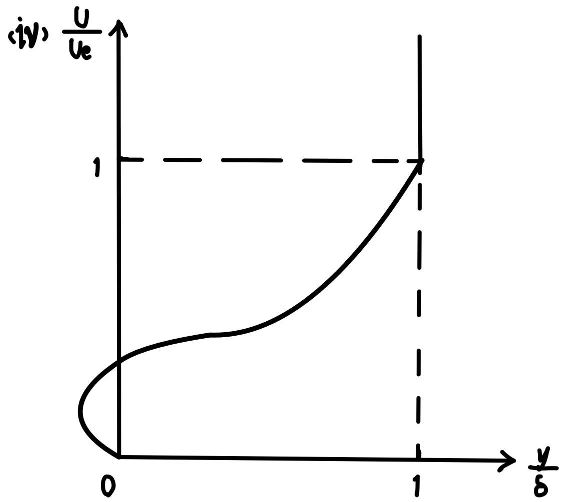
$$\therefore Re_{D2} = \frac{8}{C_{f2}} = 7846 > 2500$$ $\therefore$ The flow will be turbulent

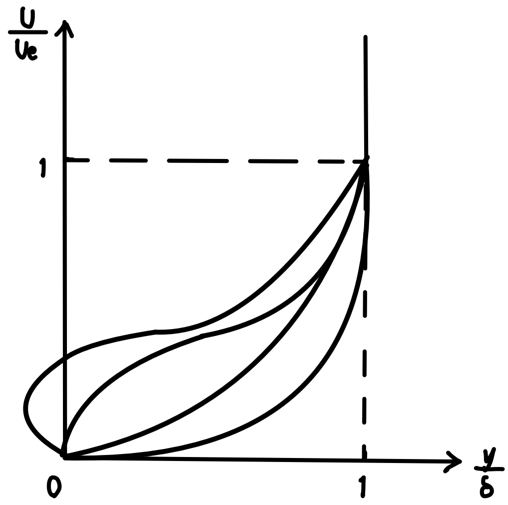
2. Draw velocity profiles, as u/u_e vs. y/δ for laminar boundary layers in
   - (i) favourable pressure gradient
   - (ii) zero pressure gradient
   - (i) adverse pressure gradient
   - (ii) separated flow











3. Calculate the thickness of the laminar boundary layer on a flat plate at distances of 1m and 2m from the leading edge, for free steam velocities of 2m/s and 4m/s. Compare the results and discuss. **(Ans at 1m: 14.6mm, 10.4mm at 2m: 20.8mm, 14.6mm)**

On a flat plane, $\frac{dp}{dx} = 0$

$$\therefore \delta = 5.3\sqrt{\frac{\nu x}{U_e}}$$

For air in normal condition,

$$\nu = \frac{\mu}{\rho} = \frac{1.82\times10^{-5}\ kg/ms}{1.19\ kg/m^3} = 1.52941\times10^{-5}\ m^2/s$$

When $x = 1m$, $U_e = 2m/s$

$\delta = 0.0147m = 14.7\ mm$

When $x = 1m$, $U_e = 4m/s$

$\delta = 0.0104\ m = 10.4\ mm$

When $x = 2m$, $U_e = 2m/s$

$\delta = 0.0207\ m = 20.7\ mm$

When $x = 2m$, $U_e = 4\ m/s$

$\delta = 0.0147\ m = 14.7\ mm$

$\therefore$ $\delta\uparrow$ when $x\uparrow$ while $\delta\downarrow$ when $U_e\uparrow$

4. The velocity U within the boundary layer on an aerofoil at a certain position from the leading edge and a distance y from a surface, is given by $\frac{U}{U_e} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$. U_e is the velocity at the edge of the boundary layer and δ is the boundary layer thickness. If the aerofoil is travelling through air at standard atmospheric conditions calculate the wall shear stress for U_e= 30m/s and δ=6mm and calculate the local pressure gradient along the aerofoil surface **(Ans 0.182 Nm^{-2}, -30.33Nm^{-3})**

$$\frac{U}{Ue} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$$

$$\therefore \frac{U}{30} = \frac{1000}{3}y - \frac{250000}{9}y^2$$

$$\therefore U = 10000y - \frac{2500000}{3}y^2$$

$$\therefore \frac{du}{dy} = 10000 - \frac{5000000}{3}y$$

$$\therefore \tau_w = \mu \frac{du}{dy}\Big|_{y=0} = 1.82\times10^{-5} \times 10000$$

$$= 0.182\ (N/m^2)$$

$$\because \frac{\partial u}{\partial t} + u\frac{du}{dx} + v\frac{du}{dy} + \frac{1}{\rho}\frac{dp}{dx} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$\therefore \frac{dp}{dx} = \mu\frac{d^2u}{dy^2}$$

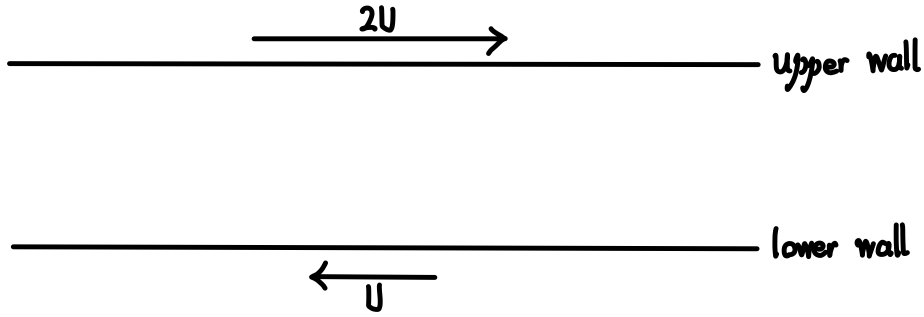
$$= 1.82\times10^{-5} \times \left(-\frac{5000000}{3}\right)$$

$$= -\frac{91}{3}\ (N/m^3)$$

$$= -30.333\ (N/m^3)$$

5. A two-dimensional, parallel- side duct of height 2h has its upper wall moving with velocity 2U and its lower wall moving in the opposite direction with velocity U. The movement of the walls causes a flow in the duct. Starting from the equations of motion for steady viscous boundary layer flow, derive an expression for the velocity profile across the duct. Noting that dp/dx = 0 determine the distribution of velocity and vorticity across the duct.

(Ans u(y)=U/2+3/2 Uy/h, ω(y) =-3U/(2h) with y=0 in the middle of the duct)



$$\because \rho \frac{\partial u}{\partial x} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{dp}{dx} = \mu \frac{d^2u}{dy^2}$$

$\therefore$ In steady viscous boundary flow

$$\frac{dp}{dx} = \mu \frac{d^2u}{dy^2} \qquad \therefore \frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}$$

$$\therefore \frac{du}{dy} = \frac{1}{\mu}\frac{dp}{dx}y + C \qquad \therefore u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + Cy + D$$

When $\frac{dp}{dx} = 0 \qquad \therefore \frac{du}{dy} = C \qquad \therefore u = Cy + D$

When $y = h,\ u = 2U \qquad \therefore 2U = Ch + D$

When $y = -h,\ u = -U \qquad \therefore -U = -Ch + D$

$$\therefore C = \frac{3U}{2h} \qquad \therefore D = \frac{1}{2}U$$

$$\therefore u(y) = \frac{3U}{2h}y + \frac{1}{2}U$$

$$\omega(y) = \frac{dv}{dx} - \frac{du}{dy} = -\frac{du}{dy} = -\frac{3U}{2h}$$