

```

x = {t, r,  $\theta$ ,  $\varphi$ }
g = DiagonalMatrix[{- (1 - (2 M) / r), (1 - 2 M / r) ^ (-1), r^2, r^2 * (Sin[ $\theta$ )] ^2}];
Print["The Schwarzschild metric is"];
g // MatrixForm

```

Out[•]=

{t, r,  $\theta$ ,  $\varphi$ }

The Schwarzschild metric is

Out[•]//MatrixForm=

$$\begin{pmatrix} -1 + \frac{2M}{r} & 0 & 0 & 0 \\ 0 & \frac{1}{1 - \frac{2M}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2[\theta] \end{pmatrix}$$

```
ig = Inverse[g];
```

```
gd = D[g, {x}];
```

```
Print["The inverse metric is"];
```

```
ig // Simplify // MatrixForm
```

The inverse metric is

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{r}{2M-r} & 0 & 0 & 0 \\ 0 & 1 - \frac{2M}{r} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{\csc^2[\theta]}{r^2} \end{pmatrix}$$

```

ChristoffelSymbol[g_, xx_] := Block[{ig, gd}, ig = Inverse[g];
  gd = D[g, {xx}];
  ig.(gd + Transpose[gd, {3, 1, 2}] - Transpose[gd, {2, 3, 1}]) / 2];
Print["The ChristoffelSymbol is"];
ChristoffelSymbol[g, x] // Simplify // MatrixForm
The ChristoffelSymbol is

```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
 \begin{pmatrix} 0 \\ -\frac{M}{2Mr-r^2} \\ 0 \\ 0 \end{pmatrix} &
 \begin{pmatrix} -\frac{M}{2Mr-r^2} \\ 0 \\ 0 \\ 0 \end{pmatrix} &
 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
 \begin{pmatrix} \frac{M(-2M+r)}{r^3} \\ 0 \\ 0 \\ 0 \end{pmatrix} &
 \begin{pmatrix} 0 \\ \frac{M}{2Mr-r^2} \\ 0 \\ 0 \end{pmatrix} &
 \begin{pmatrix} 0 \\ 0 \\ 2M-r \\ 0 \end{pmatrix} &
 \begin{pmatrix} 0 \\ 0 \\ 0 \\ (2M-r)\sin[\theta]^2 \end{pmatrix} \\
 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
 \begin{pmatrix} 0 \\ 0 \\ \frac{1}{r} \\ 0 \end{pmatrix} &
 \begin{pmatrix} 0 \\ \frac{1}{r} \\ 0 \\ 0 \end{pmatrix} &
 \begin{pmatrix} 0 \\ 0 \\ 0 \\ -(\cos[\theta]\sin[\theta]) \end{pmatrix} \\
 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} &
 \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{r} \end{pmatrix} &
 \begin{pmatrix} 0 \\ 0 \\ 0 \\ \cot[\theta] \end{pmatrix} &
 \begin{pmatrix} 0 \\ \frac{1}{r} \\ \cot[\theta] \\ 0 \end{pmatrix}
 \end{pmatrix}$$

```
RiemannTensor[g_, xx_] := Block[{ig, chr, chrd, chrdif}, chr = ChristoffelSymbol[g, xx];
  chrd = D[chr, {xx}];
  chrdif = chrd - chr.chr;
  Transpose[chrdif, {4, 1, 3, 2}] - Transpose[chrdif, {4, 2, 3, 1}]];
Print["The Riemann tensor is"];
RiemannTensor[g, x] // Simplify // MatrixForm
The Riemann tensor is
```

`Out[•]//MatrixForm=`

$$\begin{pmatrix}
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & \frac{2M(-2M-r)}{r^4} & 0 & 0 \\ \frac{2M}{(2M-r)r^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & \frac{M(-2M+r)}{r^4} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{M}{r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{M \sin[\theta]^2}{r} & 0 & 0 & 0 \end{pmatrix} \\
\begin{pmatrix} 0 & \frac{2M(-2M+r)}{r^4} & 0 & 0 \\ -\frac{2M}{(2M-r)r^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{M}{(2M-r)r^2} & 0 \\ 0 & \frac{M}{r} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{M \sin[\theta]^2}{r} & 0 & 0 \end{pmatrix} \\
\begin{pmatrix} 0 & 0 & \frac{M(2M-r)}{r^4} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{M}{r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{M}{r^2(-2M+r)} & 0 \\ 0 & -\frac{M}{r} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2M \sin[\theta]^2}{r} & 0 \end{pmatrix} \\
\begin{pmatrix} 0 & 0 & 0 & \frac{M(2M-r)}{r^4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{M \sin[\theta]^2}{r} & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{M}{r^2(-2M+r)} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{M \sin[\theta]^2}{r} & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2M}{r} \\ 0 & 0 & \frac{2M \sin[\theta]^2}{r} & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\end{pmatrix}$$

... The Riemann tensor is

[illegible]

```

RicciTensor[g_, xx_] := Block[{rie}, rie = RiemannTensor[g, xx];
  Tr[Transpose[rie, {3, 2, 4, 1}], Plus, 2]];
RicciScalar[g_, xx_] := Block[{ric}, ric = RicciTensor[g, xx];
  ric.Inverse[g] // Tr]

```

```

Print["The Ricci tensor is"];
RicciTensor[g, x] // Simplify // MatrixForm

```

```

Print["The Ricci scalar is"];
RicciScalar[g, x] // Simplify

```

The Ricci tensor is

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The Ricci scalar is

Out[ ]=

0

```

In[ ]:= M := 1;
Print["The graph of equal t-plane"];
RevolutionPlot3D[Sqrt[8 M (r - 2 M)], {r, 0, 40}, MeshStyle -> {{Cyan, Opacity[1]}},
  PlotStyle -> Directive[Black, Specularity[White, 40]]]

```

The graph of equal t-plane

Out[ ]=

