1. We have a database describing 100 examples of printer failures. Of these, 75 examples are hardware failures, and 25 examples are driver failures. Of the hardware failures, 15 had Windows operating system. Of the driver failures, 15 had Windows operating system. Show your work.

a. Calculate P(windows | hardware) using the information in the problem.

P(windows | hardware) = P(windows, hardware) / P(hardware) = 0.15 / 0.75 = 0.2

b. Calculate P(driver | windows) using Bayes' rule and the information in the problem.

P(driver | windows) = P(windows | driver) P(driver) / P(windows) = (0.15 / 0.25) * 0.25 / 0.3 = 0.5

2. After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that it is a rare disease, striking only 1 in 10,000 people of your age. What is the probability that you actually have the disease?

P(disease | test) = P(test | disease) P(disease) / P(test)

= P(test | disease) P(disease) / [P(test | disease) P(disease) + P(test | –disease) P(–disease)]

= 0.99 * 0.0001 / [0.99 * 0.0001 + 0.01 * 0.9999]

≈ 0.009804

4. Suppose you are given a bag containing n unbiased coins. You are told that 1 of these coins are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides. Show your work for the questions below.

a. Suppose you reach into the bag, pick out a coin uniformly at random, flip it, and get a head. What is the conditional probability that the coin you chose is the fake coin?

P(fake | heads) = P(heads | fake) P(fake) / P(heads)

= P(heads | fake) P(fake) / [P(heads | fake) P(fake) + P(heads | −fake) P(−fake)]

= 1 * (1/n) / [1 * (1/n) + 0.5 * (n−1) / n]

= 2 / (n+1)

b. Suppose you continue flipping the coin for a total of k times after picking it and see k heads. Now what is the conditional probability that you picked the fake coin?

P(fake | k_heads) = P(k_heads | fake) P(fake) / P(k_heads)

= P(k_heads | fake) P(fake) / [P(k_heads | fake) P(fake) + P(k_heads | −fake) P(−fake)]

= 1 * (1/n) / [1 * (1/n) + 2^{-k} * (n−1) / n]

= 2^k / (2^k+n−1)

c. Suppose you wanted to decide whether a chosen coin was fake by flipping it k times. The decision procedure returns FAKE if all k flips come up heads, otherwise it returns NORMAL. What is the (unconditional) probability that this procedure makes an error on coins drawn from the bag?

The procedure makes an error only if a normal coin is drawn and all *k* flips come up heads.

P(k_heads, −fake) = P(k_heads | −fake) P(−fake) = (n−1) / n2^k

5. Consider a production unit of a steel plant. Here Boilers B1, B2 and B3 make 30%, 30% and 40% of total product.. Of their output, let's assume that 2%, 3% and 4% are defective. A product is drawn at random and is found defective. What is the probability that the product is made by Boiler B1 or B2 or B3.

Let A1, A2 and A3 be the events that a product selected at random is produced by the Boilers B1,B2 and B3.Assume D denote that it is defective.

Prob(A1)=0.3,Prob(A2)=0.3 and Prob(A3) =0.4 and these represents the prior probabilities.

Probability of drawing a defective product made by B1 = Prob (D|A1) = 0.02

Probability of drawing a defective product made by B1 = Prob (D|A2) = 0.03

Probability of drawing a defective product made by B1 = Prob (D|A3) = 0.04

Therefore Prob(B1|D) = Prob(A1) * Prob (D|A1)/ ∑ Prob(Ai) * Prob(D|Ai) (i=1..3) =0.1935

Similarly Prob(B2|D) = 0.2903 and Prob(B3|D)= 0.5162

6. *A biased coin (with probability of obtaining a Head equal to $p > 0$) is tossed repeatedly and independently until the first head is observed. Compute the probability that the first head appears at an even numbered toss.*

Define:

a. sample space Ω to consist of all possible infinite binary sequences of coin tosses;
b. event H_1 - head on first toss;
c. event E - first head on even numbered toss.

We want P (E) : using the Theorem of Total Probability, and the partition of Ω given by $\{H_1, H_1^0\}$

$$P(E) = P(E|H_1)P(H_1) + P(E|H_1^0)P(H_1^0).$$

Now clearly, $P(E|H_1) = 0$ (given H_1, that a head appears on the first toss, E cannot occur) and also $P(E|H_1^0)$ can be seen to be given by

$$P(E|H_1^0) = P(E^0) = 1 - P(E),$$

(given that a head does **not** appear on the first toss, the required conditional probability is merely the probability that the sequence concludes after a further **odd** number of tosses, that is, the probability of E^0). Hence P (E) satisfies

$$P(E) = 0 \times p + (1 - P(E)) \times (1 - p) = (1 - p)(1 - P(E)),$$

so that

$$P(E) = \frac{1-p}{2-p}.$$