Solving Method

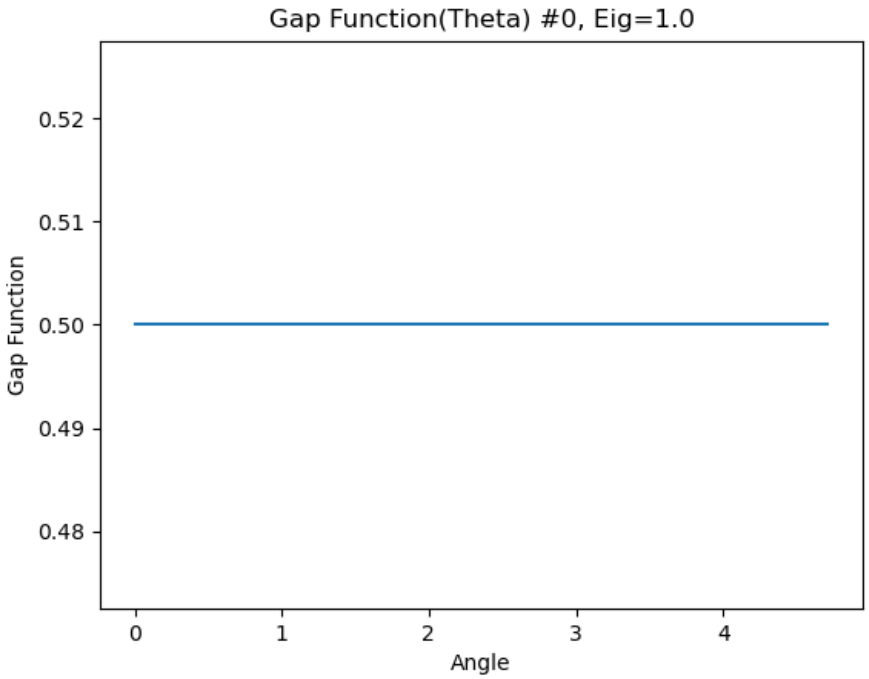
1. $(\sum_{k'} P_{kk'} f(\epsilon_{k'}) - \delta_{kk'})\, \Delta_{k'} = 0$
2. $\epsilon_k = k^2$,$\vec{k} = r(cos\theta\hat{i} + sin\theta\hat{j}) \rightarrow \epsilon_k(r)$
3. $\sum_{r',\theta'} P_{r,\theta,r',\theta'} f(\epsilon_{k'}(r'))\Delta_{r',\theta'} = \Delta_{r,\theta} \rightarrow \int P(r,\theta,r',\theta') f(\epsilon_{k'}(r'))\Delta((r',\theta')dr'd\theta' = \Delta(r,\theta)$
4. The integral is split into radial and angular components. If there is radial dependence in P and Δ then we first define the P matrix along angular components, and then integrate radially for each of those angular values. However in this instance we will assume that P and Δ are only dependent on θ.
5. Now we assume that Δ and P are only dependent upon θ, so the above equation transforms to
6. $\int P(\theta,\theta') f(\epsilon_k)\Delta(\theta')dr'd\theta' = \Delta(\theta)$
7. The radial bounds are given by $\epsilon = \pm\omega_c$, or from 0 to ω_c, since the only function that depends on r is $f(\epsilon_k)$, which is symmetric around 0. Therefore the radial bounds are from 0 to $\sqrt{\omega_c}$ and we multiply the integral by 2. This cannot be done analytically, so we do it numerically.
8. In this instance, $f(\epsilon)$ is a constant that multiplies the entire matrix P, so we find the lowest eigenvalue of P, and set $f = P^{-1}$ so that $fP = 1$.
9. $f = \int_{-\omega_c}^{\omega_c} tanh(\frac{\epsilon}{2T_c})\frac{d\epsilon}{2\epsilon}$, so we use the value of f found above and solve numerically for the value of T_c.

Note: The matrix P is normalized so that regardless of size the eigenvalues remain the same for a given function. This is done by dividing the matrix by the number of elements in it.
Note:

Checks

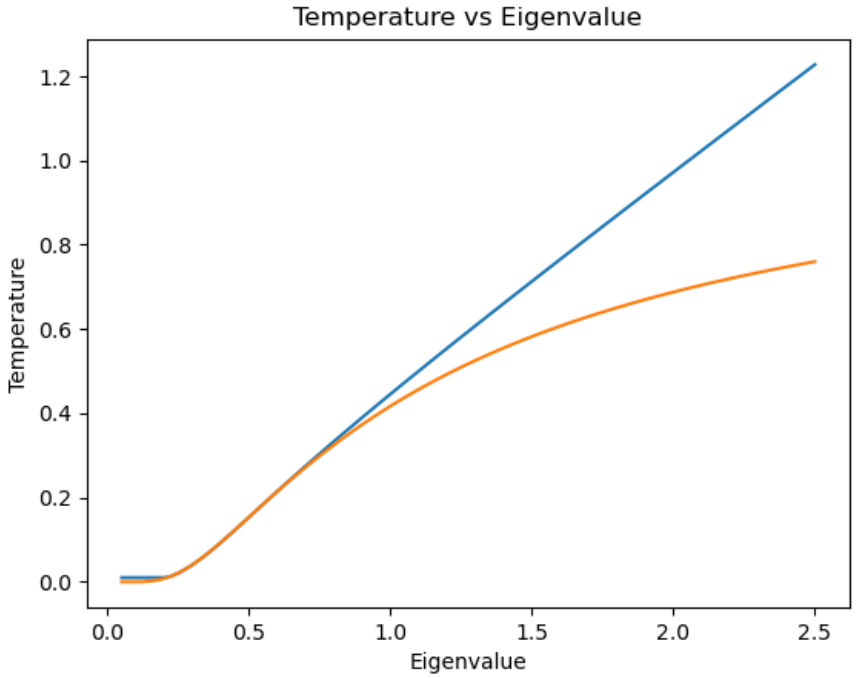
1. Constant matrix $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ results in $\Delta(\theta)$ is independent of θ
2. Check that for function $P = \frac{1}{|k-k'|}$, $\Delta(\theta)$ follows spherical harmonics (s-wave, p-wave, d-wave)
3. Not sure how to verify solution of function in paper
4. For each function P test temperature against approximation function $T_c = 1.134\omega_c e^{-1/\lambda}$, under the condition that $\omega_c >> T_c$
5. Also test that eigenvalues are stable as matrix size increases

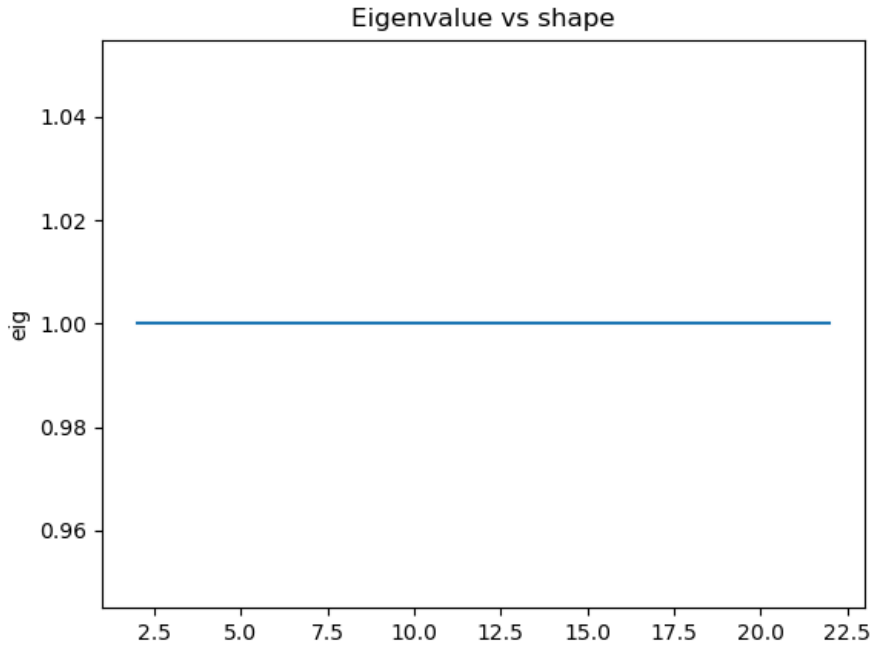
Check Results



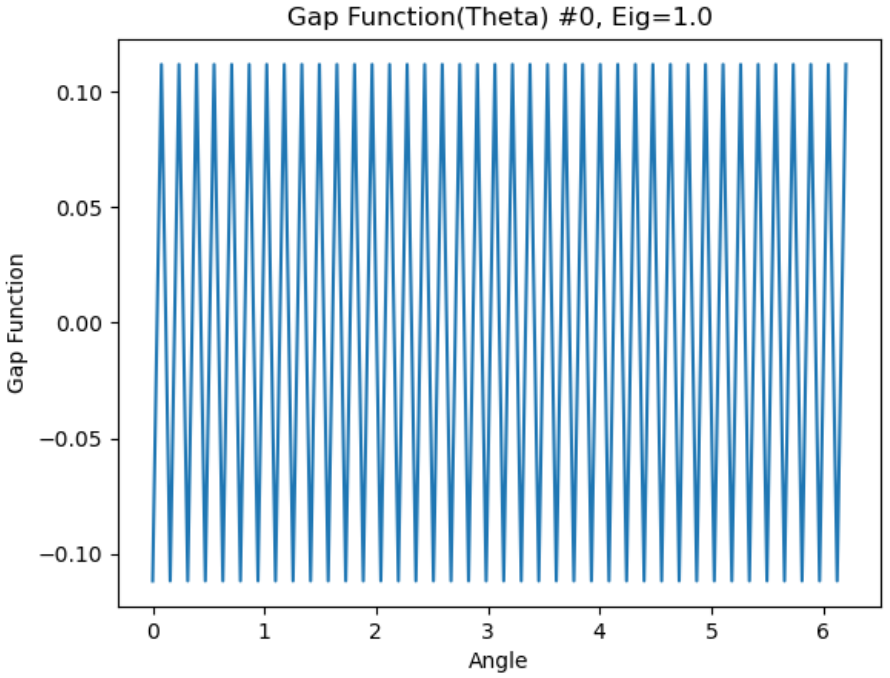
#1:

P is normalized by 1/n

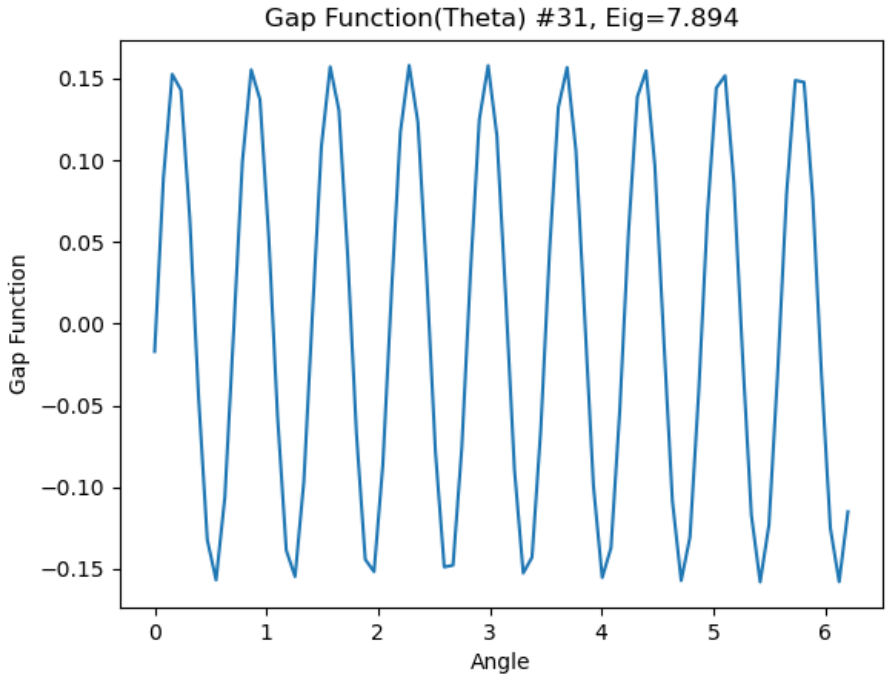


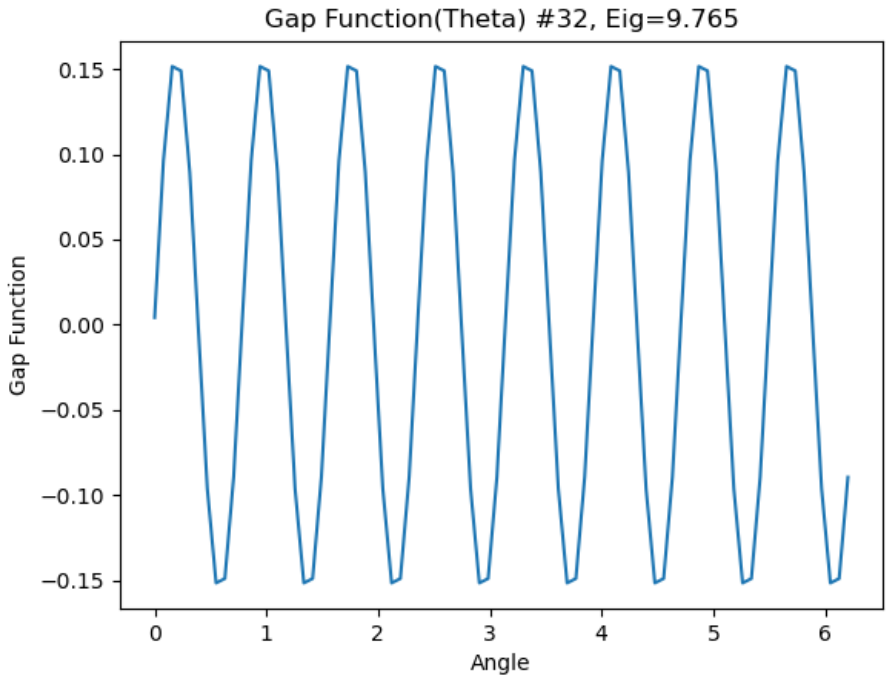


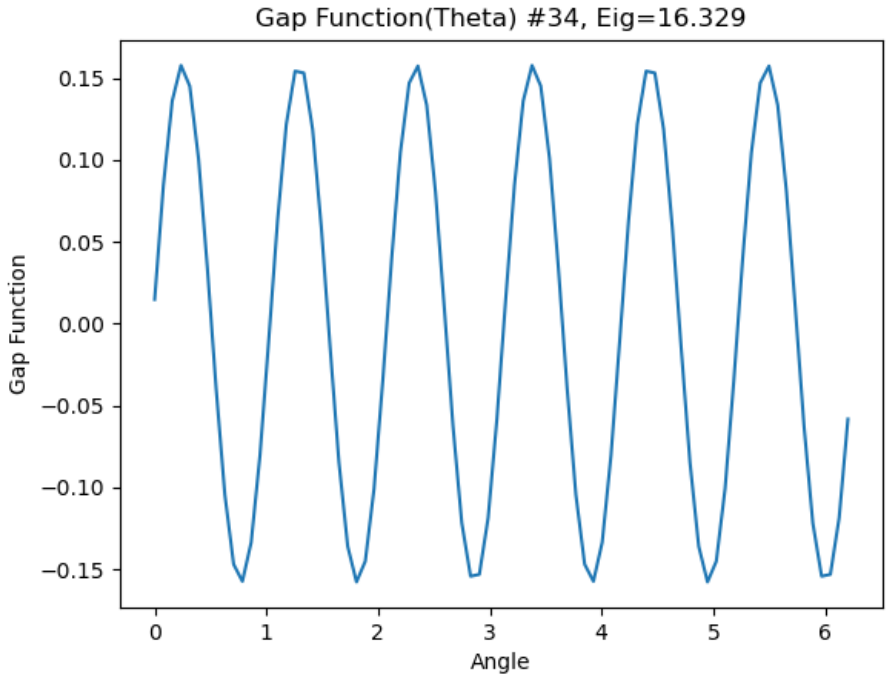
#2:
Gap function of lowest positive eigenvalue (normalized to 1)

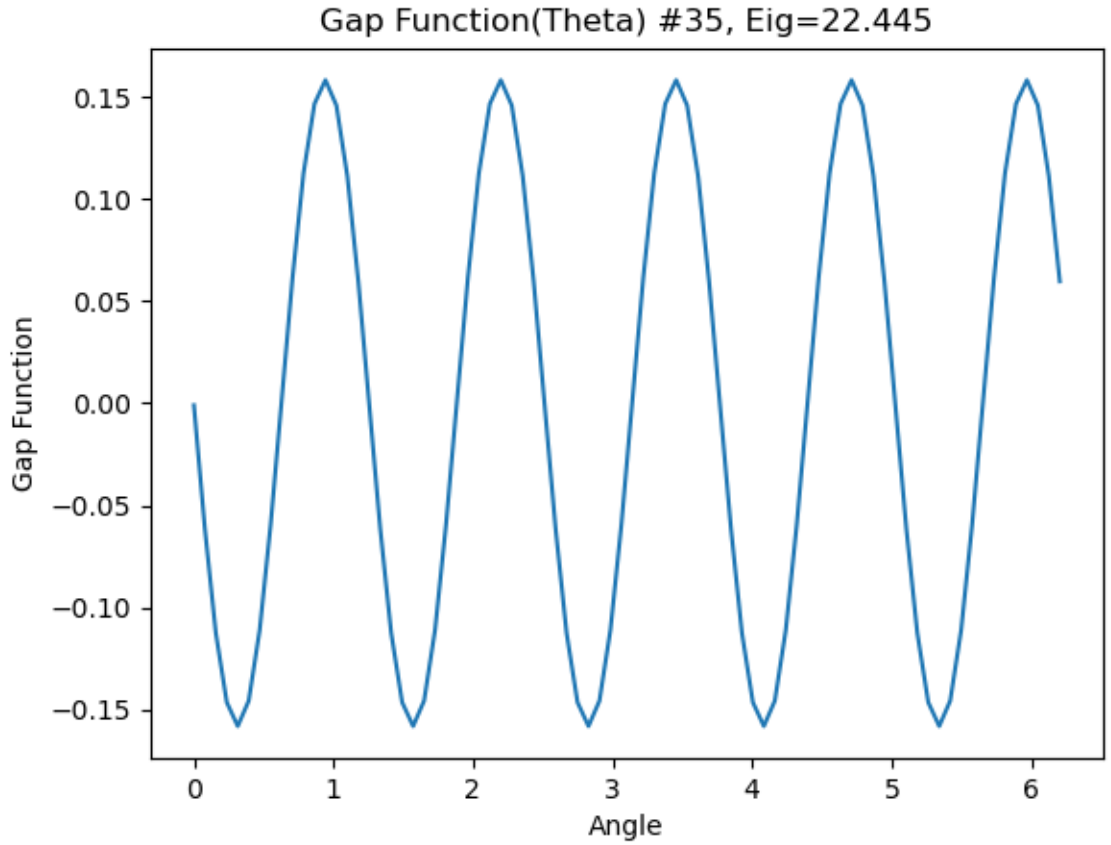


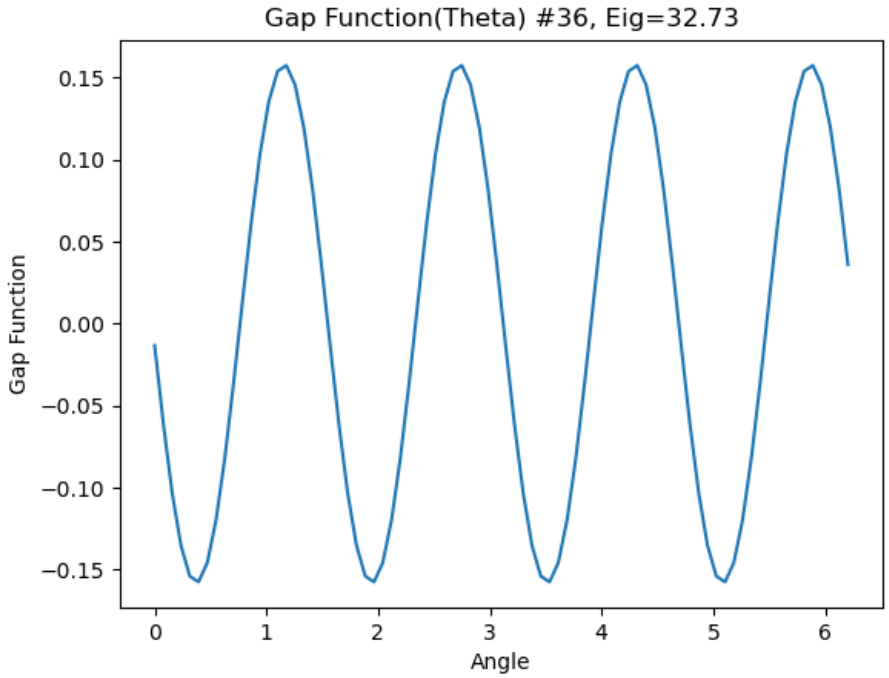
d-wave functions


Gap Function(Theta) #31, Eig=7.894
Gap Function
Angle
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0
1
2
3
4
5
6

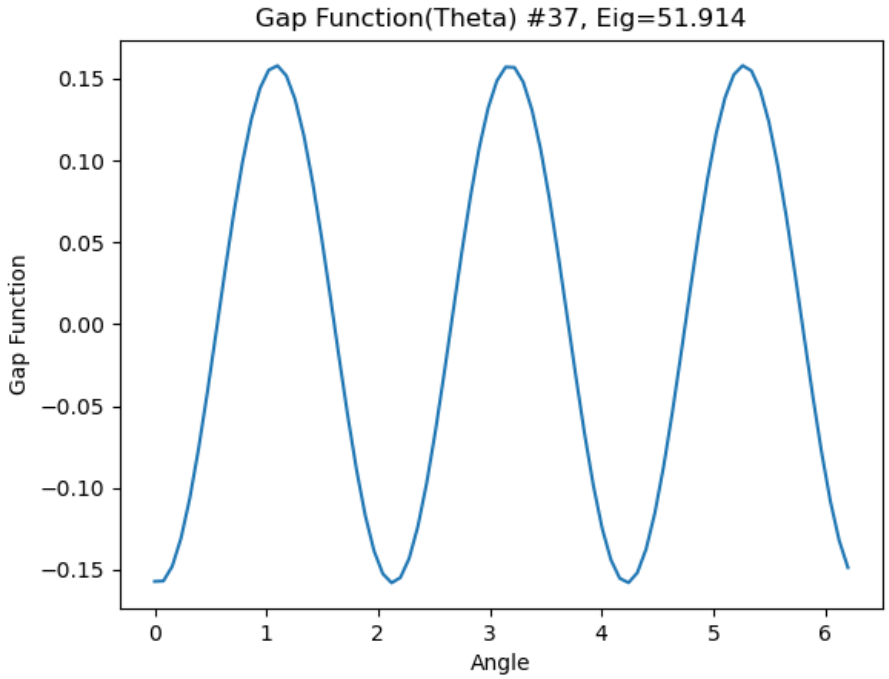

Gap Function(Theta) #32, Eig=9.765
Gap Function
Angle
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0
1
2
3
4
5
6

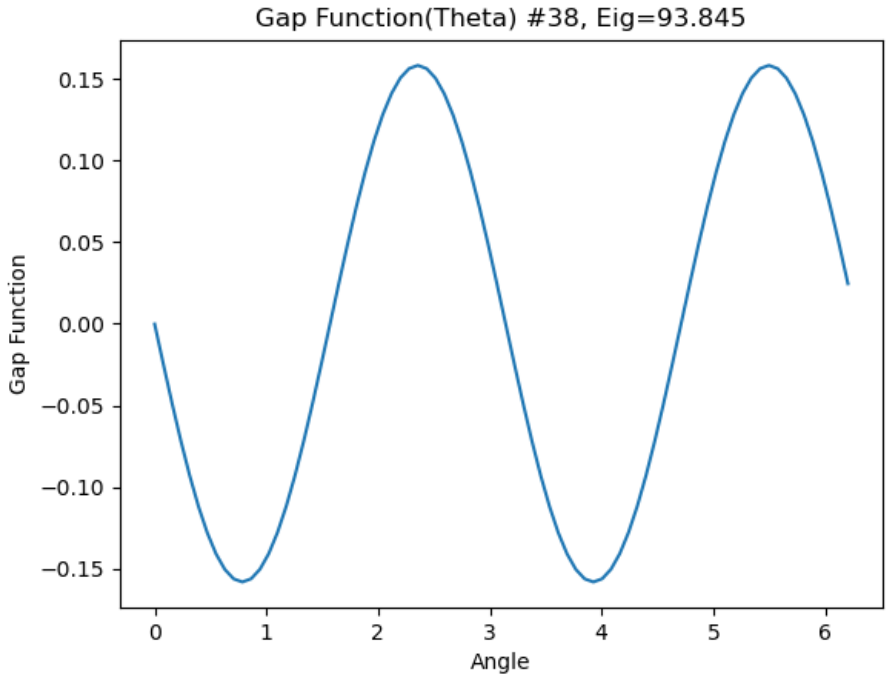

Gap Function(Theta) #34, Eig=16.329
Gap Function
Angle
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0
1
2
3
4
5
6

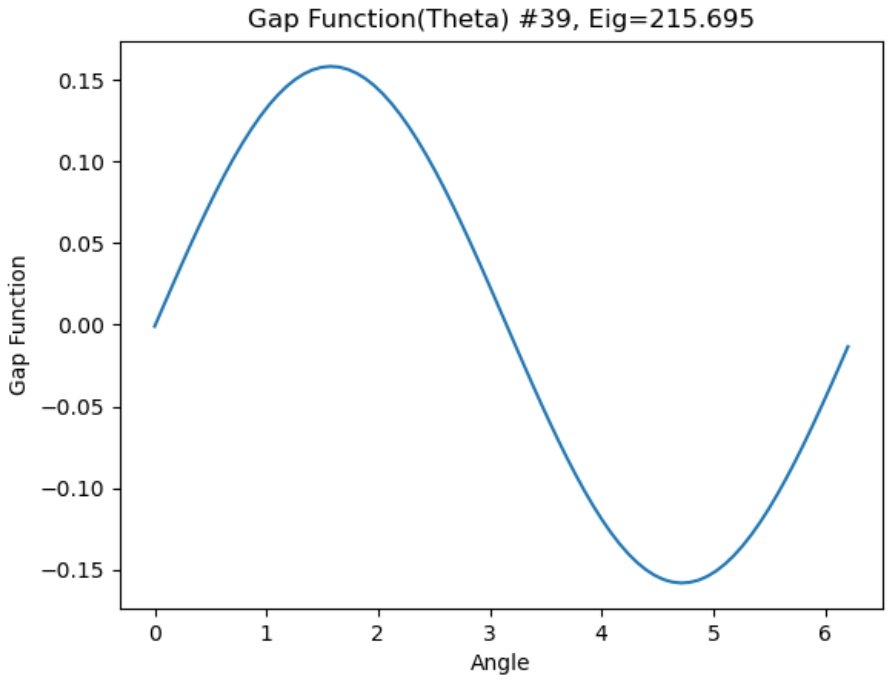

Gap Function(Theta) #35, Eig=22.445
Gap Function
Angle
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0
1
2
3
4
5
6



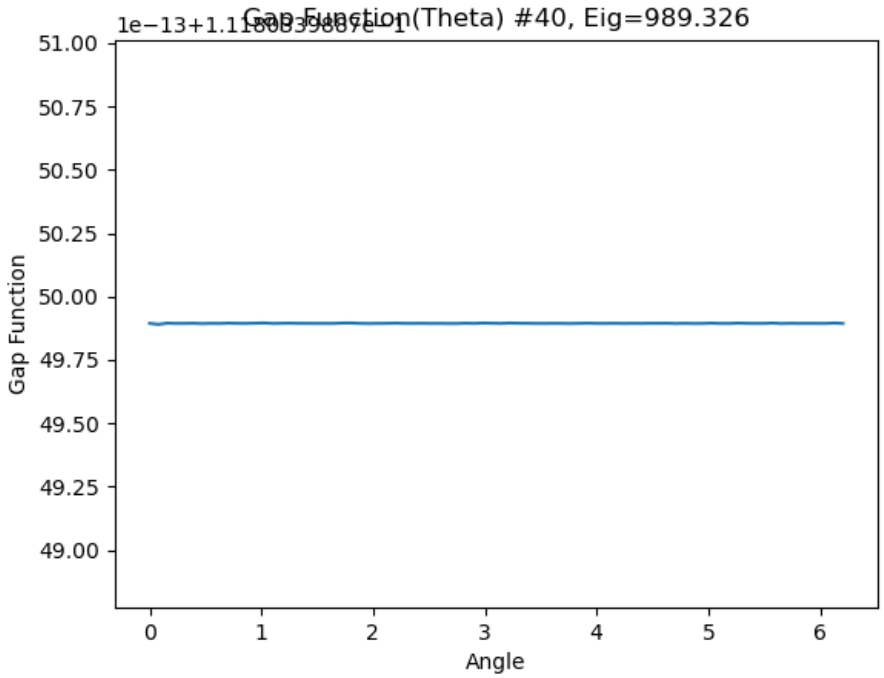
p-wave functions

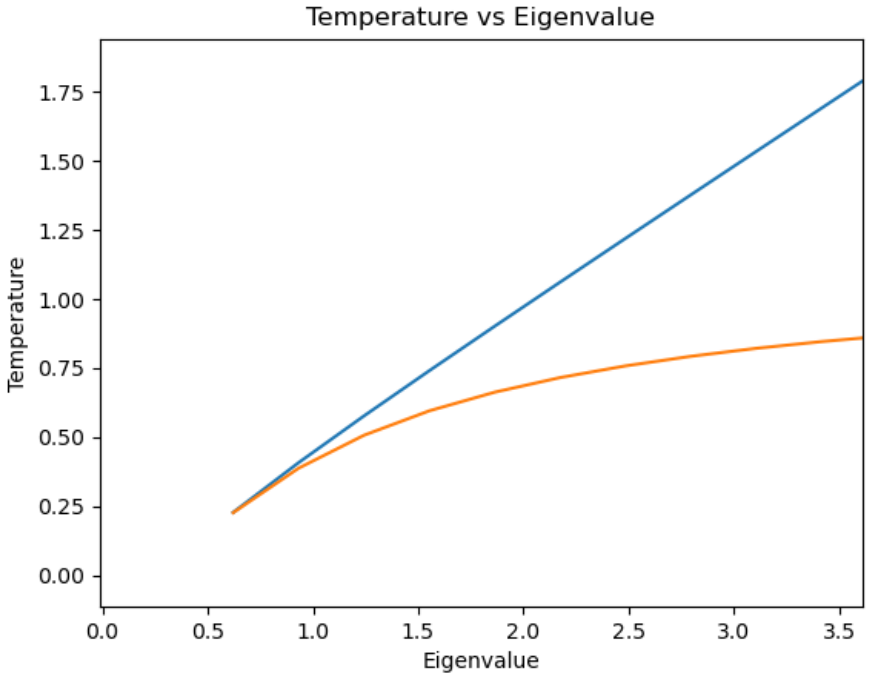


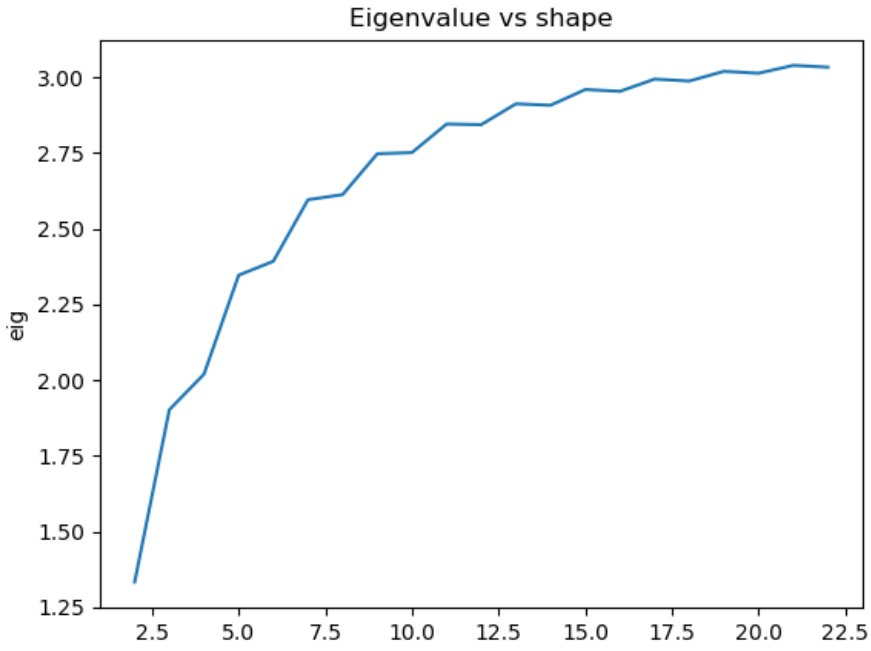




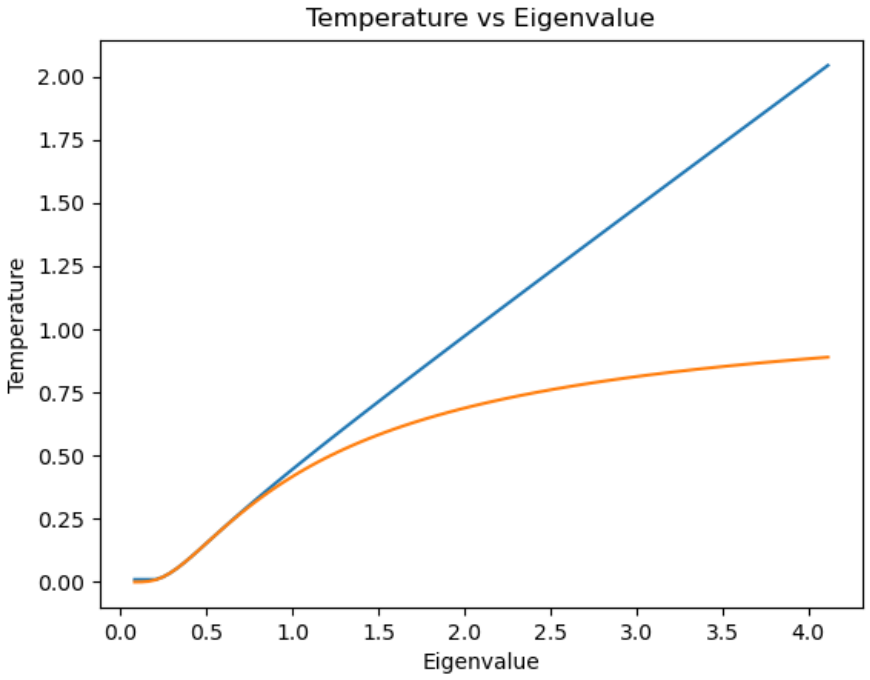
s-wave function

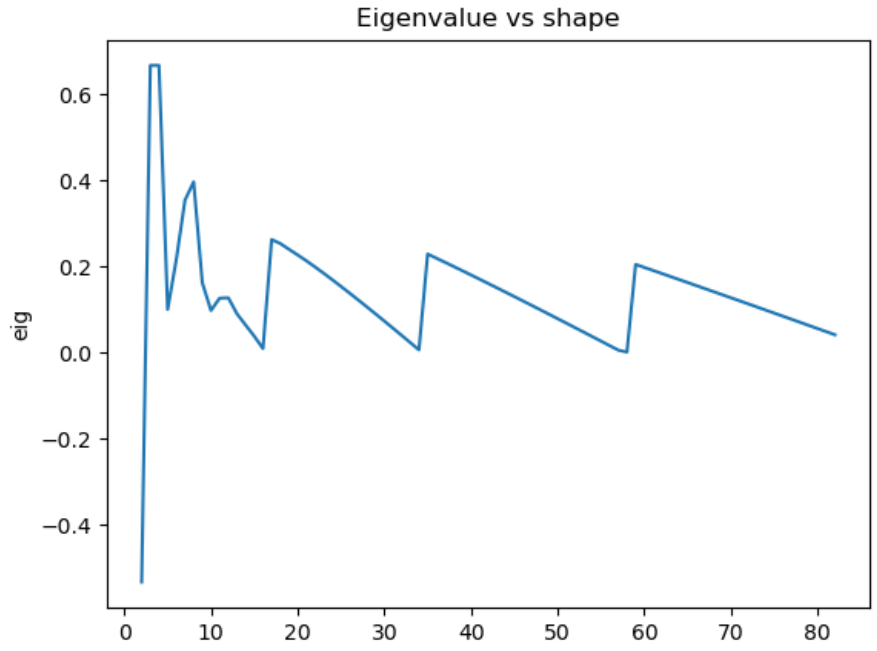

Gap Function(Theta) #40, Eig=989.326
1e−13+1.11
51.00
50.75
50.50
50.25
50.00
49.75
49.50
49.25
49.00
Gap Function
0
1
2
3
4
5
6
Angle


Temperature vs Eigenvalue
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Temperature
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Eigenvalue

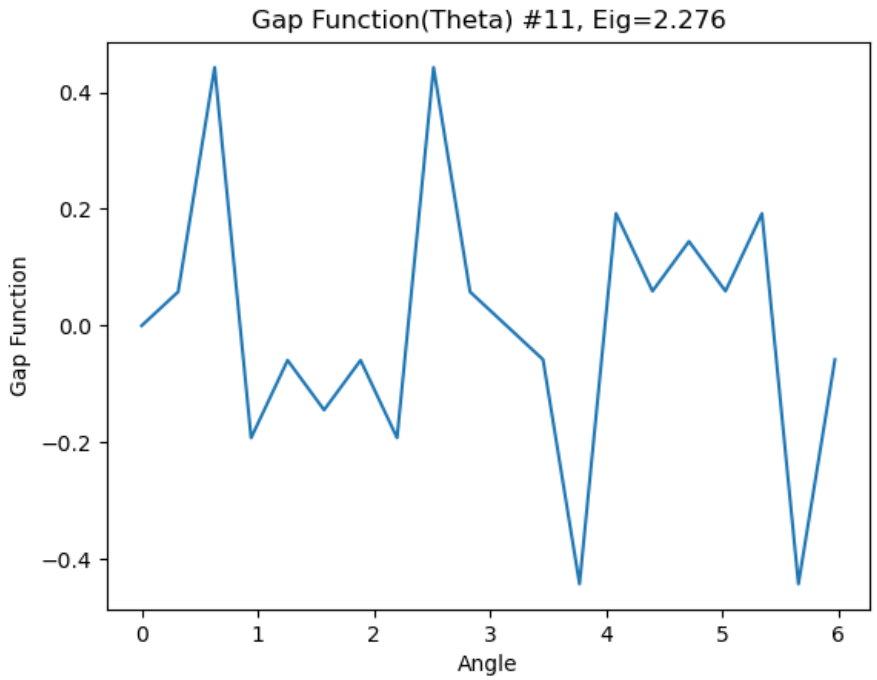


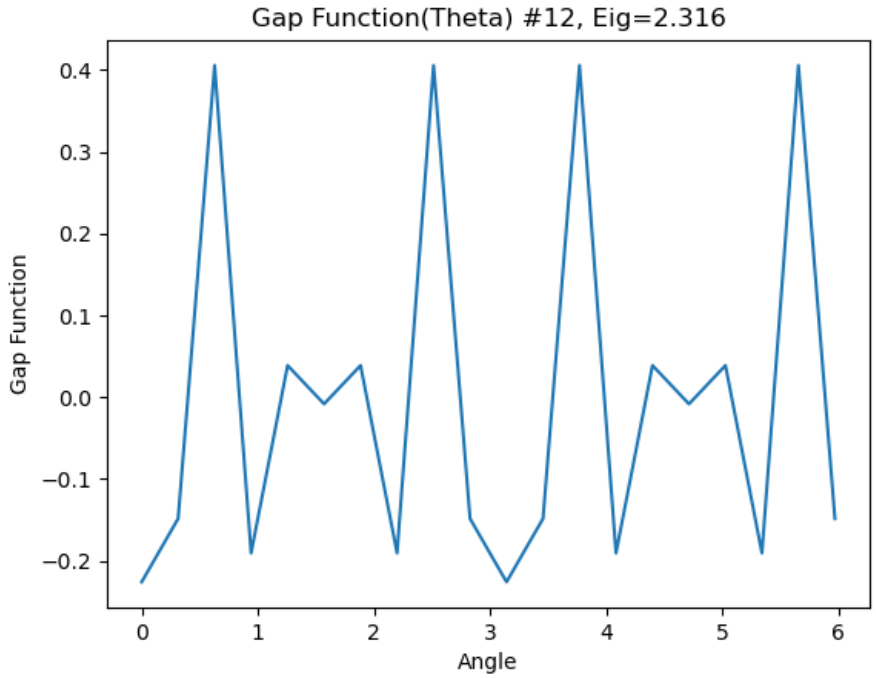
P is normalized by n
#3:

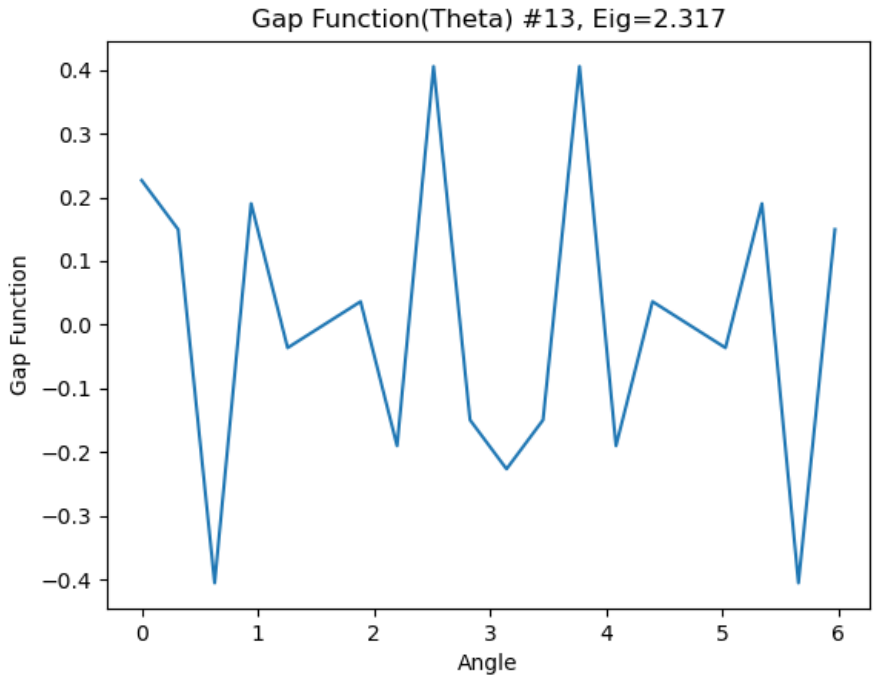


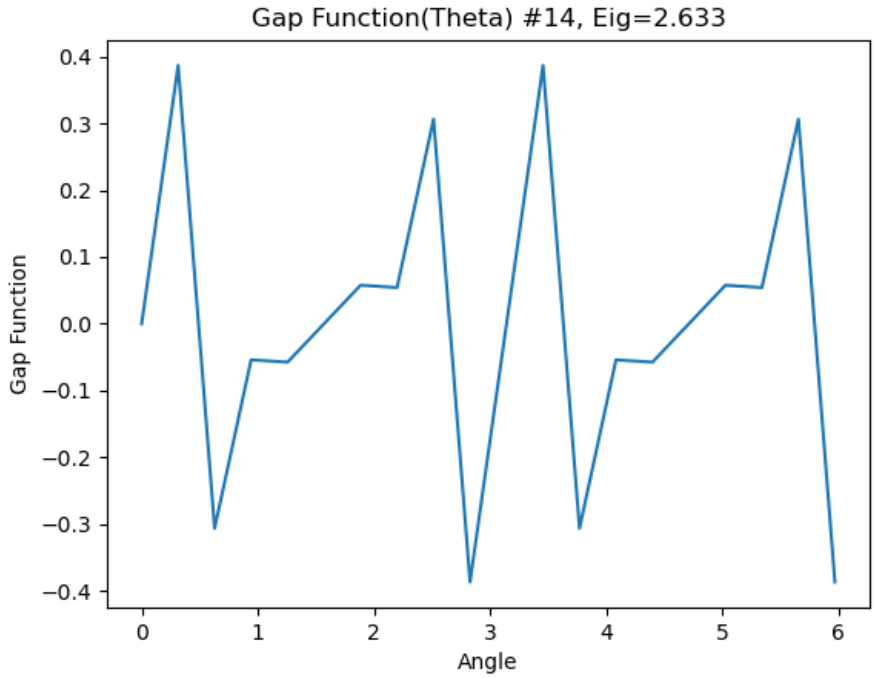


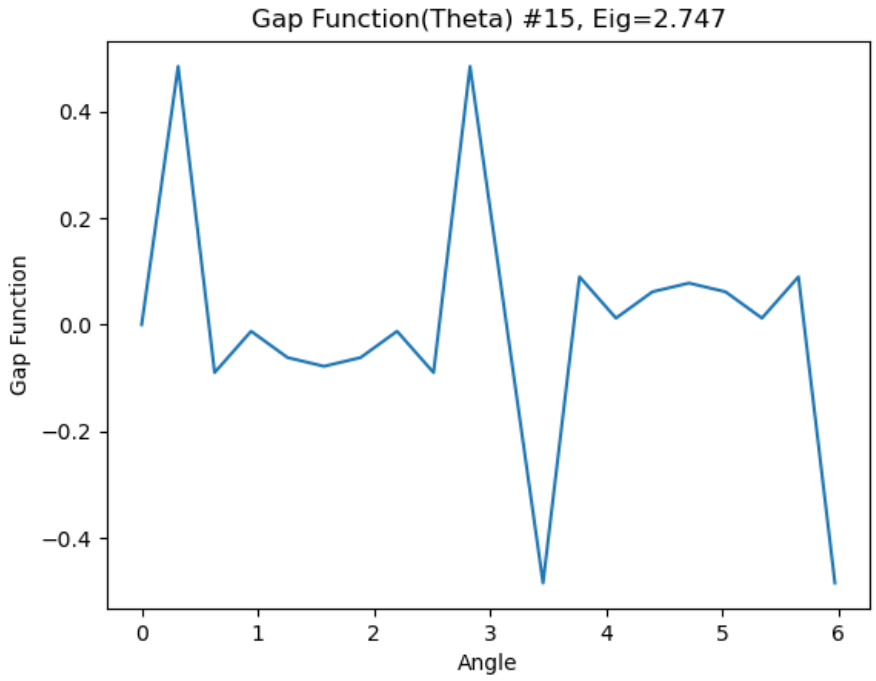
(Gap functions for highest 5-9 eigenvalues)



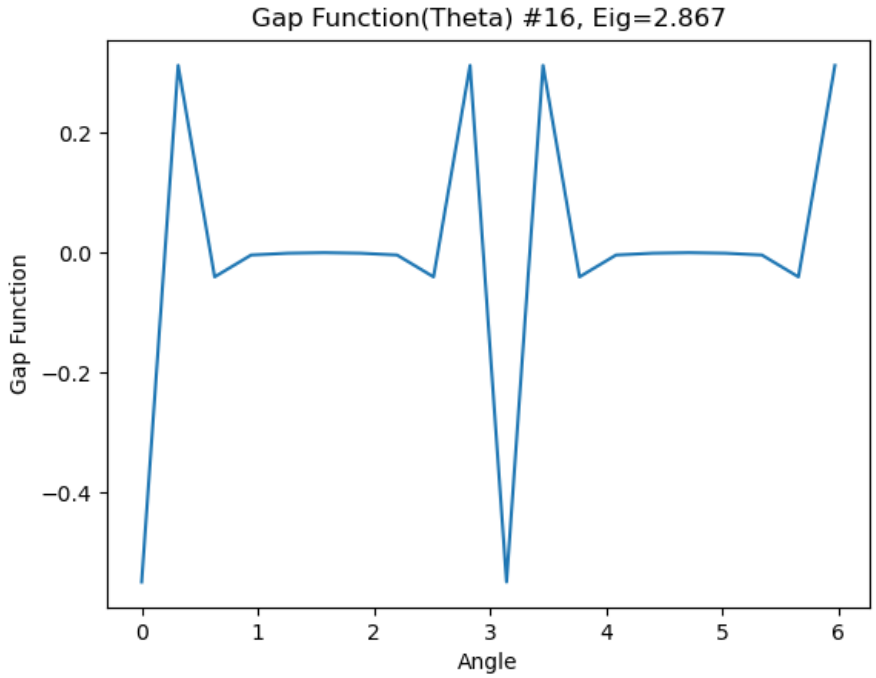

Gap Function(Theta) #12, Eig=2.316
Gap Function
Angle
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
0
1
2
3
4
5
6

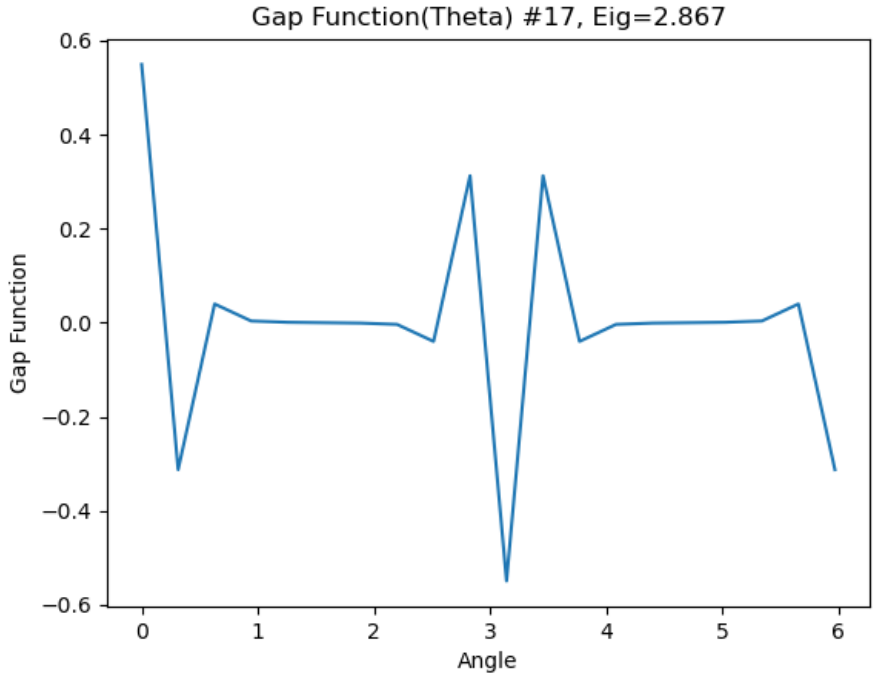

Gap Function(Theta) #13, Eig=2.317
Gap Function
Angle
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
0
1
2
3
4
5
6

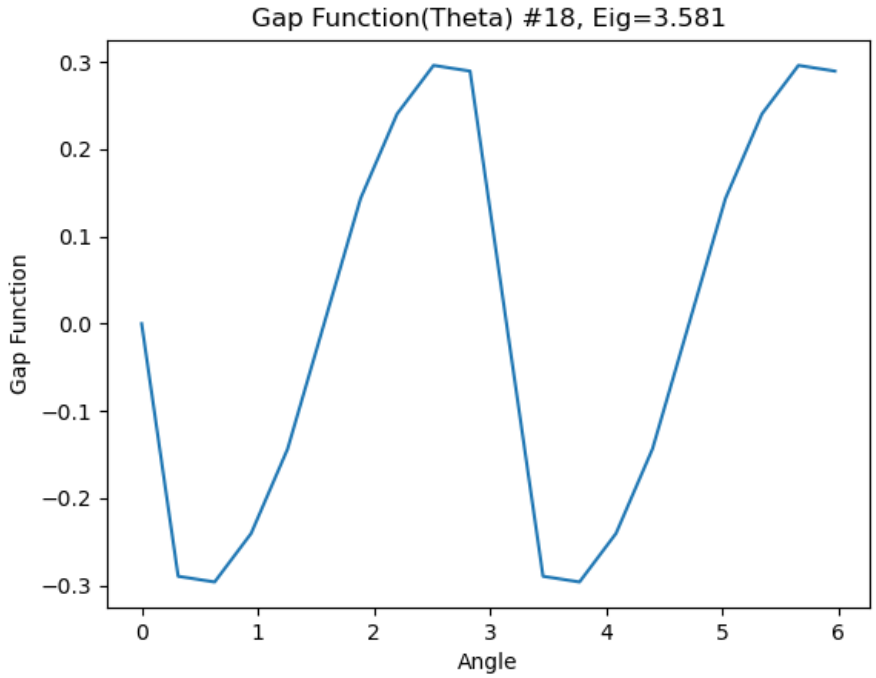




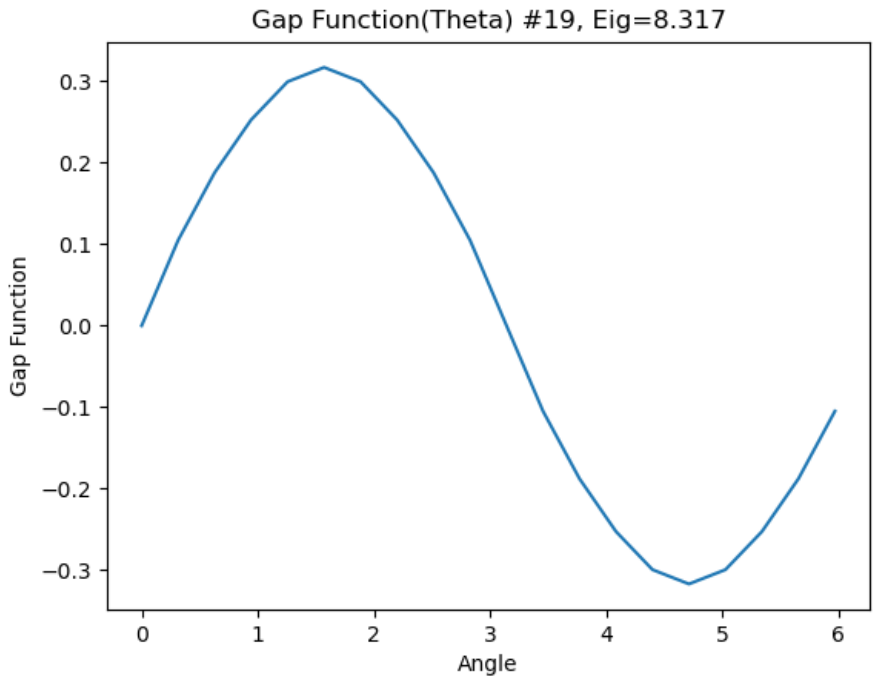
(Gap function for highest eigenvalues 2-4)


Gap Function(Theta) #16, Eig=2.867
Gap Function
Angle
0.2
0.0
−0.2
−0.4
0
1
2
3
4
5
6


Gap Function(Theta) #17, Eig=2.867
Gap Function
Angle
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
1
2
3
4
5
6



p-wave function (highest eigenvalue)



Issues

1. Sometimes (rarely) eigenvalues are identical with different eigenvectors. Must fix this by implementing more advanced computational methods when diagonalizing matrices

2. Not sure how to test results for the function in the paper

3. Normalizing the matrix is more difficult than dividing the eigenvalues by the number of elements, unfortunately. That might work for the highest eigenvalue, as that corresponds to an s-wave, and would scale with the length of the matrix, but for the lowest eigenvalue this does not work. The Eigenvalue vs. Shape graphs above show the eigenvalue changing as the size of the matrix changes. However each matrix was normalized with a different scale in order for the eigenvalues to level off. The first was normalized with $1/n$, and the second was normalized with n. I'm working on using matrix norms to stabilize the eigenvalues in general with regards to shape.

4. The paper's wavefunctions look a bit odd after the first couple.

Questions

Why is the lowest positive eigenvalue selected to become 1? The math I've done seems to indicate that the higher the eigenvalue the higher the superconducting temperature, and as shown above the higher eigenvalues also correspond to the gap functions with the lowest angular momentum.

$$Pf\psi = \psi \rightarrow pf = 1$$

$$\int_{-1}^{1} \frac{\tanh(x/0.1)}{x} dx = 6.24, \int_{-1}^{1} \frac{\tanh(x/0.2)}{x} dx = 4.86$$

Clearly the lowest positive eigenvalue is the one to use, as it perfectly corresponds to the approximate function, but why is that the case?