ADVANCED HYDRAULIC SIMULATION MODELS

1D hydraulic simulation models of channels and rivers: PART II

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RESCUER

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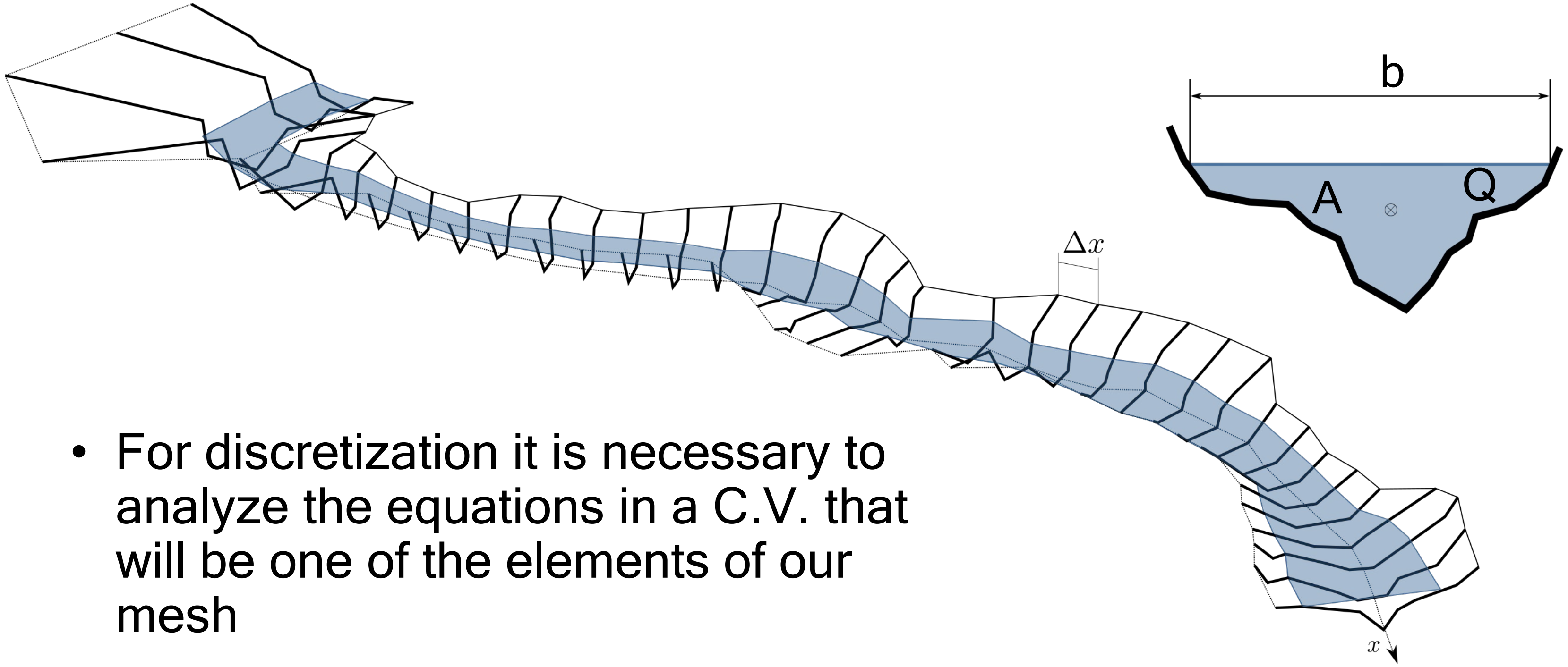
Simulation model: equations

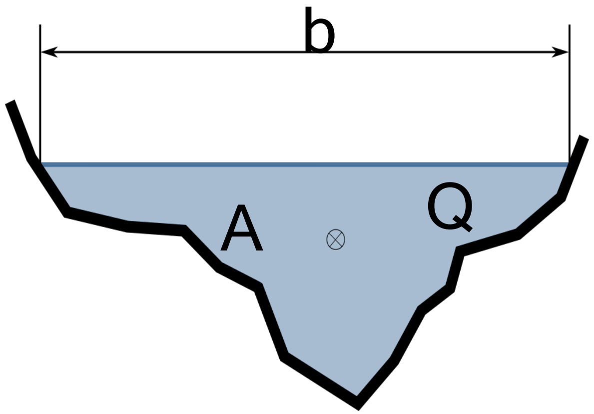
- Requirement of a numerical scheme for the resolution of the system
- Discretization in cells → mesh

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = -Q_l$$
$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A} + gI_1\right) = gA(S_0 - S_f) + gI_2$$



- For discretization it is necessary to analyze the equations in a C.V. that will be one of the elements of our mesh

Conservative formulation



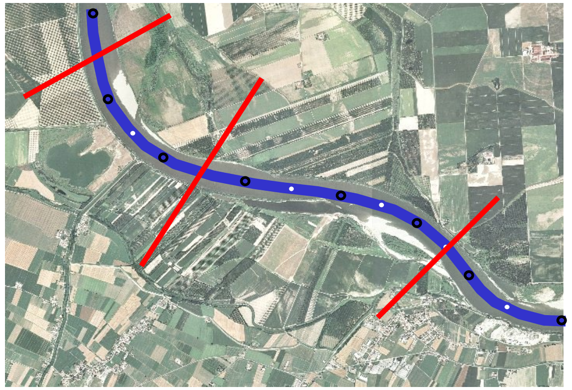
$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}(\frac{Q^2}{A} + gI_1) = gI_2 + gA(S_0 - S_f)$$

$$I_1 = \int_0^{h(x,t)} (h-\eta)b(x,\eta)d\eta \ , \ b(x,\eta) = \frac{\partial A(x,t)}{\partial \eta} \ , \ I_2 = \int_0^{h(x,t)} (h-\eta)\frac{\partial b(x,\eta)}{\partial x}d\eta$$

y

$$\frac{\partial I_1}{\partial x} = I_2 + A\frac{\partial h}{\partial x}$$



Hyperbolic system

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{x}, \mathbf{U})}{\partial x} = \mathbf{H}(\mathbf{x}, \mathbf{U}) \begin{cases} \mathbf{U} = (A, Q)^T \\ \mathbf{F} = (Q, \frac{Q^2}{A} + gI_1)^T \\ \mathbf{H} = (0, gI_2 + gA(S_0 - S_f))^T \end{cases}$$

Conserved variables

Flows (physical)

Hyperbolic system

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{x}, \mathbf{U})}{\partial x} = \mathbf{H}(\mathbf{x}, \mathbf{U}) \begin{cases} \boxed{\mathbf{U} = (A, Q)^T \quad \text{Conserved variables}} \\ \mathbf{F} = (Q, \frac{Q^2}{A} + gI_1)^T \quad \text{Flows (physical)} \\ \mathbf{H} = (0, gI_2 + gA(S_0 - S_f))^T \end{cases}$$

Hyperbolic system

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Conserved variables

Flows (physical)

The Jacobian matrix is:

$$\mathbf{J} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} = \begin{pmatrix} 0 & 1 \\ g\frac{A}{b} - \frac{Q^2}{A^2} & 2\frac{Q}{A} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ c^2 - u^2 & 2u \end{pmatrix},$$

The eigenvalues are: $\lambda^{1,2} = u \pm c$

The eigenvectors are: $\mathbf{e}^{1,2} = (1, u \pm c)^T$

Characteristic formulation

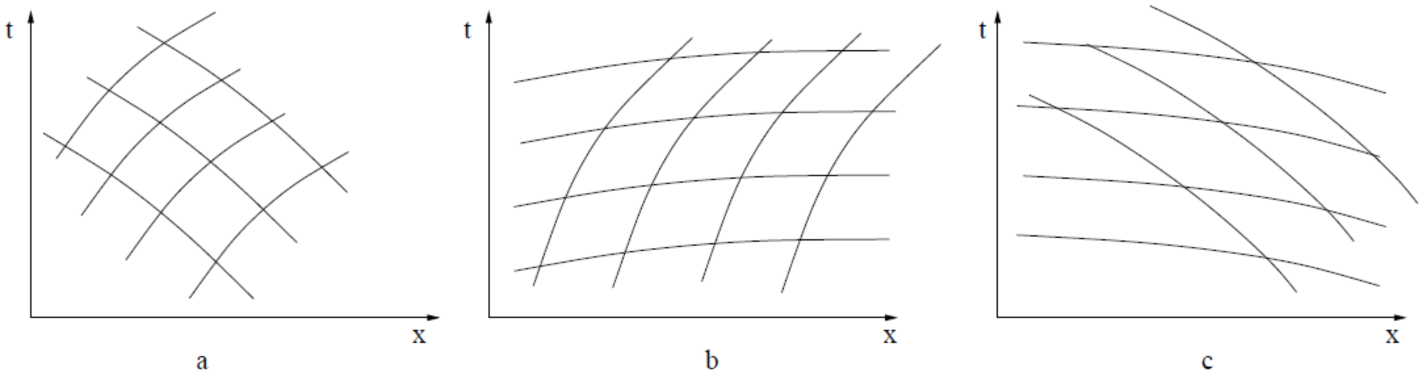
- Characteristic formulation

$$
\begin{aligned}
\frac{\partial}{\partial t}(u+2c) + (u+c)\frac{\partial}{\partial x}(u+2c) &= g(S_0 - S_f) \\
\frac{\partial}{\partial t}(u-2c) + (u-c)\frac{\partial}{\partial x}(u-2c) &= g(S_0 - S_f)
\end{aligned}
$$

therefore, in the ideal case

$$
\begin{aligned}
\frac{dx}{dt} = u + c \quad &\Rightarrow \quad d(u+2c) = 0 \Rightarrow u + 2c = \text{cte} \\
\frac{dx}{dt} = u - c \quad &\Rightarrow \quad d(u-2c) = 0 \Rightarrow u - 2c = \text{cte}
\end{aligned}
$$

Characteristic formulation



a) Subcritical flow b) supercritical flow from left to right c) supercritial flow from right to left

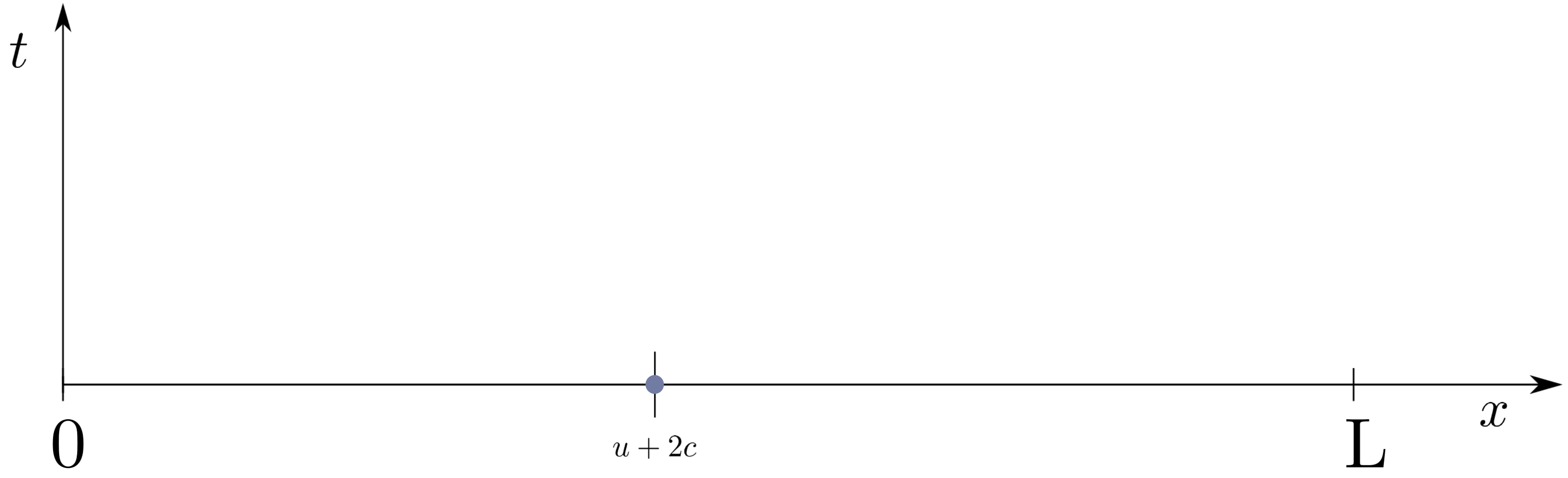
The C^+ characteristic has a slope:

$$\frac{dx}{dt} = u + c > 0$$

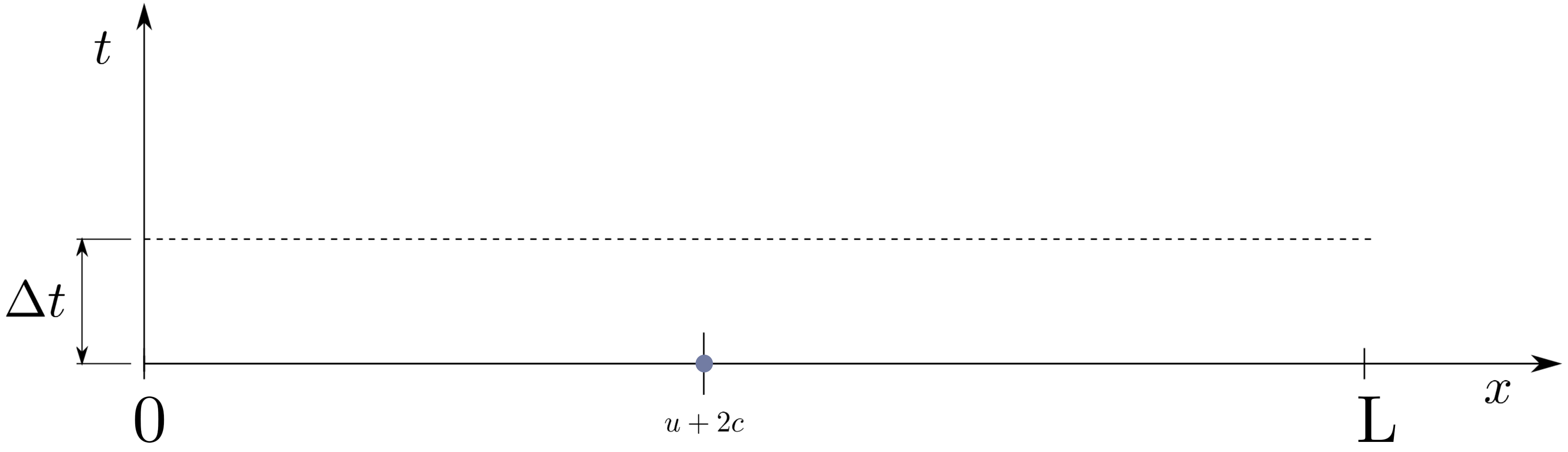
The C^- characteristic will have a slope:

$$\frac{dx}{dt} = u - c < 0$$

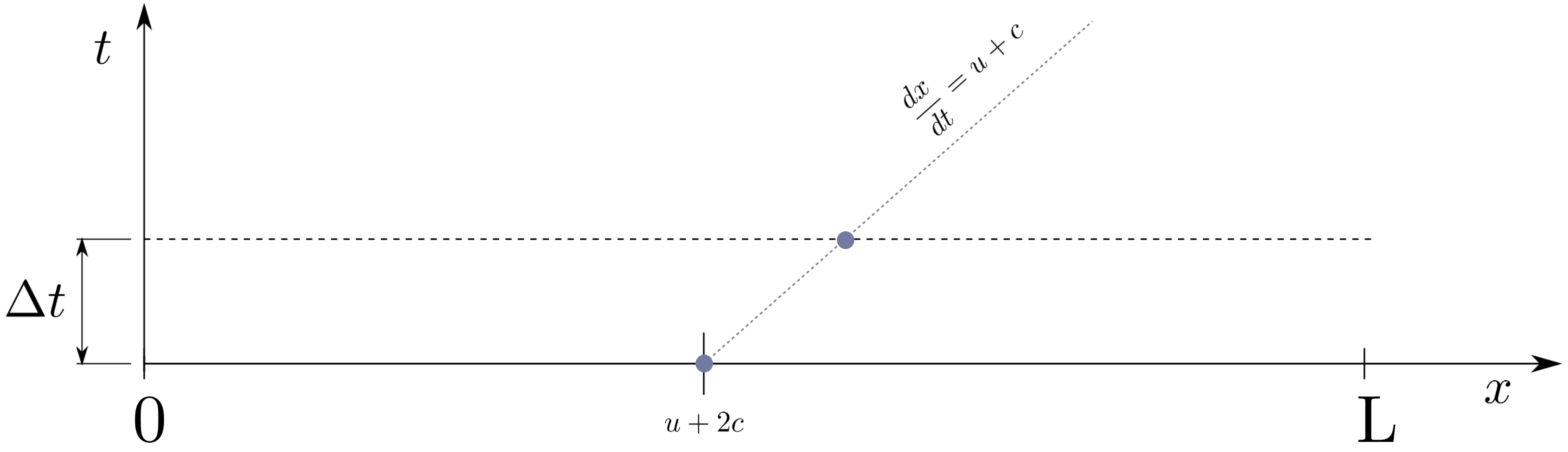
Characteristic formulation



Characteristic formulation

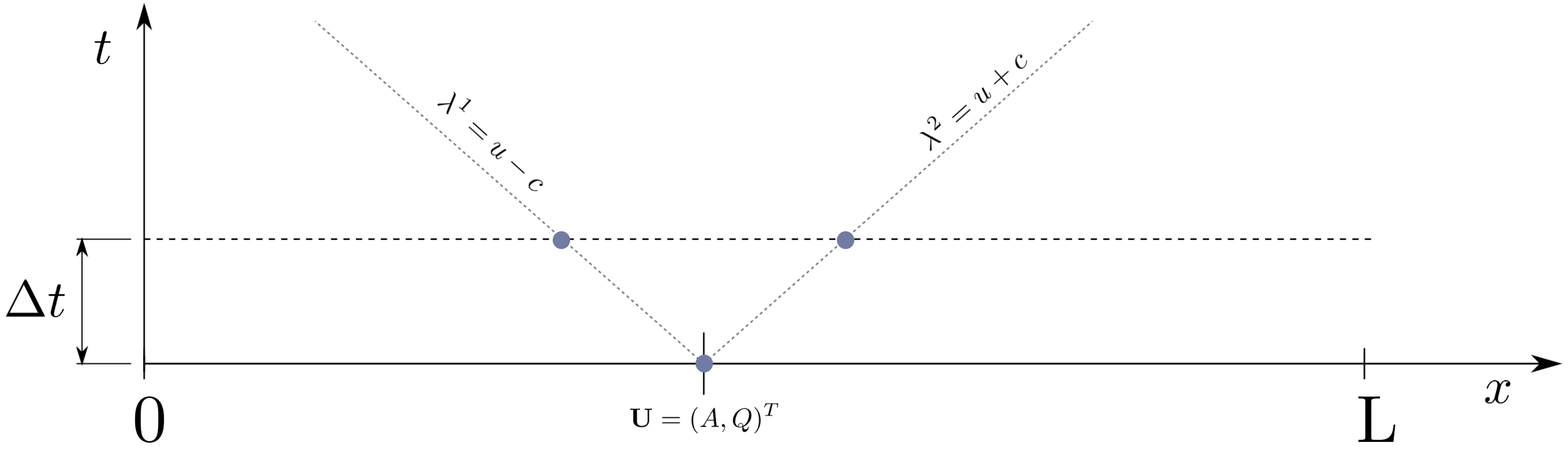


Characteristic formulation



Characteristic formulation

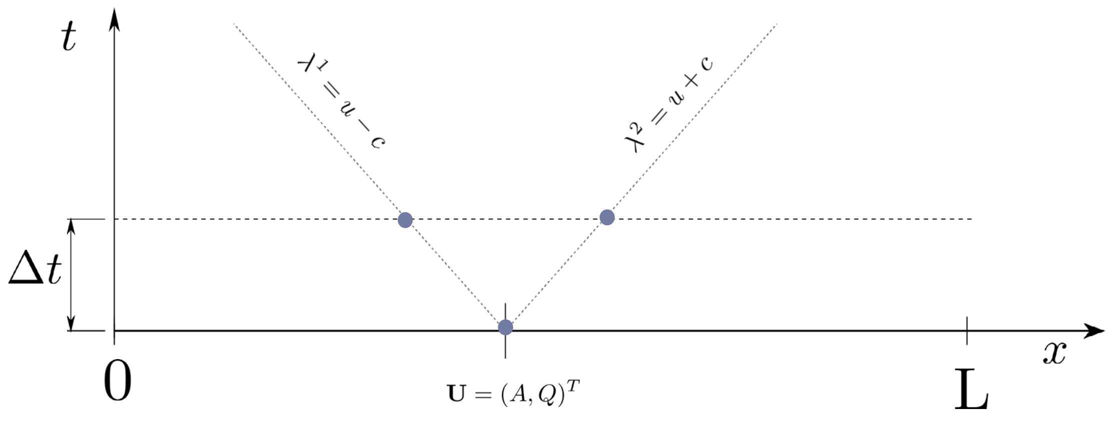
The system's eigenvalues give information about the propagation of the variables in the flow in terms of magnitude and direction



Characteristic formulation

The system's eigenvalues give information about the propagation of the variables in the flow in terms of magnitude and direction

$$Fr = \frac{u}{\sqrt{gh}} = \frac{u}{c}$$

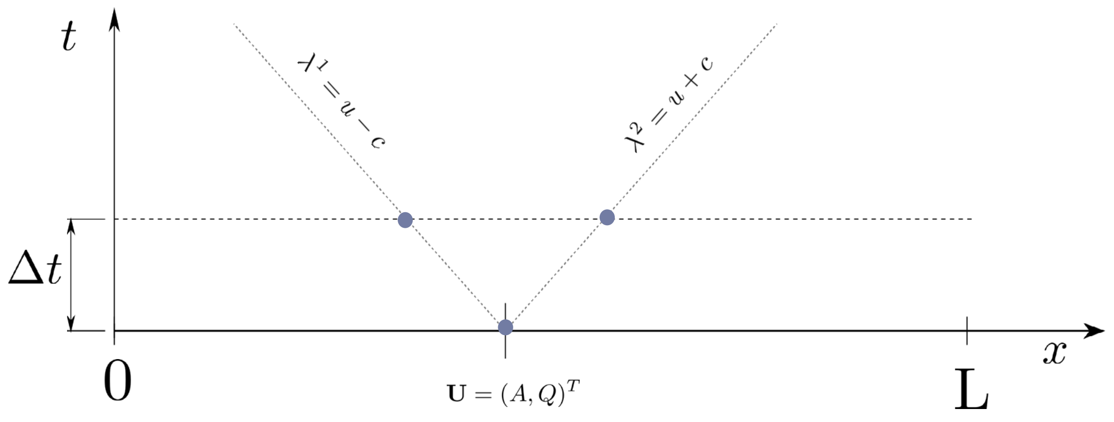


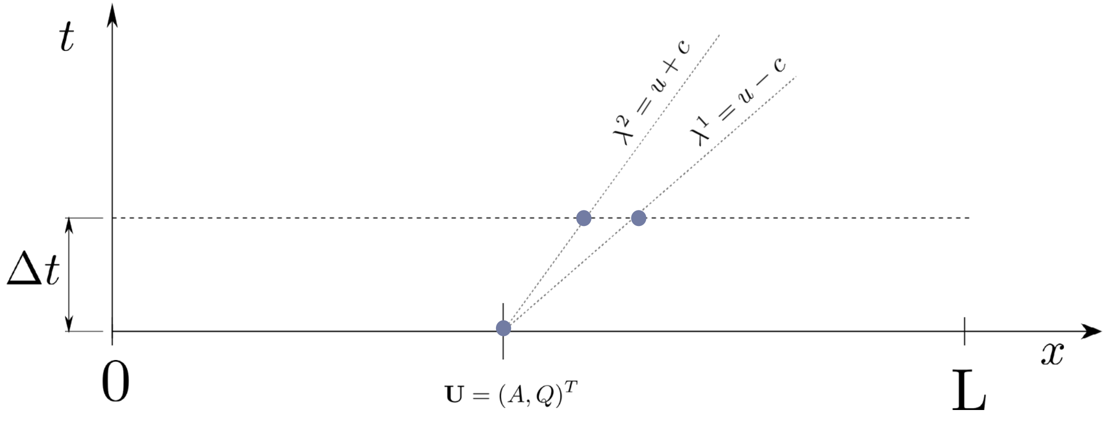
Subcritical flow (Fr<1) → u<c

Characteristic formulation

The system's eigenvalues give information about the propagation of the variables in the flow in terms of magnitude and direction

$$Fr = \frac{u}{\sqrt{gh}} = \frac{u}{c}$$





Subcritical flow (Fr<1) → u<c

Supercritical flow (Fr>1) → u>c

Numerical techniques

- Lagrangian/Eulerian
- Semi-lagrangians → Method of characteristics
- Time integration
  - Explicit: Conditionally stable. CFL condition
  - Implicit: Unconditionally stable
- Space integration: central *vs* upwind
- Conservative/ non conservative
- Desirable: Accuracy, stability, monotonicity, …

Simulation model: equations

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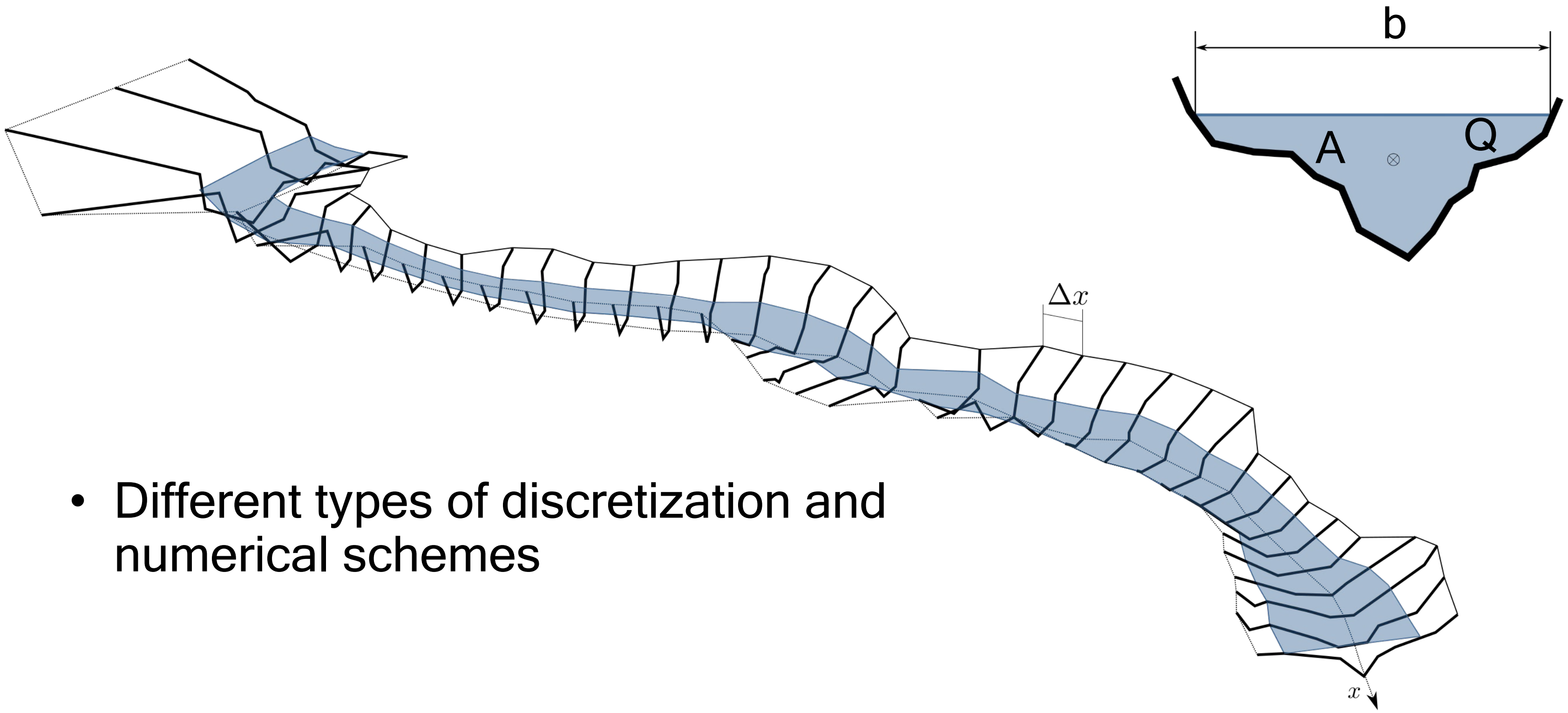
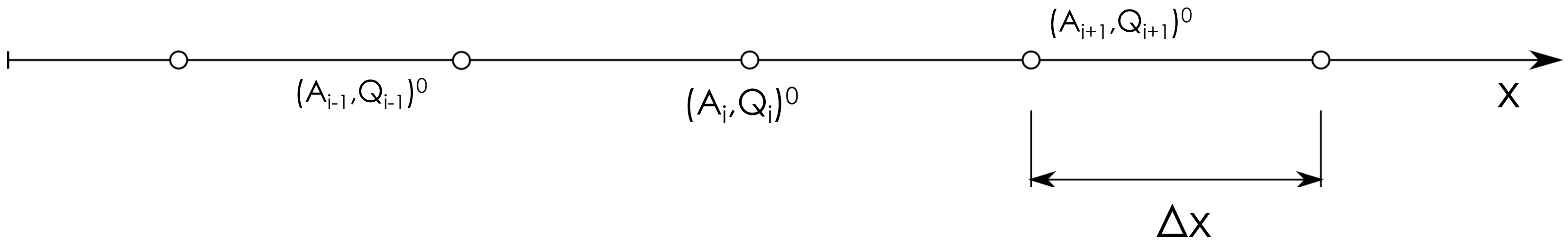


- Different types of discretization and numerical schemes

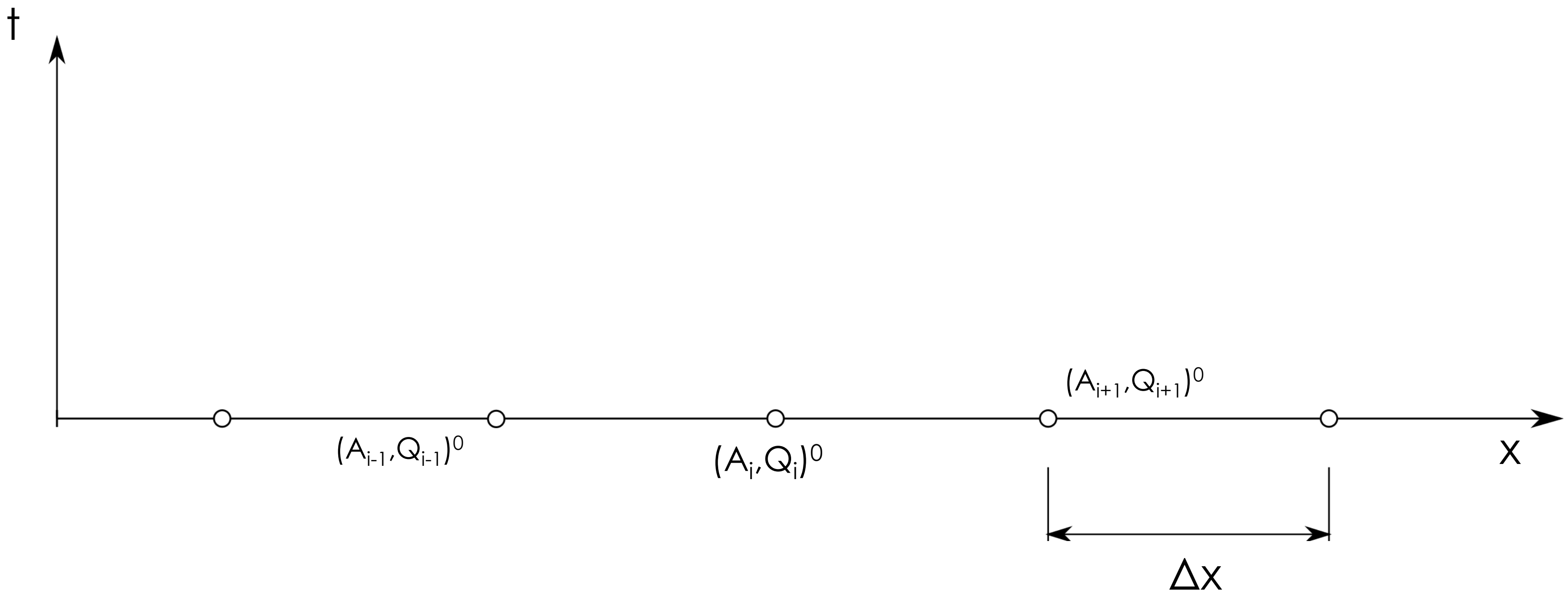
Numerical scheme

- NUMERICAL SCHEME
  - Space Discretization
    - Centered
    - Non-centered (*upwind*)
  - Time Discretization
    - Explicit
    - Implicit

Numerical scheme



Numerical scheme



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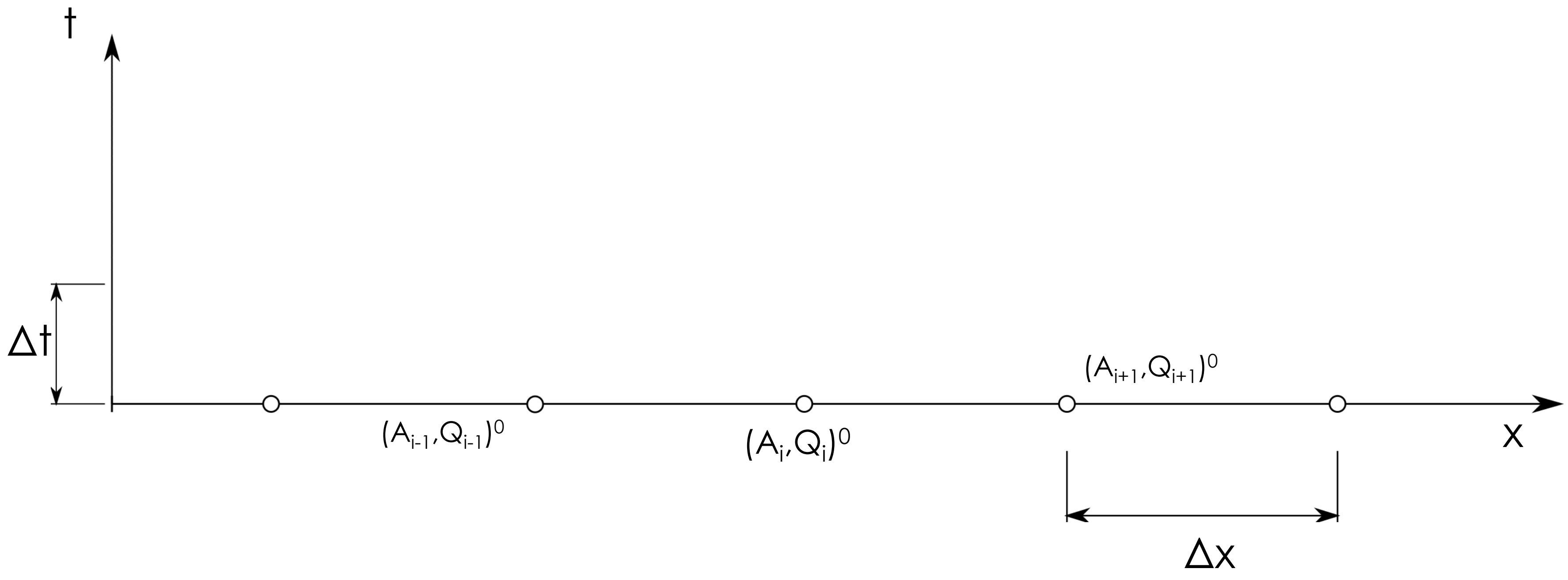

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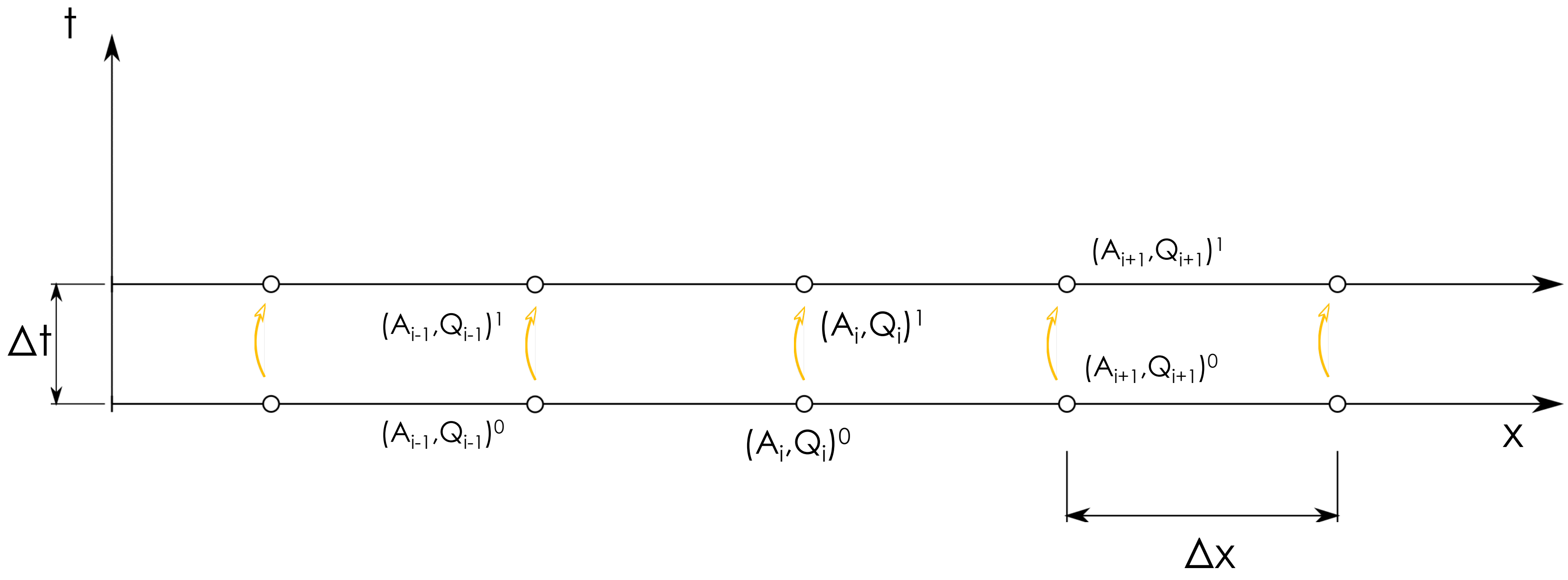
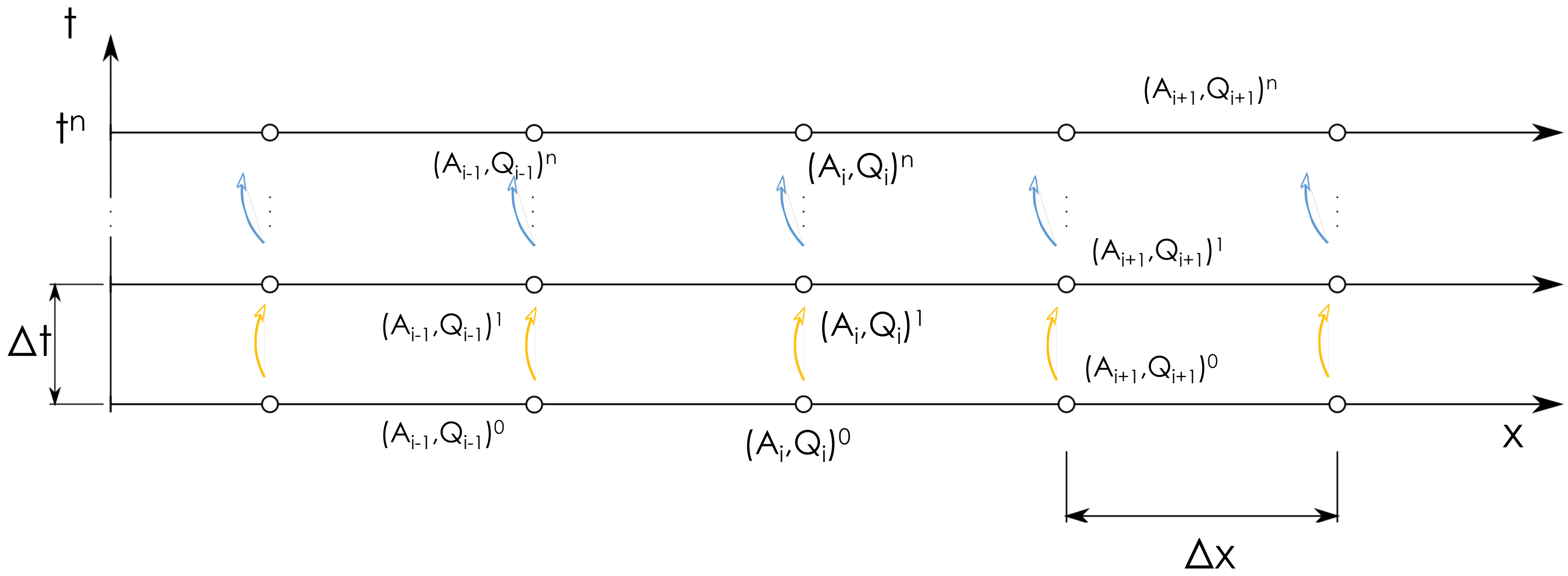
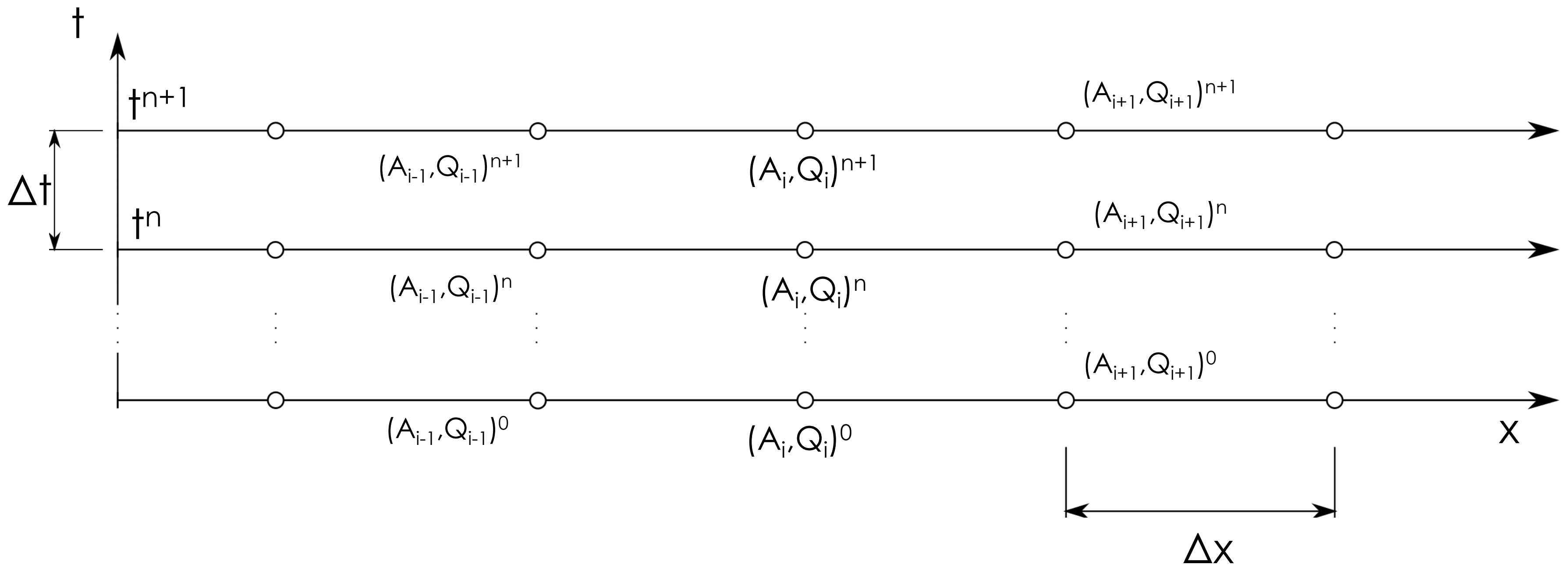


Numerical scheme

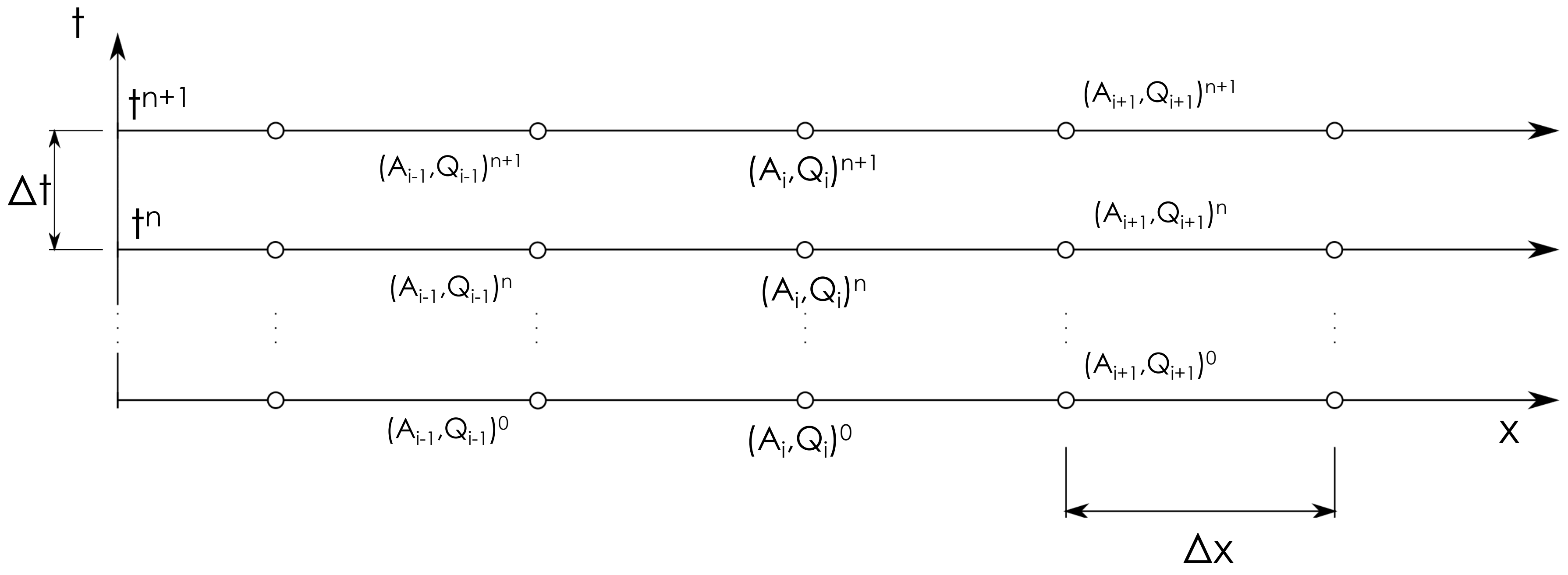


Numerical scheme



Numerical scheme

- The spatial discretization can be:
  - Centered or non-centered (upwind)
- The time discretization can be:
  - Explicit or implicit



Numerical scheme

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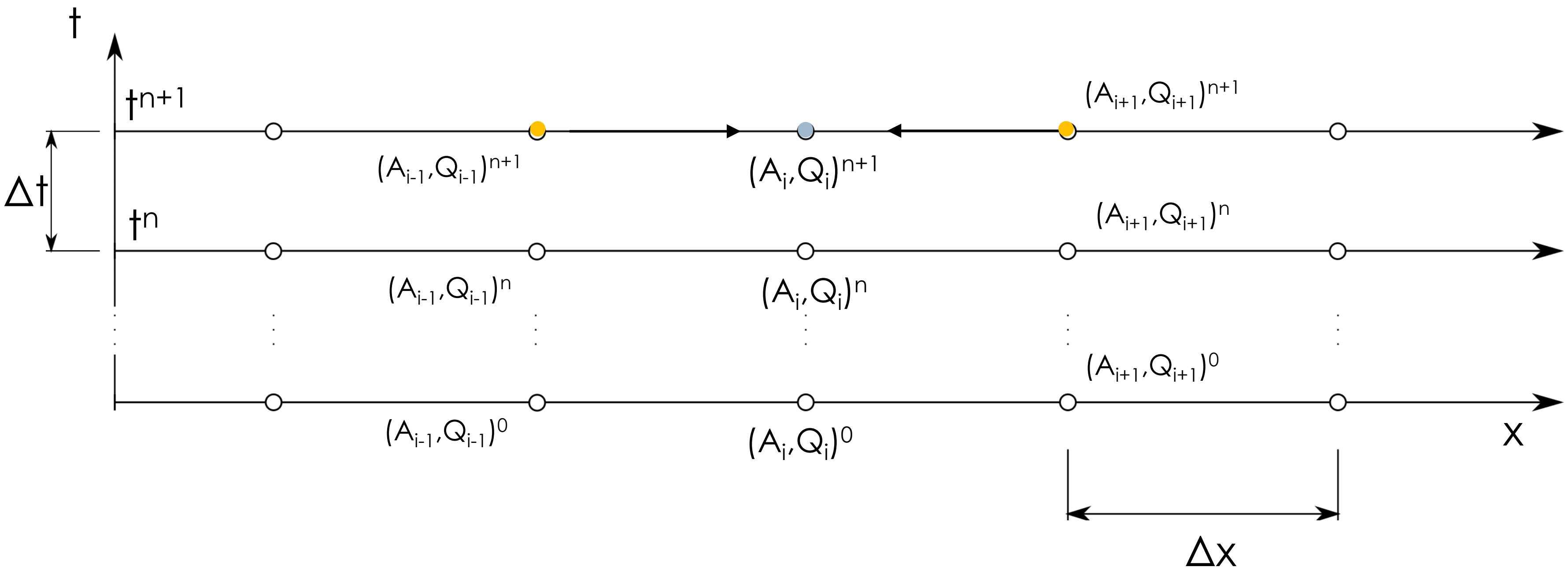

Numerical scheme

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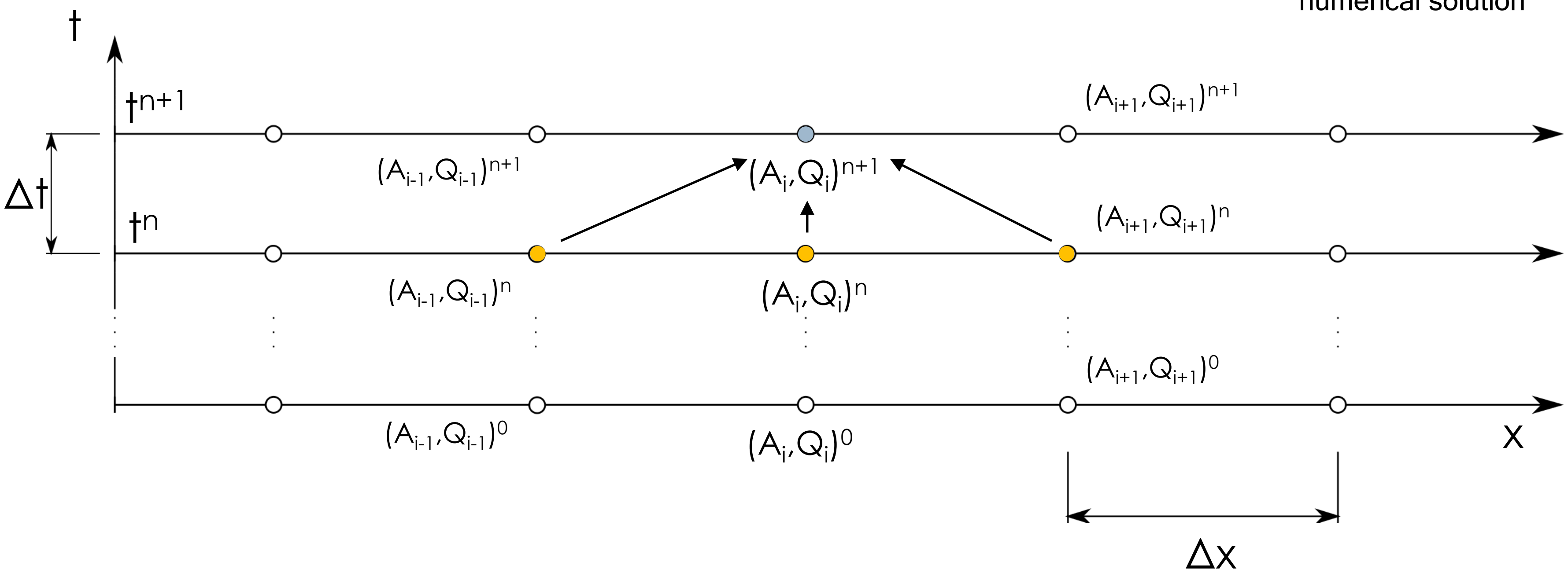
The solution at the *i*-th node time t^{n+1} depends exclusively on the known values of the variables at time t^n

Immediate calculation of numerical solution



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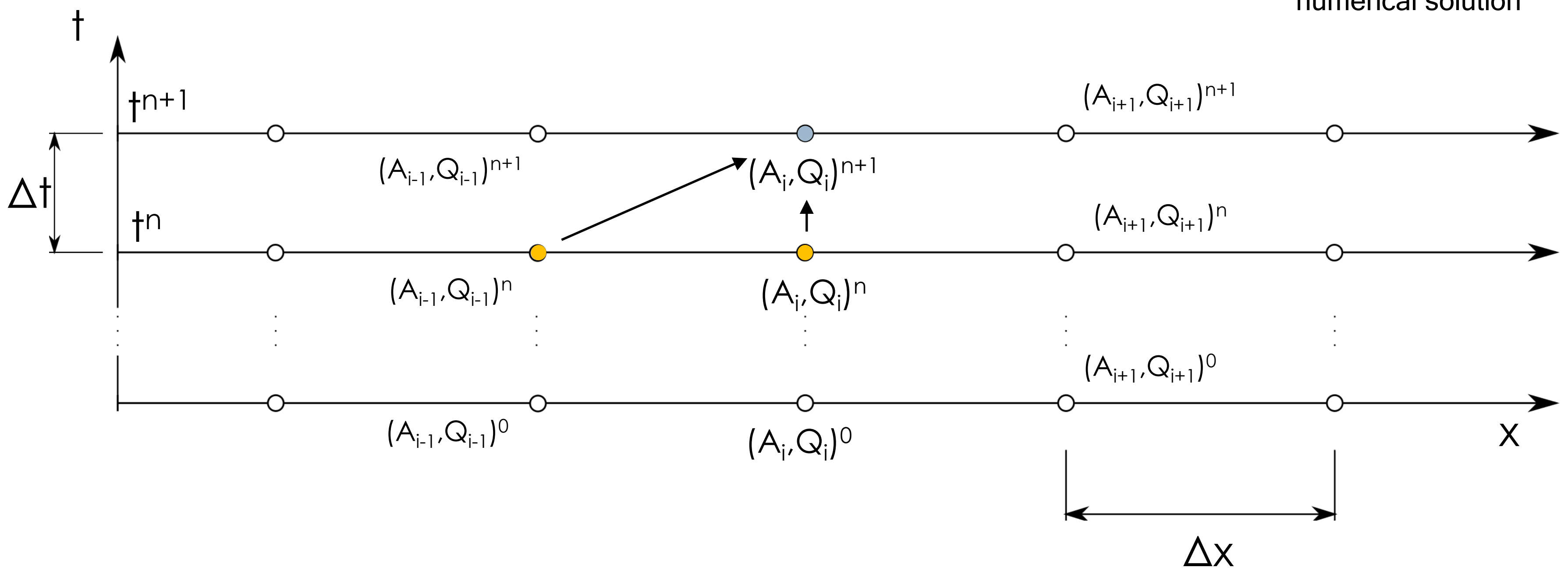
Immediate calculation of numerical solution



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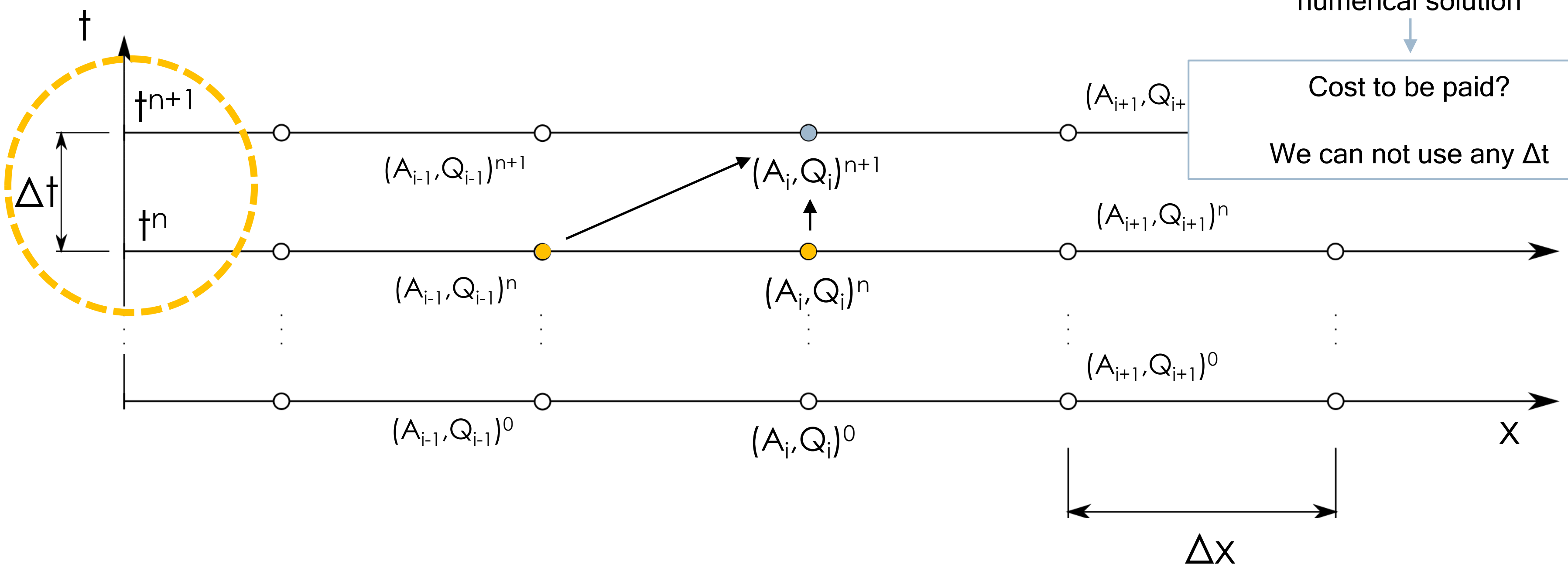
The solution at the *i*-th node time t^{n+1} depends exclusively on the known values of the variables at time t^n

Immediate calculation of numerical solution

Cost to be paid?

We can not use any Δt



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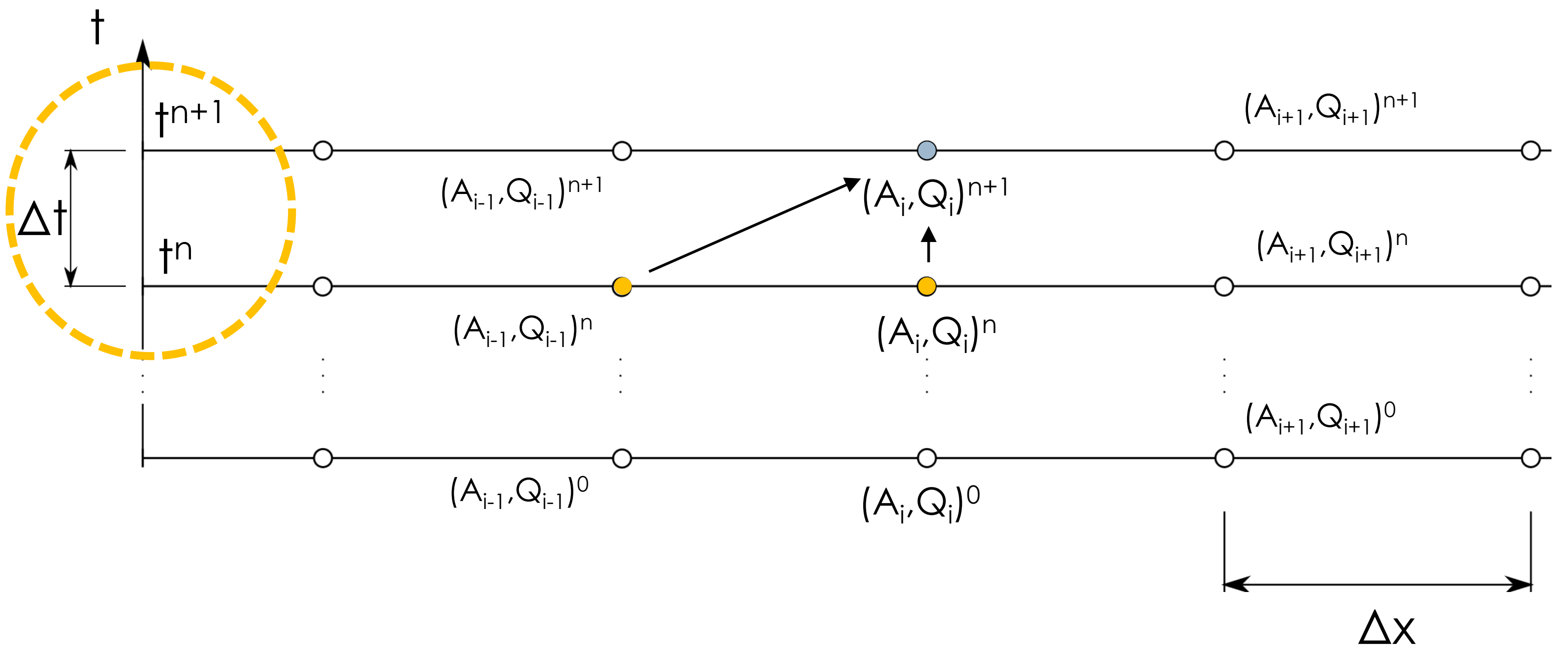
↓

Immediate calculation of numerical solution



The time step, the same throughout the domain, is governed by a non-dimensional number: CFL

$$CFL = \frac{\Delta t}{\Delta t_{max}}$$

Numerical scheme

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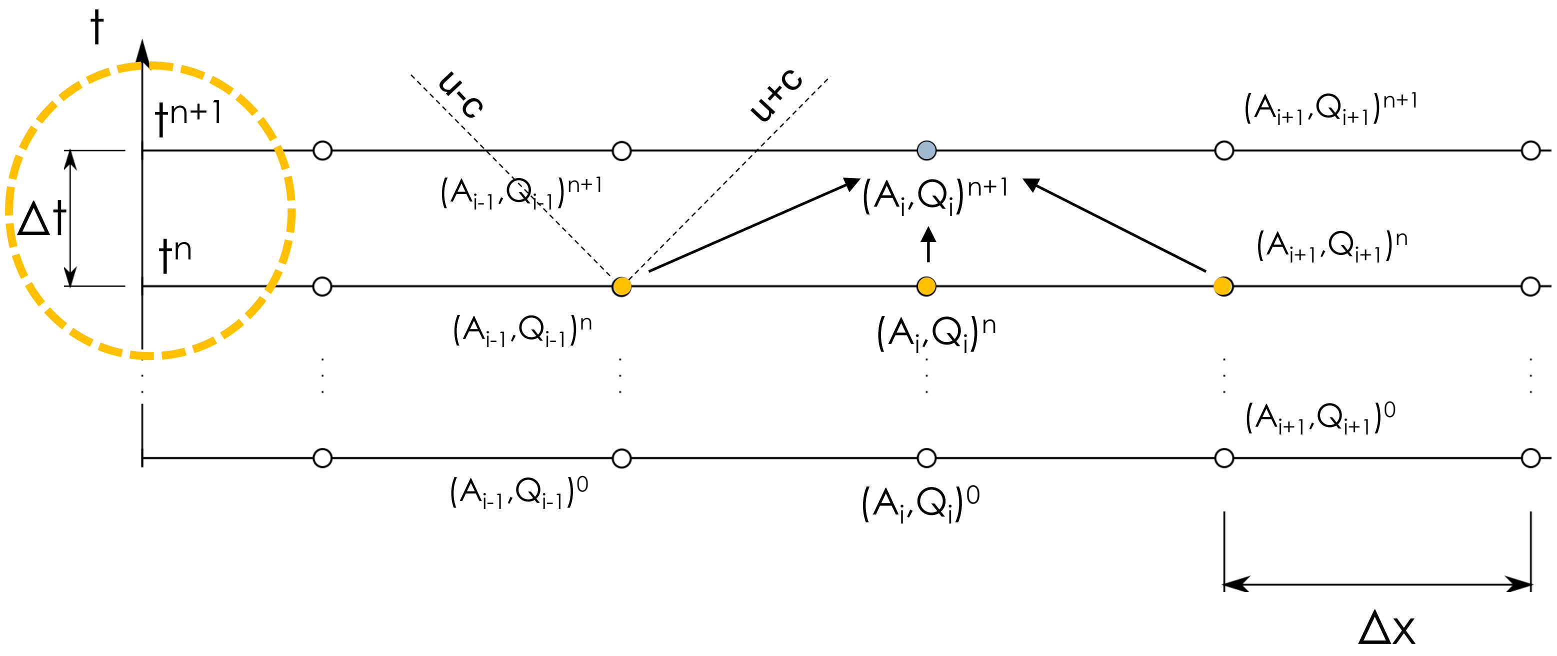

The solution at the *i*-th node time t^{n+1} depends exclusively on the known values of the variables at time t^n

Immediate calculation of numerical solution

The time step, is governed by a non-dimensional number: CFL

$$CFL = \frac{\Delta t}{\Delta t_{max}}$$

The maximum time step is a function of:

- Δx
- λ_{max}

Thus, it depends on:

- The mesh
- The flow

Numerical scheme

- The space discretization can be:
  - Centered or **non-centered (*upwind*)**
- The time discretization can be:
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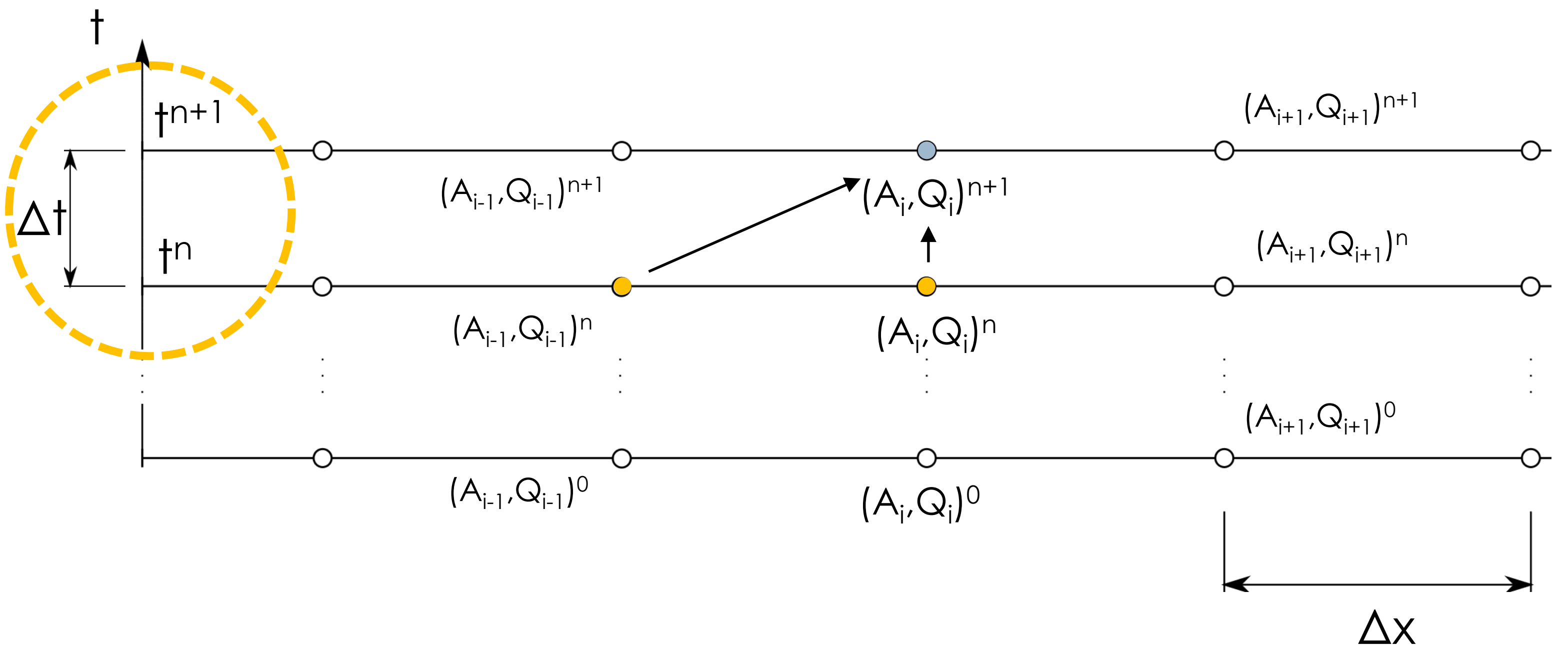


The solution at the i-th node time t^{n+1} depends exclusively on the known values of the variables at time t^n

Immediate calculation of numerical solution

The time step, the same throughout the domain, is governed by a non-dimensional number: CFL

$$CFL = \frac{\Delta t}{\Delta t_{max}}$$

Therefore:

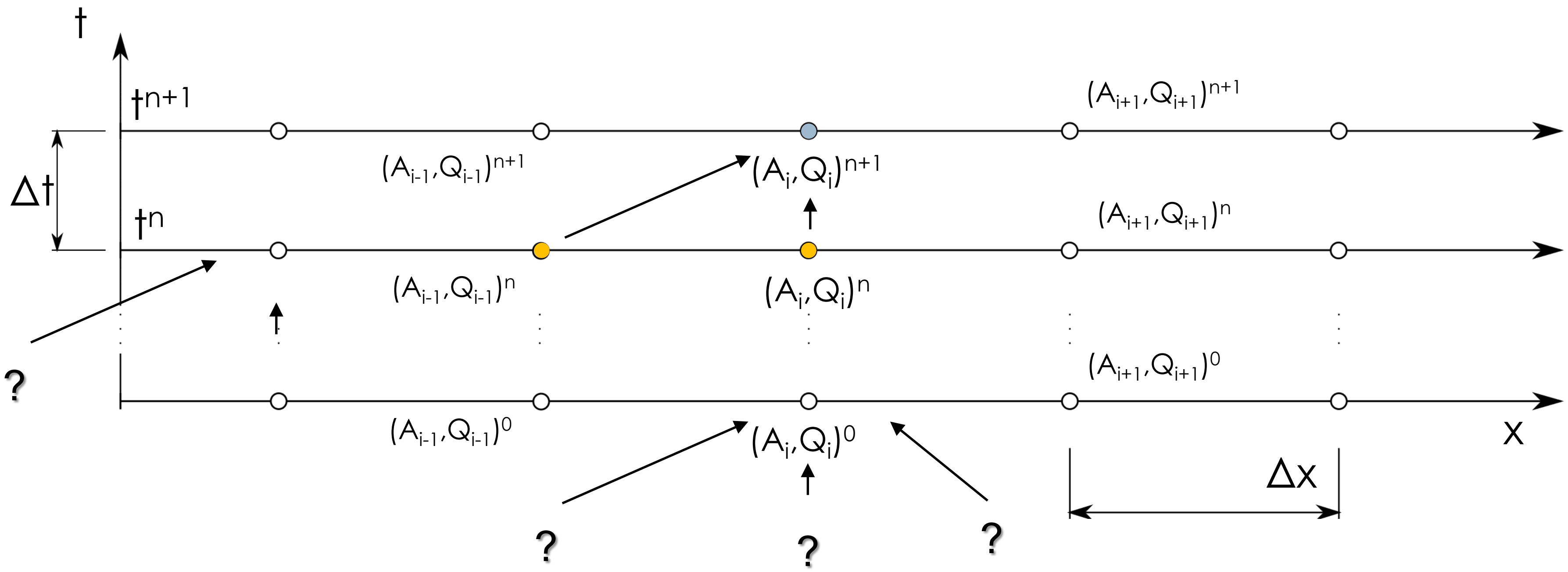
$$\Delta t = CFL \cdot \Delta t_{max}$$

$$\boxed{\Delta t = CFL \cdot \frac{\Delta x}{\lambda_{max}}}$$

Numerical scheme

- The space discretization can be:
  - Centered or non-centered (*upwind*)
- The time discretization can be:
  - Explicit or implicit

Requirement of Boundary Conditions (inlet and outlet)
Requirement of Initial Conditions: How was the canal or river at the initial time?



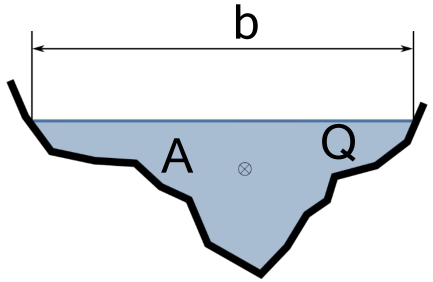
Mathematical model: Summary

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=-Q_l$$
$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}+gI_1\right)=gA(S_0-S_f)+gI_2$$

The shallow water equations (SWE) or Saint-Venant equations represent conservation of:

- Mass
- Linear momentum

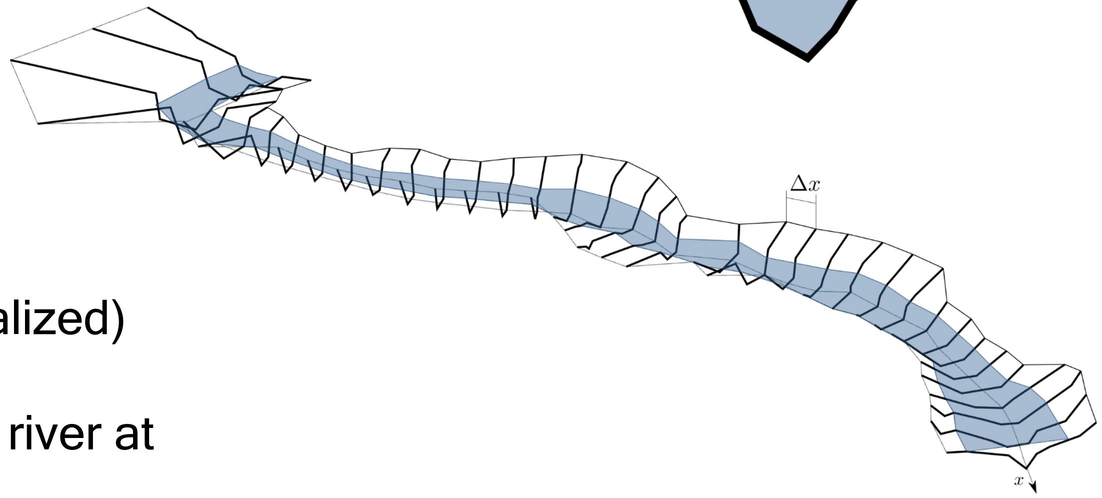
They come from the Navier-Stokes equations averaged over the section



Non-linear equation system
Complex solution
Requirement for numerical methods

- Explicit time integration
- Spatial integration using an upwind method (decentralized)

Requirement of Boundary Conditions (entry and exit)
Requirement of Initial Conditions: How was the canal or river at the initial time?



Hyperbolic system: No source terms

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0$$

$$\mathbf{J} = \frac{d\mathbf{F}}{d\mathbf{U}}$$

- $\mathbf{J}$ is strictly hyperbolic
- two real eigenvalues λ^1, λ^2
- two real eigenvectors $\mathbf{e}^1, \mathbf{e}^2$
- $\mathbf{P} = (\mathbf{e}^1, \mathbf{e}^2)$ and $\mathbf{P}^{-1}$ diagonalize the Jacobian

$$\mathbf{J} = \mathbf{P}\Lambda\mathbf{P}^{-1}$$

Roe's linearization

- $\tilde{\mathbf{J}} = \tilde{\mathbf{J}}^n = \tilde{\mathbf{J}}(\mathbf{U}_i^n, \mathbf{U}_{i+1}^n)$ is constant

$$\tilde{\mathbf{J}}_{i+1/2} = \begin{pmatrix} 0 & 1 \\ \widetilde{c}^2 - \widetilde{u}^2 & 2\widetilde{u} \end{pmatrix}_{i+1/2} \qquad \delta\mathbf{F}_{i+1/2} = \widetilde{\mathbf{J}}_{i+1/2}\delta\mathbf{U}_{i+1/2}$$

- Set of real eigenvalues and eigenvectors

$$\widetilde{\lambda}^1 = \widetilde{u} - \widetilde{c} \qquad \widetilde{\lambda}^2 = \widetilde{u} + \widetilde{c}$$

$$\widetilde{\mathbf{e}}^1 = \begin{pmatrix} 1 \\ \widetilde{u} - \widetilde{c} \end{pmatrix} \qquad \widetilde{\mathbf{e}}^2 = \begin{pmatrix} 1 \\ \widetilde{u} + \widetilde{c} \end{pmatrix}$$

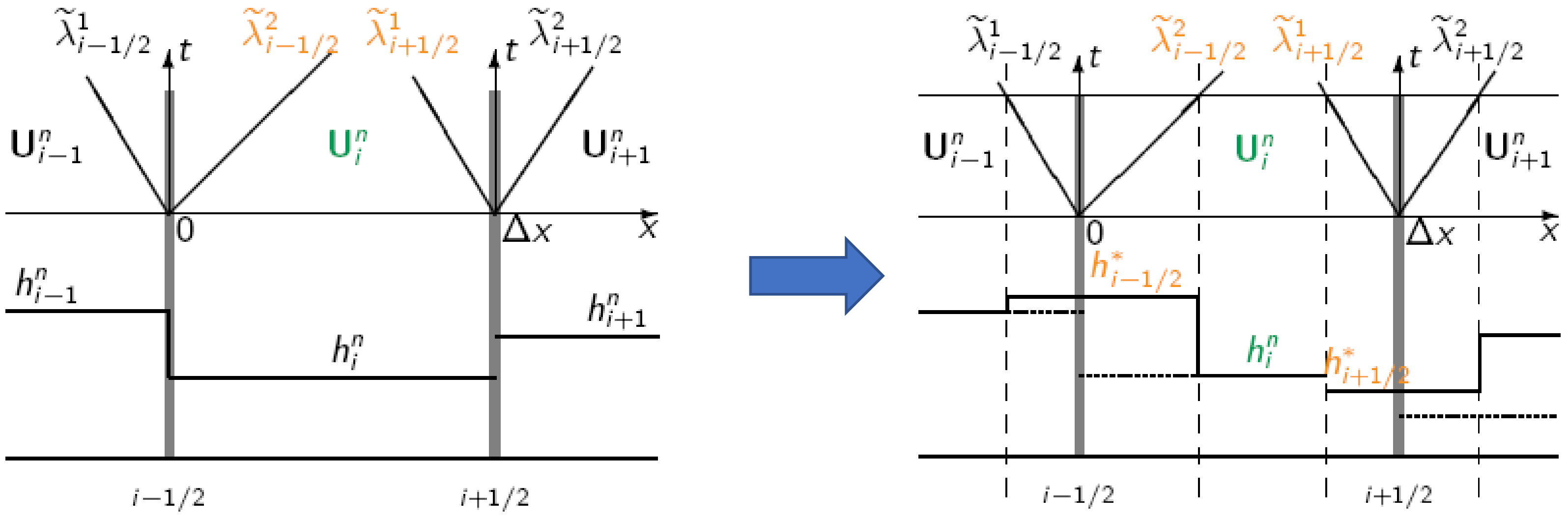
- Roe's averaged values

$$\widetilde{c} = \sqrt{g\frac{h_i + h_{i+1}}{2}} \quad \widetilde{u} = \frac{u_{i+1}\sqrt{h_{i+1}} + u_i\sqrt{h_i}}{\sqrt{h_{i+1}} + \sqrt{h_i}}$$

1D conservative schemes

Godunov method NO SOURCE TERMS

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0$$



$$\mathbf{U}^*_{i+1/2}(\mathbf{U}_{i+1}, \mathbf{U}_i) = \mathbf{U}^n_i + (\alpha\widetilde{\mathbf{e}})^1_{i+1/2}$$

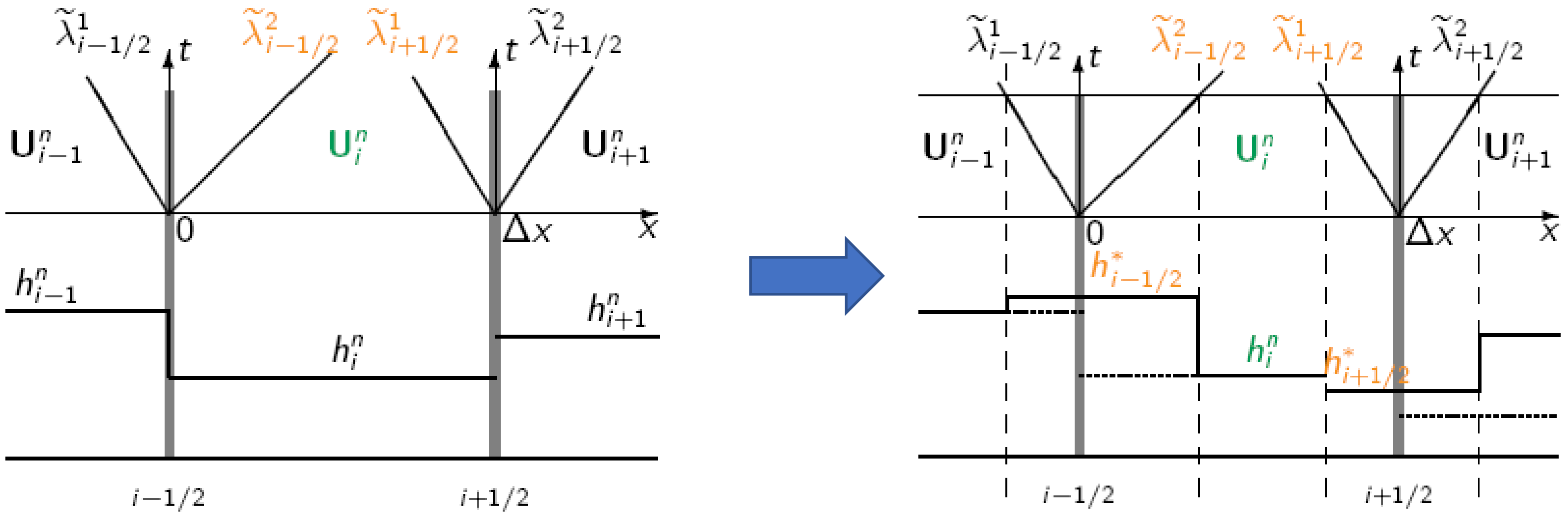
$$\mathbf{U}^*_{i+1/2}(\mathbf{U}_{i+1}, \mathbf{U}_i) = \mathbf{U}^n_{i+1} - (\alpha\widetilde{\mathbf{e}})^2_{i+1/2}$$

$$\mathbf{U}^{n+1}_i \Delta x = \mathbf{U}^*_{i-1/2}(\widetilde{\lambda}^2_{i-1/2}\Delta t) + \mathbf{U}^n_i(\Delta x - \widetilde{\lambda}^2_{i-1/2}\Delta t + \widetilde{\lambda}^1_{i+1/2}\Delta t) + \mathbf{U}^*_{i+1/2}(-\widetilde{\lambda}^1_{i+1/2}\Delta t)$$

1D conservative schemes

Godunov method NO SOURCE TERMS

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0$$



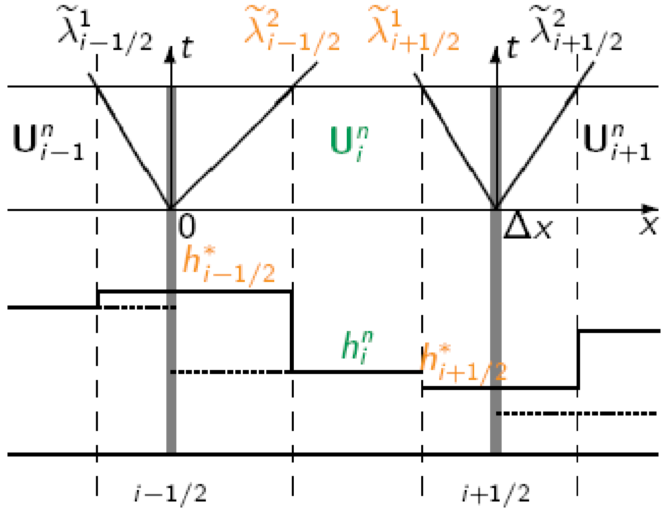
$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \left((\delta \mathbf{F})^+_{i-1/2} + (\delta \mathbf{F})^-_{i+1/2} \right) \frac{\Delta t}{\Delta x}$$

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - (\mathbf{F}^{\downarrow}_{i+1/2} - \mathbf{F}^{\downarrow}_{i-1/2}) \frac{\Delta t}{\Delta x}$$

1D conservative schemes

Godunov method NO SOURCE TERMS

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = 0$$



$$\delta \mathbf{U}_{i+1/2} = \widetilde{\mathbf{P}}_{i+1/2}\mathbf{A}_{i+1/2} = \sum_{m=1}^{N_\lambda} (\alpha \widetilde{\mathbf{e}})^m_{i+1/2}$$

$$\delta \mathbf{F}_{i+1/2} = \widetilde{\mathbf{J}}_{i+1/2}\delta \mathbf{U}_{i+1/2}$$

$$(\delta \mathbf{F})_{i+1/2} = \sum_{m=1}^{N_\lambda} \left(\widetilde{\lambda}\alpha\widetilde{\mathbf{e}}\right)^m_{i+1/2}$$

$$\boxed{\mathbf{U}^{n+1}_i = \mathbf{U}^n_i - \left((\delta \mathbf{F})^+_{i-1/2} + (\delta \mathbf{F})^-_{i+1/2}\right)\frac{\Delta t}{\Delta x}}$$

$$(\delta \mathbf{F})^\pm_{i+1/2} = \sum_{m=1}^{N_\lambda} \left(\widetilde{\lambda}^\pm\alpha\widetilde{\mathbf{e}}\right)^m_{i+1/2}$$

$$\widetilde{\lambda}^{\pm,m}_{i+1/2} = \frac{1}{2}(\widetilde{\lambda} \pm |\widetilde{\lambda}|)$$

Time step restrictions: 1st order

Classical Courant-Friedrichs-Lewy condition :

$$\boxed{\Delta t = CFL \; \Delta t_{max} \qquad CFL < 1}$$

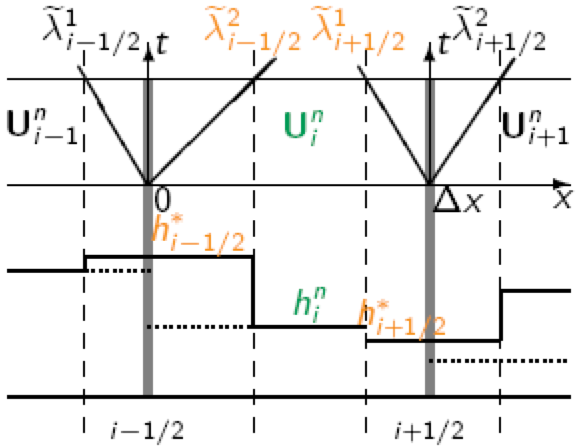
Time step restrictions

Classical Courant-Friedrichs-Lewy condition :

$$\boxed{\Delta t = CFL\ \Delta t_{max} \qquad CFL < 1}$$

For homogeneous systems:

$$\Delta t_{max} = \frac{\Delta x}{\max_m \left\{ \left| \tilde{\lambda}^m \right| \right\}}$$



1D conservative scheme with SOURCE TERMS

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{x}, \mathbf{U})}{\partial x} = \mathbf{H}(\mathbf{x}, \mathbf{U}) \Rightarrow \frac{\Delta \mathbf{U}_i^n}{\Delta t} = \left(\mathbf{H} - \frac{\delta \mathbf{F}}{\delta x}\right)_{i-1/2}^{+} + \left(\mathbf{H} - \frac{\delta \mathbf{F}}{\delta x}\right)_{i+1/2}^{-}$$

$$\frac{\Delta \mathbf{U}_i^n}{\Delta t} = \mathbf{G}_{i-1/2}^{+} + \mathbf{G}_{i+1/2}^{-} \qquad \mathbf{G}_{i+1/2} = \left(\mathbf{H} - \frac{\delta \mathbf{F}}{\delta x}\right)_{i+1/2}$$

1D conservative scheme with SOURCE TERMS

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{x},\mathbf{U})}{\partial x} = \mathbf{H}(\mathbf{x},\mathbf{U}) \Rightarrow \frac{\Delta \mathbf{U}_i^n}{\Delta t} = \left(\mathbf{H} - \frac{\delta \mathbf{F}}{\delta x}\right)^+_{i-1/2} + \left(\mathbf{H} - \frac{\delta \mathbf{F}}{\delta x}\right)^-_{i+1/2}$$

$$\frac{\Delta \mathbf{U}_i^n}{\Delta t} = \mathbf{G}^+_{i-1/2} + \mathbf{G}^-_{i+1/2} \qquad \mathbf{G}_{i+1/2} = \left(\mathbf{H} - \frac{\delta \mathbf{F}}{\delta x}\right)_{i+1/2}$$

$$\begin{aligned}(\Delta x \mathbf{H})_{i+1/2} &= \mathbf{P}\beta = \mathbf{P}\Lambda\Lambda^{-1}\beta = (\mathbf{P}\Lambda^+\Lambda^{-1}\beta + \mathbf{P}\Lambda^-\Lambda^{-1}\beta)_{i+1/2} = \\ &= \mathbf{H}^{*+}_{i+1/2} + \mathbf{H}^{*-}_{i+1/2}\end{aligned}$$

$$\Delta x \mathbf{H}_{i+1/2} = \left(\sum_{k=1,2} \beta^k \tilde{\mathbf{e}}^k\right)_{i+1/2}$$

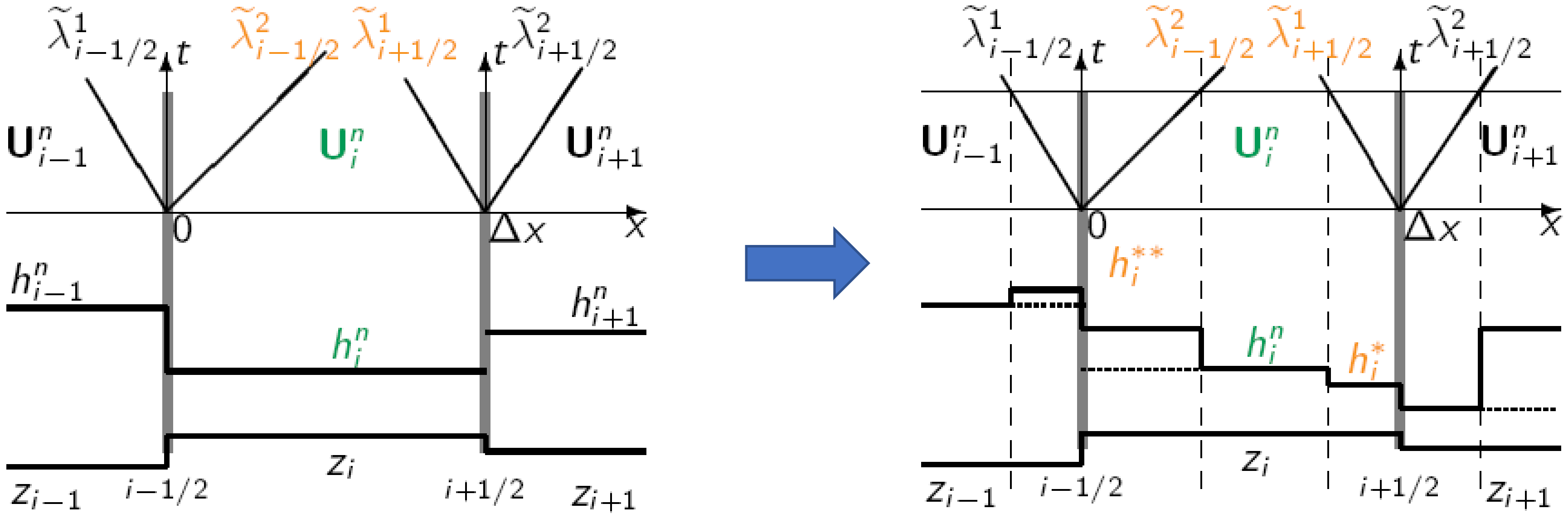
$$\mathbf{H}_{i+1/2} = \begin{pmatrix} 0 \\ -gh\frac{dz_b}{dx} \end{pmatrix}_{i+1/2} = \frac{1}{\Delta x}\left(\beta^1 \begin{pmatrix} 1 \\ \bar{u} + \bar{c} \end{pmatrix} + \beta^2 \begin{pmatrix} 1 \\ \bar{u} - \bar{c} \end{pmatrix}\right)_{i+1/2}$$

$$\beta = \frac{g\Delta x}{2\tilde{c}_{i+\frac{1}{2}}} \bar{A}\bar{S}_0 \qquad \beta^1 = -\beta^2 = \beta$$

1D conservative schemes

Godunov method WITH SOURCE TERMS

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S}$$



$$\mathbf{U}_i^{n+1}\Delta x = \mathbf{U}_i^{**}(\widetilde{\lambda}^2_{i-1/2}\Delta t) + \mathbf{U}_i^n(\Delta x - \widetilde{\lambda}^2_{i-1/2}\Delta t + \widetilde{\lambda}^1_{i+1/2}\Delta t) + \mathbf{U}_i^*(-\widetilde{\lambda}^1_{i+1/2}\Delta t)$$

$$\theta^m_{i+1/2} = \left(1 - \frac{\beta}{\widetilde{\lambda}\alpha}\right)^m_{i+1/2} \longrightarrow$$

$$\mathbf{U}_i^*(\mathbf{U}_{i+1}, \mathbf{U}_i, \mathbf{S}_{i+1/2}) = \mathbf{U}_i^n + (\theta\alpha\widetilde{\mathbf{e}})^1_{i+1/2}$$

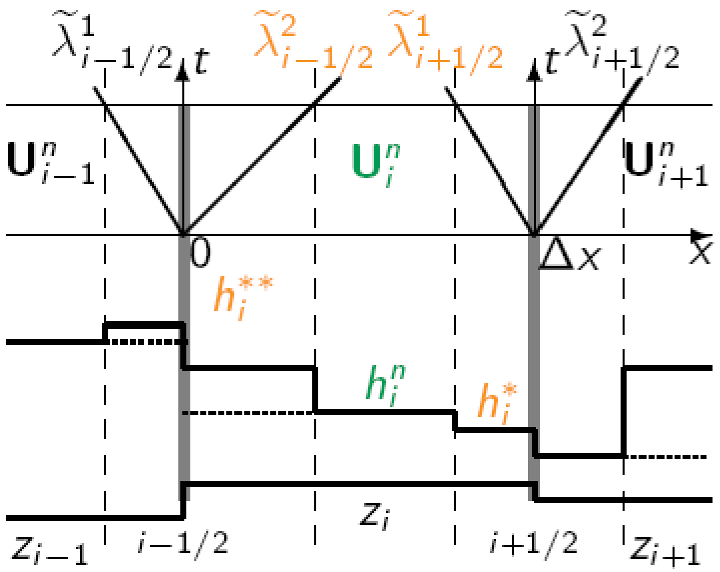
$$\mathbf{U}_{i+1}^{**}(\mathbf{U}_{i+1}, \mathbf{U}_i, \mathbf{S}_{i+1/2}) = \mathbf{U}_{i+1}^n - (\theta\alpha\widetilde{\mathbf{e}})^2_{i+1/2}$$

1D conservative schemes

Godunov method WITH SOURCE TERMS

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = S$$

$$\boxed{\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \left((\delta\mathbf{F} - \mathbf{S})^+_{i-1/2} + (\delta\mathbf{F} - \mathbf{S})^-_{i+1/2}\right)\frac{\Delta t}{\Delta x}}$$



$$(\delta\mathbf{F} - \mathbf{S})^{\pm}_{i+1/2} = \sum_{m=1}^{N_\lambda}\left(\widetilde{\lambda}^{\pm}\theta\alpha\widetilde{\mathbf{e}}\right)^m_{i+1/2}$$

$$\theta^m_{i+1/2} = \left(1 - \frac{\beta}{\widetilde{\lambda}\alpha}\right)^m_{i+1/2}$$

$$\mathbf{S}_{i+1/2} = \widetilde{\mathbf{P}}_{i+1/2}\mathbf{B}_{i+1/2} = \sum_{m=1}^{N_\lambda}(\beta\widetilde{\mathbf{e}})^m_{i+1/2}$$

Time step restrictions:

For systems with source terms:

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \sum_{k=1}^{NE} \sum_{m=1}^{N\lambda} (\widetilde{\lambda}^- \theta \alpha \widetilde{\mathbf{e}})_k^m l_k \frac{\Delta t}{A_i}$$

Definition:

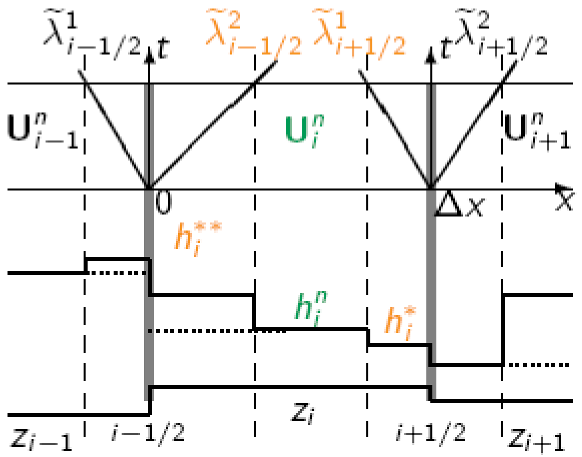
$$\begin{cases} \theta_k^m = 1 - \left(\dfrac{\beta^-}{\alpha \widetilde{\lambda}^-} \right)_k^m \\ \widetilde{\lambda}_k^{m,*} = \widetilde{\lambda}_k^{m,-} \theta_k^m \end{cases}$$

$$\Delta t_k = \frac{A_{\min,k}}{\max\limits_m \left\{ \left| \lambda_k^{m,*} \right| \right\} l_k}$$

The time step is much restricted !!

Stability revisited: Positivity fix

Murillo and Garcia-Navarro, *Journal of Comp. Physics (2012)*



IDEA: Enforcing positive values of h^*_i and h^{**}_{i+1} by modifying the source strength coefficients β

$$\mathbf{U}^*_i(\mathbf{U}_{i+1}, \mathbf{U}_i, \mathbf{S}_{i+1/2}) = \mathbf{U}^n_i + (\theta\alpha\widetilde{\mathbf{e}})^1_{i+1/2}$$

$$h^*_i = h^\star_i - \left(\frac{\beta}{\tilde{\lambda}}\right)^1_{i+1/2} \geqslant 0$$

$$h^{**}_{i+1} = h^\star_i + \left(\frac{\beta}{\tilde{\lambda}}\right)^2_{i+1/2} \geqslant 0$$

$$\boxed{\beta^1 = \begin{cases} h^\star_i \tilde{\lambda}^1_{i+1/2} & \text{if } h^*_i < 0 \\ \beta^1 & \text{otherwise} \end{cases}, \quad \beta^2 = -\beta^1}$$

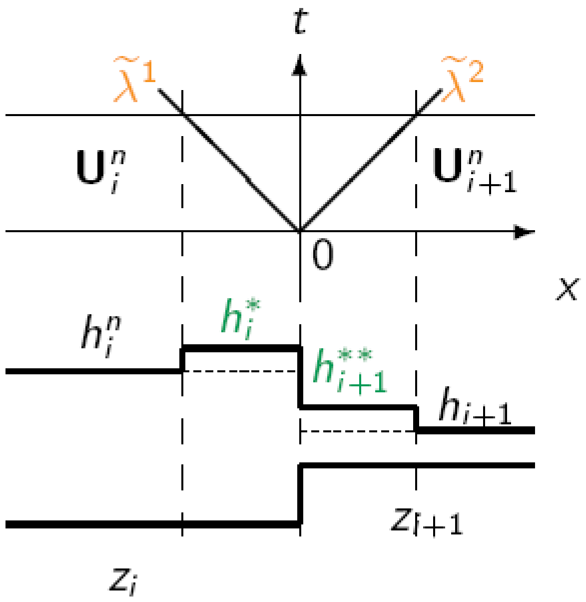
$$\boxed{\beta^2 = \begin{cases} -h^\star_i \tilde{\lambda}^2_{i+1/2} & \text{if } h^{**}_{i+1} < 0 \\ \beta^2 & \text{otherwise} \end{cases}, \quad \beta^1 = -\beta^2}$$

$$h^\star_i = h^n_i + (\alpha\tilde{e}_1)^1_{i+1/2} > 0,$$

Friction fix

Murillo and Garcia-Navarro, *Journal of Comp. Physics (2012)*

$$\mathbf{U}_i^*(\mathbf{U}_{i+1}, \mathbf{U}_i, \mathbf{S}_{i+1/2}) = \mathbf{U}_i^n + (\theta\alpha\tilde{\mathbf{e}})_{i+1/2}^1$$



$$(hu)_i^* = (hu)_i^n + (\alpha\tilde{e}_2)_{i+1/2}^1 - \beta_{i+1/2}^1$$

Bed slope and friction

$$(hu)_i^\star = (hu)_i^n + (\alpha\tilde{e}_2)_{i+1/2}^1 - \beta_{\mathbf{H},i+1/2}^1$$

Only bed slope

$$\boxed{\beta_{\mathbf{S}}^1 = \begin{cases} (hu)_i^\star & \text{if } (hu)_i^\star (hu)_i^* \leqslant 0 \\ \beta_{\mathbf{S}}^1 & \text{otherwise} \end{cases}}$$

Conservative, depth-positive and friction-bounded scheme

$$\boxed{\mathbf{U}_i^{n+1} = \mathbf{U}_i^n - \left(\delta\mathbf{M}_{i+1/2}^- + \delta\mathbf{M}_{i-1/2}^+\right)\frac{\Delta t}{\Delta x}}$$

$$\delta\mathbf{M}_{i+1/2}^- = \sum_{m=1}^{2}\left(\tilde{\lambda}^-\theta\alpha\widetilde{\mathbf{e}}\right)_{i+1/2}^m, \quad \delta\mathbf{M}_{i+1/2}^+ = \sum_{m=1}^{2}\left(\tilde{\lambda}^+\theta\alpha\widetilde{\mathbf{e}}\right)_{i+1/2}^m$$

(I) Friction source strength coefficients b following the **friction fix**.
(II) Total source strength coefficients b following the **depth positive fix**.
(III) Flux splitting ensuring positivity on the water depth both in wet/wet and in wet/dry cases.
(IV) Stability controlled by the classical CFL condition

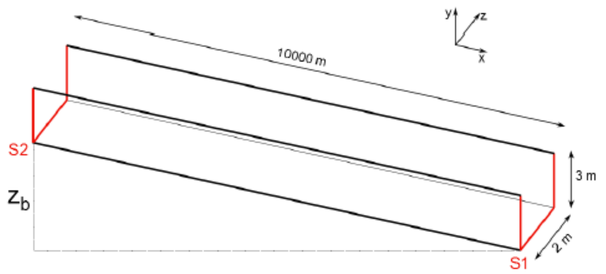
Practical activity

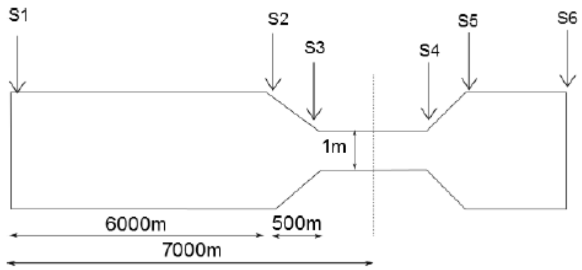
Simulation of steady and transient flow in channels with **Canalflowmodel**

Practical activities

Simulation of steady flow in channels with **Canalflowmodel.net**

1. Convergence simulation to steady states





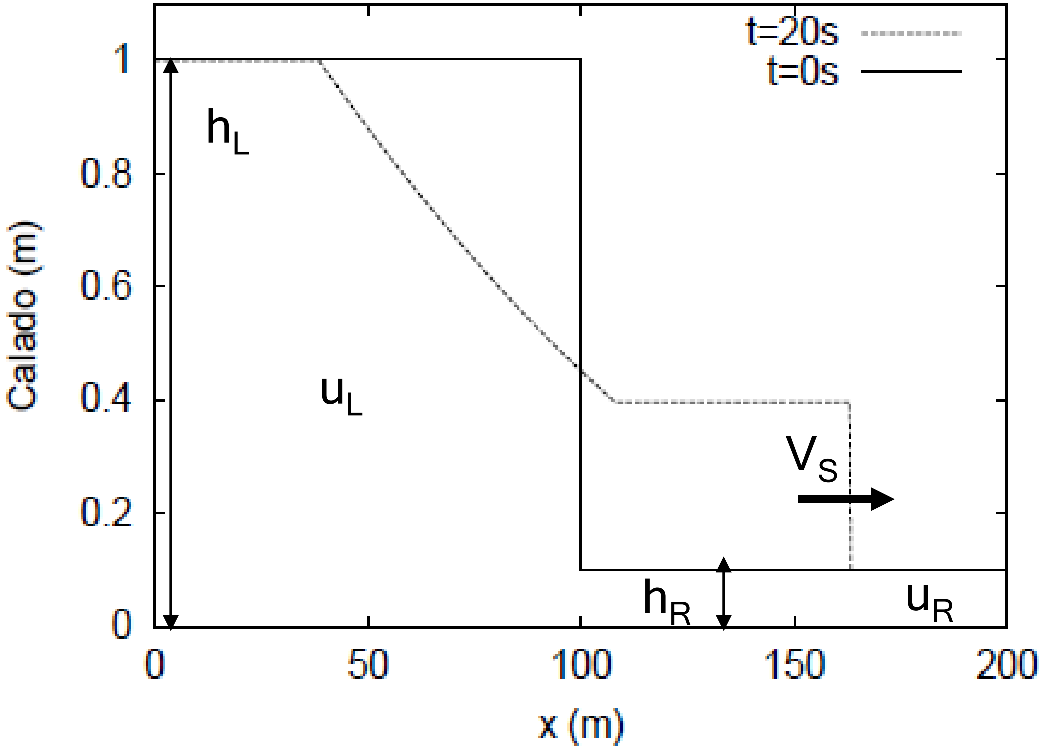
   a. Spatial Discretization (Dx): Determines the accuracy of the results
   b. Temporary Discretization (Dt): Determines the numerical stability
   c. Initial conditions: Not too important in this case
   d. Boundary conditions:
      i. Determine the state of equilibrium
      ii. 1 cc at the input + 1 cc at the output in SUBCRITICAL flow

Simulation of unsteady flow in channels with **Canalflowmodel.net**

1. Dam-break flow
2. Propagation of hydrographs
   1. Propagation over a flat bed
   2. Propagation in a sloping channel

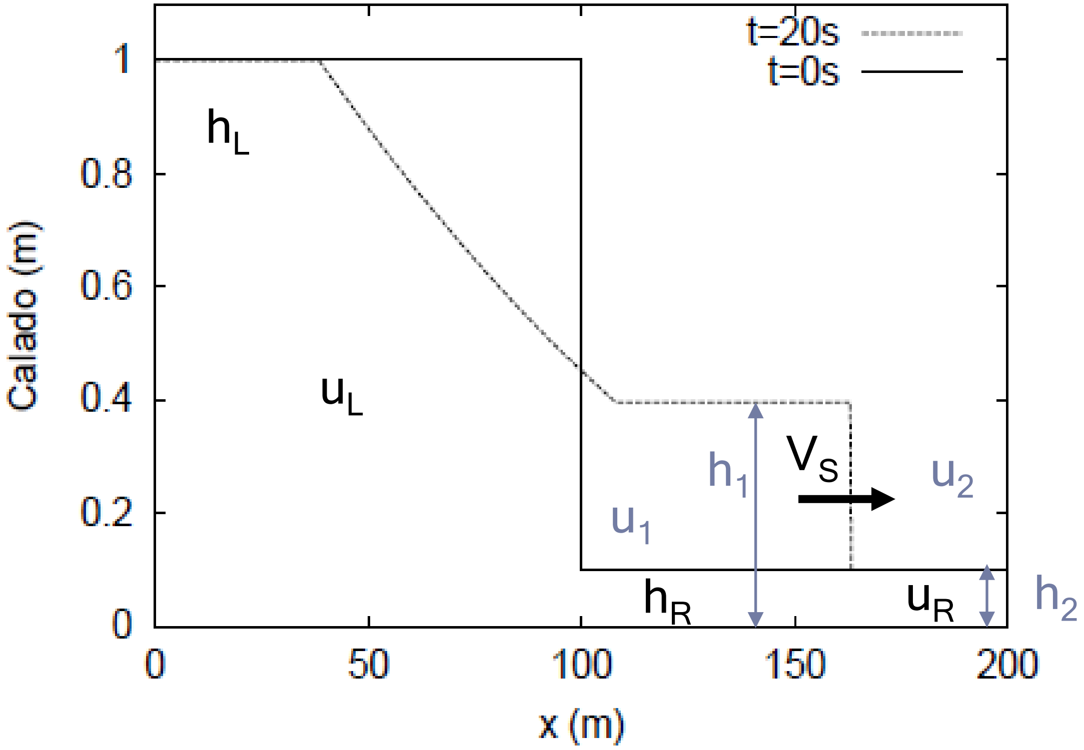
Dam-break flow

- Ideal dam break is the most typical case for the validation of transient free surface flow models.



Dam-break flow

- Ideal dam break is the most typical case for the validation of transient free surface flow models.



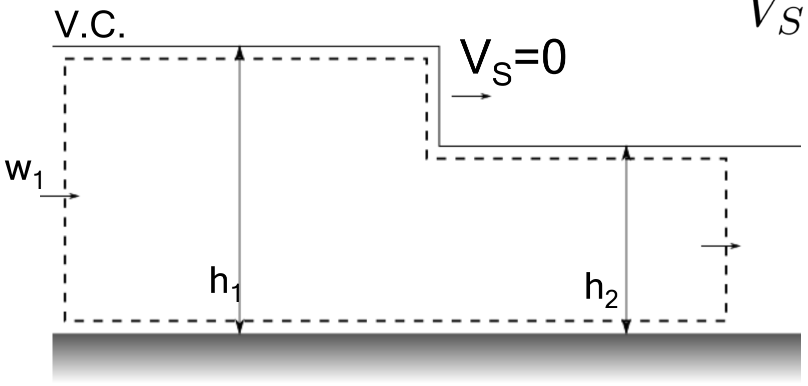
Finite amplitude waves

$$(u_1 - V_S)h_1 = (u_2 - V_S)h_2$$

$$(u_1 h_1 - u_2 h_2) = V_S(h_1 - h_2)$$

$$V_S = \frac{u_1 h_1 - u_2 h_2}{(h_1 - h_2)}$$

$$V_S = \frac{q_1 - q_2}{(h_1 - h_2)}$$



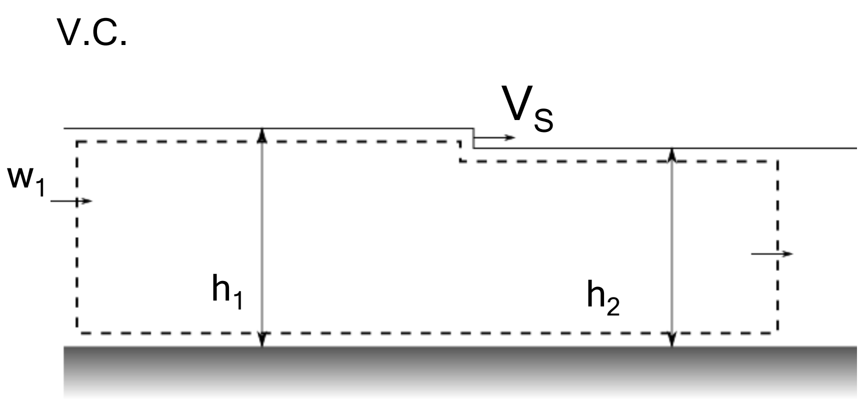
- If u_2=0
  - Mass:

$$u_1 = V_S \frac{h_1 - h_2}{h_1}$$

  - Motion:

$$V_S = \sqrt{\frac{g(h_1 + h_2)}{2} \frac{h_1}{h_2}}$$

Infinitesimal gravity waves



$$\delta h << h$$

- If the wave is at rest (u_2=0)
  - Mass:

$$u_1 = V_S \frac{h_1 - h_2}{h_1}$$

  - Motion:

$$V_S = \sqrt{\frac{g(h + \cancel{\delta h} + h)}{2} \frac{h + \cancel{\delta h}}{h}}$$

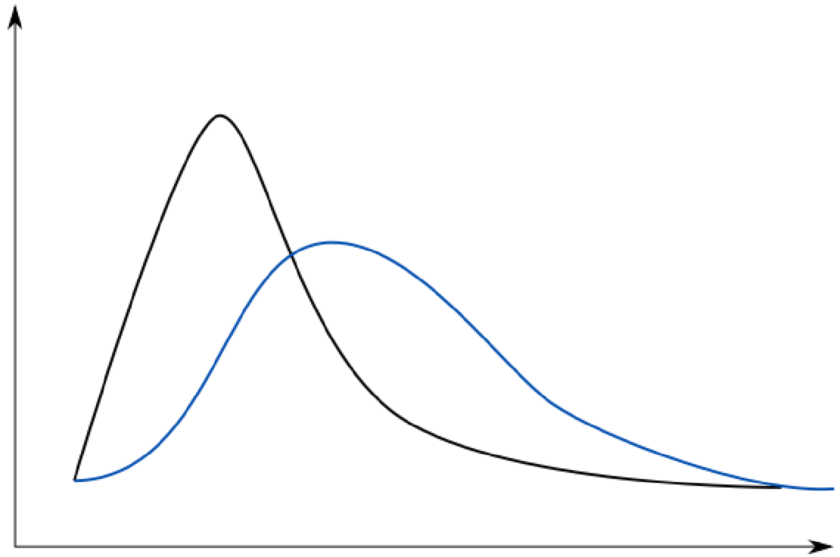
$$V_S = \sqrt{\frac{g(2h)}{2} \frac{h}{h}}$$

$$\boxed{V_S = \sqrt{gh} = c}$$

Propagation of hydrographs

- We will observe the evolution of hydrographs in frictionless channels due to non-linear effects.

- We will look at the sensitivity to friction in channels.



ADVANCED HYDRAULIC SIMULATION MODELS

1D hydraulic simulation models of channels and rivers: PART II

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RESCUER

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