

# Energy

## Units

$$E/W \text{ Joule } 1J = 1N \cdot m \quad kWh \quad 1kWh = 3.6 \times 10^6 J \quad eV \text{ electron volt } 1eV = 1.6 \times 10^{-19} J$$

$$MToe \text{ (million tonnes of oil equivalent)} \quad 1MToe = 4.19 \times 10^{16} J$$

$$\text{Power Watts } 1W = 1J/s$$

## Combustion

HHV (higher heating value)



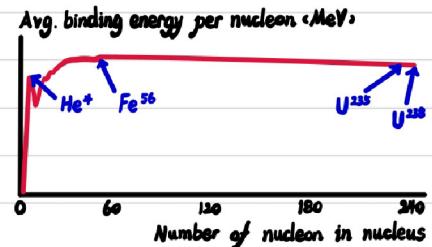
LHV (Lower heating value)



## Nuclear Power

Fusion lighter than iron  ${}^2H + {}^3H \rightarrow {}^4He + 3.5 \text{ MeV} + (n + 14.1 \text{ MeV})$

Fission heavier than iron  $U-235 + n \rightarrow Ba-144 + Kr-90 + 2n + 200 \text{ MeV}$



RTGs (Radioisotope power systems)

Activity  $A = \lambda N \quad 1Ci = 3.7 \times 10^{10} \text{ Bq} = 3.7 \times 10^{10} \text{ decay/s}$

Radio active decay  $N(t) = N_0 e^{-\lambda t}$

$$\text{half-life } \frac{1}{2} N_0 = N_0 e^{-\lambda t_{1/2}} \longrightarrow \lambda = \frac{\ln(2)}{t_{1/2}}$$

## Gas Laws and Kinetic Theory

Ideal gas law

$$\text{Ideal gas law*} \quad f(T, V, P) = 0 \longrightarrow PV = Nk_b T = MRT = n\bar{R}T \quad *M = nM_m$$

Avogadro number  $N_A = 6.023 \times 10^{26} \text{ mol}^{-1}$  number of particles in 1 kilomole \*  $N = n N_A$

Specific volume  $\gamma = \frac{V}{M} = \frac{1}{\rho} \longrightarrow P = \rho RT$

Mixtures  $P = \sum_i P_i \longrightarrow PV = \sum_i P_i V = \sum_i x_i nRT = \sum_i M_i RT$

## Kinetic Theory

Ideal gas molecules of the gas

- behave like hard spheres which move in a continuous, random motion
- are distributed uniformly throughout the container
- exert no force on each other except when they collide
- move in straight lines between collisions either with other molecules or container walls

Mean free path\* average distance travelled between collisions  $\lambda = \frac{1}{\sqrt{2} n \sigma} , n = \frac{N}{V}$

Molecular speed\* for monoatomic ideal gas  $PV = Nk_B T = \frac{1}{3} Nm\bar{v}^2 \longrightarrow \frac{1}{2} m\bar{v}^2 = \frac{3}{2} k_B T$

Viscosity  $\tau = \mu \frac{dy}{dx} \longrightarrow \mu = \frac{1}{3\sqrt{2}} n m \bar{v} \lambda = \frac{1}{3\sqrt{2}} \frac{n\bar{v}}{6}$

Diffusion in gas  $J = -D \frac{dn}{dx} \longrightarrow \text{coefficient of diffusion } D = \frac{\langle v \rangle \lambda}{3} \quad * \langle v \rangle = \text{mean, } \bar{v}$

Einstein's theory of diffusion random walk  $\langle x^2 \rangle = 2Dt$

## Definitions

Continuum 连续介质假设

continuum assumption fails when density is low  $\longrightarrow$  Knudsen number\*  $Kn = \frac{\lambda}{L} < 0.001$

System

Cycle

Control Volume

# Zeroth Law of Thermodynamics

A  $\xleftarrow{\text{thermal equilibrium}}$  B, B  $\xleftarrow{\text{thermal equilibrium}}$  C  $\longrightarrow$  A  $\xleftarrow{\text{thermal equilibrium}}$  C

## Heat

heat is the energy flow from hot object to cold object when they are brought into contact

→ heat is the form of energy transfer

## First Law of Thermodynamics

$$\Delta U \text{ internal energy} = Q - W \xrightarrow{\text{closed system}} \Delta E \text{ total stored energy} = Q - W$$

## Work

work is done by system if the sole effect on the surroundings could have been the raising of a weight

Displacement work  $W = \int P dV$  \* no friction, fully resisted & slow process  $\xrightarrow{\text{otherwise}} W < \int P dV$

## Internal Energy

$$\text{Specific heat capacity } C_v \text{ at constant volume } C_p = C_v + R \text{ at constant pressure } \gamma = \frac{C_p}{C_v}$$

Heat energy transferred to the system

$$Q = \Delta U = MC_v \Delta T \text{ at constant volume}$$

$$Q = \Delta U + W = MC_v \Delta T + P \Delta V = MC_v \Delta T + MR \Delta T = MC_p \Delta T \text{ at constant pressure}$$

## Degree of freedom, d

$$\text{for ideal gas } C_v = \frac{d}{2}R$$

Monatomic \*   $d = 3 \longrightarrow C_v = \frac{3}{2}R \longrightarrow U = \frac{3}{2}MRT$

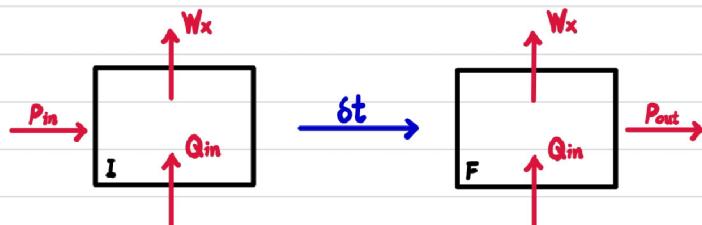
Diatomlic \*   $T_{\text{normal}} \quad d = 3 + 2 \longrightarrow C_v = \frac{5}{2}R \longrightarrow U = \frac{5}{2}MRT$

  $T_{\text{high}} \quad d = 3 + 2 + 2 * \text{vibrate}$

# Control Volume Formulation for Unsteady Flows

Specific enthalpy  $\hat{h} = u + Pv = C_p T$  for ideal gas

Specific total enthalpy  $h_o = h + \frac{1}{2}V^2$



$$\Delta E = \Delta E_{cv} + \sum M_{out} (u_{out} + \frac{1}{2}V_{out}^2) - \sum M_{in} (u_{in} + \frac{1}{2}V_{in}^2)$$

$$= M_F (u_F + \frac{1}{2}V_F^2) - M_I (u_I + \frac{1}{2}V_I^2) + \sum M_{out} (u_{out} + \frac{1}{2}V_{out}^2) - \sum M_{in} (u_{in} + \frac{1}{2}V_{in}^2)$$

$$= Q - W = Q - [W_x + (\sum P_{out} A_{out} d_{out} - \sum P_{in} A_{in} d_{in})]$$

$$= Q - [W_x + (\sum P_{out} \delta V_{out} - \sum P_{in} \delta V_{in})] = Q - [W_x + (\sum P_{out} \delta M_{out} V_{out} - \sum P_{in} \delta M_{in} V_{in})]$$

$$\rightarrow Q - W_x = M_F (u_F + \frac{1}{2}V_F^2) - M_I (u_I + \frac{1}{2}V_I^2) + \sum \delta M_{out} h_{o,out} - \sum \delta M_{in} h_{o,in}$$

# Control Volume Formulation for Steady Flows

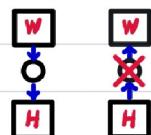
Steady flow  $M_F = M_I, U_F = U_I, V_F = V_I \rightarrow \Delta E_{cv} = 0$

$$\rightarrow Q - W_x = \sum \delta M_{out} h_{o,out} - \sum \delta M_{in} h_{o,in}$$

# Second Law of Thermodynamics

Kelvin Planck It is impossible to devise a cyclic system which produces no effect

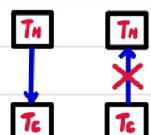
other than to deliver work and receive heat from a single reservoir



$\rightarrow$  you can't transform heat into work without some waste

Clausius No continuous cyclic process is possible, which causes heat flow from a reservoir

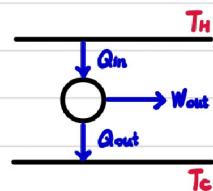
at a low temperature to one at a higher temperature with no other effect



$\rightarrow$  heat won't spontaneously flow from cold to hot

# Cyclic Heat Power Plants

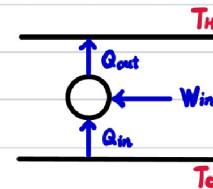
## Heat engine



$$\eta = \frac{W_{out}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \xrightarrow[\text{reversible}]{\frac{Q_{out}}{Q_{in}} = \frac{T_C}{T_H}} 1 - \frac{T_C}{T_H}$$

the carnot efficiency

## Heat pump 热泵 / Refrigerator 制冷机



$$P.E.R \text{ Performance Energy Ratio} = \frac{Q_{out}}{W_{in}} = \frac{Q_{out}}{Q_{out} - Q_{in}} \xrightarrow{\text{reversible}} \frac{T_H}{T_H - T_C}$$

$$C.O.P \text{ Coefficient of Performance} = \frac{Q_{in}}{W_{in}} = \frac{Q_{in}}{Q_{out} - Q_{in}} \xrightarrow{\text{reversible}} \frac{T_C}{T_H - T_C}$$

## Reversible and Irreversible

Irreversible process the process at each time is not in equilibrium

- heat transfer with  $\Delta T$
- friction
- non-equilibrium
- abrupt 突然的 expansion
- dissipation
- mixing

Reversible process the process which is without friction or rapid heat transfer and always in

local equilibrium  $\rightarrow$  motion is fully resisted at system boundaries

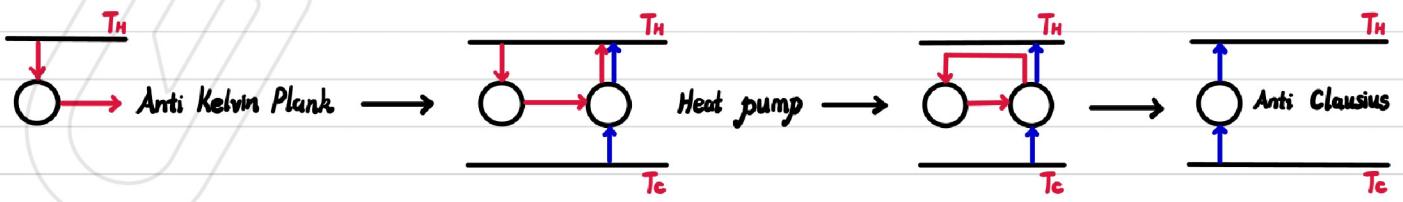
- frictionless
- heat transfer at same (near) T as reservoir
- 'slow' process
- equilibrium / fully resisted

## Equivalence Statement of the Second Law

Clausius  $\rightarrow$  Kelvin Planck

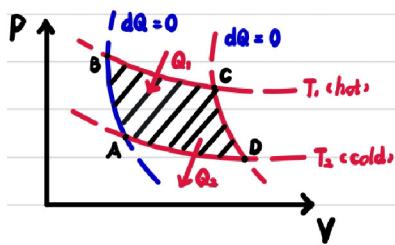


Kelvin Planck  $\rightarrow$  Clausius



# Reversible Cycles

## The Carnot cycle



\* isotropic 多方  $PV^n = \text{constant}$

isothermal  $\rightarrow PV = \text{constant}$

(adiabatic)<sub>rev</sub> = isentropic 等熵的  $\rightarrow PV^\gamma = \text{constant}$

$A \rightarrow B$  Adiabatic 绝热的 reversible compression

$$dQ = 0 \xrightarrow{PV^\gamma = \text{constant}} T_2 V_A^{\gamma-1} = T_1 V_B^{\gamma-1}$$

$B \rightarrow C$  Isothermal 等温的 reversible expansion

$$\Delta T = 0 \rightarrow \Delta U = 0 \xrightarrow{\Delta U = Q - W} Q = W \xrightarrow{PV = MRT} Q_1 = \int_{V_0}^{V_c} P dV = \int_{V_0}^{V_c} \frac{MRT_1}{V} dV = MRT_1 \ln\left(\frac{V_c}{V_0}\right)$$

$C \rightarrow D$  Adiabatic reversible expansion

$$dQ = 0 \xrightarrow{PV^\gamma = \text{constant}} T_1 V_C^{\gamma-1} = T_2 V_D^{\gamma-1}$$

$D \rightarrow A$  Isothermal reversible compression

$$\Delta T = 0 \rightarrow \Delta U = 0 \xrightarrow{\Delta U = Q - W} Q = W \xrightarrow{PV = MRT} Q_2 = \int_{V_0}^{V_A} P dV = \int_{V_0}^{V_A} \frac{MRT_2}{V} dV = MRT_2 \ln\left(\frac{V_A}{V_0}\right)$$

$$A \rightarrow B \rightarrow C \rightarrow D \rightarrow A \quad \Delta E = Q - W = 0$$

## Clausius Inequality

$$\text{Entropy } \Delta S \xrightarrow{\text{reversible}} = \left( \oint \frac{dQ}{T} = 0 \right) \xrightarrow{\text{Carnot cycle}} \frac{Q_1}{T_1} + \frac{Q_2}{T_2}$$

$$\xrightarrow{\text{irreversible}} > \left( \oint \frac{dQ}{T} < 0 \right)$$

$$* dS_{\text{universe}} = dS_{\text{system}} + dS_{\text{surroundings}} \geq 0$$

## Calculating Entropy Change

$$\text{For ideal gas } dQ = TdS = dU + dW \rightarrow \Delta S = M [C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)]$$

$$\text{Boltzmann relation} * \quad S = k \ln \Omega = k \ln \left( \frac{1}{N!} \left[ \frac{V}{\lambda \pi^2} \left( \frac{P}{kT} \right)^3 \right]^N \right) \quad * \Omega = \text{thermodynamic probability} \quad * \frac{\bar{P}^2}{2m} = \frac{U}{N}$$

Mixing mixing is irreversible  $\rightarrow$  increases the entropy

$$V_1 \text{ with } N_1 \text{ particles} + V_2 \text{ with } N_2 \text{ particles} = V = V_1 + V_2 \text{ with } N = N_1 + N_2 \text{ particles}$$

$$\Delta S_1 = k_B \ln \left( \frac{1}{N_1!} \left[ \frac{V_1}{\Delta x^3} \left( \frac{\bar{P}}{\Delta P} \right)^d \right]^{N_1} \right) - k_B \ln \left( \frac{1}{N_1!} \left[ \frac{V_1}{\Delta x^3} \left( \frac{\bar{P}}{\Delta P} \right)^d \right]^{N_1} \right) = k_B \ln \left( \frac{\frac{1}{N_1!} \left[ \frac{V_1}{\Delta x^3} \left( \frac{\bar{P}}{\Delta P} \right)^d \right]^{N_1}}{\frac{1}{N_1!} \left[ \frac{V_1}{\Delta x^3} \left( \frac{\bar{P}}{\Delta P} \right)^d \right]^{N_1}} \right) = k_B \ln \left( \frac{V_1}{V_1} \right)^{N_1}$$

$$= N_1 k_B \ln \left( \frac{V_1}{V_1} \right) \xrightarrow{V_1 \propto \frac{N_1}{N}} N_1 k_B \ln \left( \frac{N_1}{N} \right)$$

$$\Delta S_2 = N_2 k_B \ln \left( \frac{V_2}{V_2} \right) \xrightarrow{V_2 \propto \frac{N_2}{N}} N_2 k_B \ln \left( \frac{N_2}{N} \right) = N_2 k_B \ln \left( \frac{N_2}{N-N_1} \right)$$

$$\rightarrow \Delta S = \Delta S_1 + \Delta S_2 \xrightarrow{x = \frac{N_1}{N}} \Delta S = -Nk_B [x \ln(x) + (1-x) \ln(1-x)]$$

## Exergy

Exergy  $\times$  the maximum amount of useful work a system could perform

in reaching equilibrium with its surroundings  $\rightarrow$  Dead state

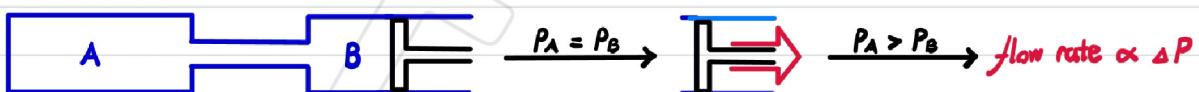
$$X = M\phi = M [ (u_i - u_a) + P_a (v_i - v_a) - T_a (s_i - s_a) ] + K.E. + P.E. + \text{etc.}$$

$$\text{Flow exergy } X_{\text{flow}} = M\psi = X + M [ P_v - P_a v ] = M [ (h_i - h_a) - T_a (s_i - s_a) + \frac{1}{2} v^2 + g z ]$$

$$\text{Carbon capture } W_{\min, \text{in}} \xrightarrow{\text{reversible}} -W_{\max, \text{out}} = -M\psi \xrightarrow{\text{constant P, T}} M T_a (S_{\text{input, mixing}} - S_{\text{output, mixing}})$$

## Compressible Flow

Choking 止動器



$$P_A = P_B \rightarrow$$



$$P_A > P_B \rightarrow \text{flow rate} \propto \Delta P$$

if flow is inviscid  $\rightarrow$    
 if flow is compressible  $\rightarrow$

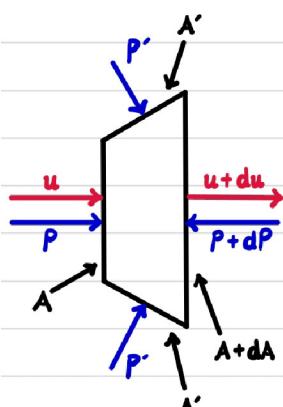
incompressible  $\rightarrow$  flow rate has no limit

compressible  $\rightarrow$  flow rate has limit  $\rightarrow$  flow choked at that limit

## 1-D Equation

\* According to the Aerodynamics 1 lecture 2-1 and 2-4

$$\text{Mass conservation } \frac{\partial \rho}{\partial t} \Delta x \Delta A + \partial_x (\rho u) \Delta A = 0 \rightarrow \frac{\partial \rho \Delta A}{\partial t} + \frac{\partial (\rho u \Delta A)}{\partial x} = 0$$



$$\text{Momentum } \frac{\partial \rho u}{\partial t} \Delta x \Delta A + \partial_x (\rho u^2) \Delta A = -\partial P \Delta A + \partial P' \Delta A' \text{ (on side walls)}$$

$$\rightarrow \frac{\partial (\rho u \Delta A)}{\partial t} + \frac{\partial (\rho u^2 \Delta A)}{\partial x} = -\frac{\partial (PA)}{\partial x} + P \frac{\partial A}{\partial x} = -A \frac{\partial P}{\partial x}$$

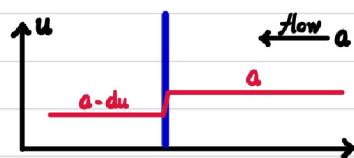
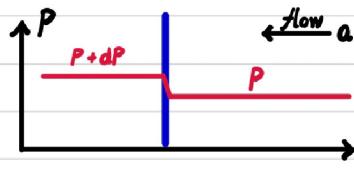
$$\rightarrow (u \frac{\partial (\rho \Delta A)}{\partial t} + \rho A \frac{\partial u}{\partial t}) + (u \frac{\partial (\rho u^2 \Delta A)}{\partial x} + \rho u A \frac{\partial u}{\partial x}) = -A \frac{\partial P}{\partial x}$$

$$\frac{\partial (\rho \Delta A)}{\partial t} + \frac{\partial (\rho u \Delta A)}{\partial x} = 0 \rightarrow \rho A \frac{\partial u}{\partial t} + \rho u A \frac{\partial u}{\partial x} = -A \frac{\partial P}{\partial x} \rightarrow \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial P}{\partial x}$$

$$\text{for steady flow, } \rho u A = \text{constant} \rightarrow u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial P}{\partial x}$$

## Speed of Sound and Mach number

Consider a weak pressure wave propagating 传播 into still air with velocity =  $a$



\* observer moving with the wave

$$\text{Mass conservation } m = \rho u A \rightarrow \frac{\dot{m}}{A} = \rho a = (\rho + d\rho)(a - du) = \rho a - \rho du + d\rho a \text{ (ignore } d\rho du) \rightarrow d\rho = \frac{\rho}{a} du$$

$$\text{Momentum } \Delta \rho u^2 A = \Delta m u = -\Delta PA \rightarrow \frac{1}{A} [(\rho + d\rho)(a - du)^2 A - \rho a^2 A] = [(\rho + d\rho) - \rho] \rightarrow d\rho = \rho adu \frac{du = \frac{a}{\rho} d\rho}{d\rho} \rightarrow \frac{dP}{d\rho} = a^2$$

$$\frac{P_v^\gamma = \frac{P}{\rho^\gamma} = \text{const}}{\frac{dP}{d\rho} = (\gamma C_p)^\gamma = \gamma C_p^{\gamma-1} = \frac{\gamma P}{\rho} = \gamma RT} \rightarrow a = \sqrt{\gamma RT}$$

## Steady inviscid flow of a perfect gas

$$u \frac{du}{dx} = -\frac{1}{\rho} \frac{dp}{dx} \rightarrow \int u du + \frac{1}{\rho} dp = 0 \rightarrow \frac{1}{2} u^2 + \frac{1}{\rho} p = \text{constant}$$

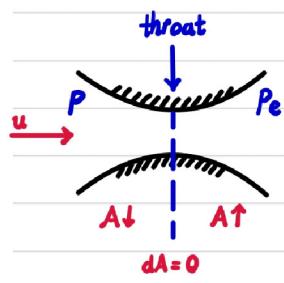
$$\frac{dp = \gamma C_p \rho^{\gamma-1} d\rho}{\frac{1}{2} u^2 + \frac{1}{\rho} p = \text{constant}} \rightarrow \frac{1}{2} u^2 + \frac{\gamma}{\gamma-1} \frac{P}{\rho} = \frac{1}{2} u^2 + C_p T = \text{constant} = h_0$$

$$\text{for adiabatic 热力学 flow } * \xrightarrow{u = aM} \frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

$$\text{for isentropic 各向同性 flow } T = \frac{P}{\rho R} \xrightarrow{\rho = C_p T^{\frac{1}{\gamma}}} \rho \left[ 1 + \frac{\gamma-1}{2} M^2 \right]^{\frac{1}{\gamma-1}} = \rho_0$$

$$\xrightarrow{\rho = C_p T^{\frac{1}{\gamma}}} P \left[ 1 + \frac{\gamma-1}{2} M^2 \right]^{\frac{\gamma}{\gamma-1}} = P_0$$

# Nozzle Flow



Mass conservation  $\rho u A = \text{const}$   $\frac{(In \rho u A)'}{\rho} = In \rho' + In u' + In A'$   $\frac{dp}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$

Momentum  $u \frac{du}{dx} = - \frac{1}{\rho} \frac{dp}{dx}$   $\rightarrow u du = - \frac{dp}{\rho}$  isentropic flow  $- \frac{\alpha^2 d\rho}{\rho}$   
 $\rightarrow \frac{dp}{\rho} = - \frac{u^2}{\alpha^2} \frac{du}{u}$

$$\rightarrow (1 - \frac{u^2}{\alpha^2}) \frac{du}{u} + \frac{dA}{A} = 0 \rightarrow (M^2 - 1) \frac{du}{u} = \frac{dA}{A}$$

$\cdot A \downarrow \rightarrow \frac{dA}{A} < 0 \rightarrow u \uparrow$  when  $M < 1$  (subsonic flow).  $u \downarrow$  when  $M > 1$  (supersonic flow)

$\cdot A \uparrow \rightarrow \frac{dA}{A} > 0 \rightarrow u \downarrow$  when  $M < 1$  (subsonic flow).  $u \uparrow$  when  $M > 1$  (supersonic flow)

$\cdot dA = 0 \rightarrow du = 0$  or  $M = 1 \rightarrow$  flow is symmetric about throat

Choking  $P_e < P \rightarrow u \uparrow$

$P_e \ll P \rightarrow$  choking  $M = 1$  at throat,  $u^*$

$P_e \ll P \rightarrow$  normal shock wave more down the nozzle  $u \uparrow \rightarrow M > 1$

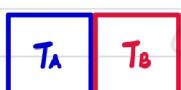
Choked Flow\*  $m^* = m_{\max} = \rho^* u^* A_T = \frac{P^*}{\sqrt{RT^*}} \sqrt{8} A_T$

$$= A_T \frac{P_e}{P_0} C \text{ where } C = \sqrt{\frac{2}{R}} \left( \frac{2}{g+1} \right)^{\frac{g+1}{2(g-1)}}$$

# Heat Transfer

Heat flux  $q = \frac{\dot{Q}}{A} = \frac{m C \Delta T}{A}$  \*  $C = C_v = C_p$  for incompressible solid/liquid

# Conduction 热传导



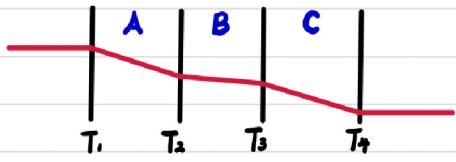
by direct contact  $\rightarrow$  gas collisions / solid vibrations 无粒子交换

Diffusion in gas coefficient of diffusion  $D = \frac{\delta^2}{2T}$  \* random jump increment  $\delta$ , time increment  $T$

thermal diffusivity  $\alpha = \frac{k}{\rho C}$  [m/s] \* thermal conductivity  $k$  [W/mK]

Einstein's theory of diffusion random walk  $x^2 = 2Dt = 2at$

Steady state conduction  $\rho C \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + \rho S \xrightarrow{\frac{\partial T}{\partial t} = 0} \dot{Q} = -kA \frac{\partial T}{\partial x} \rightarrow \dot{q} = -k \frac{\partial T}{\partial x}$



$\dot{q}$  is same through each layer

$$\dot{q} = -kaA \left( \frac{T_2 - T_1}{\Delta x_A} \right) = -kaA \left( \frac{T_3 - T_2}{\Delta x_B} \right) = -kaA \left( \frac{T_4 - T_3}{\Delta x_C} \right)$$

$$\rightarrow T_4 - T_1 = -\dot{q} \sum_{i=1}^3 \frac{\Delta x_i}{k_i} \rightarrow \dot{q} = -\frac{T_4 - T_1}{\sum_{i=1}^3 \frac{\Delta x_i}{k_i}}$$

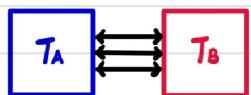
U values flow of current ~ flow of thermal energy

$$I = \frac{V}{R} \rightarrow \dot{q} = (-)k \frac{\partial T}{\partial x} = (-)U \Delta T \rightarrow U = \frac{k}{\Delta x}$$

$$R_{\text{total}} = R_1 + R_2 + \dots \rightarrow \frac{1}{U_{\text{total}}} = \frac{1}{U_1} + \frac{1}{U_2} + \dots = \frac{k_1}{\Delta x_1} + \frac{k_2}{\Delta x_2} + \dots$$

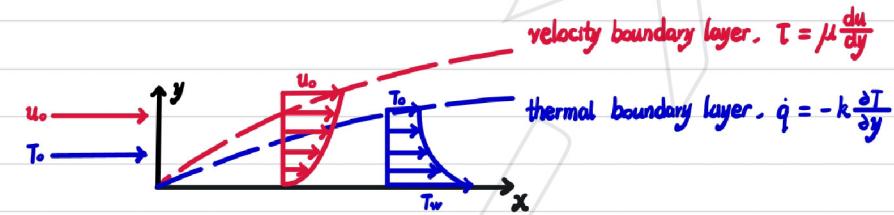
Axisymmetric conduction\*  $\rho C \frac{\partial T}{\partial t} = k \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right] + S_v$

## Convection 热对流



by fluid of energy 有粒子交换

### Thermal boundary layer



\* cannot predict

Empirical model  $\dot{q} = h \Delta \theta = h(T_f - T_w)$  \* convective heat transfer coefficient  $h$

Nusselt number\*  $Nu = \frac{hL}{k}$

Prandtl number\*  $Pr = \frac{C_p \mu}{k}$

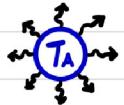
Pipe flow  $Re = \frac{\rho u D}{\mu} > 2000 \rightarrow \text{turbulent flow}$

### Reynolds analogy 类比

heat transfer between wall and fluid ~ exchange of momentum between wall and outer flow

Stanton number  $St = \frac{Nu}{Re \times Pr} = \frac{1}{2} Cf$

# Radiation 热辐射



transport of energy by electromagnetic waves

$$\alpha \text{ absorbed} + \rho \text{ reflected} + \tau \text{ transmitted (ignore since small)} = 1$$

Black body  $\alpha = 1 \rightarrow \dot{q} = \epsilon 6T^4 * \text{Stefan Boltzman constant } 6$

Grey body  $\dot{q} = \epsilon 6T^4 * \text{emissivity } \epsilon = \frac{\dot{q}}{\dot{q}_B}$

## Unsteady heat transfer

$$mc\frac{dT}{dt} = \dot{E}_{\text{net}} \text{ net energy received} = S \text{ heat generated} - hA(T - T_F) \text{ convection} - \epsilon 6(T^4 - T_e^4) \text{ radiation}$$

ignore radiation  $\rightarrow \rho V C \frac{dT}{dt} = S - hA(T - T_F) \rightarrow \frac{dT}{dt} = \frac{S}{\rho V C} - \frac{hA}{\rho V C}(T - T_F)$

$u = T - T_F$   $\frac{du}{dt} = \frac{dT}{dt} = -\frac{hA}{\rho V C} u + \frac{S}{\rho V C} = -ku + B \rightarrow u = De^{-kt} + \frac{B}{k}$

Response time constant  $t = \frac{1}{k} = \frac{\rho V C}{hA}$

Biot number indicated variation of temperature across the body

$$Bi = \frac{R_{\text{internal conduction}}}{R_{\text{external convection}}} = \frac{U_{\text{external}}}{U_{\text{internal}}} = \frac{h}{k} = \frac{h r_o}{k} \propto \text{internal temperature gradient}$$