# Bayesian Methods Exercise II

Consider an incandescent bulb manufacturing unit. Here machines M1, M2 and M3 make 30%, 30% and 40% of total bulbs. Of their output, let's assume that 2%, 3% and 4% are defective. A bulb is drawn at random and is found defective. What is the probability that the bulb is made by machine M1 or M2 or M3.

Solution – Let E1, E2 and E3 be the events that a bulb selected at random is made by machine M1, M2 and M3. Let Q denote that it is defective.

Prob(E1) = 0.3, Prob(E2) = 0.3 and Prob(E3) = 0.4 (Given Data)

Prob of drawing a defective bulb made by M1 = Prob (Q|E1) = 0.02
Prob of drawing a defective bulb made by M2 = Prob (Q|E2) = 0.03
Prob of drawing a defective bulb made by M1 = Prob (Q|E3) = 0.04

Therefore

$$\text{Prob (E1|Q)} = \frac{\text{Prob (E1) * Prob(Q|E1)}}{\text{Sum (1,3) Prob(Ei) * Prob (Q|Ei)}} = 0.3*\ 0.02 / (0.03*0.2) +(0.03*0.3) + (0.04*0.4) = 0.1935$$

$$\text{Prob (E2|Q)} = \frac{\text{Prob (E2) * Prob(Q|E2)}}{\text{Sum (1,3) Prob(Ei) * Prob (Q|Ei)}} = 0.3*\ 0.03 / (0.03*0.2) +(0.03*0.3) + (0.04*0.4) = 0.2903$$

Prob (E3|Q) = 0.5162

Marie is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the day of Marie's wedding?

*Solution:* The sample space is defined by two mutually-exclusive events - it rains or it does not rain. Additionally, a third event occurs when the weatherman predicts rain. Notation for these events appears below.

- Event $A_1$. It rains on Marie's wedding.
- Event $A_2$. It does not rain on Marie's wedding.
- Event B. The weatherman predicts rain.

In terms of probabilities, we know the following:

- $P( A_1 ) = 5/365 =0.0136985$ [It rains 5 days out of the year.]
- $P( A_2 ) = 360/365 = 0.9863014$ [It does not rain 360 days out of the year.]
- $P( B | A_1 ) = 0.9$ [When it rains, the weatherman predicts rain 90% of the time.]
- $P( B | A_2 ) = 0.1$ [When it does not rain, the weatherman predicts rain 10% of the time.]

- We want to know $P(A_1 \mid B)$, the probability it will rain on the day of Marie's wedding, given a forecast for rain by the weatherman. The answer can be determined from Bayes' theorem, as shown below.

$$P(A_1 \mid B) = \frac{P(A_1)\,P(B \mid A_1)}{P(A_1)\,P(B \mid A_1) + P(A_2)\,P(B \mid A_2)}$$

$$P(A_1 \mid B) = \frac{(0.014)(0.9)}{[\,(0.014)(0.9) + (0.986)(0.1)\,]}$$

$$P(A_1 \mid B) = 0.111$$

- Note the somewhat unintuitive result. Even when the weatherman predicts rain, it rains only about 11% of the time. Despite the weatherman's gloomy prediction, there is a good chance that Marie will not get rained on at her wedding.

After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that it is a rare disease, striking only 1 in 10,000 people of your age. What is the probability that you actually have the disease?

$$\begin{aligned}
P(\text{disease} \mid \text{test}) &= P(\text{test} \mid \text{disease})\, P(\text{disease}) \,/\, P(\text{test}) \\
&= P(\text{test} \mid \text{disease})\, P(\text{disease}) \,/\, [\, P(\text{test} \mid \text{disease})\, P(\text{disease}) + P(\text{test} \mid -\text{disease})\, P(-\text{disease})\, ] \\
&= 0.99 * 0.0001 \,/\, [\, 0.99 * 0.0001 + 0.01 * 0.9999\, ] \\
&\approx 0.009804
\end{aligned}$$