Group of Horribly Optimistic STatisticians

CV SEMINAR

ROZKŁAD SVD

09.04.2024 Computer Vision Seminar 23/24

Agenda

1. Na czym polega wizja komputerowa?
2. Przykłady i zastosowania
3. Czym jest zdjęcie?
4. Przekształcenia liniowe
5. Rozkład macierzy

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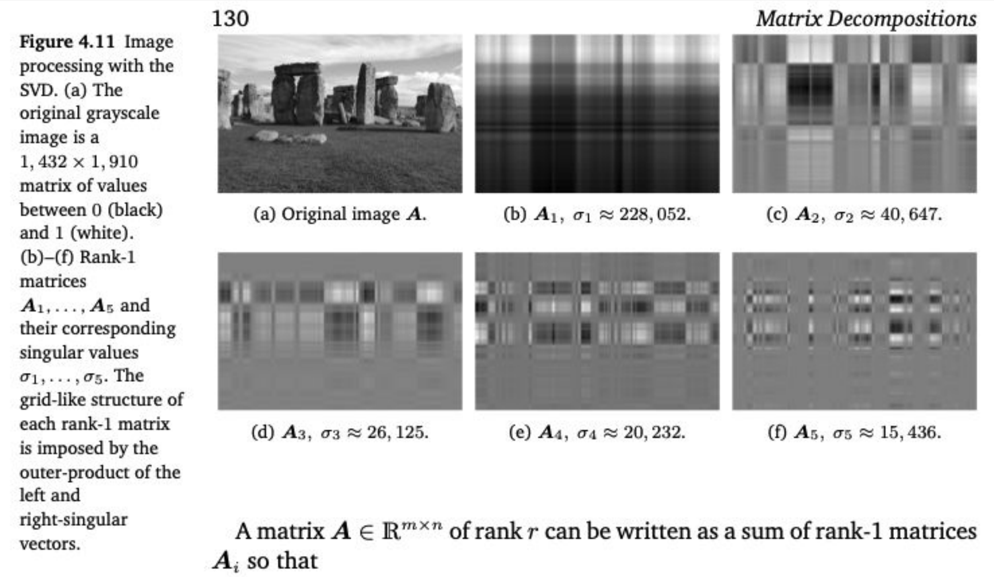
Mathematics for Machine Learning



130 *Matrix Decompositions*

Figure 4.11 Image processing with the SVD. (a) The original grayscale image is a $1,432 \times 1,910$ matrix of values between 0 (black) and 1 (white). (b)–(f) Rank-1 matrices $\boldsymbol{A}_1, \ldots, \boldsymbol{A}_5$ and their corresponding singular values $\sigma_1, \ldots, \sigma_5$. The grid-like structure of each rank-1 matrix is imposed by the outer-product of the left and right-singular vectors.

(a) Original image $\boldsymbol{A}$.

(b) $\boldsymbol{A}_1,\ \sigma_1 \approx 228,052$.

(c) $\boldsymbol{A}_2,\ \sigma_2 \approx 40,647$.

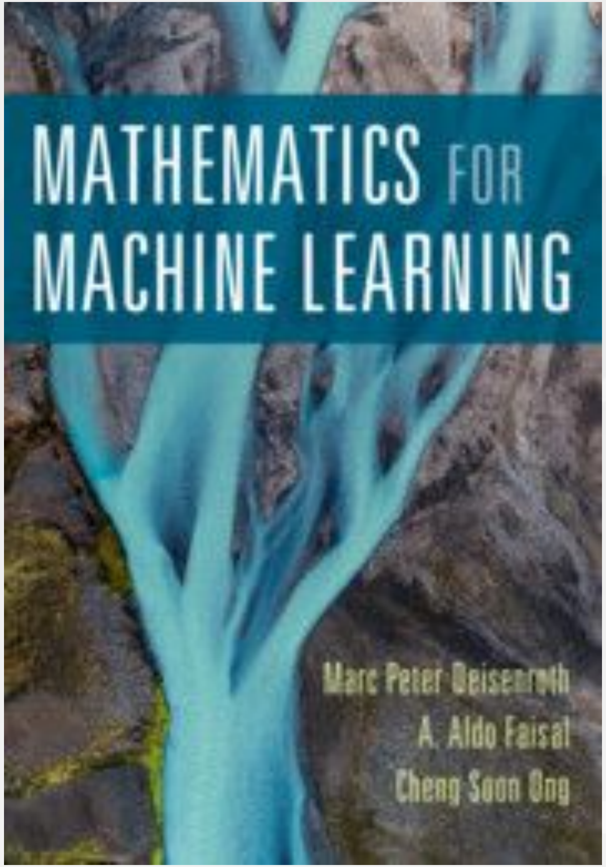
(d) $\boldsymbol{A}_3,\ \sigma_3 \approx 26,125$.

(e) $\boldsymbol{A}_4,\ \sigma_4 \approx 20,232$.

(f) $\boldsymbol{A}_5,\ \sigma_5 \approx 15,436$.

A matrix $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ of rank r can be written as a sum of rank-1 matrices $\boldsymbol{A}_i$ so that

https://mml-book.github.io/



Na czym polega computer vision?

OBRAZ → OPIS INFORMACJA

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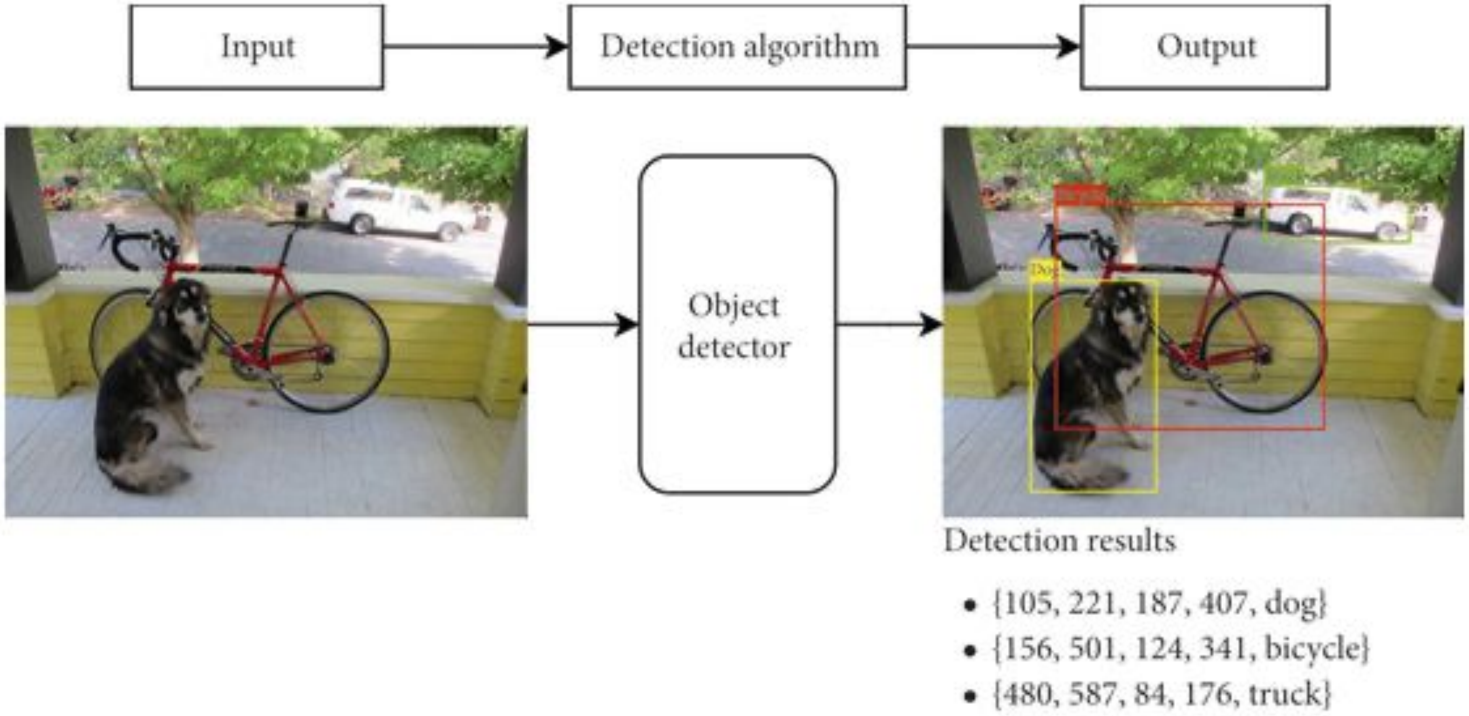
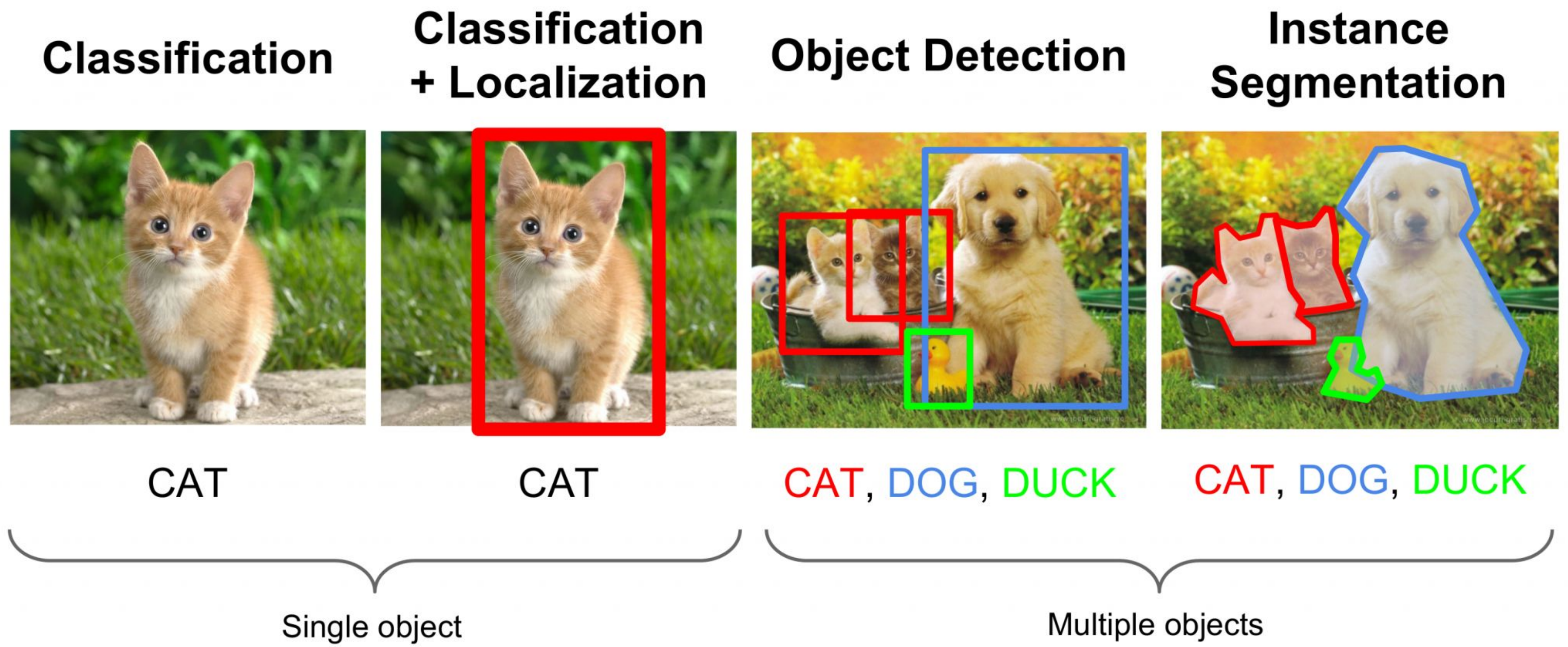



Classification
Classification + Localization
Object Detection
Instance Segmentation
CAT
CAT
CAT, DOG, DUCK
CAT, DOG, DUCK
Single object
Multiple objects

Czym jest zdjęcie cyfrowe?

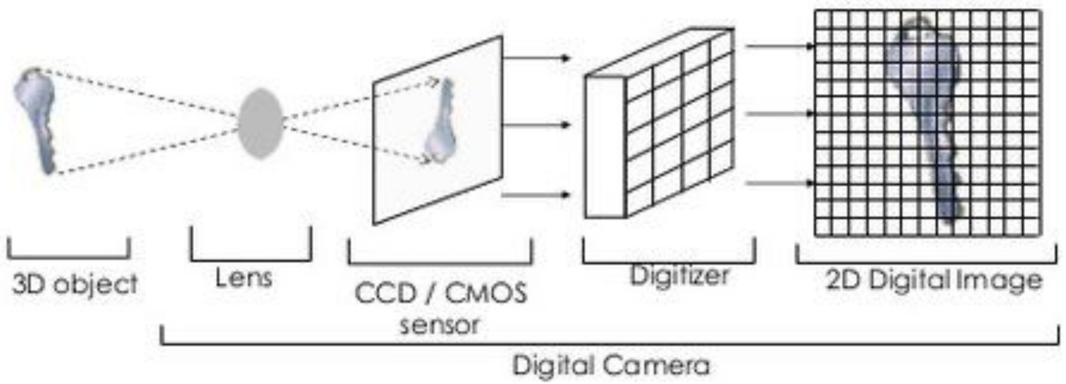
- Dwuwymiarowa funkcja f(x,y)
- x i y określają współrzędne
- Amplituda f w (x,y) jest określana jako intensywność zdjęcia w tym punkcie
- Wartości x, y i f są skończone

Lub inaczej: tablica składająca się z rzędów i kolumn pikseli o skończonych wartościach

Zdjęcie jako macierz

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & f(1,2) & \dots & f(1,N-1) \\ \vdots & \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & f(M-1,2) & \dots & f(M-1,N-1) \end{bmatrix}$$

Visual image formation-Digital Version



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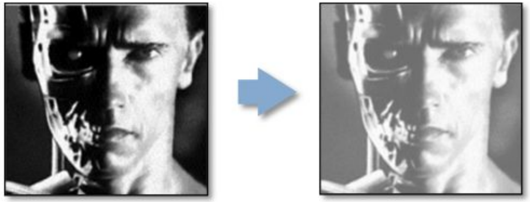
Zdjęcie jako funkcja - przekształcenia

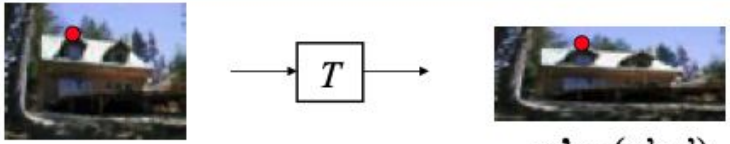
As with any function, we can apply operators to an image



$$g(x,y) = f(x,y) + 20$$

$$g(x,y) = f(-x,y)$$

Parametric (global) warping



$\mathbf{p} = (x,y)$ $\qquad$ $\mathbf{p'} = (x',y')$

- Transformation T is a coordinate-changing machine:

$$\mathbf{p'} = T(\mathbf{p})$$

- What does it mean that *T* is global?
  - Is the same for any point p
  - can be described by just a few numbers (parameters)
- Let's represent *T* as a matrix:

$$\mathbf{p'} = \mathbf{Tp}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Przesunięcie, translacja
- Skalowanie
- Obrót
- Pochylenie

Basic 2D Transformations

- Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x'} \\ \boldsymbol{y'} \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s_x} & 0 & 0 \\ 0 & \boldsymbol{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} \boldsymbol{x'} \\ \boldsymbol{y'} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh_x} & 0 \\ \boldsymbol{sh_y} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

https://sites.google.com/pjwstk.edu.pl/grk/grk/przeksztaɫ%C5%82cenia-afiniczne?pli=1

Podstawowe przekształcenia

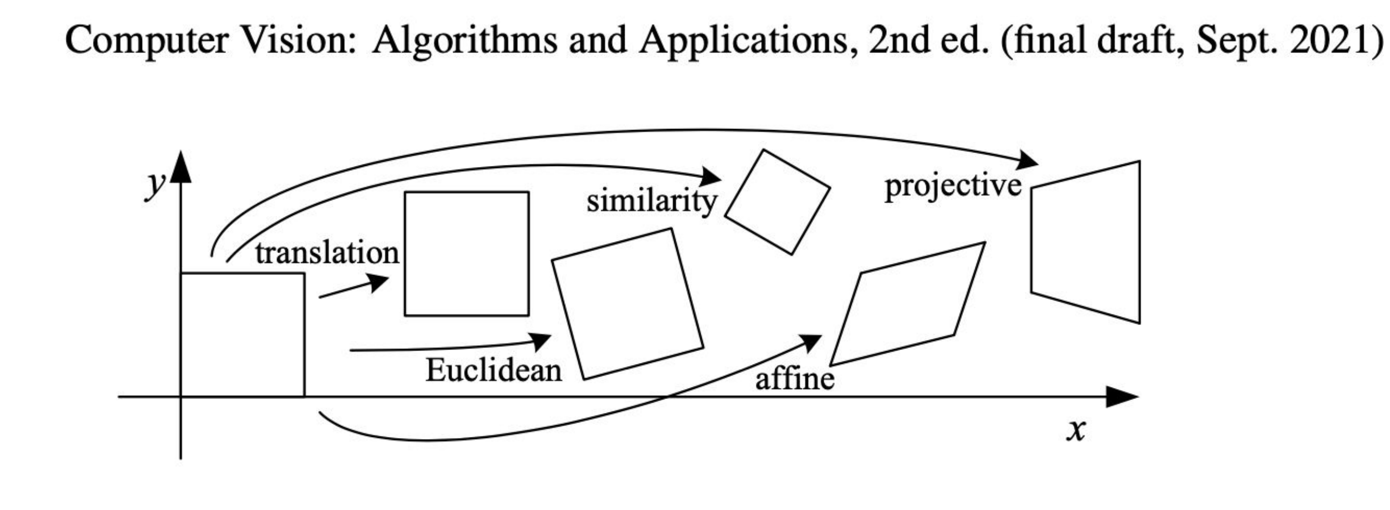


Figure 2.4 *Basic set of 2D planar transformations.*

Więcej o transformacjach: http://alumni.media.mit.edu/~maov/classes/comp_photo_vision08f/lect/08_image_warps.pdf

Macierz

- Układ liczb, symboli lub wyrażeń zapisanych w postaci prostokątnej tablicy
- Każda macierz opisuje przekształcenie liniowe

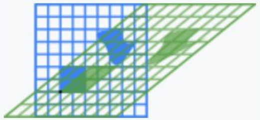
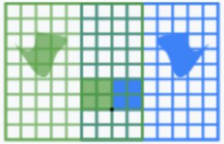
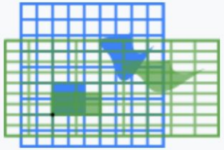
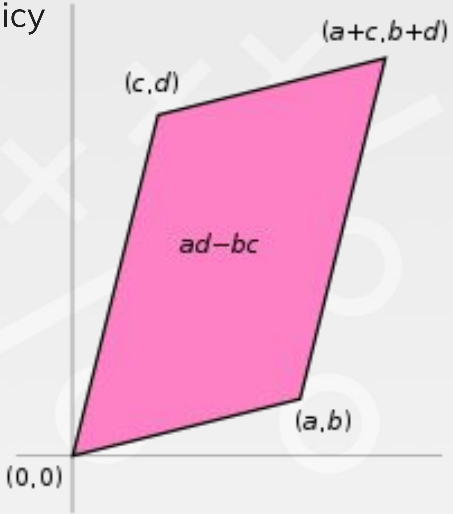
Powinowactwo względem osi poziomej o $m = \frac{5}{4}$.	Symetria względem osi pionowej	Przekształcenie ekwiafiniczne o $r = \frac{3}{2}$	Jednokładność o skali $\frac{3}{2}$	Obrót o kąt miary 30°
$\begin{bmatrix} 1 & \frac{5}{4} \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \frac{3}{2} & 0 \\ 0 & \frac{2}{3} \end{bmatrix}$	$\begin{bmatrix} \frac{3}{2} & 0 \\ 0 & \frac{3}{2} \end{bmatrix}$	$\begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix}$
				

Tabela przedstawia macierze stopnia 2 z odpowiadającymi im przekształceniami płaszczyzny: niebieska kratka zawierająca pewien kształt jest przekształcana na zieloną; czarny punkt oznacza początek przestrzeni.



https://pl.wikipedia.org/wiki/Macierz

Rozkład macierzy

- Do wielu zastosowań warto przedstawić macierz jako iloczyn kilku macierzy **o określonych właściwościach**

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

https://medium.com/mitb-for-all/singular-value-decomposition-explained-step-by-step-with-code-4ee86b12a021

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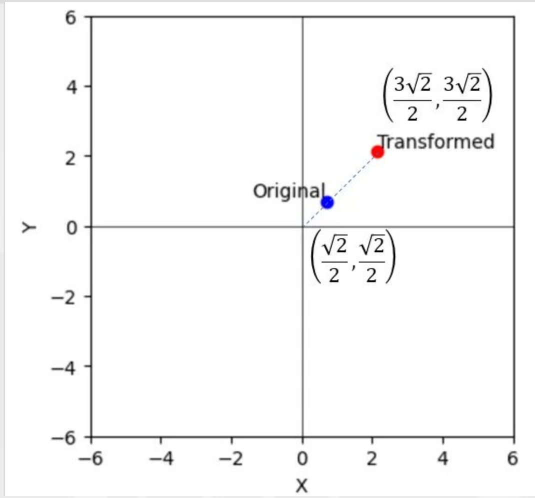
Diagonalizacja



$$\mathbf{A} = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^{\mathrm{T}}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{e}_{1,1} & \mathbf{e}_{2,1} \\ \mathbf{e}_{1,2} & \mathbf{e}_{2,2} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \mathbf{e}_{1,1} & \mathbf{e}_{1,2} \\ \mathbf{e}_{2,1} & \mathbf{e}_{2,2} \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

https://pl.wikipedia.org/wiki/Diagonalizacja

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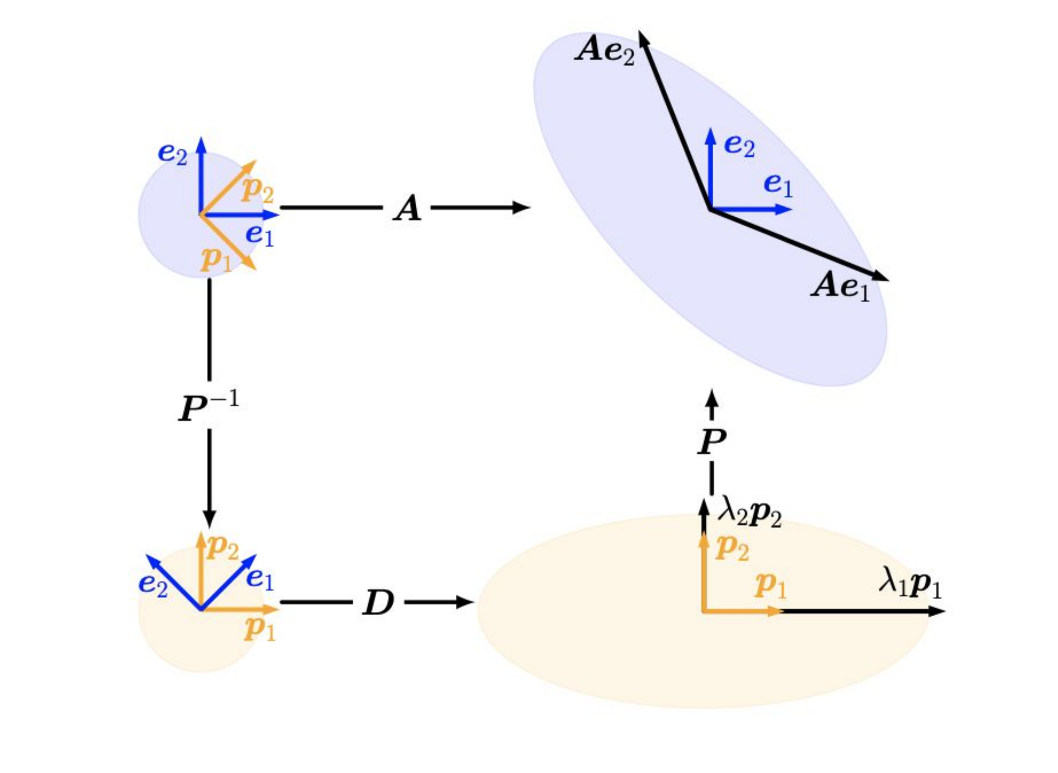
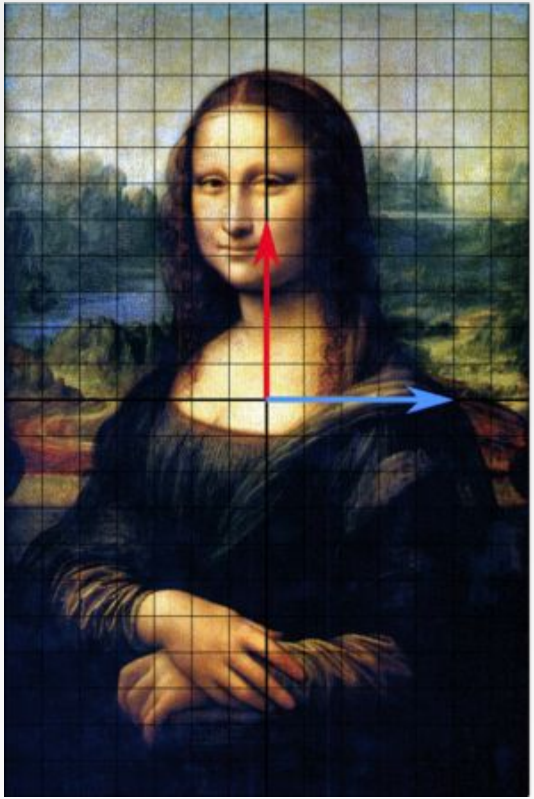
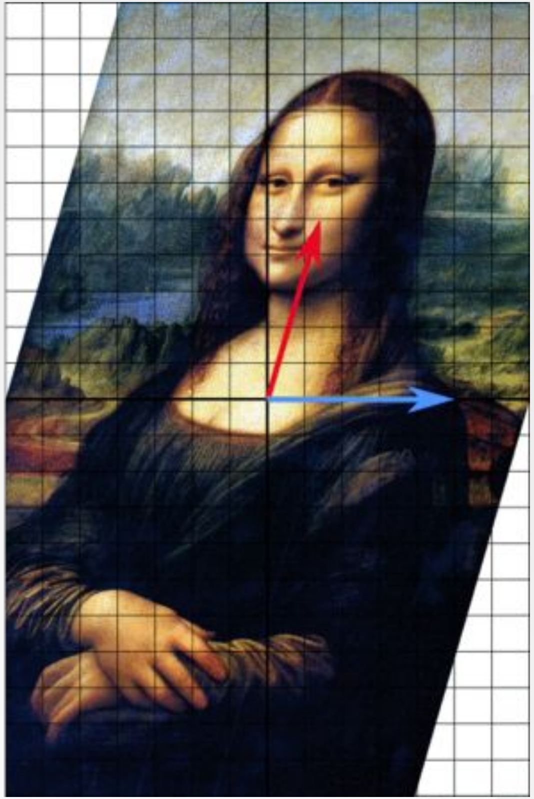


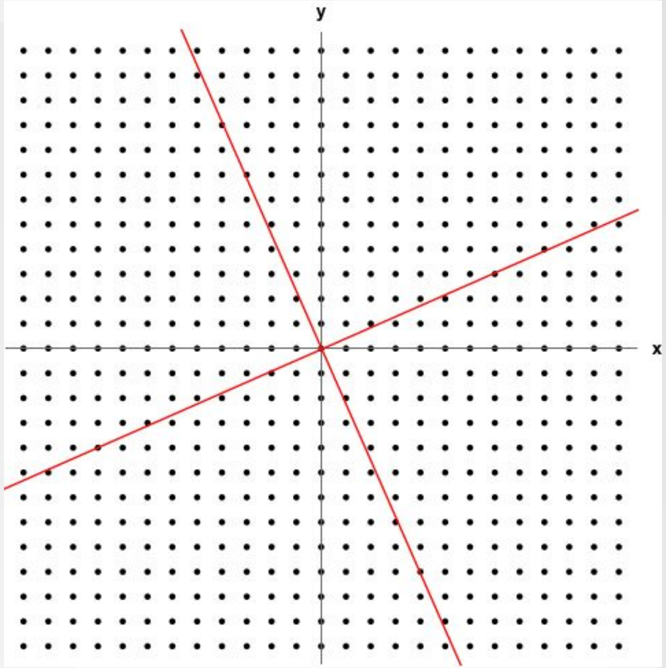
Figure 4.7 Intuition behind the eigendecomposition as sequential transformations. Top-left to bottom-left: P^{-1} performs a basis change (here drawn in $\mathbb{R}^2$ and depicted as a rotation-like operation) from the standard basis into the eigenbasis. Bottom-left to bottom-right: D performs a scaling along the remapped orthogonal eigenvectors, depicted here by a circle being stretched to an ellipse. Bottom-right to top-right: P undoes the basis change (depicted as a reverse rotation) and restores the original coordinate frame.

MML book
Page 117

https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors







Rozkład SVD

- Co w przypadku, gdy macierz nie jest kwadratowa i symetryczna?
- **SVD można zastosować do dowolnej macierzy**

$$\mathbf{A} = \begin{bmatrix} 19 & -20 & 0 \\ 6 & -15 & 1650 \end{bmatrix}$$

$$\mathbf{A}\mathbf{A}^{\mathrm{T}} = \begin{bmatrix} 19 & -20 & 0 \\ 6 & -15 & 1650 \end{bmatrix} \begin{bmatrix} 19 & 6 \\ -20 & -15 \\ 0 & 1650 \end{bmatrix} = \begin{bmatrix} 761 & 414 \\ 414 & 2722761 \end{bmatrix}$$

$$\mathbf{A}^{\mathrm{T}}\mathbf{A} = \begin{bmatrix} 19 & 6 \\ -20 & -15 \\ 0 & 1650 \end{bmatrix} \begin{bmatrix} 19 & -20 & 0 \\ 6 & -15 & 1650 \end{bmatrix} = \begin{bmatrix} 397 & -470 & 9900 \\ -470 & 625 & -24750 \\ 9900 & -24750 & 2722500 \end{bmatrix}$$

Rozkład SVD

$$\mathbf{A} = \mathbf{U} \quad \boldsymbol{\Sigma} \quad \mathbf{V}^{\mathbf{T}}$$

$$\underset{2\text{x}3}{\begin{bmatrix} & & \end{bmatrix}} = \underset{2\text{x}2}{\begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{bmatrix}} \underset{2\text{x}3}{\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \end{bmatrix}} \underset{3\text{x}3}{\begin{bmatrix} \mathbf{v}_1{}^{\mathrm{T}} \\ \mathbf{v}_2{}^{\mathrm{T}} \\ \mathbf{v}_3{}^{\mathrm{T}} \end{bmatrix}}$$

https://medium.com/mitb-for-all/singular-value-decomposition-explained-step-by-step-with-code-4ee86b12a021

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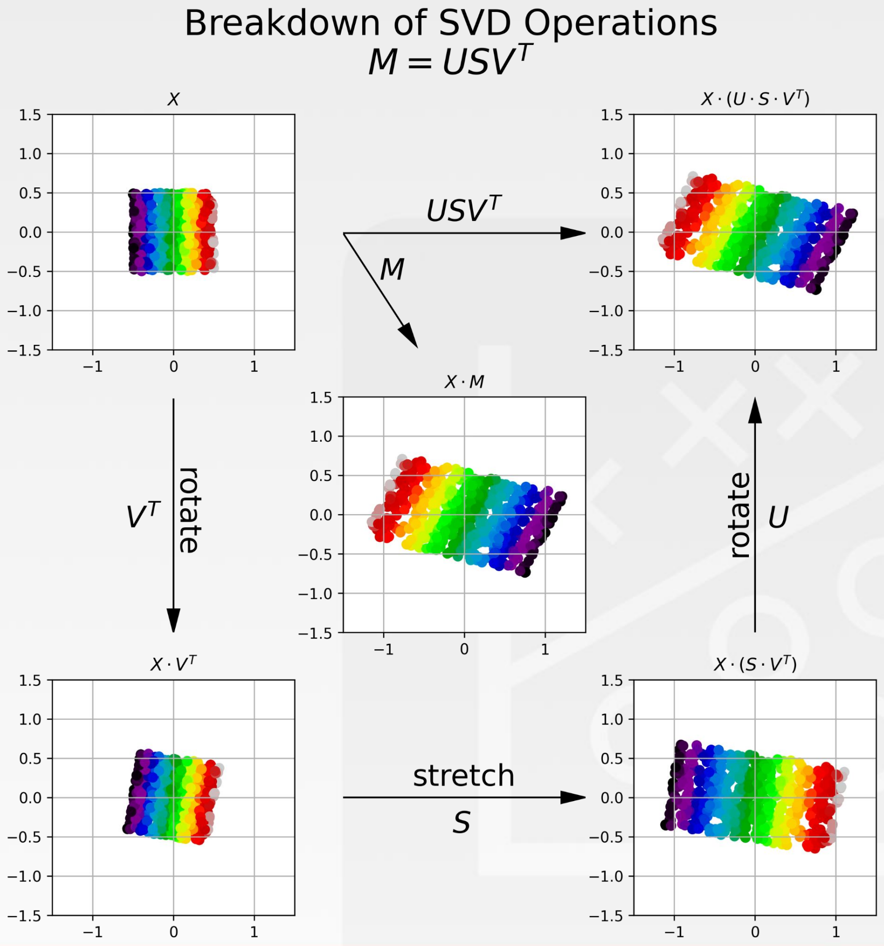
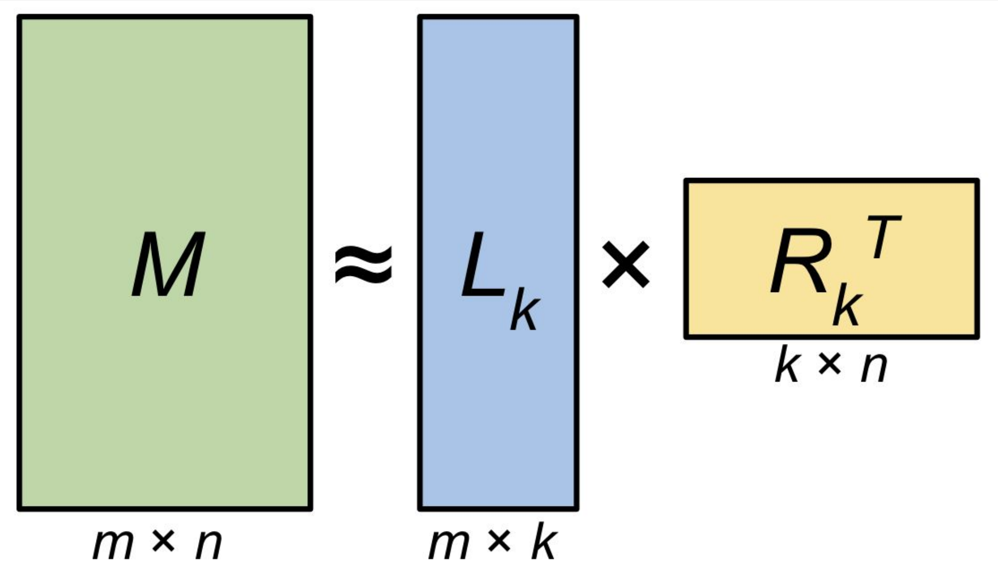
https://dustinstansbury.github.io/theclevermachine/singular-value-decomposition



Aproksymacja macierzy



A matrix $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ of rank r can be written as a sum of rank-1 matrices $\boldsymbol{A}_i$ so that

$$\boldsymbol{A} = \sum_{i=1}^{r} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^\top = \sum_{i=1}^{r} \sigma_i \boldsymbol{A}_i \,, \tag{4.91}$$

https://dustinstansbury.github.io/theclevermachine/svd-data-compression

Materiały

- Introduction to Basic Computer Vision & Image Processing

https://bishalbose294.medium.com/introduction-to-basic-computer-vision-image-processing-f692aa1a4f18

- Macierz przekształcenia liniowego

https://pl.wikipedia.org/wiki/Macierz_przekszta%C5%82cenia_liniowego

- Singular Value Decomposition — Explained with code

https://medium.com/mitb-for-all/singular-value-decomposition-explained-step-by-step-with-code-4ee86b12a021

- SVD and Data Compression Using Low-rank Matrix Approximation

https://dustinstansbury.github.io/theclevermachine/svd-data-compression

- The colab

https://colab.research.google.com/drive/1I-zSx_ApWP71dAZvJVzgXQyLC22Gy7QT?usp=sharing