Universidad Zaragoza

Workshop 2 – RESCUER MSCA DOCTORAL NETWORK 2024-2028
Universidad de Zaragoza

2D models for erodible bed and sediment transport

Practice 2
Bedload transport models for rivers and estuaries: Equilibrium slopes

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Module 3 – 2D models and numerical techniques for erodible bed and sediment transport simulation.

Practice 1 - Bedload transport models of rivers and estuaries

Equilibrium slopes

The objective of this practice is to gain experience with the usage of transient 1D cross-section integrated numerical bedload transport models based on the Saint-Venant equations, flow modelling and the Exner equation for the time solution of the evolution of the erodible bed.

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A} + gI_1\right) = gA(S_0 - S_f) + gI_2$$

$$\frac{\partial A_b}{\partial t} + \frac{1}{1-p}\frac{\partial Q_b}{\partial x} = 0$$

where:

A, Q are the area and the liquid flow rate in the cross section,

A_b, Q_b are the bed area below the free surface and the solid bedload transport flow rate in the cross section,

$S_0 = -\dfrac{\partial z_b}{\partial x}$ is the bed slope of the channel, being z_b the minimum level of the cross-section,

$S_f = \dfrac{n^2|Q|Q}{R^{4/3}A^2}$ is the slope of friction losses at the perimeter of the cross section, where n is the Manning's roughness coefficient and R the hydraulic radius,

g is the gravitational acceleration,

p is the porosity of the bed,

I_1, I_2 are the pressure integrals in the cross section.

Considering a river section with a rectangular cross-section of constant width, the areas and flow rates integrated in the cross-section can be written as

$$A = Bh \qquad Q = Bhu$$

$$A_b = B(z_b - z_r) \qquad Q_b = Bq_b$$

being h the depth of the flow in the cross section, z_r the reference level for the elevations, q_b the unitary rate of bedload transport and u the average velocity in the cross section. The hydraulic radius can be approximated as

$$R \approx h$$

and the pressure integrals are reduced to

$$I_1 = \int_0^h (h-\eta)\, B\, \mathrm{d}\eta = \frac{1}{2}Bh^2 = \frac{1}{2}\frac{A^2}{B}$$

$$I_2 = \int_0^h (h-\eta)\frac{\partial B}{\partial x}\, \mathrm{d}\eta = 0$$

The equilibrium states of the transient system are reached when the conserved quantities A, Q, A_b stop fluctuating in time, leading to a reduced system.

$$\frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{Q^2}{A} + \frac{1}{2} g \frac{A^2}{B}\right) = gA(S_0 - S_f)$$

$$\frac{\partial Q_b}{\partial x} = 0$$

In order to fulfil the first and third condition, since

$$Q = B\,hu = const \qquad Q_b = B\,q_b(h,u) = const$$

the trivial solution is

$$\frac{\partial h}{\partial x} = \frac{\partial u}{\partial x} = 0$$

leading to the second condition being fulfilled

$$S_0 = S_f$$

i.e. the uniform flow condition.

This analysis leads to the conclusion that, under constant hydrodynamic conditions for the flow, the riverbed temporarily evolves until it reaches an equilibrium state corresponding to the uniform flow slope for those hydrodynamic conditions. Consequently, when the river regime changes, the river slope evolves to a new equilibrium state.

During the practice, the student should become familiar with the influence of the different boundary conditions imposed on the ends of the domain on the equilibrium (stationary) solution obtained.

Application case

A 10 km long river section on a non-cohesive alluvial bed, with a rectangular cross-section of 30 m constant width is studied. The bed is mainly consisting of sand with an average size of 1 mm, a natural depositional porosity of 0.4 and an internal friction angle of 30°. The overall Manning's roughness coefficient for the bed is assumed to be 0.040 $sm^{-1/3}$. Under normal meteorological conditions, the average flow rate is 100 m^3/s, with a bed slope of 0.1% and a measured depth at the final section of 2.371173005 m. Under these conditions, an estimate is made for the solid bedload transport flow (not including porosity swelling) of 1.000863498 m^3/s at the headwater cross-section, located at an elevation of 200 m above sea level.

The river section undergoes seasonal regime changes due to the lack of rainfall in summer and the snowmelt of the nearby mountains in spring. The known data on mean flow, flow depth at the outlet of the section and estimated solid bedload flow for these seasonal regimes are given in Table 1.

	Q (m^3/s)	h_{out} (m)	Q_b (m^3/s)
Spring	200	4.424765284	0.898747197
Summer	50	1.385217173	0.815129488

Table 1: Seasonal regimes for the section.

Furthermore, it is planned to build a reservoir downstream of the studied section, which would raise the elevation at the outlet of the section to 195 m above sea level, keeping the filling of the reservoir constant throughout the year.

This practice requires the following:

1) Starting from the average regime conditions described above, calculate the new equilibrium slopes in the summer and spring seasonal regimes. It must be proven that the calculated slopes correspond to a stationary situation.
   The following must be provided:
   a) Initial and final bed and free surface profile plots.
   b) New equilibrium slope data and comparison with the expected analytical value.
   c) Graphs of the temporal evolution of inflow and outflow for both liquid flow and bedload transport.

2) Determine whether seasonal changes cause net erosion or deposition in the studied river section. Comment on the results.
   The following must be provided:
   a) Graphs of the temporal evolution of the sediment mass in the domain.

3) Determine how the construction of the reservoir affects the evolution of the bed section for the three regimes described.
   The following must be provided:
   a) Initial and final bed and free surface profile plots.
   b) Graphs of the temporal evolution of inflow and outflow for both liquid flow and bedload transport.
   c) Graphs of the temporal evolution of the sediment mass in the domain.

Simulation software

A C++ code developed ad-hoc for this practice will be used. The algorithm solves the Saint-Venant equations and the Exner equation using an explicit 1D Finite Volume method. The method uses a first-order upwind numerical scheme for the decoupled determination of the hydrodynamic and bedload flows on the walls of the 1D slices of the domain.

The software allows the student to experiment with the different possibilities of imposing conditions at the boundaries of the domain in a simple way.

The operation of the software and the representation of results will be explained during the practice.