Wave direction check

$$Q_k = \begin{pmatrix} a_k + b_k(g_{kx}^2 - 1) & b_k g_{kx} g_{ky} & b_k g_{kx} g_{kz} \\ b_k g_{kx} g_{ky} & a_k + b_k(g_{ky}^2 - 1) & b_k g_{ky} g_{kz} \\ b_k g_{kx} g_{kz} & b_k g_{ky} g_{kz} & a_k + b_k(g_{kz}^2 - 1) \end{pmatrix}$$

$$d(k) = -k_B T \sum_{nk'} V_{kk'} \{(G_+ G_+ + G_- G_-) d(k') + 2G_- G_- [\hat{g}_{k'}(\hat{g}_{k'} \cdot d(\hat{k'}) - d(k')]\}$$

$$d(k) = \sum_{k'} V_{kk'}(a_{k'}(d(k') + b_{k'}(\hat{g}_{k'}(\hat{g}_{k'} \cdot d(k')) - d(k'))))$$

$$d_x(k) = \sum_{k'} V_{kk'}\{a_{k'} d_x(k') + b_{k'}(g_{k'x}^2 - 1) d_x(k') + b_{k'} g_{k'x} g_{k'y} d_y(k') + b_{k'} g_{k'y} g_{k'z} d_z(k')\}$$

$$d_x(k) = \sum_{k'} V_{kk'}(q_{xx}(k') d_x(k') + q_{xy}(k') d_y(k') + q_{xz}(k') d_z(k'))$$

$$d(k) = \sum_{k'} V_{kk'} Q(k') d(k')$$

$$d(k) = \begin{pmatrix} d_x(k) \\ d_y(k) \\ d_z(k) \end{pmatrix} = \begin{pmatrix} \sum_{k'} V_{kk'}(q_{xx}(k') d_x(k') + q_{xy}(k') d_y(k') + q_{xz}(k') d_z(k')) \\ \sum_{k'} V_{kk'}(q_{yx}(k') d_x(k') + q_{yy}(k') d_y(k') + q_{yz}(k') d_z(k')) \\ \sum_{k'} V_{kk'}(q_{zx}(k') d_x(k') + q_{zy}(k') d_y(k') + q_{zz}(k') d_z(k')) \end{pmatrix}$$

$$d(k) = P(k, k') d(k')$$

$$P(k,k') = \begin{pmatrix} q_{xx}(0)V_{00} & q_{xy}(0)V_{00} & q_{xz}(0)V_{00} & q_{xx}(1)V_{01} & q_{xy}(1)V_{01} & q_{xz}(1)V_{01} & ... \\ q_{yx}(0)V_{00} & q_{yy}(0)V_{00} & q_{yz}(0)V_{00} & q_{yx}(1)V_{01} & q_{yy}(1)V_{01} & q_{yz}(1)V_{01} & ... \\ q_{zx}(0)V_{00} & q_{zy}(0)V_{00} & q_{zz}(0)V_{00} & q_{zx}(1)V_{01} & q_{zy}(1)V_{01} & q_{zz}(1)V_{01} & ... \\ q_{xx}(0)V_{10} & q_{xy}(0)V_{10} & q_{xz}(0)V_{10} & q_{xx}(1)V_{11} & q_{xy}(1)V_{11} & q_{xz}(1)V_{11} & ... \\ q_{yx}(0)V_{10} & q_{yy}(0)V_{10} & q_{yz}(0)V_{10} & q_{yx}(1)V_{11} & q_{yy}(1)V_{11} & q_{yz}(1)V_{11} & ... \\ q_{zx}(0)V_{10} & q_{zy}(0)V_{10} & q_{zz}(0)V_{10} & q_{zx}(1)V_{11} & q_{zy}(1)V_{11} & q_{zz}(1)V_{11} & ... \\ ... & ... & ... & ... & ... & ... & ... \end{pmatrix}$$

Unfortunately, P is not Hermitian, even though $V_{kk'}$ is. We could modify P to make it Hermitian, but that would require modifying the left side of the equation too, so the answer will remain unchanged. This means that just like in the spin-singlet case, for a constant potential the only solution is an s-wave, where all values of Δ are equal. In this case, all values of d_x and d_y are equal.

However, there's something else interesting occuring here. Starting at the intial equation, outside of the matrix we have

$$d(k) = \sum_{k'} V_{kk'}(a(k')(d(k') + b(k')(\hat{g}_{k'}(\hat{g}_{k'} \cdot d(k')) - d(k'))))$$

$$a(k) = \sum_n G_+G_+ + G_-G_-, b(k) = \sum_n 2G_-G_-$$

$$b(k) = a(k) + s(k)$$

$$a(k) = \frac{1}{2}\left(\frac{\tanh(\epsilon_+/2T_c)}{2\epsilon_+} + \frac{\tanh(\epsilon_-/2T_c)}{2\epsilon_-}\right), s(k) = \frac{1}{4E_k}\left(\tanh(\epsilon_+/2T_c) + \tanh(\epsilon_-/2T_c)\right)$$

$a \geq s$ and $a \geq b$, for all values of k, T, α, g

Take the 2D example for simplicity, so our Q matrix is

$$Q(k) = \begin{pmatrix} a + b(g_x^2 - 1) & bg_xg_y \\ bg_xg_y & a + b(g_y^2 - 1) \end{pmatrix}, g = (\cos(\theta_k), \sin(\theta_k))$$

$$Q(k) = \begin{pmatrix} a + b(\cos^2(\theta_k) - 1) & b\cos(\theta_k)\sin(\theta_k) \\ b\cos(\theta_k)\sin(\theta_k) & a + b(\sin^2(\theta_k) - 1) \end{pmatrix}$$

$$Q(k) = \begin{pmatrix} a - b(\sin^2(\theta_k)) & b\cos(\theta_k)\sin(\theta_k) \\ b\cos(\theta_k)\sin(\theta_k) & a - b(\cos^2(\theta_k)) \end{pmatrix}$$

$$d(k) = \begin{pmatrix} \cos(\theta_k) \\ \sin(\theta_k) \end{pmatrix}, Qd = ad, (a - b)d$$

a is the high eigenvalue, corresponding to the highest temperature, just like the paper says. However, when taking the eigenstate of P, where multiple values of $Q_k$are included, we no longer get that d follows the g vector.

$$P = \begin{pmatrix} Q_0V_{00} & Q_1V_{01} \\ Q_0V_{10} & Q_1V_{11} \end{pmatrix}$$

Let $V(k, k') = 1$, and then test the 2D case to see if $d(k)$ still follows the g vector, ie

$$d(k) = \begin{pmatrix} d_x(k_0) \\ d_y(k_0) \\ d_x(k_1) \\ d_y(k_1) \end{pmatrix} = \begin{pmatrix} \cos(\theta_0) \\ \sin(\theta_0) \\ \cos(\theta_1) \\ \sin(\theta_1) \end{pmatrix}$$

$$Pd \neq \lambda d$$

d is not an eigenvector of P, meaning that it does not follow a changing g vector, as d is unable to change direction.

Let's test to see if this result lines up with the equations given. In the initial equation, let's take the instance where $d(k)||g(k)$. Take both vectors to be normalized, making them equal in magnitude, and for this instance make $V(k, k') = 1$, and we get

$$d(k) = \sum_{k'} V_{kk'}(a_{k'}(d(k') + b_{k'}(\hat{g}_{k'}(1) - d(k'))))$$

$$d(k) = \sum_{k'} (a_{k'} - b_{k'})d(k') + b_{k'}\hat{g}_{k'}$$

$$d(k) = \hat{g}_k \rightarrow d(k) = \sum_{k'} (a_{k'} - b_{k'})d(k') + b_k d(k') = \sum_{k'} a_{k'} d(k')$$

This is the same form as the singlet equation, since for $\alpha = 0$, $a_k = \frac{\tanh(E_k/2T)}{2E_k}$. This implies

$$\cos(\theta) = \int \frac{\tanh(E_k/2T)}{2E_k} \cos(\theta_k) dk$$

The same is true for $\sin(\theta)$ as well. Because of the separation of θ, and the fact that $E_k = k^2$, this equation actually holds true in the case where we only change the magnitude of k and not the direction. This agrees with the above conclusion that d follows g, but only when the d vector does not change direction.