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Fourier Transform and its Application in Radar Imaging

A look into calculating Fourier Transform and application

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Introduction

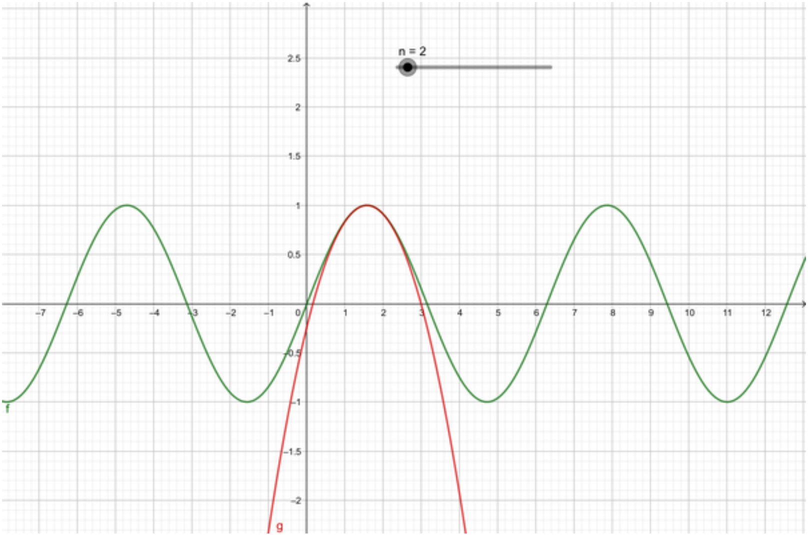
What is Fourier Transform and Why use it?

Functions in mathematics are commonly approximated using Taylor and Maclaurin Transform to create polynomial function of x^n. Some functions are better approximated by sine and cosine rather than a polynomial function. The goal is to create an approximation of input data using Fourier Transformthat will create a more accurate and faster approximation than a Taylor or Maclaurin series could.

What is Fourier Transform and Why use it?

Lets take a look at why polynomial series approximations are not always as useful as sine and cosine approximation

What is Fourier Transform and Why use it?



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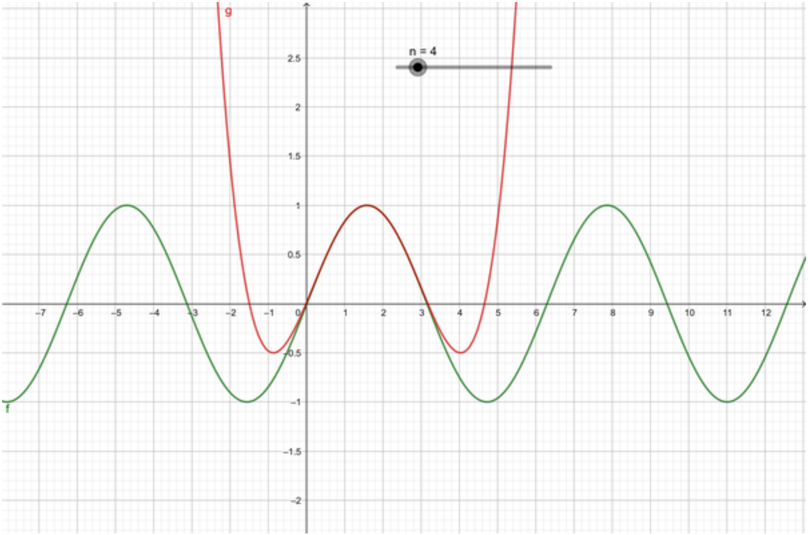
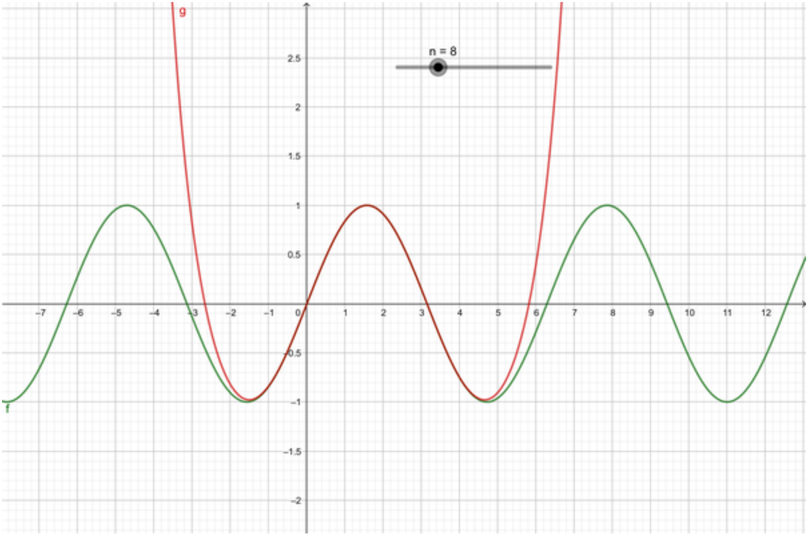
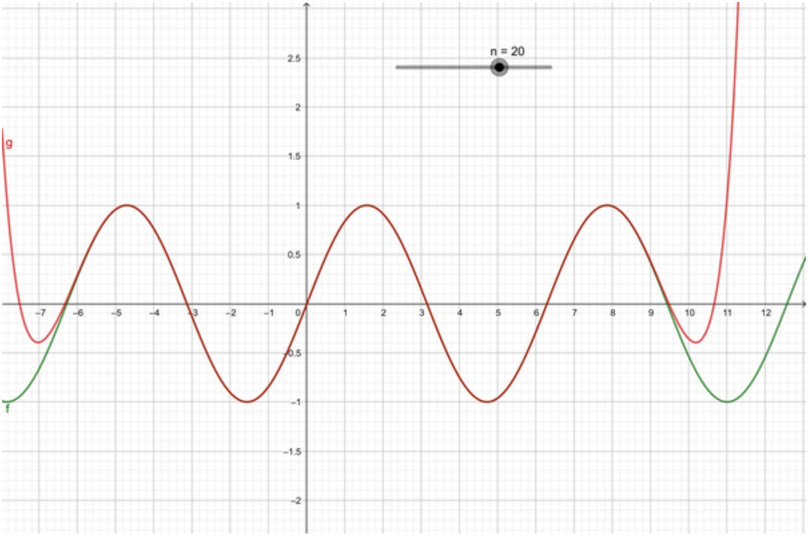


What is Fourier Transform and Why use it?



What is Fourier Transform and Why use it?



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Discrete and Continuous

It is important that when looking at Fourier Transform that we discuss which kind we will be dealing with. DFT is a transform that actually acts on a sequence of values, rather than an interval of values as the case with Fourier Transform. However, FT requires knowing the function on an interval (a,b).

- Discrete:

$$F_N(x)(k) = X(k) = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x(j)\, e^{2\pi ij\frac{k}{N}}\ ,\ k = 0, 1, .., N-1\ ,\ i = \sqrt{-1}$$

- Continuous:

$$\hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-iwx} \mathrm{dx}, -\infty < x < \infty$$

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Lets look at an example As an example, we compute the 4-point DFT of $x = (0, 1, 1, -1)$: From the equation

$$F_4(x)(k) = X(k) = \frac{1}{\sqrt{4}} \sum_{j=0}^{3} x(j)\, e^{2\pi i j \frac{k}{4}} \;,\; k = 0, 1, 2, 3$$

Using Discrete Fourier Transform

At k=0

$$F_4\left(x\right)\left(0\right) = X\left(0\right) = \frac{1}{2}\sum_{j=0}^{3} x\left(j\right)1 = \frac{1}{2}\left[0+1+1-1\right] = \frac{1}{2}$$

Using Discrete Fourier Transform

At k=1

$$F_4(x)(1) = X(1) = \frac{1}{2}\sum_{j=0}^{3} x(j)\, e^{2\pi i j \frac{1}{4}}$$

$$= \frac{1}{2}\sum_{j=0}^{3} x(j)\, e^{\frac{\mathbf{ij}}{\mathbf{2}}}$$

$$= \frac{1}{2}[x(0)\, e^{i\left(\frac{0\pi}{2}\right)} + x(1)\, e^{i\left(\frac{1\pi}{2}\right)} + x(2)\, e^{i\left(\frac{2\pi}{2}\right)} + x(3)\, e^{i\left(\frac{3\pi}{2}\right)}]$$

$$= \frac{1}{2}[0 + e^{i\frac{\pi}{2}} + e^{\mathrm{i}} - e^{i\frac{3\pi}{2}}]$$

Using Discrete Fourier Transform

Using the Euler Formula

$$e^{\mathrm{i}} = cos\theta + isin\theta$$

We get

$$e^{i\frac{\pi}{2}} = i, e^{i\pi} = -1, e^{i\frac{3\pi}{2}} = -i$$

$$F_4\left(x\right)\left(1\right) = \frac{1}{2}[0 + i - 1 + i]$$

$$= \frac{1}{2}\left[-1 + 2i\right] = -\frac{1}{2} + i$$

Using Discrete Fourier Transform

Similarly,

for k=2

$$F_4(x)(2) = X(2) = \frac{1}{2}\sum_{j=0}^{3} x(j)\, e^{2\pi i j \frac{2}{4}} = \frac{1}{2}$$

and k=3

$$F_4(x)(3) = X(3) = \frac{1}{2}\sum_{j=0}^{3} x(j)\, e^{2\pi i j \frac{3}{4}} = -\frac{-1}{2} - i$$

Using Discrete Fourier Transform

We have shown an example of computing the Fourier transform for a set of discrete point.

Of course, the reverse is possible with the use of the Inverse Discrete Fourier Transform (IDFT) which is defined by:

$$F_N^{-1}(X)(k) = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} X(j)\, e^{-2\pi j i \frac{k}{N}}, k = 0, 1, \ldots, N-1$$

Using Discrete Fourier Transform

also we consider the property:

$$F_N^{-1}\left(F_N\left(x\right)\right)\left(k\right) = x\left(k\right),\ \ k = 0, \ldots, N-1$$

This is true do to the orthonormality property of $e^{2\pi ij}$

i.e.

$$\frac{1}{N}\sum_{n=0}^{N-1} e^{2\pi i\left(\frac{j-k}{N}\right)} = \begin{Bmatrix} 0 & ,j \neq k \\ 1 & ,j = k \end{Bmatrix}$$

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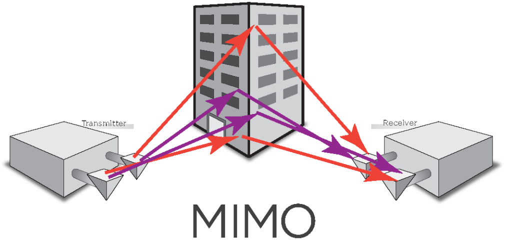
Application of Fourier Transform in Radar Imaging

Now that we have taken a look into the use of Discrete Fourier Transform, we can begin to look at its use in MIMO Radar imaging.

What is MIMO Radar Imaging?

Multi-Input Multi-Ouput (MIMO) radar imaging is a technique that utilizes multiple antennas and receivers to generate a image. MIMO radar systems transmit, through antennas, multiple probing signals. It is superior than a standard single phased array radar as it provides a higher resolution image and has higher detecting sensitivity.



Using MATLAB

When the number of data point for a set becomes very high it becomes difficult to solve a Fourier Transformproblem by hand. Thus we will use the aid of MATLAB code to generate a randomized imaging of data using Fourier Transform.

Using MATLAB

```
clear all; close all; clc
%fast discrete fourier transform FDFT
N=500;

f= randn(N,1);
fhat=fft(f);

w=exp(-i*2*pi/N);

for i=1:N
    for j=1:N
        DFT(i,j) = w^((i-1)*(j-1));
    end
end

fhat2=DFT*f; %should be equal to fhat

plot(real(fhat),'g')
hold on
plot(real(fhat2),'r--')
legend('fhat','fhat2')
axis([10 20 -30 30]);
%Notice that fhat and fhat2 overlap perfectly
```

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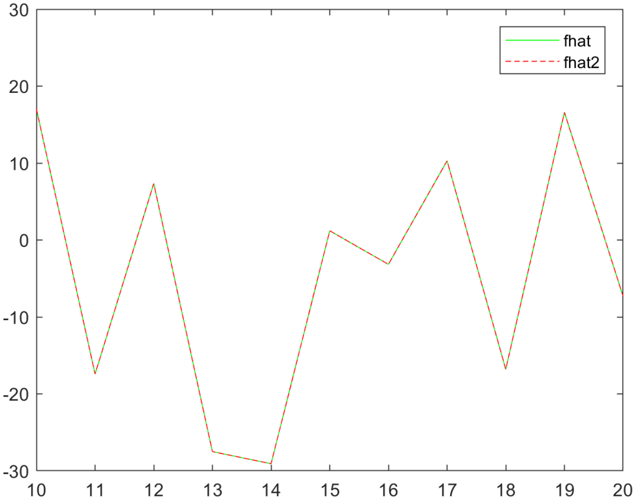
30
20
10
0
-10
-20
-30
10
11
12
13
14
15
16
17
18
19
20
fhat
fhat2

Conclusion

The application of Fourier Transformin Radar Imaging serves as a realistic usage of higher level mathematics in a real world application that is applied in cell phones, computers, networking, and programming. In addition, we saw why the usage of polynomial interpolations or approximations like Taylor or Maclaurin Series are not always a great or viable means of approximating a function.

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Thank you

Questions?