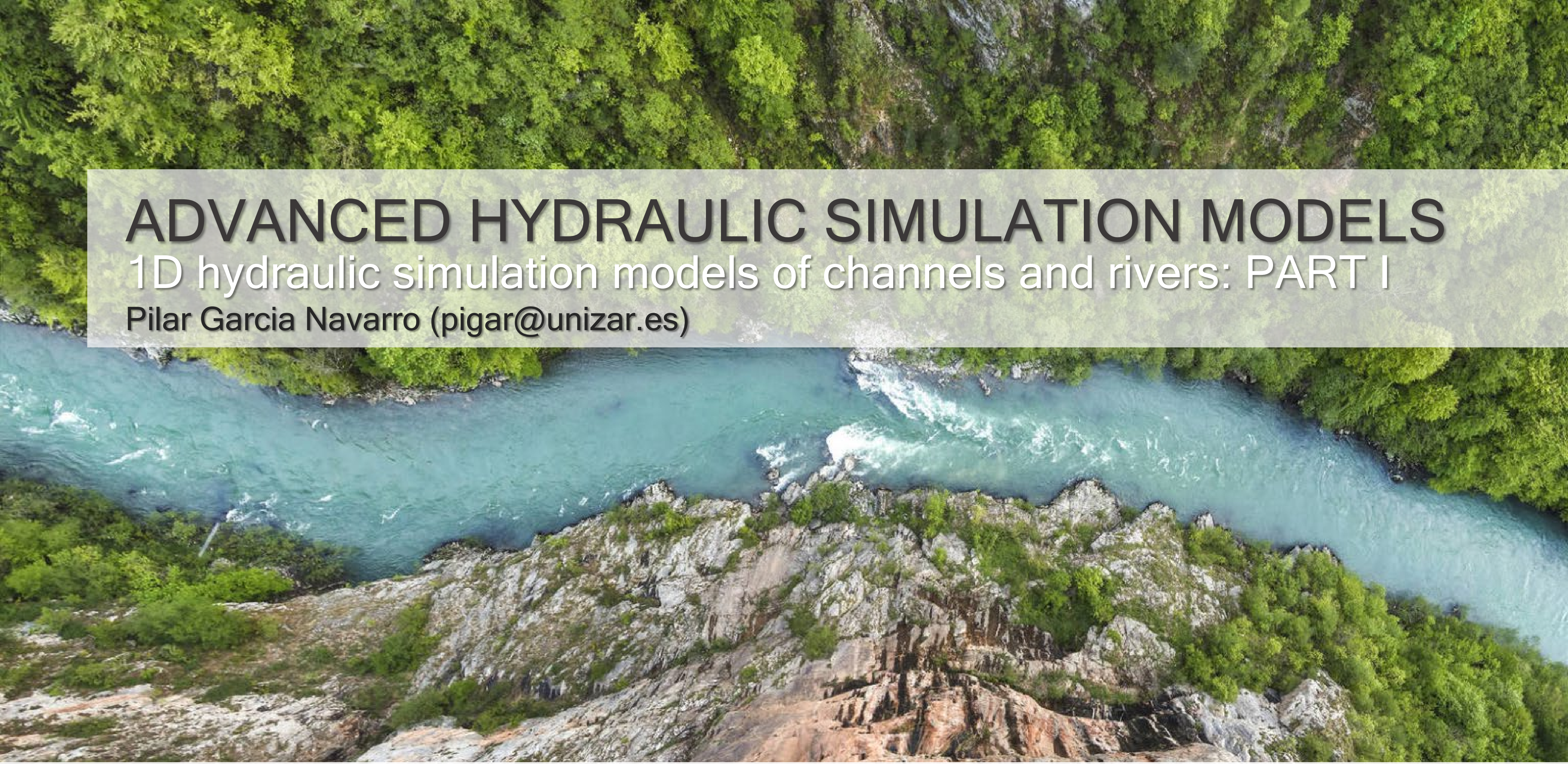

ADVANCED HYDRAULIC SIMULATION MODELS

1D hydraulic simulation models of channels and rivers: PART I

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RESCUER

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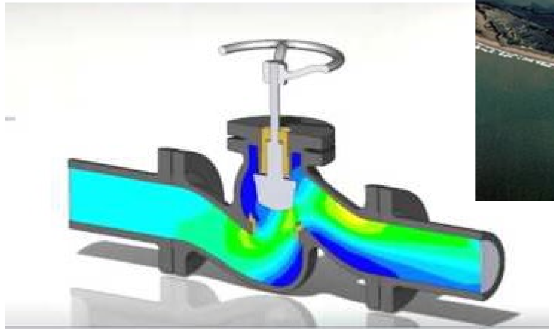
Content

Content

What type of flows are we going to simulate

- Pressure flow/free surface flow
- Flow rate, average velocity
- Piezometric level, depth, wetted section
- Stationary/transient flow
- Laminar/turbulent flow
- Ideal/viscous flow



Use of models for flows

REALITY

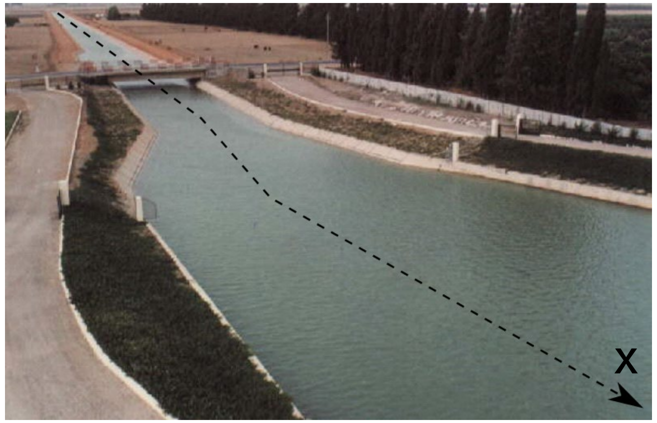
Three-dimensional		Many propulsion effects	
Bed obstacles and friction	Vertical changes (temperature or density)	Gravity	Thermodynamics

Mathematical model for channels and rivers → Analyze important effects

Characterization of flow in channels and rivers

- One-dimensional movement
- Gravity is the driving force
- Average velocity in section. Q=uA
- Hydrostatic pressure distribution p=p(h)
- Mild slopes: $\sin(\theta) \approx \tan(\theta)$

Characterization of flow in channels and rivers



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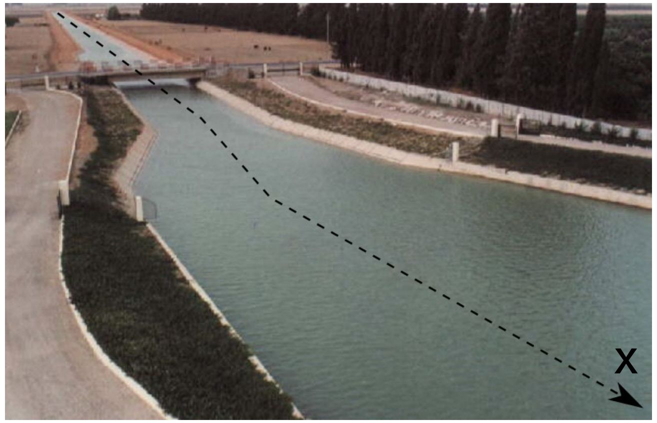

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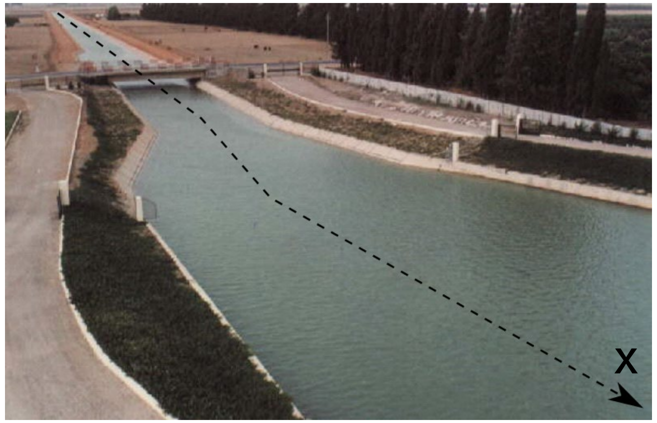

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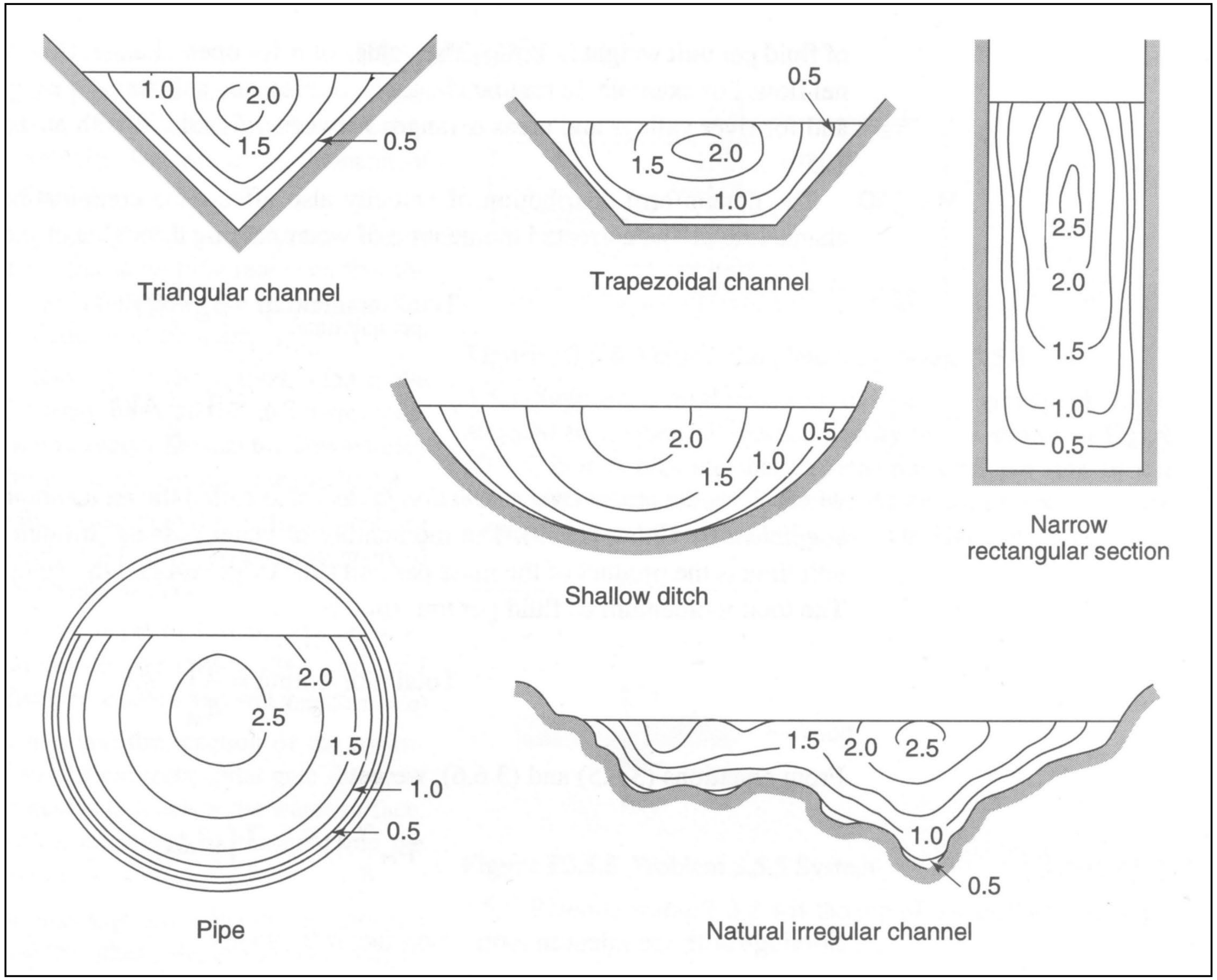
Velocity distribution across the section



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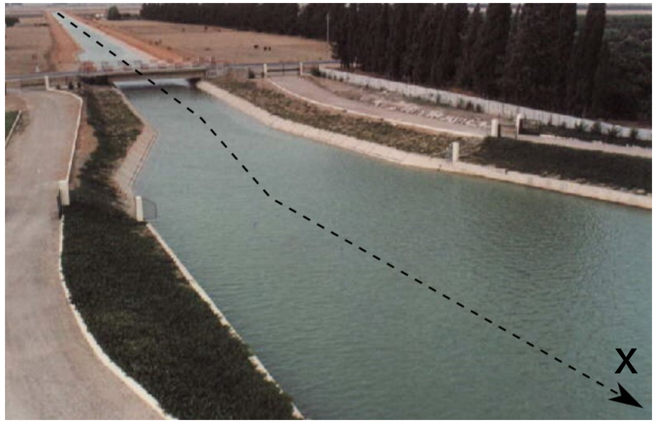

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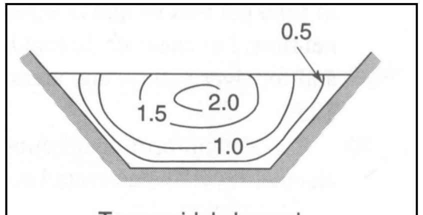
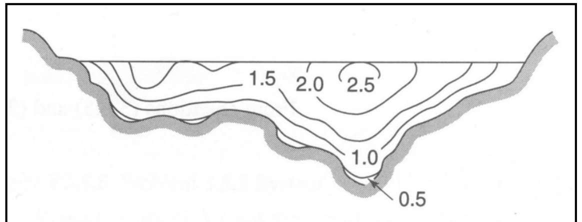
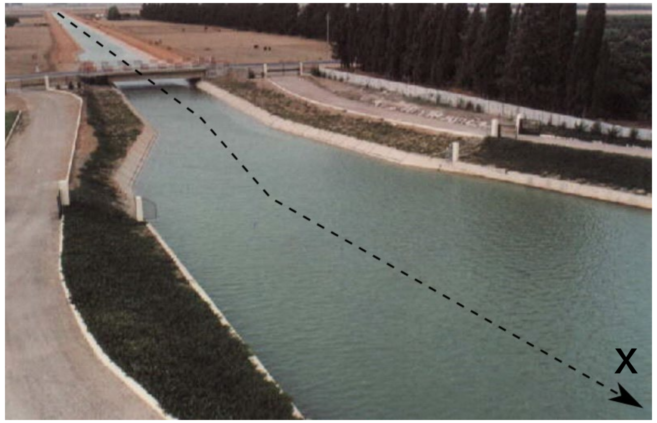


Trapezoidal channel



Natural irregular channel

Characterization of flow in channels and rivers



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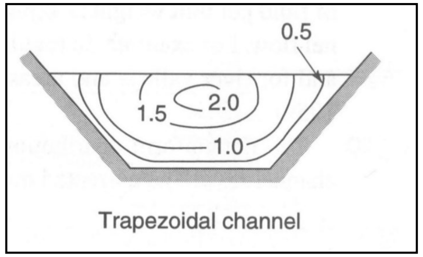
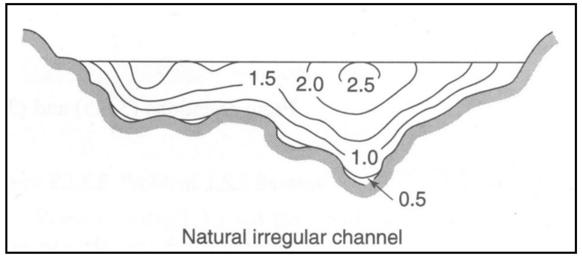




Hydrostatic pressure

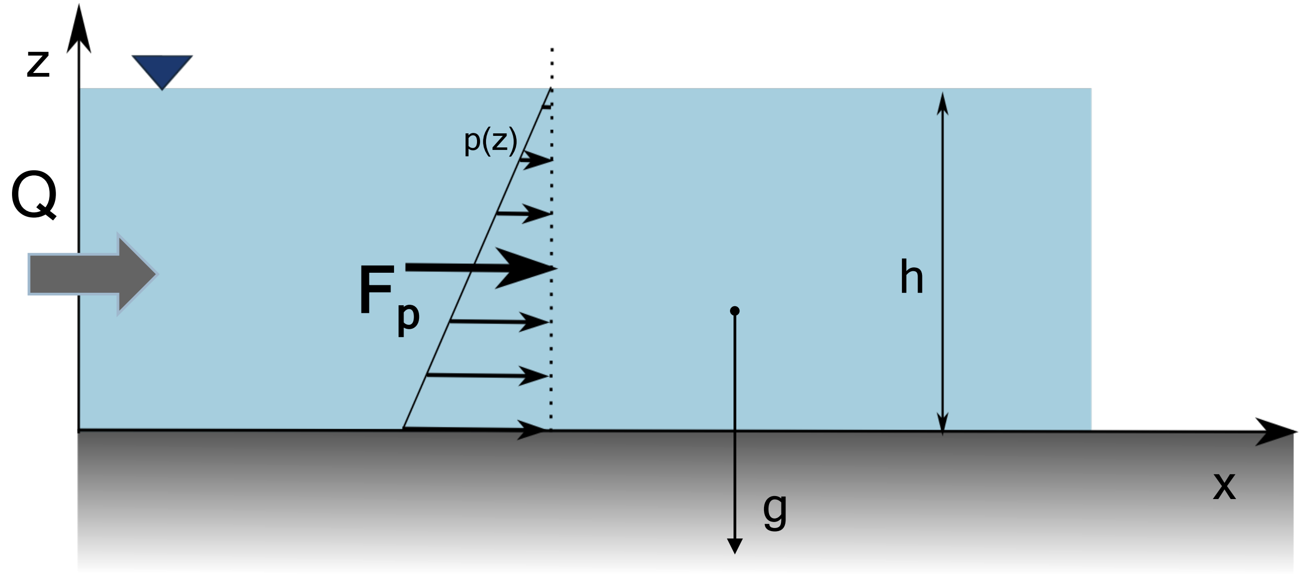
- Typical property of shallow flows: hydrostatic z-pressure

$$p(z) = p(h) + \rho g(h - z) = \rho g(h - z)$$

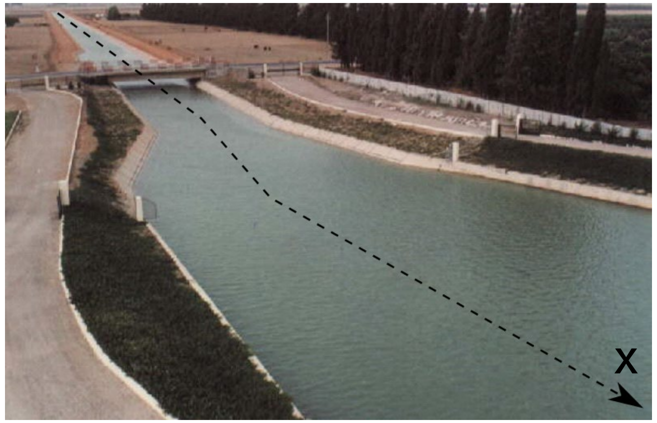
- The pressure at position x: p(x) = p(h(x))

- By integrating it, we obtain the total pressure force

$$\boxed{F_p = \frac{1}{2}\rho g h^2}$$



Characterization of flow in channels and rivers



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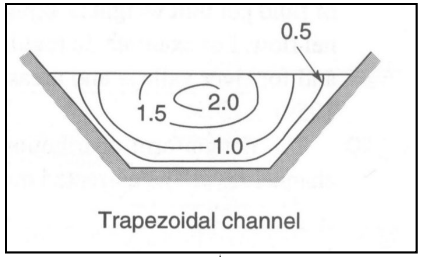
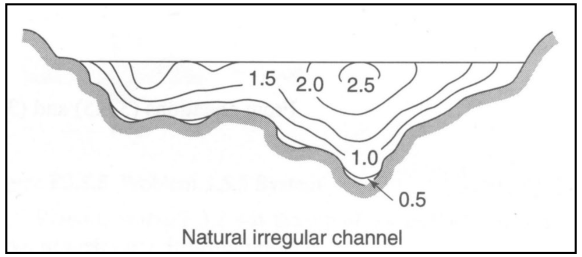
- **Mild slopes: sin(θ) ≈ tan(θ), cos(θ) ≈ 1**



Trapezoidal channel



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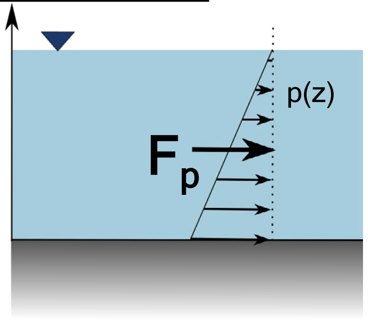


Basic concepts on channels

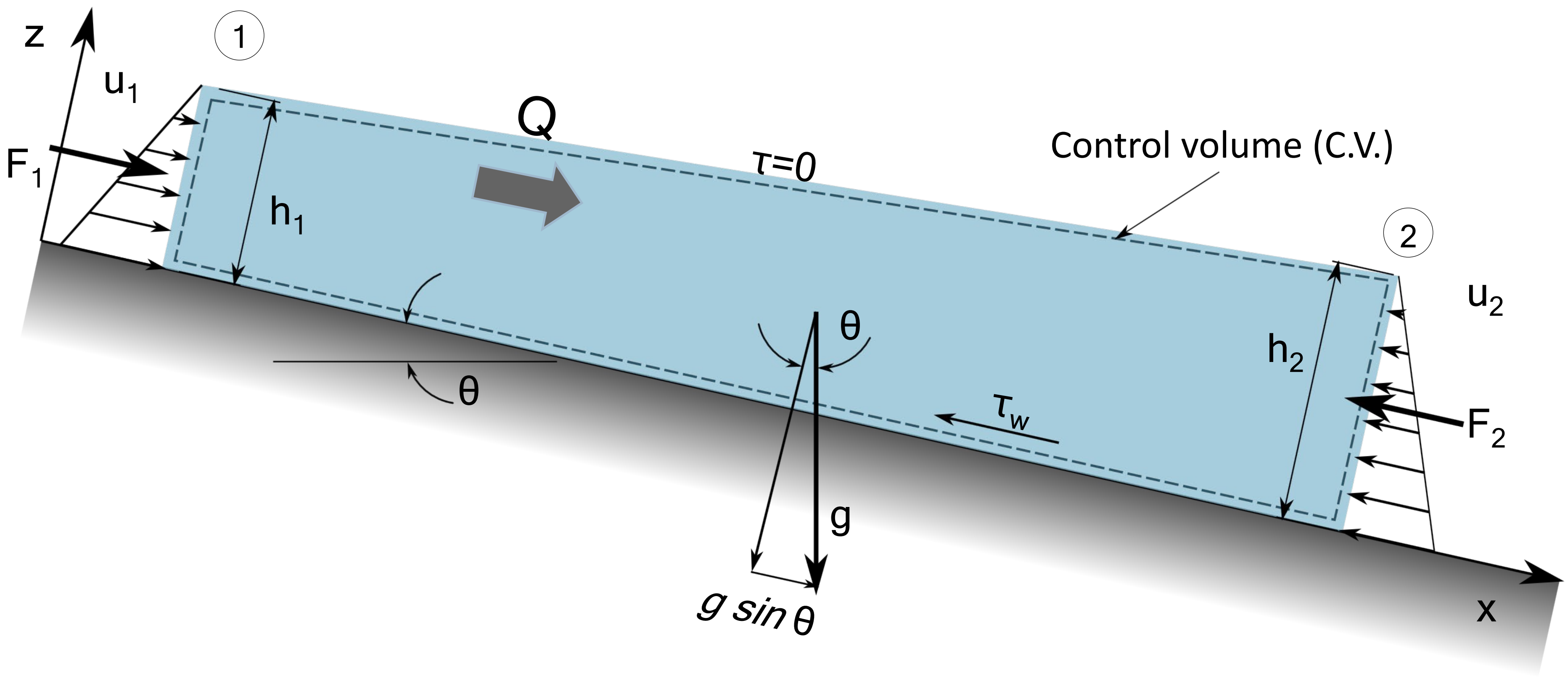
Why does the water flow in the rivers?



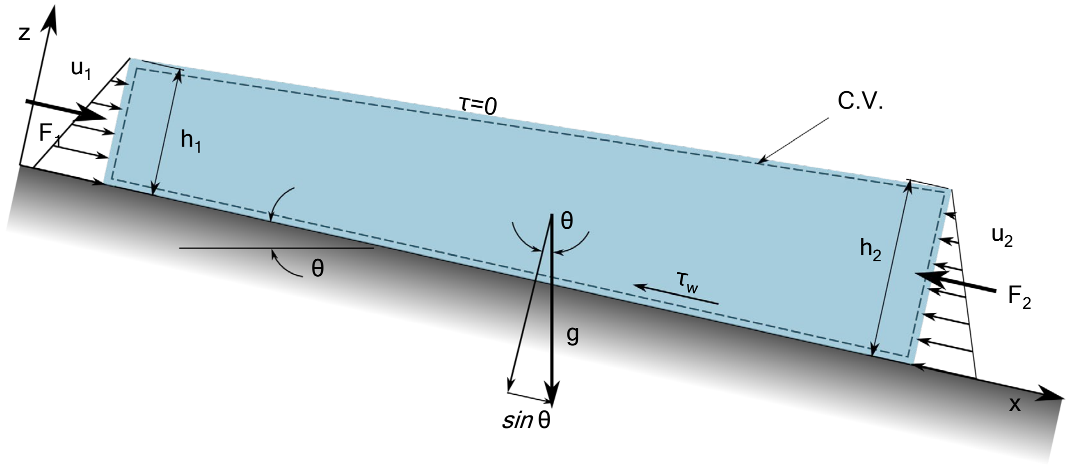

Content

1. Model hypothesis: basic concepts in channels
2. Steady flow analysis
    1. Energy balance
    2. Uniform flow: Normal Depth
    3. Energy: Critical Depth
    4. Boundary conditions
3. Complete flow equations
    1. Conservation laws
    2. Resolution

Main forces applied: control volume (C.V.)



Integral energy balance of a C.V.



$Q = cte.$

Total pressure:

$P^* = p - \rho\ g\ x\ sin\ \alpha + \rho\ g\ z \cos\alpha$

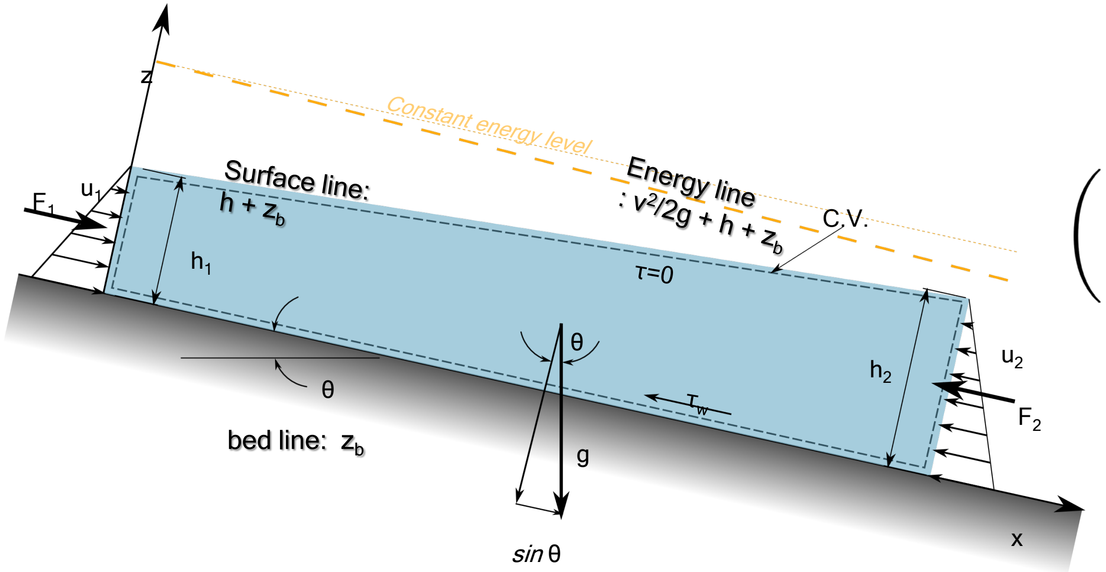
$$\int_1 \rho \frac{\vec{u}^2}{2}(\vec{u}\cdot\hat{n})\,dS + \int_2 \rho \frac{\vec{u}^2}{2}(\vec{u}\cdot\hat{n})\,dS = -\int_1 P^*(\vec{u}\cdot\hat{n})\,dS - \int_2 P^*(\vec{u}\cdot\hat{n})\,dS - \int_V \phi_{vol} dV$$

$$\left(\frac{u^2}{2g} + h + z_b\right)_1 - \left(\frac{u^2}{2g} + h + z_b\right)_2 = \frac{\int_V \phi_{vol} dV}{\rho\, g\, Q}$$

Viscous losses
In units of length

$$H_1 - H_2 = \Delta H$$

Formulation of friction losses



$$\frac{\Delta H}{\Delta x} = S_f = \frac{n^2}{R_h^{4/3}} u^2 = \frac{n^2}{R_h^{4/3}} \frac{Q^2}{A^2}$$

n: Manning's Coefficient

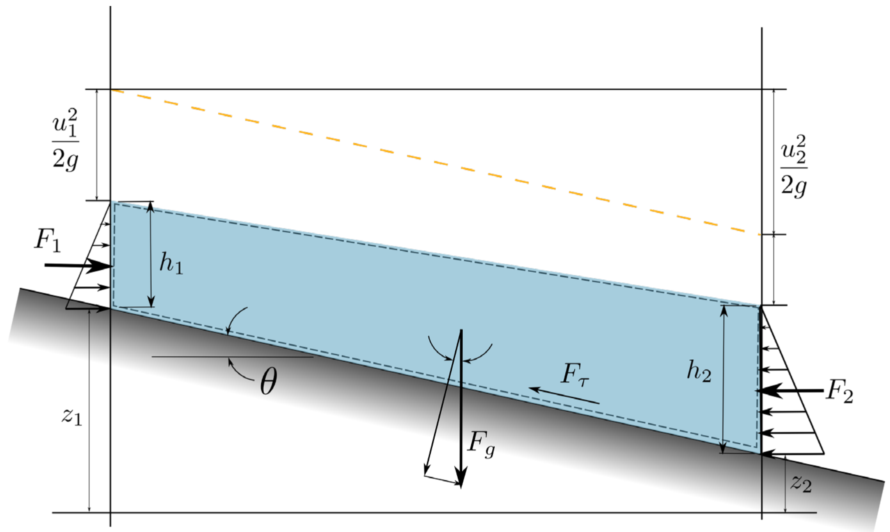
$$\left(\frac{u^2}{2g} + h + z_b\right)_1 - \left(\frac{u^2}{2g} + h + z_b\right)_2 = \Delta H = S_f \Delta x$$

R_h: Hydraulic radius
S_f: Friction slope

Uniform/non-uniform flow

General case: non-uniform flow

$$\left(\frac{u^2}{2g} + h + z_b\right)_1 - \left(\frac{u^2}{2g} + h + z_b\right)_2 = S_f \Delta x$$

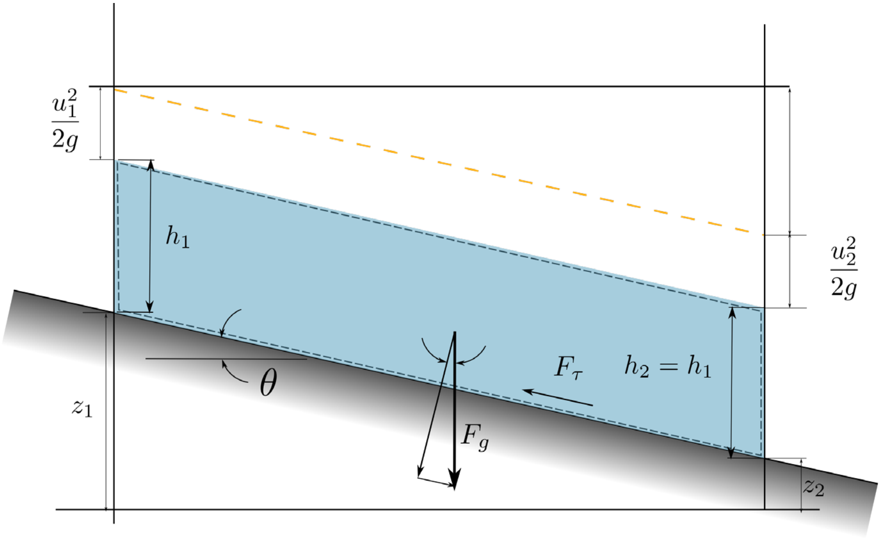


Particular case: uniform flow

$$\left(\cancel{\frac{u^2}{2g}} + \cancel{h} + z_b\right)_1 - \left(\cancel{\frac{u^2}{2g}} + \cancel{h} + z_b\right)_2 = S_f \Delta x$$

$$z_{b1} - z_{b2} = S_f \Delta x$$

$$\frac{z_{b1} - z_{b2}}{\Delta x} = S_f$$



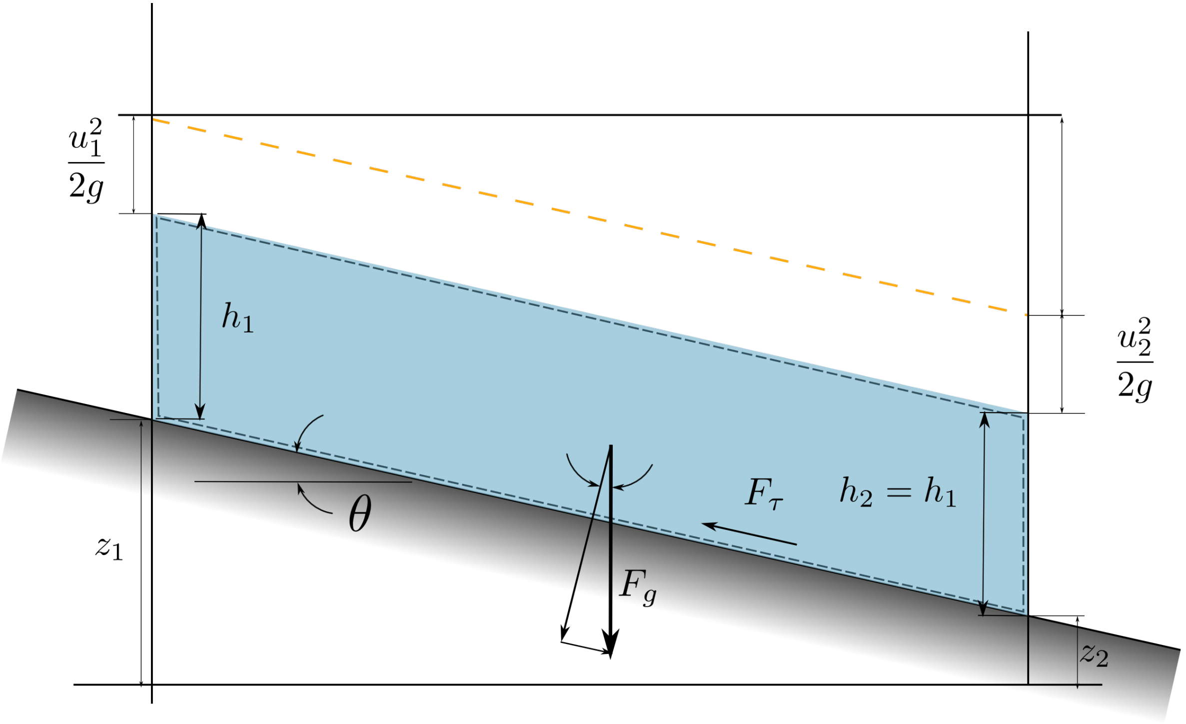
Uniform flow: normal depth

In the uniform flow

$$\left(\frac{u^2}{2g}+h+z_b\right)_1-\left(\frac{u^2}{2g}+h+z_b\right)_2=S_f\Delta x$$

$$z_{b1}-z_{b2}=S_f\Delta x$$

$$\frac{z_{b1}-z_{b2}}{\Delta x}=S_f \qquad \rightarrow \qquad S_0=S_f$$



Normal depth, h_N, ensures equality

$$S_0=S_f=\frac{n^2Q|Q|}{R_h^{4/3}A^2}$$

Where $A=A(h)$, $R_h=R_h(h)$

Uniform flow: normal depth

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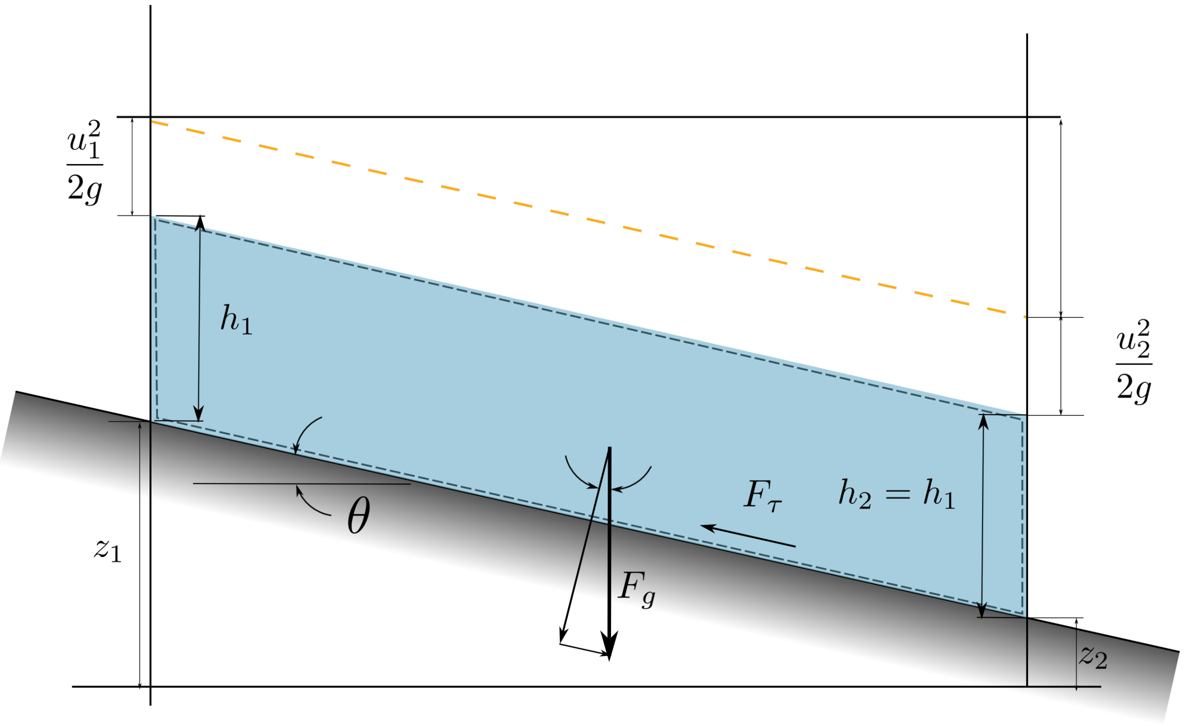


Normal depth, h_N, ensures equality

$$S_0=S_f=\frac{n^2Q|Q|}{R_h^{4/3}A^2}$$

Note: If h ↑↑ → S_f↓↓

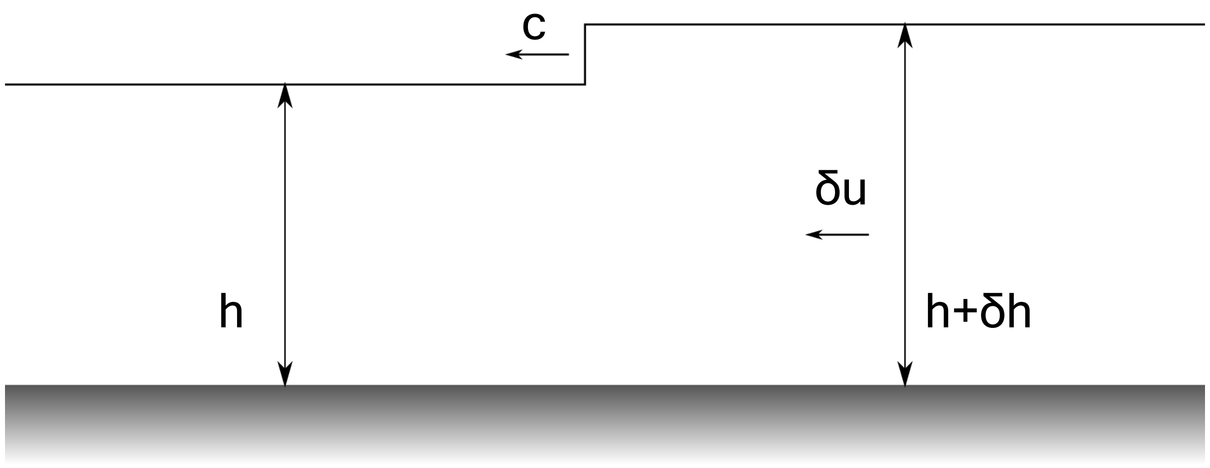
Therefore:
If $h > h_N \rightarrow S_0 > S_f$
If $h < h_N \rightarrow S_0 < S_f$

Where $A=A(h)$, $R_h=R_h(h)$

Gravity waves

- The velocity of infinitesimal surface waves

$$\delta h << h \qquad \Rightarrow \qquad \boxed{c \approx \sqrt{gh}}$$



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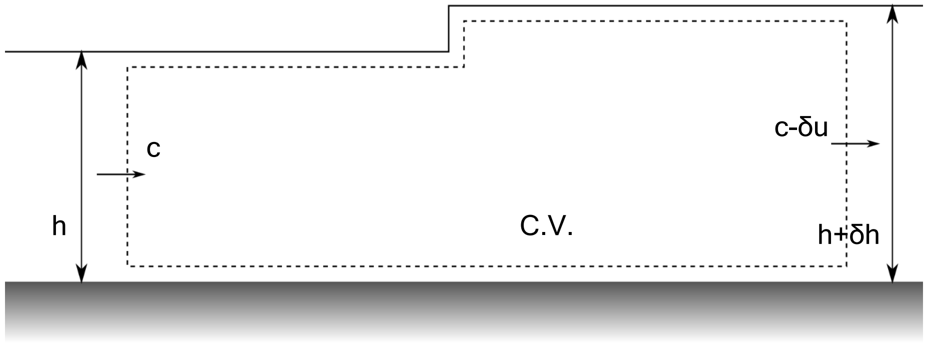

Performing an analysis of mass and linear momentum conservation in x results in:

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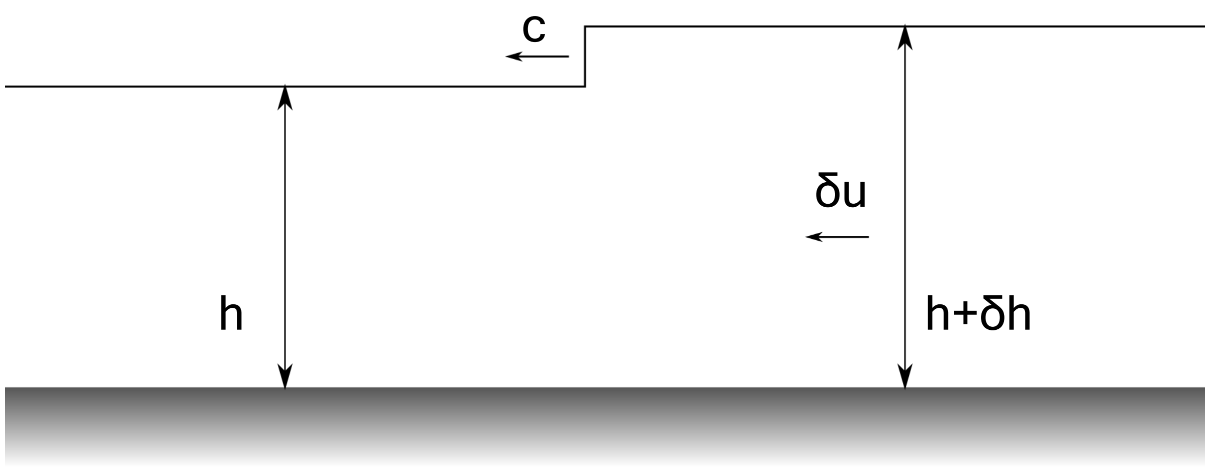

Performing an analysis of mass and linear momentum conservation in x results in:

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C is not the velocity of the flow

Finite amplitude waves

- The velocity of surface waves

$$V_S = \frac{Q_1 - Q_2}{B(h_1 - h_2)}$$

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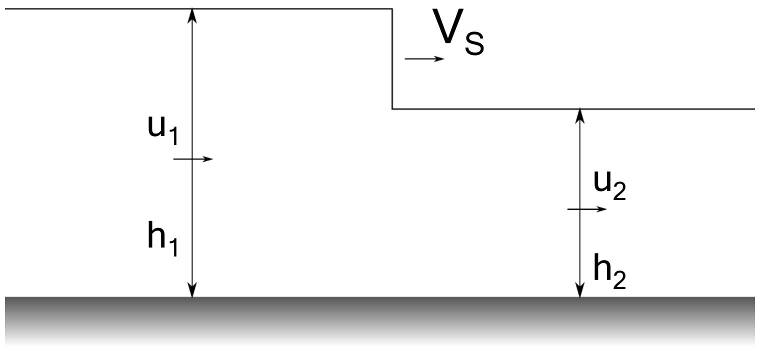

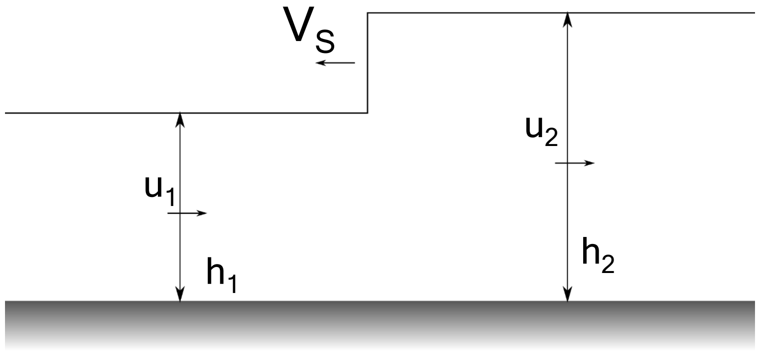

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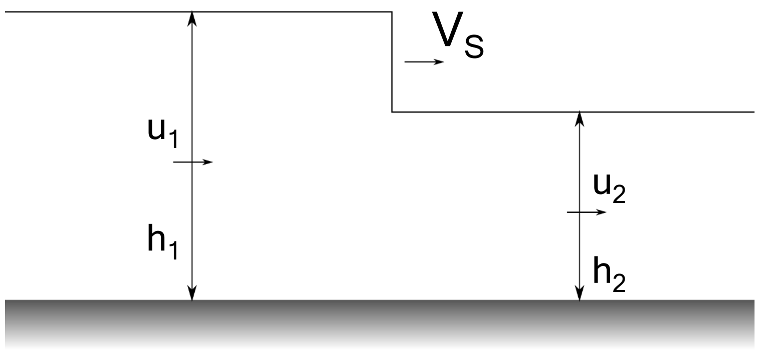
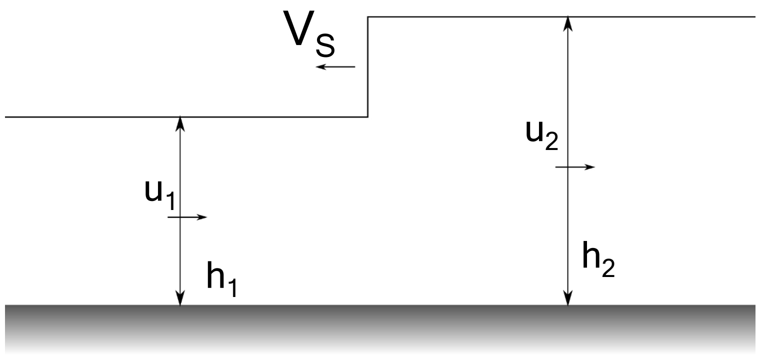




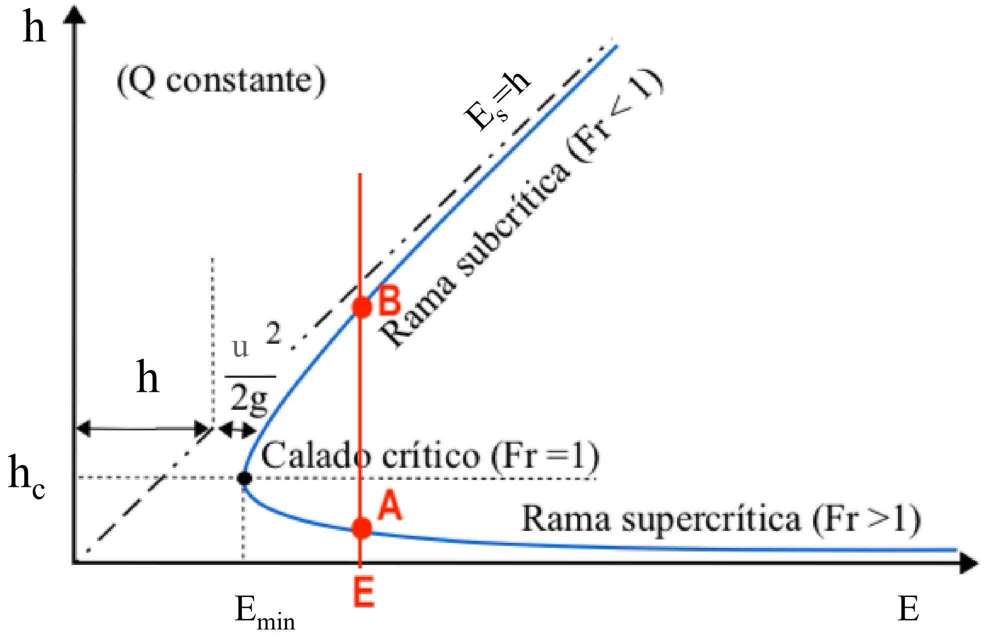
V_S is not the velocity of the flow

Specific energy

$$E = \frac{u^2}{2g} + h = \frac{Q^2}{2gB^2h^2} + h = \frac{q^2}{2gh^2} + h$$

- Total energy: $H = E + z_b$
- E is the sum of the kinetic energy and the pressure energy (represented by the depth)
- In a rectangular channel:

  A=Bh
  Q=qB



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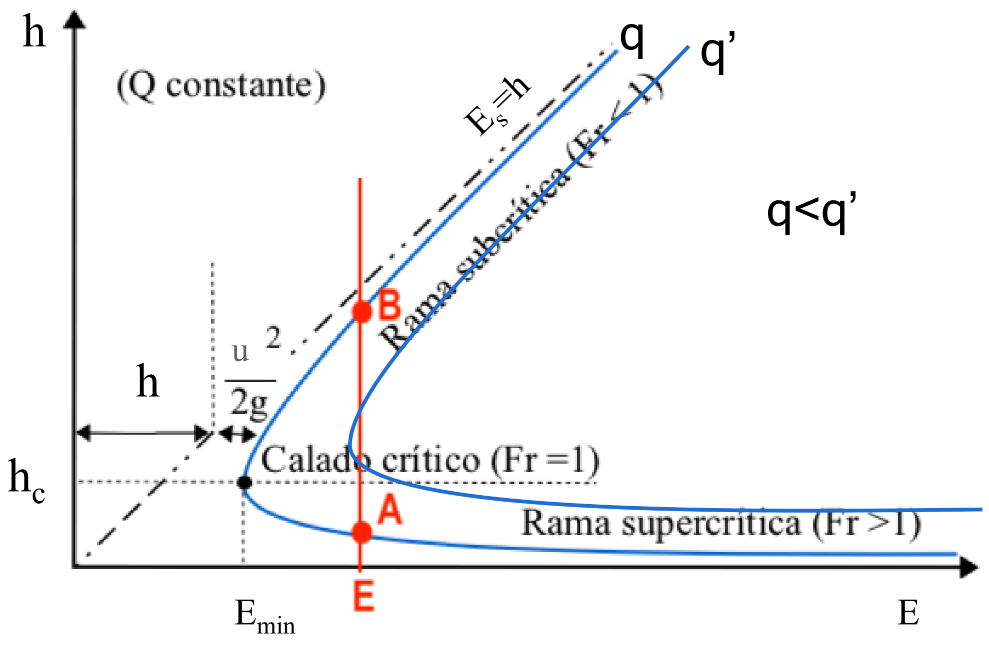
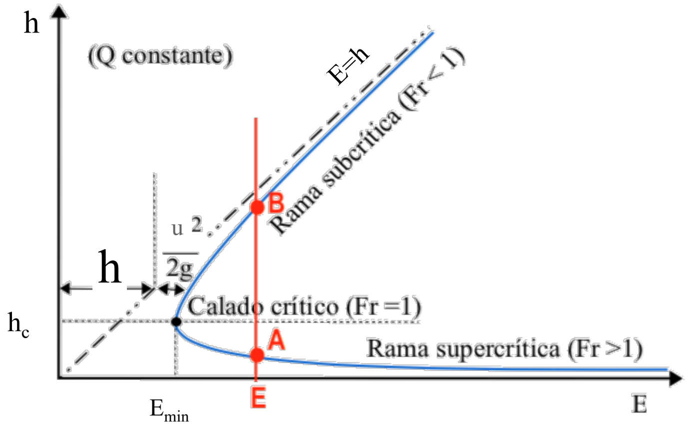


Critical depth

Minimum point of the curve E=E(h): $\frac{dE}{dh} = 0$



$$h_c = \left(\frac{Q^2}{gB^2}\right)^{1/3} \qquad E_c = \frac{3}{2}h_c = \frac{3}{2}\left(\frac{Q^2}{gB^2}\right)^{1/3}$$

When dE/dh=0

$$\frac{dE}{dh} = \frac{-Q^2 2h}{2gB^2h^4} + 1 = 0$$

$$\frac{Q^2h}{gB^2h^4} = 1$$

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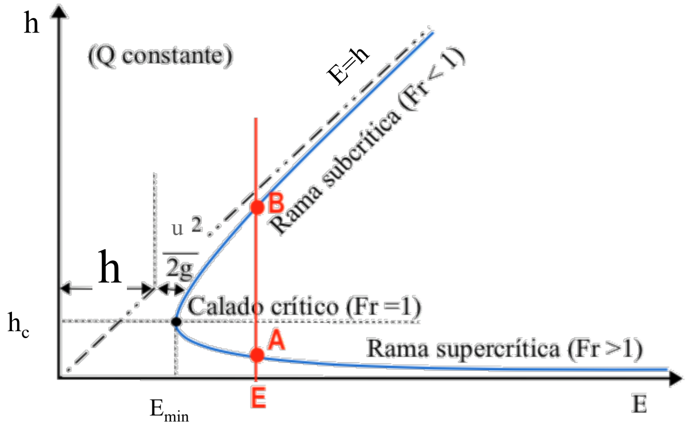


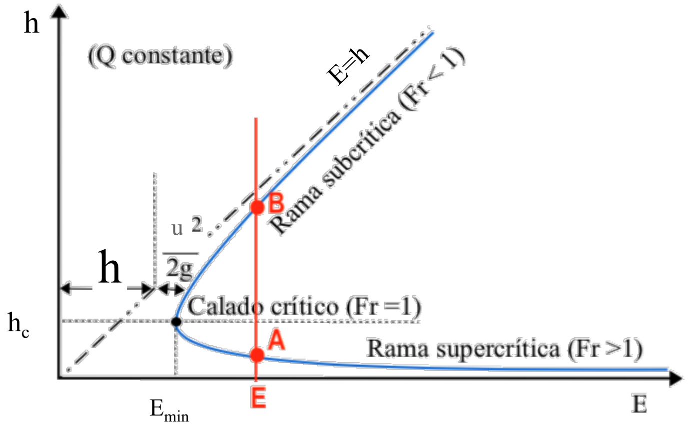
When dE/dh = 0

$$\frac{dE}{dh} = \frac{-Q^2 2h}{2gB^2h^4} + 1 = 0$$

$$\frac{Q^2 h}{gB^2h^4} = 1 \longrightarrow \frac{Q^2/B^2h^2}{gh} = 1 \longrightarrow \frac{u^2}{gh} = 1$$

Critical depth

Minimum point of the curve E=E(h): $\dfrac{dE}{dh} = 0$



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When dE/dh = 0

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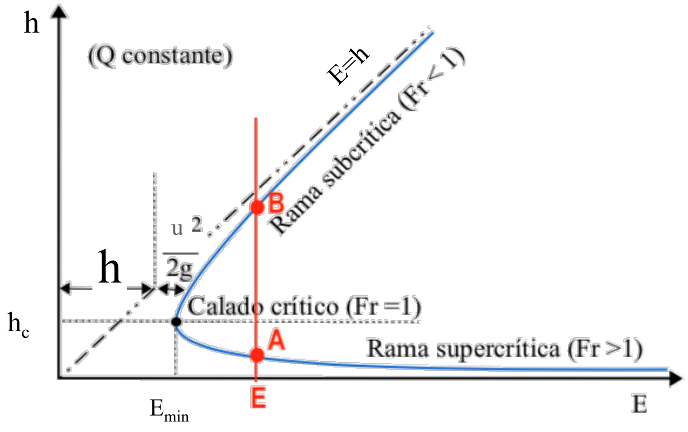

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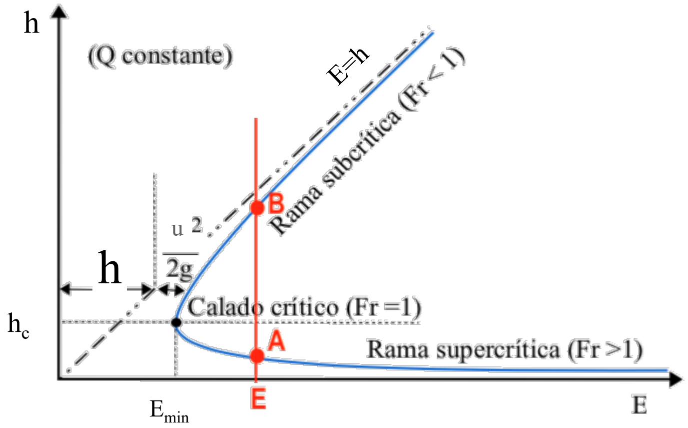

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Froude number, $Fr = \dfrac{u}{\sqrt{gh}} = \dfrac{u}{c} = 1$

Fr>1 supercritical

Fr<1 subcritical

- Subcritical flow, $u < c$ (Fr < 1) controlled by downstream conditions
- Supercritical flow, $u > c$ (Fr >1) controlled by upstream conditions

Equilibrium profiles

The behaviour of the free surface can be calculated
Starting from the energy balance between two points

$$\left(\frac{u^2}{2g} + h + z_b\right)_1 - \left(\frac{u^2}{2g} + h + z_b\right)_2 = S_f \Delta x$$

Equilibrium profiles

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Knowing : Q=uA
In rectangular channel: A=bh

$$\frac{d}{dx}\left(\frac{Q^2}{2gB^2h^2}+h+z_b\right)=-S_f$$

Equilibrium profiles

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Knowing : Q=uA
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$$\frac{d}{dx}\left(\frac{Q^2}{2gB^2h^2}+h+z_b\right)=-S_f \qquad \rightarrow \qquad \frac{Q^2}{\cancel{2}gB^2}\left(\frac{-\cancel{2}}{h^3}\right)\frac{dh}{dx}+\frac{dh}{dx}+\frac{dz_b}{dx}=-Sf$$

Equilibrium profiles

The behaviour of the free surface can be calculated
Starting from the energy balance between two points

$$\left(\frac{u^2}{2g}+h+z_b\right)_1-\left(\frac{u^2}{2g}+h+z_b\right)_2=S_f\Delta x \qquad \rightarrow \qquad \frac{d}{dx}\left(\frac{u^2}{2g}+h+z_b\right)=-S_f$$

Knowing : Q=uA
In rectangular channel: A=bh

$$\frac{d}{dx}\left(\frac{Q^2}{2gB^2h^2}+h+z_b\right)=-S_f \qquad \rightarrow \qquad \frac{Q^2}{\cancel{2}gB^2}\left(\frac{-\cancel{2}}{h^3}\right)\frac{dh}{dx}+\frac{dh}{dx}+\frac{dz_b}{dx}=-Sf$$

And rearranging and replacing definitions

$$\frac{dh}{dx}(1-Fr^2)+\frac{dz_b}{dx}=-Sf \qquad \rightarrow \qquad \boxed{\frac{dh}{dx}=\frac{S_0-S_f}{1-Fr^2}}$$

Equilibrium profiles

Variation of the flow surface:

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}} \longrightarrow \frac{\Delta h}{\Delta x} = \frac{S_0 - S_f}{1 - Fr^2}$$

Equilibrium profiles

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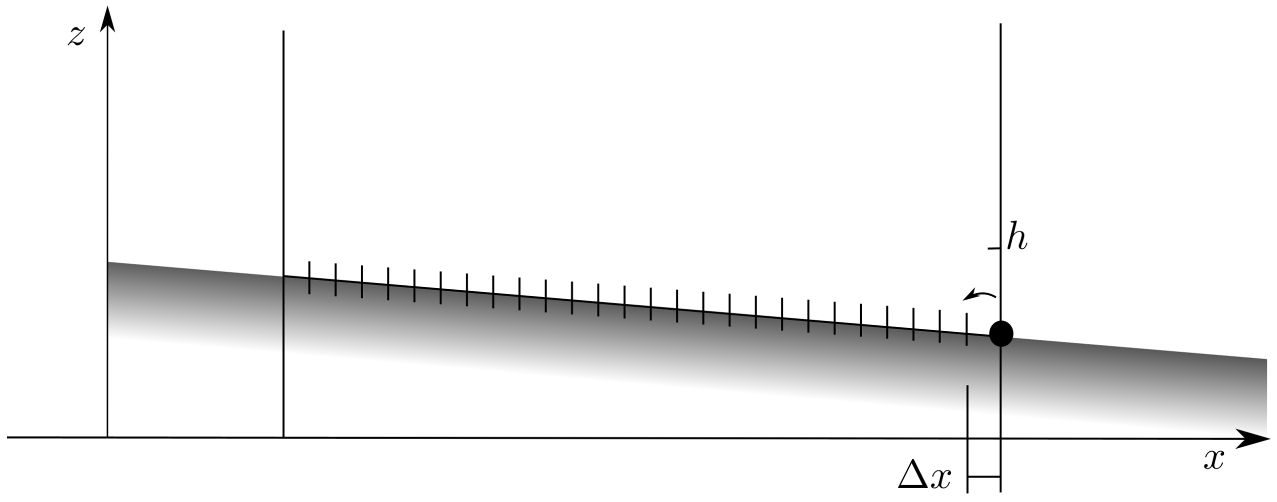
Trend analysis of the flow surface slopes



Equilibrium profiles

Variation of the flow surface:

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

There are two borderline cases:

- $h = h_N \rightarrow S_0 = S_f \rightarrow \dfrac{dh}{dx} = 0$

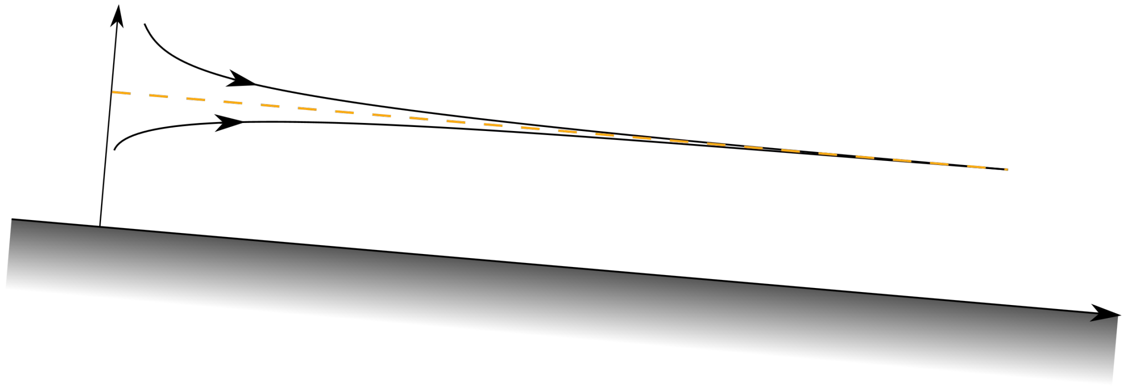
Equilibrium profiles

Variation of the flow surface:

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

There are two limit cases:

- $h = h_N \rightarrow S_0 = S_f \rightarrow \frac{dh}{dx} = 0$



Equilibrium profiles

Variation of the flow surface:

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

There are two limit cases:

- h = h_N → S_0=S_f → $\dfrac{dh}{dx} = 0$

$$\frac{dh}{dx} = \infty$$

Equilibrium profiles

Variation of the flow surface:

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

There are two limit cases:

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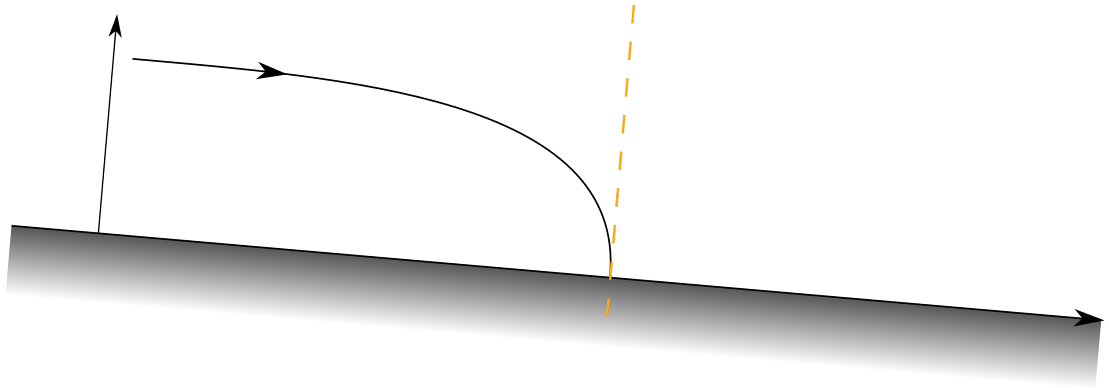
$$\frac{dh}{dx} = \infty$$



Equilibrium profiles

Variation of the flow surface:

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

There are two limit cases:

- $h = h_N \rightarrow S_0 = S_f \rightarrow \frac{dh}{dx} = 0$

$$\frac{dh}{dx} = \infty$$

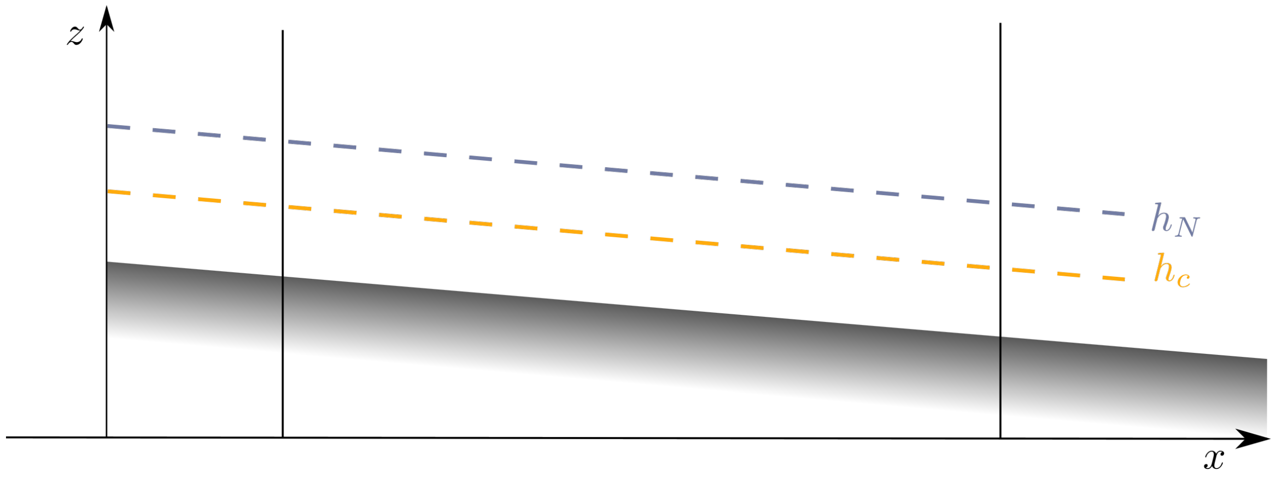
Two situations to be analyzed:

- Mild slopes: $h_c < h_N$
- Steep slopes: $h_c > h_N$

Profiles with $h_c < h_N$

Surface variation of the flow on a **mild slope**

$$\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$$



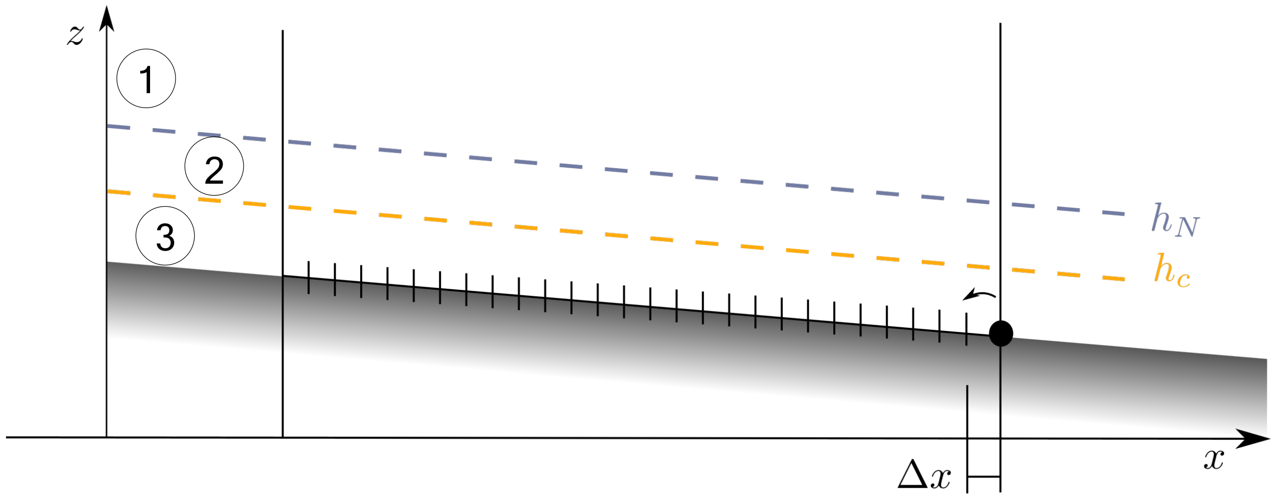
Profiles with $h_c<h_N$

Surface variation of the flow on a **mild slope**

The equation is solved for each depth region

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

$$\frac{\Delta h}{\Delta x} = \frac{S_0 - S_f}{1 - Fr^2}$$



Profiles with $h_c < h_N$

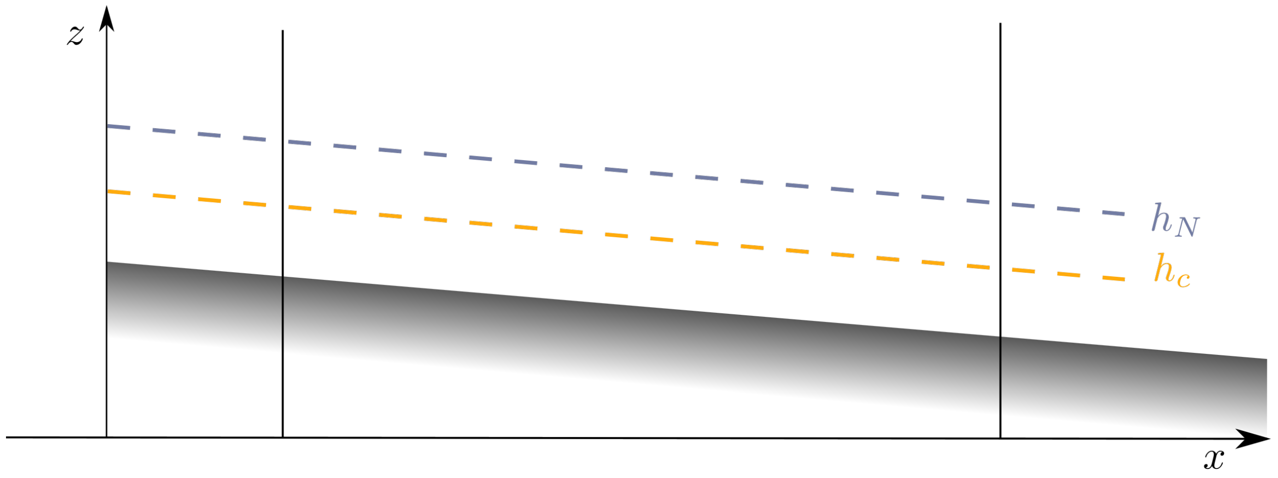
Surface variation of the flow on a **mild slope**

The equation is solved for each depth region

The evolution of the surface profile is analyzed

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

$$\frac{\Delta h}{\Delta x} = \frac{S_0 - S_f}{1 - Fr^2}$$



Profiles with $h_c<h_N$

Surface variation of the flow on a **mild slope**

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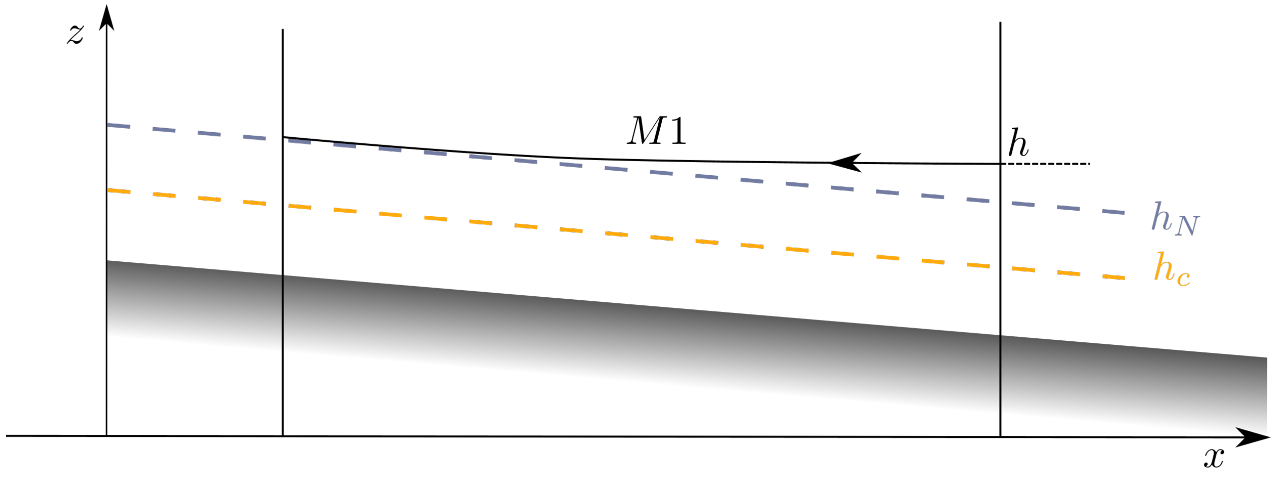
$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

$$\frac{\Delta h}{\Delta x} = \frac{S_0 - S_f}{1 - Fr^2}$$

- **M1:** If $h>h_N>h_c$
    - $S_0>S_f$, $Fr<1 \rightarrow \dfrac{dh}{dx} > 0$



Profiles with $h_c < h_N$

Surface variation of the flow on a **mild slope**

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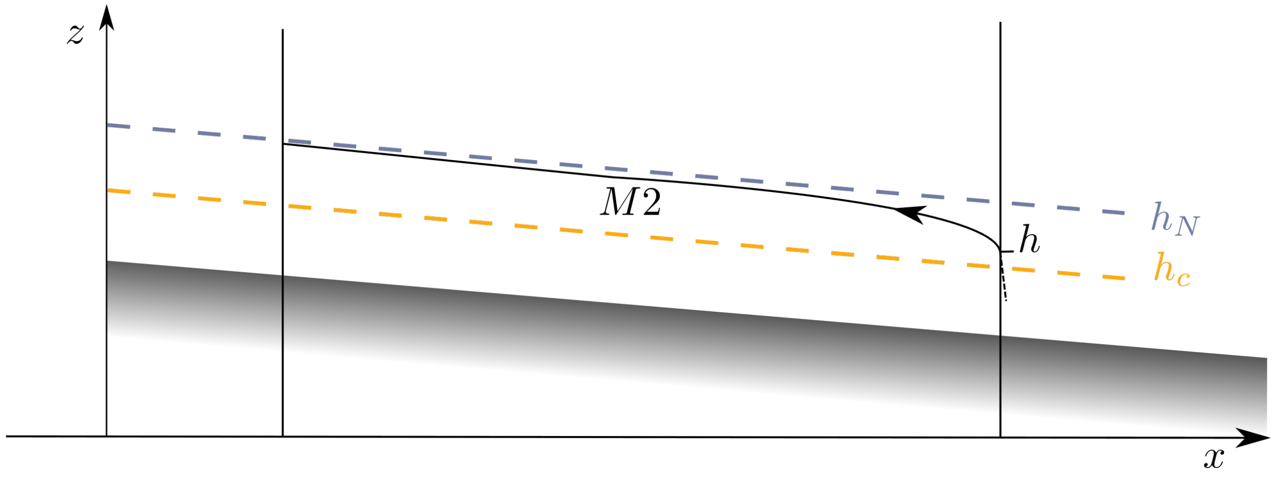
The evolution of the surface profile is analyzed

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

$$\frac{\Delta h}{\Delta x} = \frac{S_0 - S_f}{1 - Fr^2}$$

- **M1:** If $h > h_N > h_c$
  - $S_0 > S_f$, $Fr < 1 \rightarrow \frac{dh}{dx} > 0$
- **M2:** If $h_N > h > h_c$
  - $S_0 < S_f$, $Fr < 1 \rightarrow \frac{dh}{dx} < 0$



Profiles with $h_c<h_N$

Surface variation of the flow on a **mild slope**

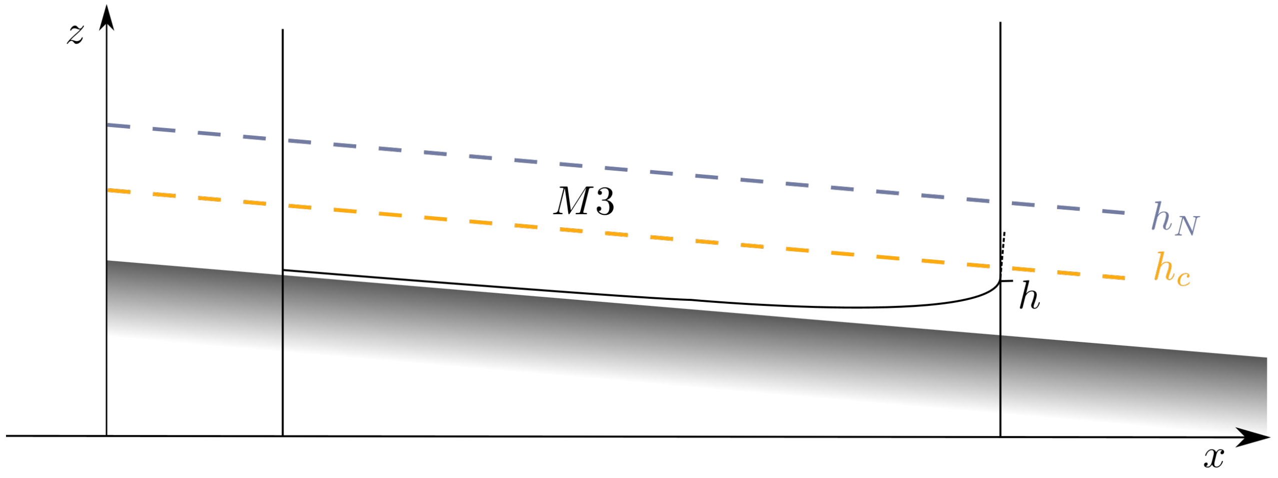
The equation is solved for each depth region

The evolution of the surface profile is analyzed

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

$$\frac{\Delta h}{\Delta x} = \frac{S_0 - S_f}{1 - Fr^2}$$

- **M1:** If $h>h_N>h_c$
    - $S_0>S_f$, $Fr<1 \rightarrow \frac{dh}{dx} > 0$
- **M2:** If $h_N>h>h_c$
    - $S_0<S_f$, $Fr<1 \rightarrow \frac{dh}{dx} < 0$
- **M3:** If $h_N > h_c >h$
    - $S_0<S_f$, $Fr>1 \rightarrow \frac{dh}{dx} > 0$



Profiles with $h_c > h_N$

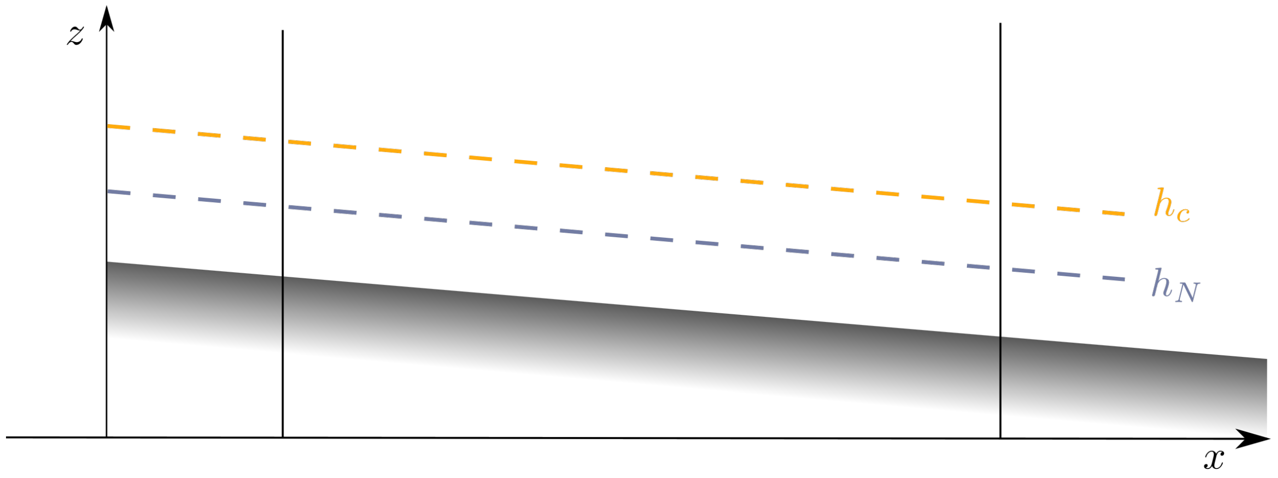
Surface variation of the flow on a **steep slope**

The equation is solved for each depth region

The evolution of the surface profile is analyzed

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

$$\frac{\Delta h}{\Delta x} = \frac{S_0 - S_f}{1 - Fr^2}$$



Profiles with $h_c > h_N$

Surface variation of the flow on a **steep slope**

The equation is solved for each depth region

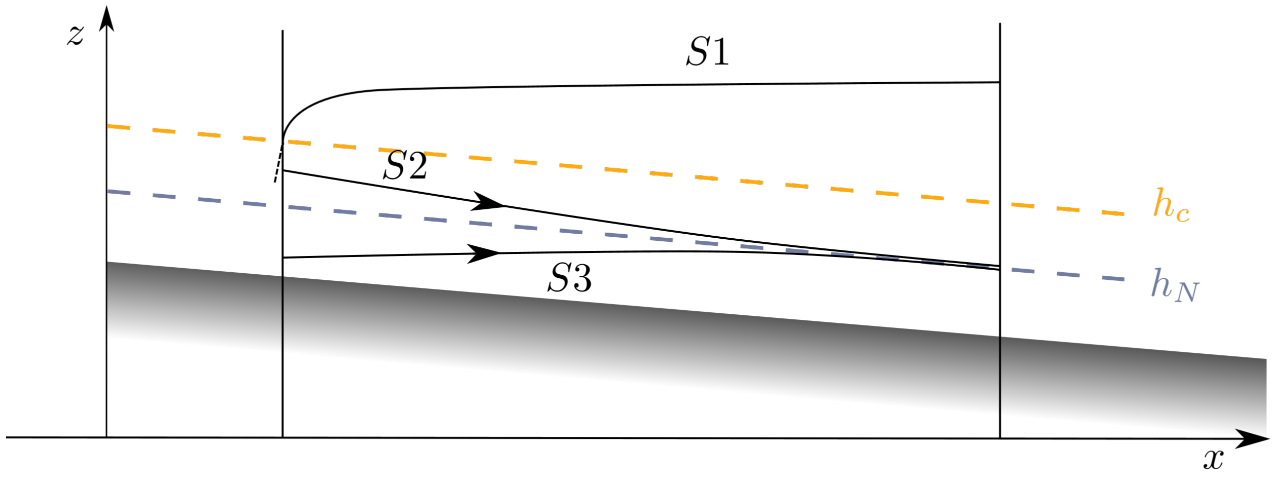
The evolution of the surface profile is analyzed

$$\boxed{\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2}}$$

$$\downarrow$$

$$\frac{\Delta h}{\Delta x} = \frac{S_0 - S_f}{1 - Fr^2}$$

- **S1:** If $h > h_c > h_N$
  - $S_0 > S_f$, $Fr < 1 \rightarrow \dfrac{dh}{dx} > 0$
- **S2:** If $h_c > h > h_N$
  - $S_0 > S_f$, $Fr > 1 \rightarrow \dfrac{dh}{dx} < 0$
- **S3:** If $h_c > h_N > h$
  - $S_0 < S_f$, $Fr > 1 \rightarrow \dfrac{dh}{dx} > 0$



Hydraulic jump

- Is the physical phenomenon that occurs when the regime changes from supercritical to subcritical.
- The balance profile gives an infinite slope that shows up as a discontinuity in the depth.
- Energy losses can be quantified according to the depths on either side of the jump.

Steady case: the hydraulic jump

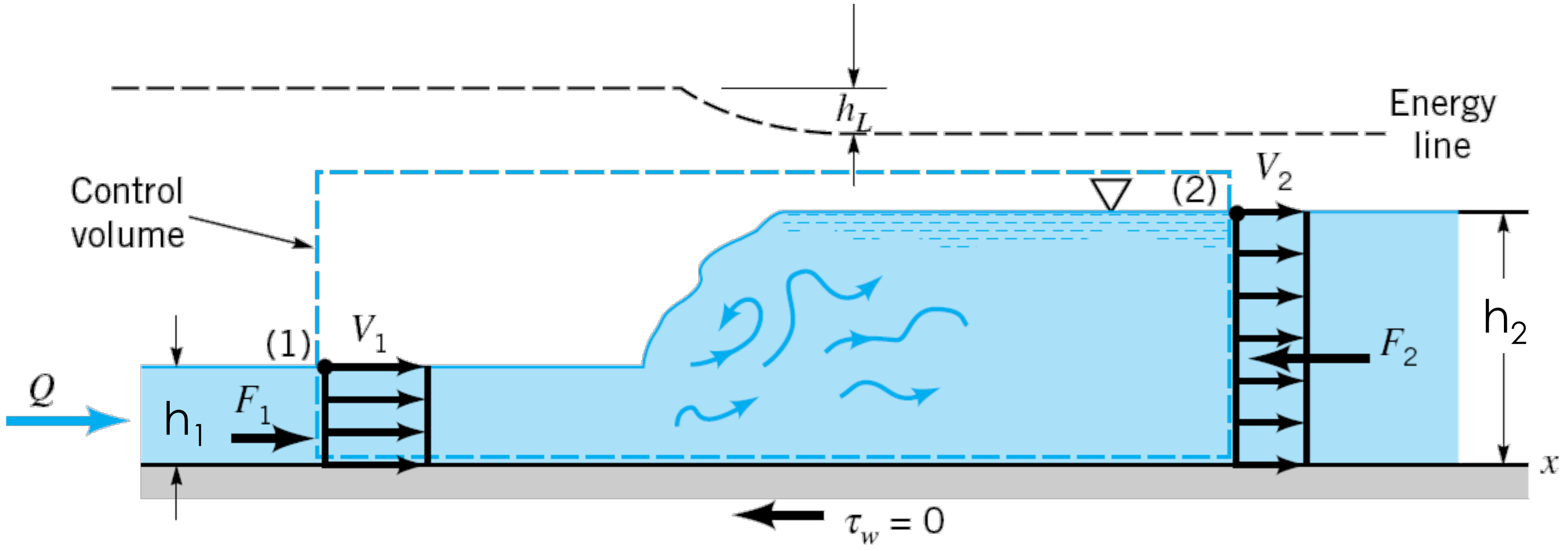
$$Q = V_1 h_1 B = V_2 h_2 B$$

$$Q(V_2 - V_1) = \left(\frac{1}{2} g h_2^2 - \frac{1}{2} g h_1^2\right) B$$

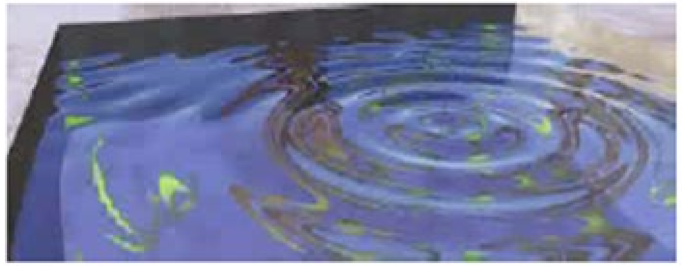
$$\left(\frac{V^2}{2g} + h\right)_1 = \left(\frac{V^2}{2g} + h\right)_2 + \Delta H$$

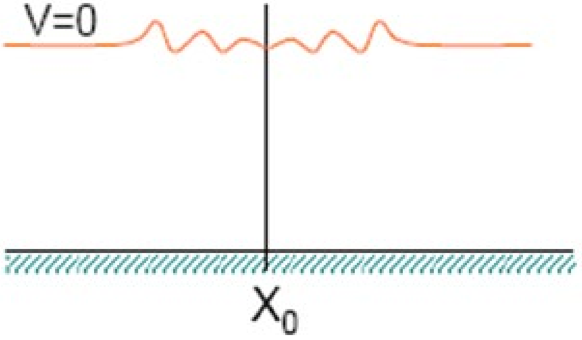
$$\boxed{\frac{h_2}{h_1} = \frac{1}{2}\left(\sqrt{1 + 8Fr_1^2} - 1\right)}$$

$$\boxed{\Delta H = \frac{(h_2 - h_1)^3}{4 h_1 h_2}}$$



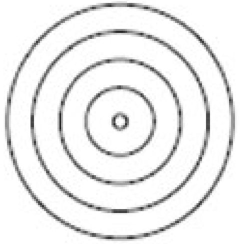
Subcritical/critical flow

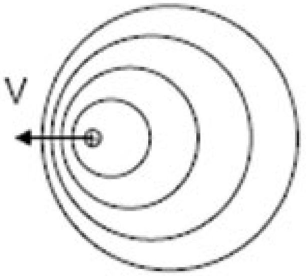


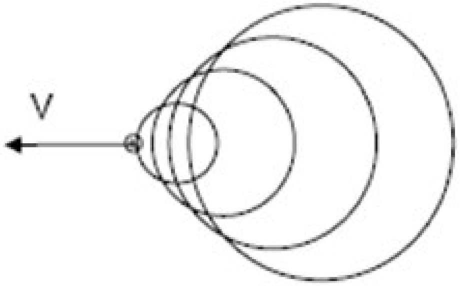


Marque urbaine (Roubaix)

http://www.eng.vt.edu/fluids/msc/gallery/gall.htm

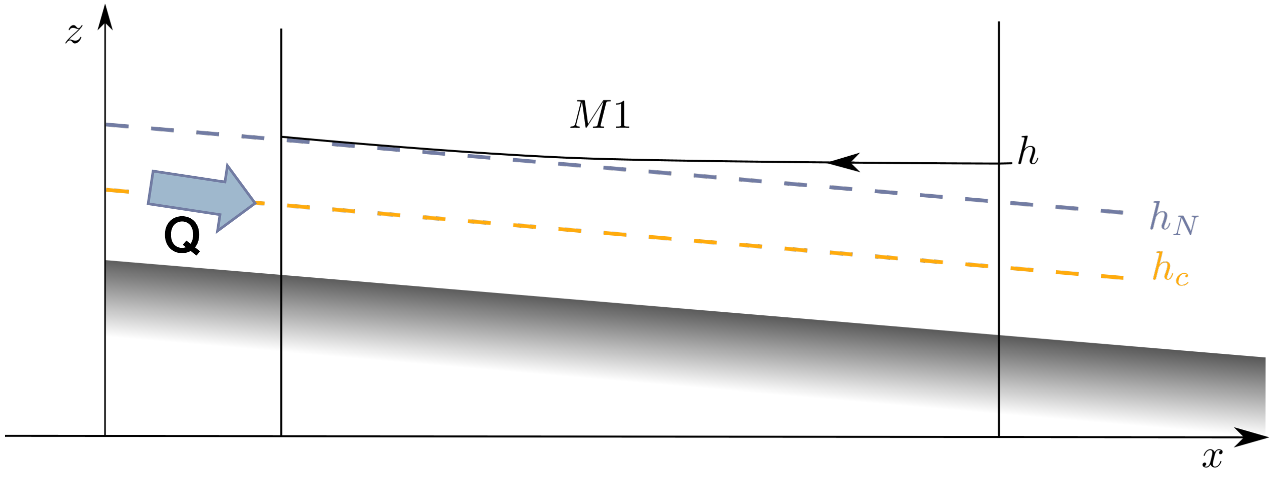






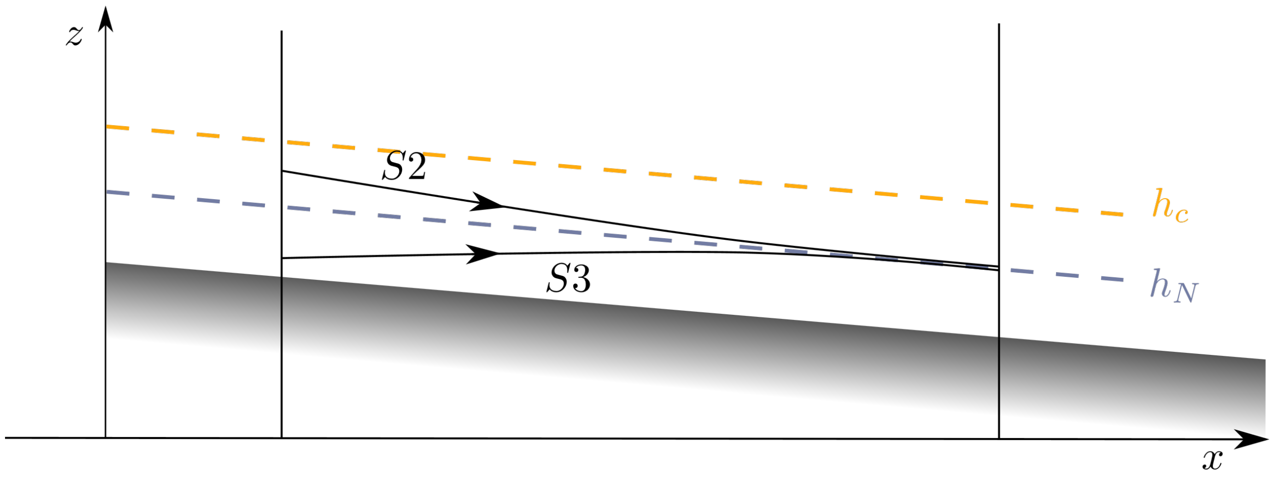
Subcritical flow

- **Subcritical flow:**
- Decelerated flow
- Low velocity
- Under the influence of the downstream condition
- Backwater curves



Supercritical flow

- **Supercritical flow:**
- Accelerated flow
- High velocity
- Under the influence of the upstream condition



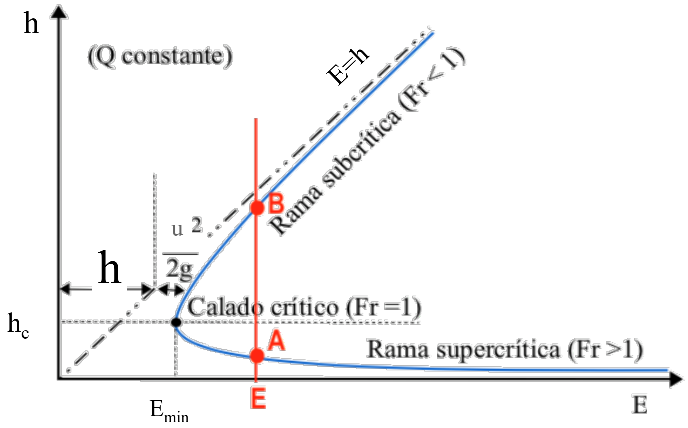
doc.Brett Sanders – http://www.eng.uci.edu/~bfs/oblique.jpg

Variations of h with E = cte

Constant flow rate:

$$Q = qB = cte \quad \Rightarrow \quad 0 = qdB + Bdq$$

Constant energy:



$$E = \frac{q^2}{2gh^2} + h = cte \quad \Rightarrow \quad 0 = \frac{q}{gh^2}dq + \left(1 - \frac{q^2}{gh^3}\right)dh$$

$$\boxed{\left(1 - Fr^2\right)\frac{dh}{dx} = Fr^2\,\frac{h}{B}\,\frac{dB}{dx}}$$

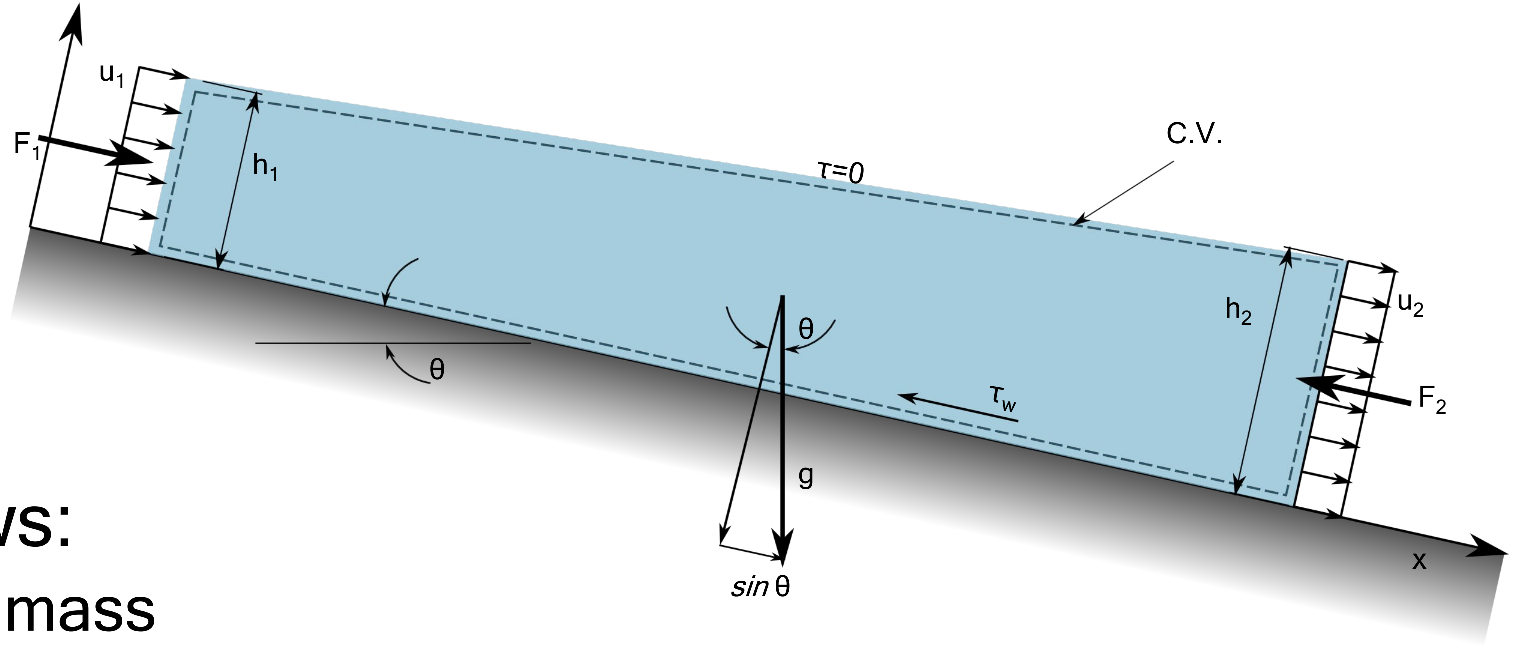
Slow flows (Fr < 1):
- If B decreases h also decreases
- If B increases, h also increases

Fast flows (Fr > 1):
- If B decreases h increases
- If B increases h decreases

Content

Motion equations: conservation laws

- Applied forces:



- Conservation laws:
  - Conservation of mass
  - Conservation of linear momentum
  - Conservation of energy

Conservation of mass

- The volume of liquid passing through the inlet section per unit of time equals the volume leaving the section,

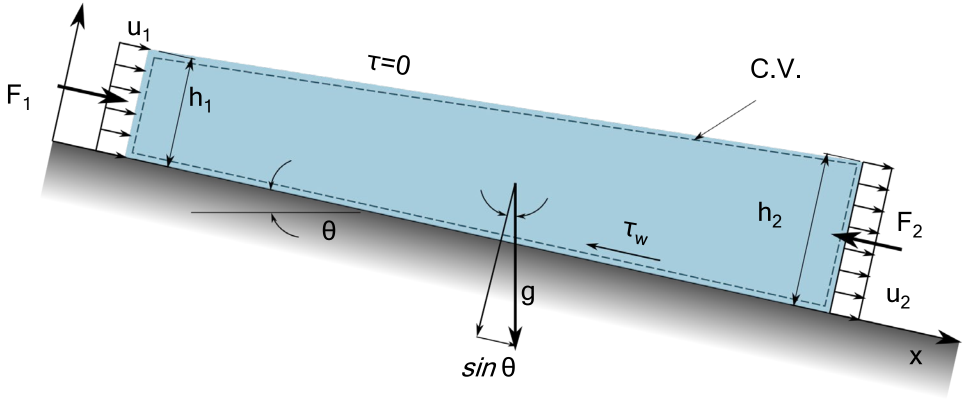
$$Q = \int_{A_e} u \, dA = \int_{A_s} u \, dA$$

- The volumetric flow or flow rate through a section is often described as a function of an average velocity over the section

$$Q = u_e A_e = u_s A_s$$

- In general, if the movement is transient

$$\boxed{\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0}$$



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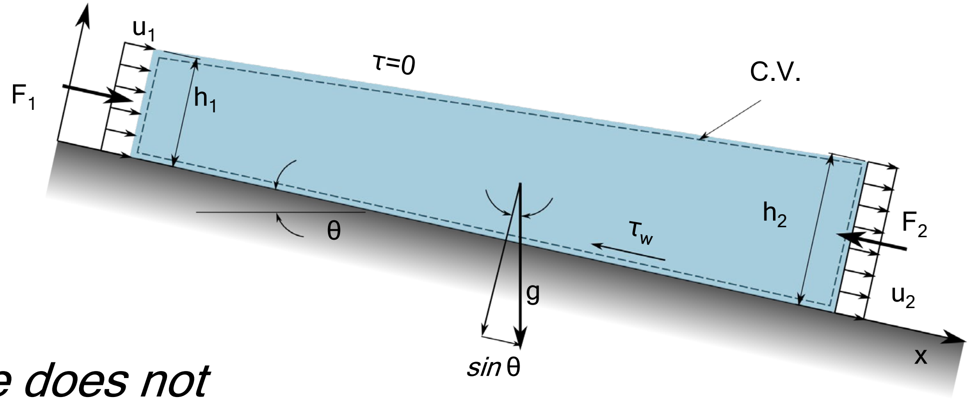
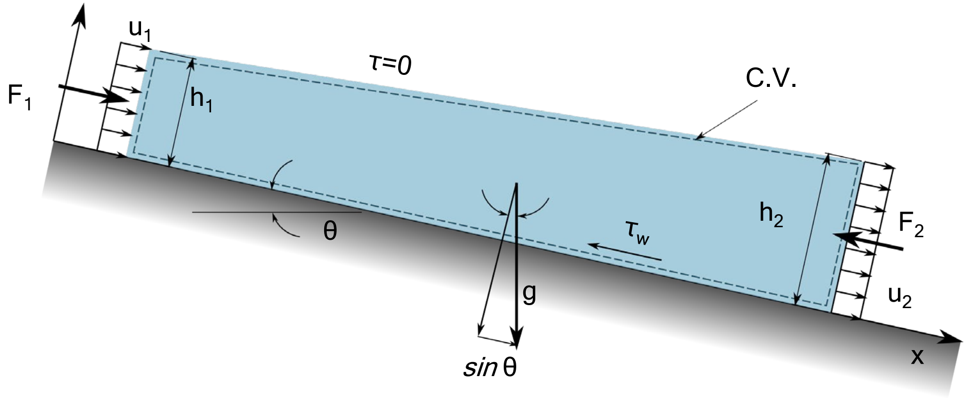


The flow rate does not vary along the channel

Conservation of linear momentum

- Newton's second law:

$$\rho \mathbf{a} = \frac{D(\rho \mathbf{V})}{Dt} = \rho \mathbf{F}_V - \nabla p + \mathbf{F}_{vis}$$



- Expressed in the direction of the channel:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) = \underbrace{-\rho g \frac{\partial z}{\partial x}}_{\text{gravity}} \underbrace{- \frac{\partial p}{\partial x}}_{\text{pressure}} + \underbrace{F_{vis}}_{\text{friction}}$$

- Integrated over the section A:

$$\boxed{\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{Q^2}{A} + g I_1 \right) = g A \left(S_0 - S_f \right) + g I_2}$$

Conservation of linear momentum

- Newton's second law states:

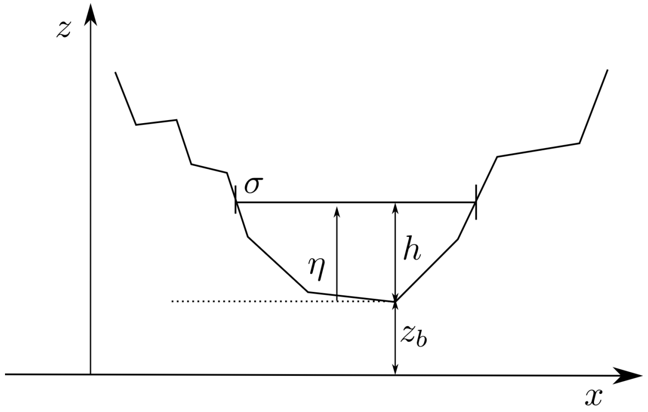
$$\rho \mathbf{a} = \frac{D(\rho \mathbf{V})}{Dt} = \rho \mathbf{F}_V - \nabla p + \mathbf{F}_{vis}$$

- Expressed in the direction of the channel:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) = \underbrace{-\rho g \frac{\partial z}{\partial x}}_{\text{gravity}} \underbrace{- \frac{\partial p}{\partial x}}_{\text{pressure}} + \underbrace{F_{vis}}_{\text{friction}}$$

- Integrated over the section A:

$$\boxed{\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{Q^2}{A} + g I_1 \right) = g A \left(S_0 - S_f \right) + g I_2}$$



Where:

$$S_0 = -\frac{\partial z_b}{\partial x} = tg\theta$$ bed slope

$$S_f = \frac{n^2 Q |Q|}{R_h^{4/3} A^2}$$ Slope of the energy line

$$I_1 = \int_0^h (h - \eta) \cdot \sigma(x, \eta) d\eta$$

$$I_2 = \int_0^h (h(x) - \eta) \frac{\partial \sigma(x, \eta)}{\partial x} d\eta$$

Conservation of energy

$$\boxed{\frac{1}{g}\frac{\partial u}{\partial t} + \frac{u}{g}\frac{\partial u}{\partial x} = -\frac{\partial h}{\partial x} + S_0 - S_f}$$

$$S_0 = -\frac{\partial z_b}{\partial x} = tg\theta$$

bed slope

$$S_f = \frac{n^2 Q|Q|}{R_h^{4/3} A^2}$$

Slope of the energy line

Conservation of energy

In steady state there are no temporary variations

$$\boxed{\frac{1}{g}\cancel{\frac{\partial u}{\partial t}} + \frac{u}{g}\frac{\partial u}{\partial x} = -\frac{\partial h}{\partial x} + S_0 - S_f}$$

Applying the definition of S_0: $S_0 = -\frac{\partial z_b}{\partial x}$

And rearranging... $$\frac{1}{2g}\frac{\partial (u^2)}{\partial x} + \frac{\partial h}{\partial x} + \frac{\partial z_b}{\partial x} = -S_f$$

Conservation of energy: stationary

The mechanical energy per weight unit varies due to viscous losses

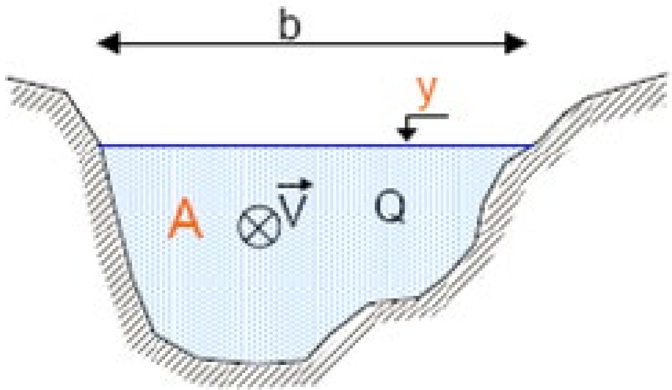
$$\boxed{\frac{d}{dx}\left(\frac{u^2}{2g}+h+z_b\right)=-S_f}$$

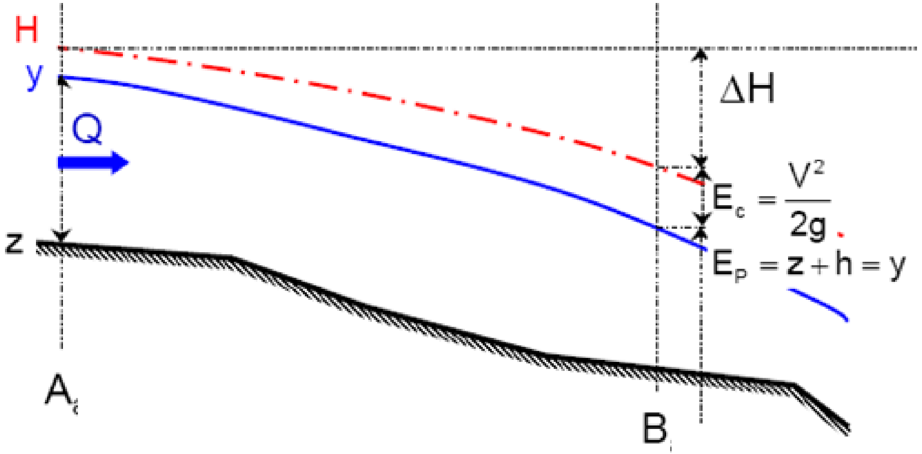
Evaluated between points 1 and 2

$$\left(\frac{u^2}{2g}+h+z_b\right)_1-\left(\frac{u^2}{2g}+h+z_b\right)_2=h_f=S_f\Delta x$$

Saint-Venant equations

- Describe the behaviour of the fluid
- Hypothesis for one-dimensional flow:
  - Gentle bed slope
  - Depth and speed only function of (x,t)
  - Uniform cross-sectional velocity





Simulation model: equations

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A} + gI_1\right) = gA\left(S_0 - S_f\right) + gI_2$$

$$S_0 = -\frac{\partial z_b}{\partial x} = tg\theta$$

$$S_f = \frac{n^2 Q|Q|}{R_h^{4/3} A^2}$$

$$I_1 = \int_0^h (h-\eta)\cdot\sigma(x,\eta)d\eta$$

$$I_2 = \int_0^h (h(x)-\eta)\frac{\partial\sigma(x,\eta)}{\partial x}d\eta$$

- We only need two equations to solve A and Q:
  - We choose mass and linear momentum
- Although in 1D it does not* matter which one you choose, it is common to choose the momentum one for when the models extend to more dimensions

**In reality the losses in the energy equation are more complicated to quantify and therefore even in 1D it is sometimes more convenient to choose the equation of linear momentum*

Simulation model: equations

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A} + gI_1\right) = gA\left(S_0 - S_f\right) + gI_2$$

- Without analytical solution
- Computer Simulation
- Discretization method of finite volumes

$$S_0 = -\frac{\partial z_b}{\partial x} = tg\theta$$

$$S_f = \frac{n^2 Q|Q|}{R_h^{4/3} A^2}$$

$$I_1 = \int_0^h (h - \eta) \cdot \sigma(x, \eta) d\eta$$

$$I_2 = \int_0^h (h(x) - \eta) \frac{\partial \sigma(x, \eta)}{\partial x} d\eta$$

ADVANCED HYDRAULIC SIMULATION MODELS

1D hydraulic simulation models of channels and rivers: PART I

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RESCUER

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