Wave direction check

The singlet-triplet pairing terms is dropped, as is done in the paper, leaving us with the following equation, which I will use to construct a matrix.

$$\vec{d}(k) = -k_B T \sum_{nk'} V_{kk'} \{(G_+ G_+ + G_- G_-)\vec{d}(k') + 2G_- G_- (\hat{g}_{k'}(\hat{g}_{k'} \cdot \vec{d}(k')) - \vec{d}(k'))\}$$

$$\vec{d}(k) = \sum_{k'} V_{kk'} \{a_{k'}(\vec{d}(k') + b_{k'}(\hat{g}_{k'}(\hat{g}_{k'} \cdot \vec{d}(k')) - \vec{d}(k'))\}, a_{k'} = -(G_+ G_+ + G_- G_-), b_{k'} = -2G_- G_-$$

Taking the x-component as an example, the equation reduces to

$$d_x(k) = \sum_{k'} V_{kk'} \{a_{k'} d_x(k') + b_{k'}(g_{k'x}^2 - 1) d_x(k') + b_{k'} g_{k'x} g_{k'y} d_y(k') + b_{k'} g_{k'y} g_{k'z} d_z(k')\}$$

We replace the coefficients with q_{ij}

$$d_x(k) = \sum_{k'} V_{kk'}(q_{xx}(k') d_x(k') + q_{xy}(k') d_y(k') + q_{xz}(k') d_z(k'))$$

And then use this as an example for each d component, giving us

$$\vec{d}(k) = \sum_{k'} (V_{kk'} \otimes Q(k')) d(k'), Q(k') = \begin{pmatrix} q_{xx}(k') & q_{xy}(k') & q_{xz}(k') \\ q_{yx}(k') & q_{yy}(k') & q_{yz}(k') \\ q_{zx}(k') & q_{zy}(k') & q_{zz}(k') \end{pmatrix}$$

$$\vec{d}(k) = \begin{pmatrix} d_x(k) \\ d_y(k) \\ d_z(k) \end{pmatrix} = \begin{pmatrix} \sum_{k'} V_{kk'}(q_{xx}(k') d_x(k') + q_{xy}(k') d_y(k') + q_{xz}(k') d_z(k')) \\ \sum_{k'} V_{kk'}(q_{yx}(k') d_x(k') + q_{yy}(k') d_y(k') + q_{yz}(k') d_z(k')) \\ \sum_{k'} V_{kk'}(q_{zx}(k') d_x(k') + q_{zy}(k') d_y(k') + q_{zz}(k') d_z(k')) \end{pmatrix}$$

It is easy to see that for a constant potential $V_{kk'}$ that the components of $\vec{d}(k)$ will be independent of k, each being an s-wave as in the singlet case. This will prevent $\vec{d}(k)$ from changing direction, or being parallel to a changing $\hat{g}$ vector.

$$\vec{d}(k) = \hat{P}(k, k')\vec{d}(k')$$

$$\hat{P}(k, k') = \begin{pmatrix} q_{xx}(0)V_{00} & q_{xy}(0)V_{00} & q_{xz}(0)V_{00} & q_{xx}(1)V_{01} & q_{xy}(1)V_{01} & q_{xz}(1)V_{01} & \dots \\ q_{yx}(0)V_{00} & q_{yy}(0)V_{00} & q_{yz}(0)V_{00} & q_{yx}(1)V_{01} & q_{yy}(1)V_{01} & q_{yz}(1)V_{01} & \dots \\ q_{zx}(0)V_{00} & q_{zy}(0)V_{00} & q_{zz}(0)V_{00} & q_{zx}(1)V_{01} & q_{zy}(1)V_{01} & q_{zz}(1)V_{01} & \dots \\ q_{xx}(0)V_{10} & q_{xy}(0)V_{10} & q_{xz}(0)V_{10} & q_{xx}(1)V_{11} & q_{xy}(1)V_{11} & q_{xz}(1)V_{11} & \dots \\ q_{yx}(0)V_{10} & q_{yy}(0)V_{10} & q_{yz}(0)V_{10} & q_{yx}(1)V_{11} & q_{yy}(1)V_{11} & q_{yz}(1)V_{11} & \dots \\ q_{zx}(0)V_{10} & q_{zy}(0)V_{10} & q_{zz}(0)V_{10} & q_{zx}(1)V_{11} & q_{zy}(1)V_{11} & q_{zz}(1)V_{11} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

$$\hat{P}(k,k') = \begin{pmatrix} Q_0V_{00} & Q_1V_{01} & ... \\ Q_0V_{10} & Q_1V_{11} & ... \\ ... & ... & ... \end{pmatrix}$$

The math below shows that for a 2D case with a constant potential and a single value of k, $P = Q$, and in this case $\vec{d}(k)||\hat{g}_k$, but that fails to be the case as the $\hat{P}$ matrix grows.

$$\hat{P}(k) = \begin{pmatrix} a + b(g_x^2 - 1) & bg_xg_y \\ bg_xg_y & a + b(g_y^2 - 1) \end{pmatrix}, \hat{g} = (\cos(\theta_k), \sin(\theta_k))$$

$$\hat{P}(k) = \begin{pmatrix} a + b(\cos^2(\theta_k) - 1) & b\cos(\theta_k)\sin(\theta_k) \\ b\cos(\theta_k)\sin(\theta_k) & a + b(\sin^2(\theta_k) - 1) \end{pmatrix}$$

$$\hat{P}(k) = \begin{pmatrix} a - b\sin^2(\theta_k) & b\cos(\theta_k)\sin(\theta_k) \\ b\cos(\theta_k)\sin(\theta_k) & a - b\cos^2(\theta_k) \end{pmatrix}$$

$$\vec{d}(k) = \begin{pmatrix} \cos(\theta_k) \\ \sin(\theta_k) \end{pmatrix}, \hat{P}\vec{d} = a\vec{d}, (a-b)\vec{d}$$

a is the high eigenvalue, corresponding to the highest temperature, just like the paper says. This means that in this case the $\vec{d}||\hat{g}$. However, when taking the eigenstate of $\hat{P}$, where multiple values of k are included, we no longer get that $\vec{d}(k)$ follows the $\hat{g_k}$ vector. Let's take an example with only two values of k

$$\hat{P} = \begin{pmatrix} \hat{Q}_0V_{00} & \hat{Q}_1V_{01} \\ \hat{Q}_0V_{10} & \hat{Q}_1V_{11} \end{pmatrix}$$

Let $V_{kk'} = 1$, and then test the 2D case to see if $\vec{d}(k)$ still follows the $\hat{g}$ vector, ie

$$\hat{P} = \begin{pmatrix} \hat{Q}_0 & \hat{Q}_1 \\ \hat{Q}_0 & \hat{Q}_1 \end{pmatrix}, Q(k') = \begin{pmatrix} q_{xx}(k') & q_{xy}(k') \\ q_{yx}(k') & q_{yy}(k') \end{pmatrix}$$

$$\hat{g_k} = \begin{pmatrix} \hat{g_0} \\ \hat{g_1} \end{pmatrix}, \vec{d}(k) = \begin{pmatrix} d_x(k_0) \\ d_y(k_0) \\ d_x(k_1) \\ d_y(k_1) \end{pmatrix} = \begin{pmatrix} \cos(\theta_0) \\ \sin(\theta_0) \\ \cos(\theta_1) \\ \sin(\theta_1) \end{pmatrix}$$

$$\hat{P}\vec{d}(k) \neq \lambda\vec{d}(k)$$

$\vec{d}(k)$ is not an eigenvector of $\hat{P}$, meaning that it does not follow a $\hat{g}$ vector that changes direction.