Solving Method

$$\vec{d}(k) = -k_B T \sum_{k'} V_{kk'}((G_+G_+ + G_-G_-)\vec{d}(k') + 2G_-G_-(\hat{g}_{k'} \cdot \vec{d}(k')) - \vec{d}(k')))$$

$$\vec{d}(k) = -\sum_{k'} V_{kk'}(a\vec{d}(k') + b(\hat{g}_{k'}(\hat{g}_{k'} \cdot \vec{d}(k')) - \vec{d}(k'))), a(\epsilon_k, T) = G_+G_+ + G_-G_-, b(\epsilon_k, T) = 2G_-G_-$$

$$\vec{d}(k) = -\sum_{k'} V_{kk'}(a(\vec{d_x}(k') + d_y(k') + d_z(k')) + b((g_x d_x + g_y d_y + g_z d_z)(\hat{g}_{k'} - \vec{d}(k'))))$$

$$d_i(k) = -\sum_{k'} V_{kk'}(a d_i(k') + b(d_x g_x + d_y g_y + d_z g_z)(g_i - d_i(k'))$$

$$\begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} = \sum_{k'} V_{kk'} \begin{pmatrix} (a + b(g_x^2 - 1))d_x + bg_x(g_y d_y + g_z d_z) \\ (a + b(g_y^2 - 1))d_y + bg_y(g_x d_x + g_z d_z) \\ (a + b(g_z^2 - 1))d_z + bg_z(g_y d_y + g_x d_x) \end{pmatrix}$$

$$\begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} = \sum_{k'} V_{kk'} Q^{-1}(\epsilon_{k'}, T) \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix}, \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} = Q \begin{pmatrix} (a + b(g_x^2 - 1))d_x + bg_x(g_y d_y + g_z d_z) \\ (a + b(g_y^2 - 1))d_y + bg_y(g_x d_x + g_z d_z) \\ (a + b(g_z^2 - 1))d_z + bg_z(g_y d_y + g_x d_x) \end{pmatrix}$$

This is significant, because it means that $\vec{d}$ at a given value of k' has the same direction regardless of $V(k, k')$, since it is a constant at that value of k'. Therefore, $\vec{d}(k')$ is an eigenvector of Q.
The Q matrix is shown below:

```
In[22]:= Q = {{q11, q12, q13}, {q21, q22, q33}, {q31, q32, q33}}
         Simplify[Inverse[Q]]
```

Out[22]=

$$\left\{\left\{-\frac{-a+b-b\,gy^2-b\,gz^2}{(a-b)\,(a-b+b\,gx^2+b\,gy^2+b\,gz^2)}, -\frac{b\,gx\,gy}{(a-b)\,(a-b+b\,gx^2+b\,gy^2+b\,gz^2)}, \frac{b\,gx\,gz}{(-a+b)\,(a-b+b\,gx^2+b\,gy^2+b\,gz^2)}\right\},\right.$$

$$\left\{\frac{b\,gx\,gy}{-a^2+2\,a\,b-b^2-a\,b\,gx^2+b^2\,gx^2-a\,b\,gy^2+b^2\,gy^2+a\,b\,gy\,gz-b^2\,gy\,gz-a\,b\,gz^2+b^2\,gz^2-b^2\,gy^2\,gz^2+b^2\,gy\,gz^3}, -\frac{-a+b-b\,gx^2-b\,gz^2}{a^2-2\,a\,b+b^2+a\,b\,gx^2-b^2\,gx^2+a\,b\,gy^2-b^2\,gy^2-a\,b\,gy\,gz+b^2\,gy\,gz+a\,b\,gz^2-b^2\,gz^2+b^2\,gy^2\,gz^2-b^2\,gy\,gz^3},\right.$$

$$\left.-\frac{-a+b-b\,gx^2-b\,gy^2}{(a-b)\,(a-b+b\,gx^2+b\,gy^2+b\,gz^2)}\right\}, \left\{\frac{b\,gx\,gz}{(-a+b)\,(a-b+b\,gx^2+b\,gy^2+b\,gz^2)}, \frac{b\,gy\,gz}{(-a+b)\,(a-b+b\,gx^2+b\,gy^2+b\,gz^2)}, \left.\left.-\frac{-a+b-b\,gx^2-b\,gy^2}{(a-b)\,(a-b+b\,gx^2+b\,gy^2+b\,gz^2)}\right\}\right\}$$

Results

$$g = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \to d = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$g = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \to d = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0.524 \\ 0.852 \end{pmatrix}$$

$$g = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \to d = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0.359 \\ 0.933 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \rightarrow d = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$g = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \rightarrow d = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0.117 \\ 0.993 \end{pmatrix}, \begin{pmatrix} 0 \\ 0.745 \\ -0.667 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \rightarrow d = \begin{pmatrix} 0.0769 \\ -0.926 \\ 0.0769 \end{pmatrix}, \begin{pmatrix} 0.0769 \\ -0.926 \\ 0.0769 \end{pmatrix}, \begin{pmatrix} 0.0044 \\ -0.709 \\ 0.7048 \end{pmatrix}$$

All values of 1 and 0 are constant as $\epsilon_k$and T change, the other numbers change with them. This doesn't make sense to me, it's weird that the Q matrix wouldn't be symmetric, which is why the g vectors aren't spatially invariant either. I've quintuple checked my math so I have no idea where/if I'm going wrong.

Checks

1. Confirm that critical temperature optimization code gives the same result for both integration and matrix diagonalization
2. Confirm that the same result is given for singlet states and triplet states as $g \rightarrow 0$

Check Results

Having an issue with #1 at the moment, it's close but there's a slight discrepancy. There must be a bug somewhere, I just have to find it.

Issues

Questions

How to handle matrix scaling factors?