Rescuer Worshop 2: Urban flooding and Porous shallow water equations

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Overview

- Motivation: Ubran flooding
- Historical overview of models
- Description of the physical model
- $C-$ property
- Description of a well-balanced scheme for the porous shallow water equations
- Implementation of the scheme

Motivation: Urban flooding

This photograph taken on October 17, 2024, shows cars are submerged in a commercial zone on October 16, 2024, in Givors, central-eastern France, following heavy rainfall in the area. (Photo by JEAN-PHILIPPE KSIAZEK / AFP)

A flooded area following heavy rainfall in Annonay, central France, on October 17, 2024. (Photo by JEFF PACHOUD / AFP)

Storm on the Merlimont-Plage seafront (February 2022).

Motivation: Urban flooding

Definition:

- ”Urban flooding is a major problem in many parts of the world and is one of the most natural disastrous event which takes place every year, especially in the coastal cities..The coastal urban flooding is a complex phenomenon which may occur in various forms such as: urban flooding due to high intensity rainfall; due to inadequate drainage and flooding caused by overtopping in the channels or rivers; flooding due to high tides, etc.”, Chapter 12 - Urban Flood Management in Coastal Regions Using Numerical Simulation and Geographic Information System, Integrating Disaster Science and Management,2018,205-219

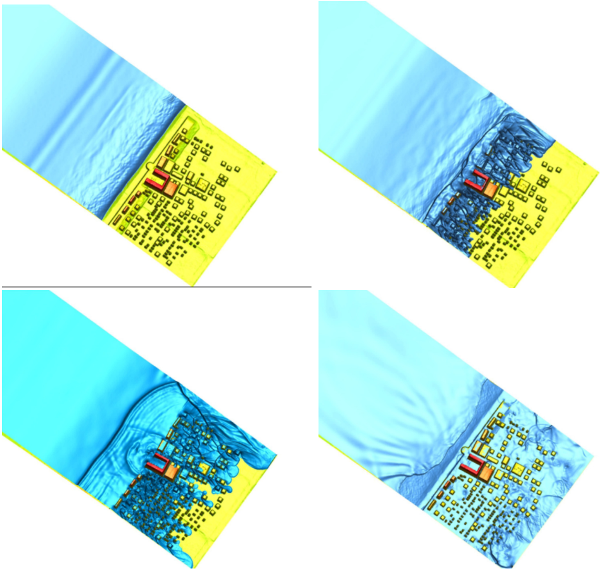
- ” Urban flooding is the leading cause of global flood losses and can arise as a result of pluvial, fluvial, or coastal flooding” ,Chapter 5- Urban flood modeling: Perspective, challenges and opportunities, Coastal Flood Risk, reduction, 2022, 47-60

- ”Floods can be categorized into three main types, river flooding, coastal flooding, and urban flooding. Flood is defined as an inundation of a normally dry zone due to an increased water level of the river, sea (coastal flooding), lake, or a surcharged urban drainage system. ” Chapter25-Uncertainty analysis using fuzzy models in hydroinformatics, Handbook of Hydroinformatics, 2023, 423-424

Mitigation of the catastrophic results!

Modelling

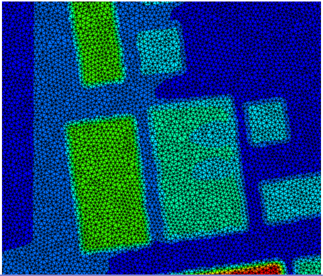
Coastal Urban flooding case: https://www.youtube.com/watch?v=nj98sHcTGOo



Pictures taken from Kazolea et al. Ocean Modelling, 2023

Urban flooding and modelling

- Urban flooding is an increasing issue due to climate change and population growth.
- Traditional numerical models for flood simulations require high computational resources.
- **Porosity-based shallow water models** incorporate subgrid-scale effects, improving efficiency. Promising tool to account for the efects exerted by buildings, without impacting on the mesh resolution and hence on the computational times.
- These models reduce computational time while maintaining accuracy in flood hazard prediction.



Why Use Porosity Models?

- Urban environments contain complex structures (buildings, streets, vegetation) that affect water flow.
- High-resolution numerical models explicitly resolving these structures are computationally prohibitive.
- Porosity models introduce effective parameters to account for subgrid-scale effects.
- They allow coarser grids while preserving critical flow dynamics in urban flood simulations.

Presence of an obstacle → effect on the flow:

- Reduce the volume available for water storage
- Channelize the flow along directional pathways
- Induce flow resistance.

Review based on Dewals et al. Porosity Models for Large-scale Urban Flood Modelling, Water, 2021

Control Volume and Porosity Parameters

- A control volume in porous shallow water models represents an area containing both water and solid obstacles.
- **Storage porosity** (ϕ): Represents the fraction of the volume available for water storage: $\phi = V_{water} / V_{total}$ (statistical descriptor). First introduced by Defina et. al (1994).
  - The porosity is the same in all directions - the storage capacity does not vary with orientation.
  - Often used in macroscopic flood models where fine-scale directional variations in porosity are ignored.
  - Urban flooding-fraction of space available for water retention within a coarse-resolution cell, Natural porous media (soil,gravel beds)- represents the uniform capacity of the medium to retain water.
- Several different isotropic formulations have been presented though the years, see for eg. Guinot et Frazao (2006), Cea and Vazquez-Cendon (2010), Ferrari et. al (2017), Viero (2019)

Control Volume and Porosity Parameters

- **Conveyance porosity** (Ψ): Represents the fraction of space available for flow exchange.
- These parameters determine how water moves and is stored within the computational domain.

- Isotropic porosity assumes uniformity in all directions
- Anisotropic porosity means that storage capacity differs in different directions due to structural variations (e.g., buildings, terrain slopes, street orientations).

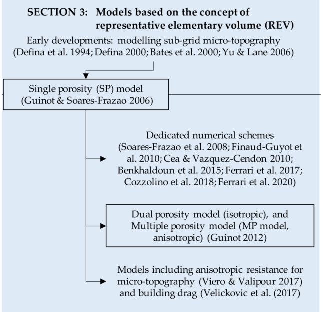
Storage and Conveyance porosity

Property	Storage Porosity (ϕ)	Conveyance Porosity (Ψ)
Definition	Fraction of space available for water storage	Fraction of space available for flow movement
Effect	Affects water retention	Affects flow velocity and discharge
Directionality	Can be isotropic	Often anisotropic (direction-dependent)
Influencing Factors	Topography, obstacles	Flow pathways, obstacles, street orientation
Urban Example	Ponds, basements, open courtyards	Streets, alleys, channels

Comparison of Storage Porosity (ϕ) and Conveyance Porosity (Ψ)

Representative Elementary Volume (REV)

- The representative elementary volume (REV) is defined as the smallest control volume for which the statistical properties of the porous medium become independent of the size of this control volume.
- A REV can be defined around any arbitrary point (x, y), irrespective of the positioning of this specific point in water or an obstacle.



Picture taken from Dewals et al., Porosity models for large-scale Urban Flood Modelling: A Review, Water, 2021

Single Porosity Model (SPM)

- First derived by Guinot and Soarez-Frazao (2006, by phase-averaging the standard SWE over a REV.
- Uses a single porosity parameter: $\phi = \Psi$. (Assumes that the available storage and conveyance space are identical).
- A uniform porosity value is assigned to the computational cells in the urban area.
- Effective for large-scale modeling but lacks directional flow information.
- Less accurate in cases where anisotropy (directional flow resistance) is significant.

$$
\begin{aligned}
\frac{\partial \phi h}{\partial t} + \frac{\partial \phi h u}{\partial x} &= 0 \\
\frac{\partial \phi h u}{\partial t} + \frac{\partial}{\partial x}\left(\phi u^2 h + \frac{1}{2} g \phi h^2\right) &= \frac{1}{2} g h \frac{\partial \phi}{\partial x} - g \phi h \frac{\partial b}{\partial x}
\end{aligned}
$$

Multiple Porosity Model (MPM)

- Introduces separate parameters for storage porosity (ϕ) and conveyance porosity (Ψ).
- Accounts for preferential flow pathways in urban areas.
- More accurate in capturing directional flow effects compared to SPM.
- Commonly used in high-resolution flood modeling where directional flow plays a role.

Integral Porosity Model (IPM)

- Developed using Reynolds Transport Theorem, leading to an *integral form* of the shallow water equations. Sanders et al.2008
- Computes porosity parameters locally at the computational cell level. In these models, the complex structure of urban environments is represented by a binary indicator function $I(x, y)$, $I(x, y) = 0$ in buildings and $I(x, y) = 1$ in points of the domain that allow flood storage and conveyance.
- Allows for anisotropic effects but is sensitive to mesh resolution and design.
- Works well with unstructured grids.
- Dual Integral Porosity Model Guinot et al. (2017)
    - Enhances IPM by introducing separate treatment for control volumes and boundaries.
    - Introduces a new momentum dissipation mechanism for transient waves.
    - Captures complex anisotropic flow effects better than previous models.
    - Reduces sensitivity to computational mesh design.

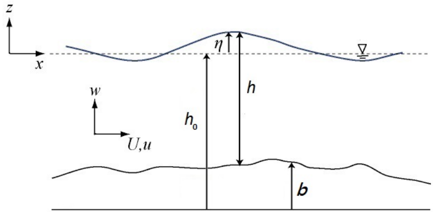
Binary Single Porosity Models (BSPM)

- Derived very recently by Varra et al. 2020,2024.
- Theoretical connection between IPM and SPM.
- Allow for local definition of the urban fabric geometric characteristics. The BSP model inherits the structure of the SWE by replacing porosity $\phi(x, y)$ with a binary function.

We will work with

- A single porosity model (SP) of Guinot and Soares-Frazao (2006) - Porosity as a statistical descriptor
- Depth-independent porosity
- Model expressed in differential form
- Isotropic porosity effects
- Shock-capturing scheme

The physical problem



η: **free surface elevation**;
h_0: **steel water level**;
$b(x)$: **bottom's topography variation**;
$h(x,t) = \eta(x,t) - b(x)$: **total water depth**;
$u(x,t)$: **flow velocity**;
$\boxed{\phi(x): \quad \textbf{porosity};}$

Mathematical model: The Shallow Water equations with porosity

$$
\begin{aligned}
\frac{\partial \phi h}{\partial t} + \frac{\partial \phi h u}{\partial x} &= 0 \\
\frac{\partial \phi h u}{\partial t} + \frac{\partial}{\partial x}\left(\phi u^2 h + \frac{1}{2} g \phi h^2\right) &= \frac{1}{2} g h \frac{\partial \phi}{\partial x} - g \phi h \frac{\partial b}{\partial x}
\end{aligned}
$$

where g is the gravitational acceleration.

The source terms at the right hand side represent the reaction due to the porosity variation and to the bed slope.
When $\phi = 1$ we retrieve the classical shallow water equations.

Mathematical model: The Shallow Water equations with porosity

$$
\begin{aligned}
\frac{\partial \phi h}{\partial t} + \frac{\partial \phi h u}{\partial x} &= 0 \\
\frac{\partial \phi h u}{\partial t} + \frac{\partial}{\partial x}\left(\phi u^2 h + \frac{1}{2} g \phi h^2\right) &= \frac{1}{2} g h \frac{\partial \phi}{\partial x} - g \phi h \frac{\partial b}{\partial x}
\end{aligned}
$$

Jacobian matrix:

$$
\mathbf{F}'(\mathbf{q}) = \begin{bmatrix} 0 & 1 \\ -u^2 + gh & 2u \end{bmatrix}
$$

Eigenvalues and eigenvectors:

$$
\lambda^1 = u - c, \quad \lambda^2 = u + c \quad \text{where } c = \sqrt{gh}
$$

$$
\mathbf{r}_1 = \begin{bmatrix} 1 \\ u - c \end{bmatrix}, \quad \mathbf{r}_2 = \begin{bmatrix} 1 \\ u + c \end{bmatrix}
$$

Same as the classical shallow water model!

The C-property

We write the solution of the PSW when at rest:

$$u = 0, \quad \eta = h + b = C \tag{1}$$

For this lake at rest condition we have $\mathbf{U}_t = 0$ so this means that the fluxes and the source terms are in balance.

$$\frac{\partial}{\partial x}\left(\frac{1}{2}g\phi h^2\right) = \frac{1}{2}gh\frac{\partial \phi}{\partial x} - g\phi h\frac{\partial b}{\partial x}$$

Our numerical scheme should keep in balance the fluxes with the source therm in this special state.

Definition: *A numerical scheme satisfies the C-property if it solves exactly the steady state for the lake at rest, i.e.*

$$u = 0, \quad h = D - b. \tag{2}$$

where D is a constant such as $D > max b(x), \;\; x \in [0, \; L]$

For the porous shallow water system :

$$\begin{aligned}\frac{\partial \phi h}{\partial t} + \frac{\partial \phi hu}{\partial x} &= 0 \\ \frac{\partial \phi hu}{\partial t} + \frac{\partial}{\partial x}\left(\phi u^2 h + \frac{1}{2} g\phi h^2\right) &= \frac{1}{2} gh^2 \frac{\partial \phi}{\partial x} - g\phi h \frac{\partial b}{\partial x}\end{aligned}$$

For a stationary solution $q = Constant = C$ and

$$\frac{\partial}{\partial x}(\phi u^2 h + \frac{1}{2}\phi h^2) = \frac{1}{2} gh^2 \frac{\partial \phi}{\partial x} - ghb_x$$

$$\frac{\partial}{\partial x}(\frac{q^2}{(\phi h)} + \frac{1}{2}\phi h^2) = \frac{1}{2} gh^2 \frac{\partial \phi}{\partial x} - ghb_x$$

$$\left(\frac{q^2}{(\phi h)}\right)_x + \frac{1}{2}\phi_x h^2 g + g\phi h h_x = \frac{1}{2} gh^2 \phi_x - g\phi h b_x$$

$$C^2 (\phi h)_x^{-1} + g(\phi h) h_x + g(\phi h) b_x = 0$$

$$-C^2(\phi h)_x^{-3} + gh_x + gb_x = 0$$

so

$$\frac{1}{2}\frac{C^2}{(\phi h)^2} + gh + gb = C_2$$

so for the shallow water system with porosity

$$q = C_1, \quad \frac{1}{2}\frac{q^2}{(\phi h)^2} + gh + gb = C_2$$

and you have the physical restriction that $0 < \phi(x) < 1$

Trivial solution (lake at rest)

$$q = 0, \quad h + b = Constant$$

Well balanced schemes

A numerical scheme for a system of balance laws is said to be **well-balanced** if it preserves all (or a specific class of) stationary equilibrium solutions of the system. This means that when a steady-state equilibrium is perturbed, the numerical method correctly captures the resulting waves propagating at finite speed, while ensuring that the equilibrium remains unchanged in regions unaffected by the perturbation.

- **Preserves Steady States:** The method must maintain exact or near-exact equilibrium solutions without introducing numerical artifacts.
- **Correctly Captures Perturbations:** When a perturbation is introduced, the scheme generates the expected wave structure, without disturbing the equilibrium in unperturbed regions.
- **Consistency with Physical Behavior:** The numerical solution must mimic the physical system's ability to maintain equilibrium where no disturbances exist.

Existing FV methods for the PSWEs

Difficulties:

- Addition of spatially varying topography
- Additional source terms to the modified equations.
- Preservation of nontrivial equilibrium due to the presence of extra source terms.

We need specific treatment! Some existing works:

- A modified HLL approximate Riemann solver that accounts for the presence of porosity. Guinot and Soarez-Frazao, (2006).
- A modified Rusanov scheme for the shallow water equations with porosity Mohamed, Computer and Fluids, 2014.
- A new approximate solver (PorAs) Finaus-Guyot et al. Int J. Numer Methods Fluids, 2010.
- A Roe-type approximate solver Wang et al. Chin J Thor Appl Mech, 2008.
- An exact Riemann solver for the PSWEs and the dam break problem Cozzolino et al, Adv. Water Resour, 2018
- A non-homogeneous Riemann solver for shallow water equations in porous media Benkhaldoun et al., Applicable Analysis, 2015
- A 1D-2D Augmented Roe solver for porosity without bathymetry. Ferrari et al. Adv. Water Resour., 2017
- A second order numerical scheme for the PSWE based on a DOT ADER augmented Riemann solver Ferrari et al., Adv Water Resour. 2020

Augmented model

Reminder:

$$
\begin{aligned}
\frac{\partial \phi h}{\partial t} + \frac{\partial \phi h u}{\partial x} &= 0 \\
\frac{\partial \phi h u}{\partial t} + \frac{\partial}{\partial x}\left(\phi u^2 h + \frac{1}{2} g \phi (\eta^2 - 2\eta b)\right) &= \frac{1}{2} g (\eta^2 - 2\eta b) \frac{\partial \phi}{\partial x} - g \phi h \frac{\partial b}{\partial x}
\end{aligned}
$$

that can be written in the compact form:

$$
\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \mathbf{H} \frac{\partial \mathbf{U}}{\partial x} = 0
$$

with

$$
\mathbf{U} = \begin{bmatrix} \phi\eta \\ \phi h u \\ \phi \\ b \end{bmatrix}, \ \mathbf{F} = \begin{bmatrix} \phi h u \\ \phi h u^2 + \frac{g}{2}\phi(\eta^2 - 2\eta b) \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} g (\eta^2 - 2\eta b) & g\phi\eta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

Augmented model

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \mathbf{H}\frac{\partial \mathbf{U}}{\partial x} = 0$$

with

$$\mathbf{U} = \begin{bmatrix} \phi\eta \\ \phi hu \\ \phi \\ b \end{bmatrix}, \ \mathbf{F} = \begin{bmatrix} \phi hu \\ \phi hu^2 + \frac{g}{2}\phi(\eta^2 - 2\eta b) \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}g(\eta^2 - 2\eta b) & g\phi\eta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- We add two fictitious additional conserved variables
- This means two extra equations that state the invariance for porosity and topography in time

Augmented model

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \mathbf{H}\frac{\partial \mathbf{U}}{\partial x} = 0$$

with the correct initial conditions is a Generalized Riemann problem.

The source terms now influence the eigen-structure of the system.

Not the same eigen-structure as the shallow water anymore!

Augmented model

We write the system in the following quasi-linear form:

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}\frac{\partial \mathbf{U}}{\partial x} = 0$$

where

$$\mathbf{A} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} + \mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ gh - u^2 & 2u & u^2 b - g\eta^2 + g\eta b & u^2\phi \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

with eigenvalues:

$$\lambda_1 = u - \sqrt{gh}, \quad ,\lambda_2 = 0, \quad \lambda_3 = 0, \quad \lambda_4 = u + \sqrt{gh} \tag{3}$$

Augmented model

And eigenvectors:

$$\mathbf{R}_1 = \begin{bmatrix} 1 \\ u - \sqrt{gh} \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{R}_2 = \begin{bmatrix} \frac{-u^2 b + g\eta^2 - g\eta b}{gh - u^2} \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{R}_3 = \begin{bmatrix} \frac{-u^2 \phi}{gh - u^2} \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{R}_4 = \begin{bmatrix} 1 \\ u + \sqrt{gh} \\ 0 \\ 0 \end{bmatrix}$$

Non-conservative methods

We are concerned with systems of PDEs written in the non-conservative form

$$\partial_t \mathbf{U} + \mathbf{A}(\mathbf{U})\partial_x \mathbf{U} = \mathbf{S}(\mathbf{U})$$

where $\mathbf{U}$ are the conserved variables. A class of schemes for solving PDEs in non-conservative form is the path-conservative schemes . These schemes depend on the choice of the path Ψ in phase space.

For a first order scheme we have:

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta t}{\Delta x}\left(\mathbf{D}^-_{i+1/2} + \mathbf{D}^+_{i-1/2}\right) + \Delta t \mathbf{S}_i$$

$\mathbf{D}^-_{i+1/2}, \mathbf{D}^+_{i-1/2}$ are called fluctuations and are related to numerical flux in the case of a conservative system.

Non-conservative methods

The fluctuations, accounting for the absence of a flux function are expected to satisfy the

- consistency condition $\mathbf{D}^{\pm}_{i+1/2}(\mathbf{U}, \dots, \mathbf{U}) = 0$
- compatibility condition

$$\mathbf{D}^{-}_{i+1/2} + \mathbf{D}^{+}_{i+1/2} = \int_0^1 \mathbf{A}(\Psi(s; \mathbf{U}_i^n, \mathbf{U}_{i+1}^n)) \frac{\partial}{\partial s} \Psi(s; \mathbf{U}_i^n, \mathbf{U}_{i+1}^n).$$

The path function should satisfy

$$\Psi(0; \mathbf{U}_i^n, \mathbf{U}_{i+1}^n) = \mathbf{U}_i^n \text{ and } \Psi(1; \mathbf{U}_i^n, \mathbf{U}_{i+1}^n) = \mathbf{U}_{i+1}^n$$

Simplest path is the canonical path :

$$\boldsymbol{\Psi}(s) = \mathbf{U}_i + s\,(\mathbf{U}_{i+1} - \mathbf{U}_i)$$

Dot Riemann solver

For non-conservative hyperbolic systems

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \frac{\Delta t}{\Delta x}\left(\mathbf{D}_{i+1/2}^{-} + \mathbf{D}_{i-1/2}^{+}\right)$$

in a control volume $[x_i,\ x_{i+1}] \times [t^n,\ t^{n+1}]$.
DOT Riemann solver for the fluctuations: $\mathbf{D}_{i+1/2}^{-}, \mathbf{D}_{i-1/2}^{+}$

$$\mathbf{D}_{i+1/2}^{\pm} = \frac{1}{2}\int_0^1 \left[\mathbf{A}(\boldsymbol{\Psi}(s)) \pm |\mathbf{A}(\boldsymbol{\Psi}(s))| \frac{\boldsymbol{\Psi}(s)}{\partial s}\right] ds$$

- C. Pares, Numerical methods for nonconservative hyperbolic systems: a theoretical framework., SIAM J. Numer. Anal. 44(1), 300-321, 2006.
- Castro et al. High order finite volume schemes based on reconstruction of states for solving hyperbolic systems with nonconservative products. applications to shallow water system. Math. Comput. 75 (255), 1103-1134, 2006.
- Dumbser and Toro, A simple extension to of the Osher Riemann solver to non conservative hyperbolic systems. J.Sc. Comput. 48, 2011.
- Ferrari A., A second order numerical scheme for the porous shallow water equations based on a DOT ADER augmented Riemann solver.Advances in Water Resources, 140, 2020.

Dot Riemann solver

For non-conservative hyperbolic systems

The matrices $\mathbf{A}$ and $|\mathbf{A}|$ are evaluated along the path $\mathbf{\Psi}(s)$ that is a Lipschitz continuous function

$$\mathbf{\Psi}(s) = \mathbf{U}^{-}_{i+1/2} + s\left(\mathbf{U}^{+}_{i+1/2} - \mathbf{U}^{-}_{i+1/2}\right)$$

so, for a first order scheme

$$\mathbf{\Psi}(s) = \mathbf{U}_i + s\left(\mathbf{U}_{i+1} - \mathbf{U}_i\right)$$

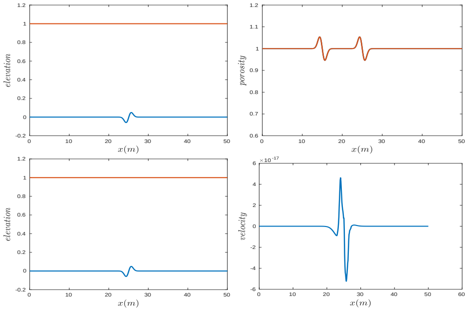
and we compute the integral with a 3-point Gauss-Legandre quadrature :

$$\mathbf{D}^{\pm}_{i+1/2} \approx \frac{1}{2}\sum_{j=1}^{3} \omega_j \left[\mathbf{A}(\mathbf{\Psi}(s_j)) \pm |\mathbf{A}(\mathbf{\Psi}(s_j))|\right]\left(U_{i+1} - U_i\right)$$

with $s_{1,3} = \frac{5\pm\sqrt{15}}{10},\ s_2 = 0.5, \quad \omega_{1,3} = \frac{5}{18},\ \omega_2 = \frac{8}{18}$

Test cases

Lake at rest



Topography:

$$b(x) = 0.05 * \sin(x - 12.5) * e^{1-(x-25)*(x-25)}$$

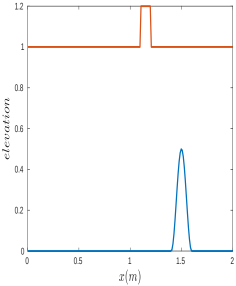
Porosity:

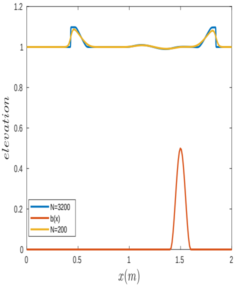
$$\begin{aligned} \phi(x) &= 0.5 - 0.05 * \sin(x - 25) * e^{1-(x-25)*(x-25)} \\ &+ 0.05 * \sin(x - 15) * e^{1-(x-15)*(x-15)} \end{aligned}$$

Initial conditions and porosity function for the lake at rest test case (up). Solution after 7sec (down)

Test cases

Pertrubation over a lake at rest





Topography:

$b(x) = -0.25(\cos(10\pi(x-1.5))-1)$ if $1.4 \leq x \leq 1.6$

Porosity:

$$\phi(x) = 0.5 - 0.05 * \sin(x-5) * e^{1-(x-5)*(x-5)}$$

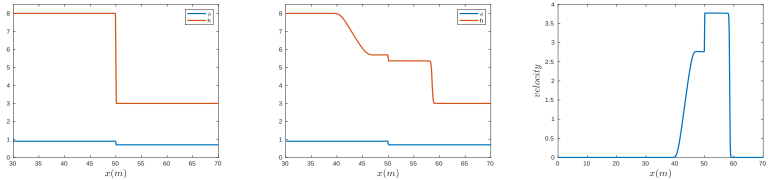
Initial conditions (left) and solution at time $t = 0.2sec$ for N=200 and N=3200

Test cases

Riemann problems: Rarefantion- Shock

h_L	u_L	ϕ_L	b_L	h_R	u_R	ϕ_R	b_R
8	0	0.9	0	3	0	0.7	0

Initial conditions



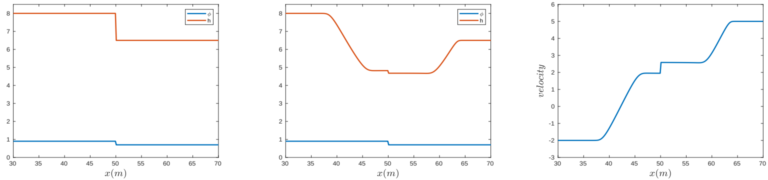
Initial conditions for h and ϕ (left). Solution for h at $t = 1$ sec. (middle). Solution for u at $t = 1$ sec. (right).

Test cases

Riemann problems: Rarefaction-Rarefaction

h_L	u_L	ϕ_L	b_L	h_R	u_R	ϕ_R	b_R
8	-2	0.9	0	6.5	5	0.7	0

Initial conditions



Initial conditions for h and ϕ (left). Solution for h at $t = 1$ sec. (middle). Solution for u at $t = 1$ sec. (right).

Test cases

Riemann problems: To do

Test number	h_L	u_L	ϕ_L	b_L	h_R	u_R	ϕ_R	b_R
1	6	-18	0.9	0	15	0	0.7	0
2	5	0	1	0	1	0	1	0.5
3	8	-2	1	0	5	7	1	0.5
4	6	-16	1	0	10	0	1	0.5

Initial conditions