Universidad Zaragoza

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2D models for erodible bed and sediment transport

Practice 1
Bedload transport models for rivers and estuaries: Analysis of erosion by dambreak

Sergio Martínez Aranda sermar@unizar.es

Module 3 – 2D models and numerical techniques for erodible bed and sediment transport simulation.

Practice 1 - Bedload transport models of rivers and estuaries: Analysis of erosion by dambreak

The objective of this practice is to gain experience programing a robust 1D numerical bedload transport model for highly energetic transient flows.

1. Governing equations

We base our approach on the Saint-Venant equations and the Exner equation for solving the flow motion and the erodible bed evolution, respectively, in a rectangular cross-section channel of constant width:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A} + \frac{1}{2} g \frac{A^2}{B}\right) = gA(S_0 - S_f)$$

$$\frac{\partial A_b}{\partial t} + \frac{1}{1-p}\frac{\partial Q_b}{\partial x} = 0$$

where:

A, Q are the area and the liquid flow rate in the cross section,

B is the constant width of the channel,

$S_0 = -\frac{\partial z_b}{\partial x}$ is the bed slope of the channel, being z_b the bed surface level for the cross-section,

$S_f = \frac{n^2\ |Q|\ Q}{A^2\ R^{4/3}}$ is the slope of friction losses at the perimeter of the cross section, where n is the Manning's roughness coefficient and R the hydraulic ratio, which can be approximated by $R \approx h$ for large-width rectangular channels,

g is the gravitational acceleration,

A_b, Q_b are the bed area below the channel bottom and the solid bedload transport rate in the channel cross-section, and

p is the porosity of the erodible bed layer.

The integrated conservative variables in the rectangular cross-section can be written as:

$$A = Bh \qquad Q = Bhu$$

$$A_b = B(z_b - z_R) \qquad Q_b = B\, q_b$$

being h and u the flow depth and averaged velocity in the cross-section, z_R the reference level for the bed and z_b the bed surface elevation.

The term q_b denotes the unitary bedload transport rate, with can be computed by different empirical relationships:

Fixed-bed:

$$q_b = 0$$

Grass formula:

$$q_b = G\,|u|^2\,u$$

being G the local Grass interaction factor between the flow and the bed layer, which is given by the modeler.

Meyer-Peter-Müller formula:

$$q_b = \beta_B\, 8\,(\theta - \theta_c)^{3/2}\,\sqrt{\left(\frac{\rho_s}{\rho_w} - 1\right) g\, d_s^3}$$

where ρ_s and ρ_w are the sediment and the water density, respectively, ρ_s is the sediment diameter, θ is the dimensionless Shields stress, θ_c is the critical value of the Shield stress for the incipient motion, and β_B is an user-defined parameter for tuning the magnitude of the bedload rate.

The dimensionless Shields parameter is written as:

$$\theta = \frac{|\tau_b|}{(\rho_s - \rho_w)\,g\,d_s}$$

being τ_b the shear stress at the bed surface, which can be calculated for pure-turbulent flows in rectangular cross-sections as:

$$\tau_b = \rho_w g h\, S_f = \rho_w g\,\frac{n^2\,|u|\,u}{h^{1/3}}$$

2. Numerical scheme

The system of governing equations is solve using a Finite Volume (FV) method. For this practice, the water flow and the bed evolution components of the system are solved separately, following a weakly-coupled strategy. The 1D shallow water equations are solved using a first-order Roe scheme (as in the previous exercises of the W2), which allows to update the local value of the conservative flow variables A and Q for each time step. In this practice, we focus on the resolution of the Exner equation for the bedload transport.

The Exner equation is solved using Finite Volume method. The value of the conservative solid area A_b at the i cell is updated as:

$$(A_b)_i^{n+1} = (A_b)_i^n - \frac{\Delta t}{\Delta x}\frac{1}{1-p}\left[(Q_b)_{i+1/2}^{\downarrow} - (Q_b)_{i-1/2}^{\downarrow}\right]$$

being $(Q_b)_{i+1/2}^{\downarrow}$ the numerical bedload rate at the right wall of the i cell. The numerical bedload flux is computed following an upwind scheme based on the virtual bedload celerity as:

$$(Q_b)_{i+1/2}^{\downarrow} = \begin{cases} (Q_b)_i^n & \text{if} \quad \left(\tilde{\lambda}_b\right)_{i+1/2} > 0 \\ (Q_b)_{i+1}^n & \text{if} \quad \left(\tilde{\lambda}_b\right)_{i+1/2} < 0 \end{cases}$$

where bedload celerity is computed as:

$$\left(\tilde{\lambda}_b\right)_{i+1/2} = \frac{1}{1-p}\frac{(q_b)_{i+1}^n - (q_b)_i^n}{(z_b)_{i+1}^n - (z_b)_i^n}$$

Note that a threshold in case of flat bed surface $(z_b)_{i+1}^n - (z_b)_i^n \approx 0$. Furthermore, time step should be additionally limited by the virtual bedload celerity following the expression:

$$\Delta t_b = \min\left(\frac{\Delta x}{|\tilde{\lambda}_b|_{i+1/2}}\right)$$

Application case

We consider a 2000 m long channel with a rectangular cross-section of 10 m constant width and an erodible flat bed of elevation 1 m respect to the reference level. We set an initial dambreak condition at the middle of the channel, with 4 m depth on the left side and 1 m depth on the right side. We compute the numerical solution of this transient erodible flow during the first 180 seconds of movement.

This test is intended to analyze the sensibility of the bedload model to the variation of the main parameters defining the bedload rate. Maintaining the specified initial conditions, complete the following sets of simulations varying each parameter within the range indicated in Table 1.

	Bedload rate formula	*Grass factor* $G\ [s^2/m]$	*MPM tuning par.* $\beta_B\ [-]$	*Sed. diameter* $d_s\ [mm]$	*Manning factor* $n\ [s/m^{1/3}]$	Sed. density $\rho_s\ [kg/m^3]$	Bed porosity $p\ [-]$
Set 1	Grass	0.001 0.01 0.1	-	1	0.05	2650	0.4
Set 2	MPM	-	1 10 100	1	0.05	2650	0.04
Set 3	MPM	-	10	0.01 1 100	0.05	2650	0.04
Set 4	MPM	-	10	1	0.01 0.05 0.10	2650	0.4
Set 5	MPM	-	10	1	0.05	1450 2650 3850	0.4
Set 6	MPM	-	10	1	0.05	2650	0.2 0.4 0.8

Table 1: Sets of simulations for testing the model sensibility.

Perform a sensibility analysis using the obtained results. For example, the final bed level profiles can be plotted for each set of simulations in order to identify the most relevant parameter influencing the bedload model results.

Simulation software

A C++ code developed ad-hoc for this practice will be used. The algorithm solves the Saint-Venant equations and the Exner equation using an explicit 1D Finite Volume method. The method uses a first-order upwind numerical scheme for the decoupled determination of the hydrodynamic and bedload flows on the walls of the 1D slices of the domain.

The software allows the student to experiment with the different possibilities of imposing conditions at the boundaries of the domain in a simple way.

The operation of the software and the representation of results will be explained during the practice.