

Supplementary Material of “Tensor Ring Decomposition Guided Dictionary Learning for OCT Images Denoising”

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I. Overview

This supplementary material includes the following sections. In Section II, we investigate the computational complexity of our proposed TRGDL model. In Section III, we have provided an empirical examination to depict the convergence behavior of the suggested TRGDL denoising method. To show the effect of the number of the B-scans in our algorithm, we try to use our algorithm on 3, 4, and 5 B-scans, in Section IV. In Section V, we provided the quantitative and qualitative results for denoising OCT images of dataset-3. Section VI presents a comparison of the results obtained by the proposed TRGDL method with those obtained using the averaging method

II. Computational Complexity

In each iteration, the computational complexity of the suggested TRGDL model mainly contains two main parts. 1) Considering fixed dictionaries and solving four sub-problems including \mathcal{L}^k , \mathcal{A}^k , \mathcal{Q}^k , and \mathcal{M}^k subproblems for each group tensor ($k=1, 2, \dots, K$). 2) Dictionary updating. In the first part, the main computational complexity lies in the solving the \mathcal{L}^k sub-problem and updating \mathcal{Z}^k . Assuming that we have a tensor $\mathcal{Y}^k \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ where $I_1=I_2=I_3=I$, TR-rank is considered as $R_1=R_2=R_N=R$, the computational cost of updating the \mathcal{L}^k is $O(R^6 + R^4 I^2 + R^2 I^3)$, and therefore the overall complexity for K group tensors is about $O(K(R^6 + R^4 I^2 + R^2 I^3))$. In the second part, the complexity of updating \mathbf{D}_S and \mathbf{D}_T is $O(P_{\text{total}}(\omega_S \mathbf{p}\mathbf{p})^2 + P_{\text{total}}(\omega_T \mathbf{V})^2)$.

III. Convergence Analysis

In this section, we have provided an empirical examination to depict the convergence behavior of the suggested TRGDL denoising method. For this purpose, we conducted our experiments on three datasets and recorded the value of objective function in Eq. (6) (in the paper) at each iteration for all subjects. Fig. S1 illustrates the evaluations of objective function values versus the iterations for a sample subject in each dataset. In the three curves, we can clearly observe that with iteration numbers increasing, the functional energy value monotonically decreases, which proves a good convergence property in our suggested TRGDL model. We conduct this examination on all subjects in three datasets and the same behavior is observed.

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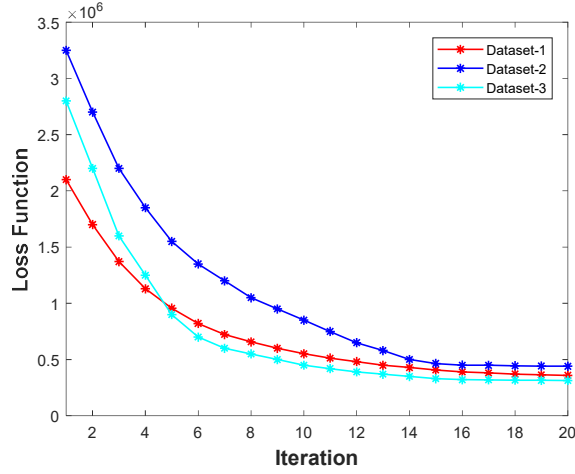


Fig.S1 Empirical convergence curve of objective function against iterations for a sample subject in three OCT datasets i.e., Dataset-1, Dataset-2, Dataset-3

IV. Analysis of the number of OCT slices of Dataset-2 and Dataset-3

To show the effect of the number of the B-scans in our algorithm, we try to use our algorithm on 3, 4, and 5 B-scans. We run these experiments for dataset-2 and dataset-3. Tables S1 and S2 present the average quantitative results on dataset-2 and dataset-3 for CNR and MSR, obtained from our proposed TRGDL model using 3, 4, and 5 nearby B-scans.

Table.S1 Mean of the CNR, MSR results for denoising retinal images in dataset-2 using 3, 4, and 5 B-scans in our proposed method

	3-Bscans as input	4-Bscans as input	5-Bscans as input
CNR	5.18	5.71	5.96
MSR	10.99	12.90	14.06

Table.S2 Mean of the CNR, MSR results for denoising retinal images in dataset-3 using 3, 4, and 5 B-scans in our proposed method

	3-Bscans as input	4-Bscans as input	5-Bscans as input
CNR	4.67	5.02	5.13
MSR	11.37	13.26	14.29

As Tables S1 & S2 show, the quantitative results of our proposed model using 5 nearby B-scans is better than the TRGDL model with 3- and 4-Bscans in terms of CNR, and MSR.

V. Results for OCT Images Denoising of Dataset-3

This section provides qualitative and quantitative results for the noise suppression experiment of dataset-3. Fig.S2 and Table. S3 show results of all compared techniques. As Fig.S2 shows, the proposed TRGDG has the most pleasing performance in maintaining layer structures and reducing the noise.

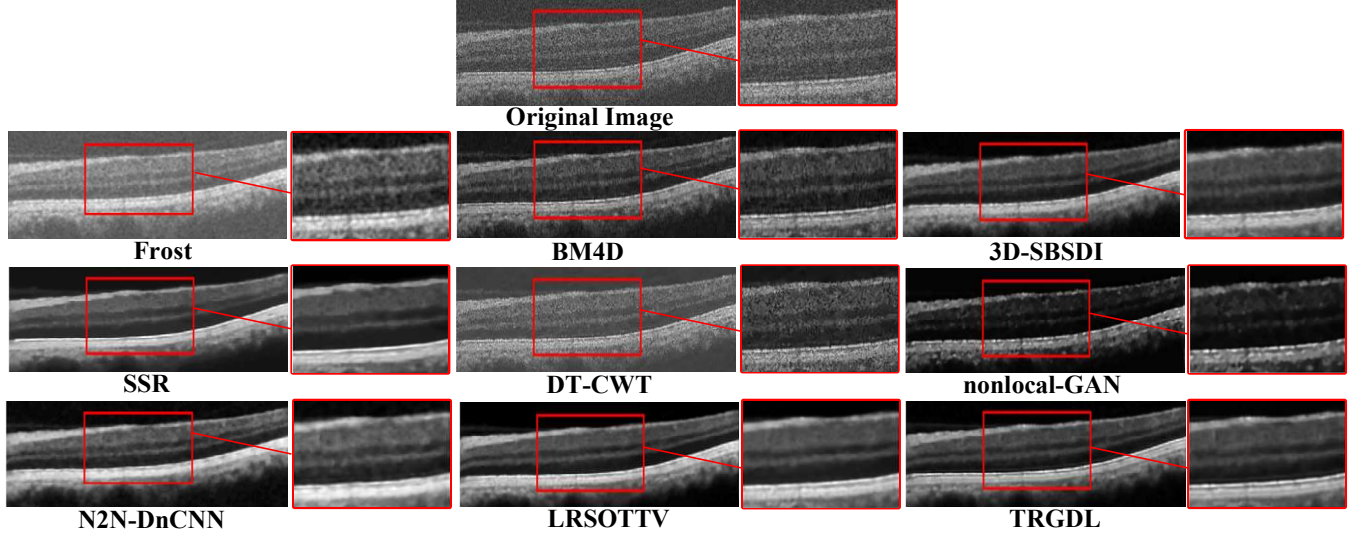


Fig.S2 One retinal OCT image in the dataset-3 and their denoising results utilizing the Frost [1], BM4D [2], 3D-SBSDI [3], SSR [4], DT-CWT [5], nonlocal-GAN [6], N2N-DnCNN [7], LRSOTTV [8], and the suggested TRGDG approach.

Moreover, this method outperforms other rival methods in terms of CNR, and MSR. In addition, the result of all examined approaches is close to each other in EP, and our method's EP value holds the third position among all approaches, closely approaching the top score attained by BM4D. This underscores the effectiveness of the suggested TRGDG in preserving layer structures in OCT images. Table. S3 further illustrates a significant improvement in all metrics when comparing the suggested TRGDG model to other techniques.

Table.S3 Mean & Standard deviation of the MSR, CNR, and EP results for denoising dataset-3 by various approaches. Best results are bolded. Where $p < 0.05$, the measures for each test approach are statistically significant and characterized by a* symbol.

	CNR		MSR		EP	
	Mean± SD	P-value	Mean± SD	P-value	Mean± SD	P-value
Noisy	2.24±0.29	4.00E-05*	4.66±0.31	4.00E-05*	1.0000± 0.0000	4.00E-05*
Frost	4.12±0.84	4.00E-05*	9.34±1.24	4.00E-05*	0.9298±0.0394	4.00E-05*
BM4D	3.93±0.70	4.00E-05*	8.59± 0.96	4.00E-05*	0.9673 ±0.0220	4.00E-05*
3D-SBSDI	5.01±1.26	1.93E-03*	12.82±1.99	2.01E-04*	0.9426±0.0340	8.31E-04*
SSR	4.71±1.27	9.83E-04*	13.23±2.13	6.08E-03*	0.9433±0.0345	8.94E-03*
DT-CWT	3.47±0.55	4.00E-05*	6.99±0.68	4.00E-05*	0.9641±0.0142	4.00E-05*
nonlocal-GAN	2.91±0.64	4.00E-05*	13.37± 2.23	4.95E-02*	0.9425±0.0347	9.27E-04*
LRSOTTV	5.04±1.27	4.06E-03*	13.30± 2.23	1.73E-03*	0.9359±0.0396	3.13E-04*
N2N-DnCNN	4.96±1.24	1.54 E-03*	13.09± 2.58	3.30E-03*	0.9345±0.0366	4.00E-05*
TRGDG	5.13 ±1.26	-	14.29 ±1.99	-	0.9460±0.0320	-

VI. Comparison of the proposed TRGDL method with the averaging method

To compare the result of our TRGDL method with the averaging method, we utilized one additional dataset, namely, dataset-4. This dataset is a large public dataset which is introduced in [9]. This dataset contains several volumes of size $640 \times 400 \times 400$ voxels that were acquired using the RTVueXR SD-OCT system.

First, we randomly chose 30 normal volumes from the large dataset. Then, to implement the averaging strategy, in each volume, we selected one B-scan and 19 noisy adjacent B-scans around each chosen B-scan. Next, we obtained the averaged image from these 20 overlapping B-scans using the averaging method.

To implement our TRGDL method, we considered each chosen B-scan in above and considered 4 noisy adjacent B-scans around each chosen B-scan. Accordingly, 30 normal volumes of size $640 \times 400 \times 5$ voxels are considered as input of our algorithm.

Fig. S3 & S4 display the visual denoising results of the proposed TRGDL method and the averaging method on two representative OCT images of dataset-4.

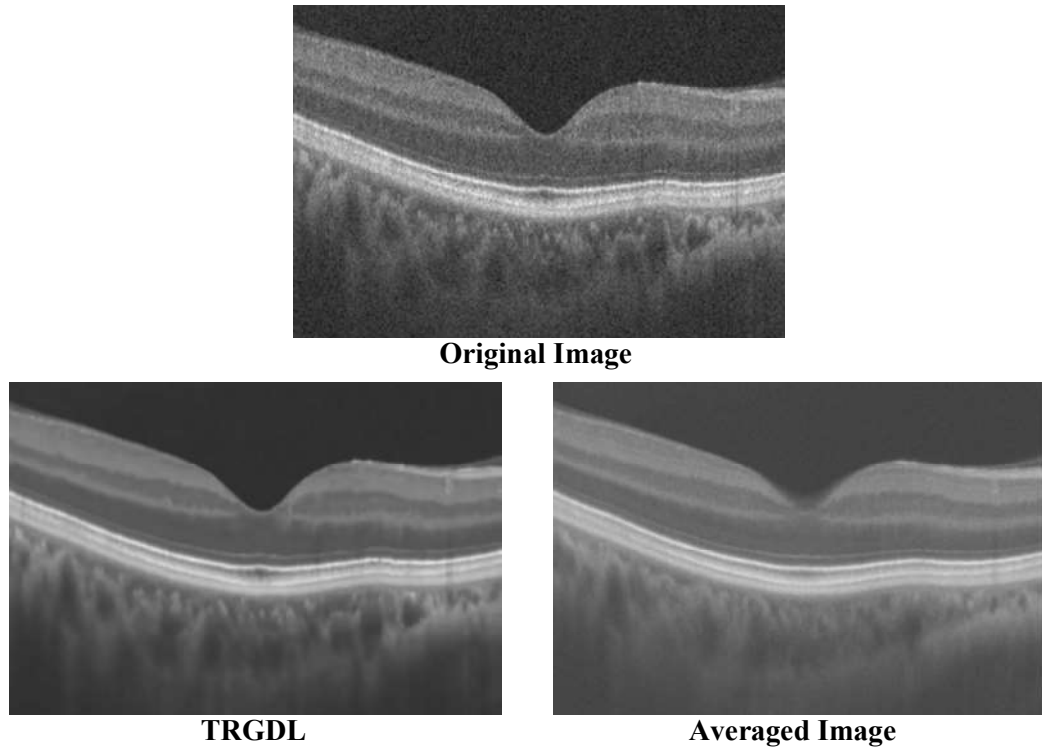


Fig.S3 One retinal OCT image in dataset-4 and their denoising results utilizing the suggested TRGDL approach and the averaging method

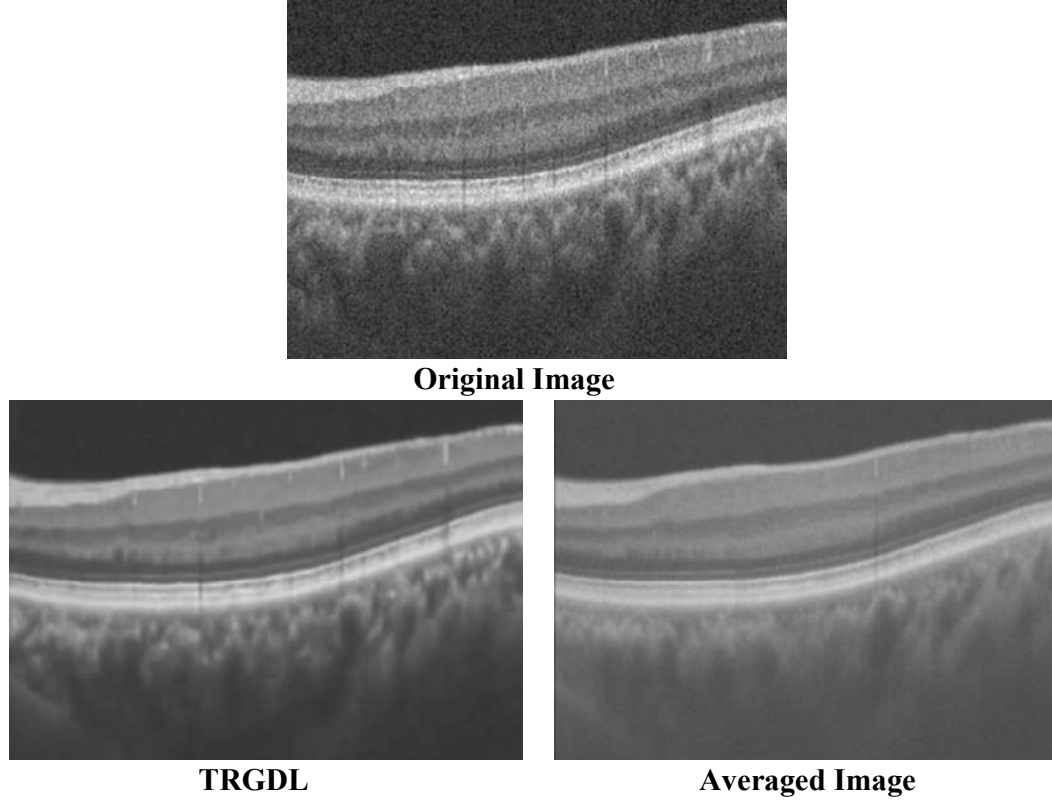


Fig.S4 One retinal OCT image in dataset-4 and their denoising results utilizing the suggested TRGDL approach and the averaging method

A visual comparison of the results of our suggested TRGDL model and the average method reveals that both approaches remove the speckle noise but the average method vanishes the details and meaningful structures. Moreover, Table S4 shows the quantitative results. As Table S4 shows, our TRGDL method is better than the average method in terms of CNR. And the average method has a higher MSR value than our method.

Table S4 Mean & Standard deviation of the MSR, CNR results for denoising 30 retinal images in the dataset-4 by TRGDL method and the averaging method

	CNR	MSR
	Mean± SD	Mean± SD
Noisy	4.25± 0.47	6.60±0.60
Average Image	5.72±0.95	10.10±1.41
TRGTDL	6.13±0.97	9.55±1.28

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