Ice Fracture Analytic Model

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October 15, 2023

Abstract

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1 Introduction

We are developing an analytic model to describe how increases in pressure due to the lesser density of ice than liquid water result in eventual ice fracture.

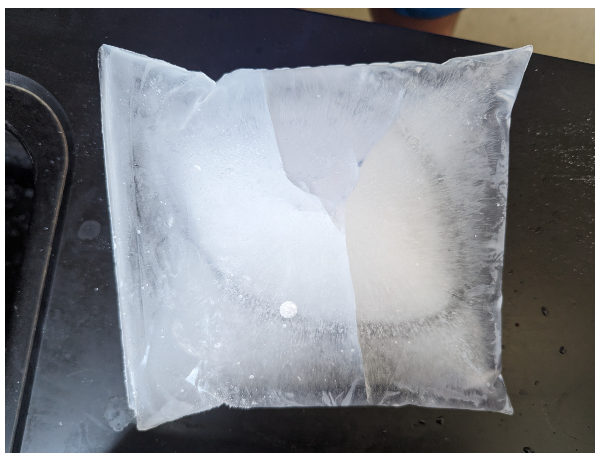

Figure 1: Ice fractures present in a freshwater sample frozen at -30C

2 Analytic Model

2.1 Pressure via Incremental Change in Volume (?)

P: Pressure
K_w: Bulk Modulus of Water
V: Volume
V_0: Initial Volume of Water
V_w: Volume of water
V_i: Volume of ice
A: Unit Area
M: Mass
ρ_w: Density of Water
ρ_i: Density of Ice
t: time
x: horizontal along which ice formation occurs
Initial identity:

$$\frac{P}{K_w} = \frac{\frac{dV}{A}}{\frac{V}{A}}] = \frac{dx}{X}$$

With each increment of freezing:

$$\frac{\Delta M}{\rho_w}$$

$$\dot{m} = \frac{\Delta M}{\Delta t}$$

$$V_w(t) = V_0 - \frac{\dot{m}}{\rho_w}\Delta t$$

$$V_i(t) = \frac{\dot{m}}{\rho_i}\Delta t$$

$$\frac{V(t)}{V_0} = \frac{V_0 + \dot{m}(\rho_i^{-1} - \rho_w^{-1})\Delta t}{V_0}$$

$$\frac{dV}{V_0} = \frac{V_0 + \dot{m}(\rho_i^{-1} - \rho_w^{-1})\Delta t}{V_0} = \frac{P}{K_f}$$