Solving Method

$$H_{ASOC} = \sum_{\mathbf{k}} \sum_{\alpha\beta=\uparrow,\downarrow} \gamma(\mathbf{k}) \cdot \tilde{\boldsymbol{\sigma}}_{\alpha\beta} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\beta}$$

The full Hamiltonian, ASOC included, is the following:

$$\mathcal{H} = \sum_{\mathbf{k},s} \varepsilon(\mathbf{k}) c^{\dagger}_{\mathbf{k}s} c_{\mathbf{k}s} + \frac{1}{2} \sum_{\mathbf{k},\mathbf{k}',s_1,s_2,s_3,s_4} V_{s_1 s_2 s_3 s_4}(\mathbf{k},\mathbf{k}')\, c^{\dagger}_{-\mathrm{k}s_1} c^{\dagger}_{\mathbf{k}s_2} c_{\mathbf{k}'s_3} c_{-\mathbf{k}'s_4} + \sum_{\mathbf{k}} \sum_{\alpha\beta=\uparrow,\downarrow} \gamma(\mathbf{k}) \cdot \tilde{\boldsymbol{\sigma}}_{\alpha\beta} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\beta}$$

Without the ASOC portion of the Hamiltonian, the gap function is defined as such:

$$\Delta_{\sigma s}(\mathbf{k}j) = \sum_{\mathbf{k}'j'}^{FS} \sum_{\sigma s} V^{(\sigma s s' \sigma')}_{\mathbf{k}j\mathbf{k}'j'} \left\langle c_{-\mathbf{k}'j'\sigma'} c_{\mathbf{k}'j's'} \right\rangle$$

If we keep the same definition in the ASOC case then the resultant Hamiltonian looks like this:

$$\mathcal{H} = \sum_{\mathbf{k},s} \varepsilon(\mathbf{k}) c^{\dagger}_{\mathbf{k}s} c_{\mathbf{k}s} + \frac{1}{2} \sum_{\mathbf{k},,\sigma,s} \Delta_{\sigma s}(\mathbf{k}j) c^{\dagger}_{k\sigma} c^{\dagger}_{-\mathbf{k}s} - \Delta^{*}_{\sigma s}(-\mathbf{k}j) c_{-\mathbf{k}\sigma} c_{\mathbf{k}s} + \sum_{\mathbf{k}} \sum_{\alpha\beta=\uparrow,\downarrow} \gamma(\mathbf{k}) \cdot \tilde{\boldsymbol{\sigma}}_{\alpha\beta} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\beta}$$

The energy matrix in this basis, without the ASOC term, looks like this:

$$\widehat{\mathscr{E}}_{\mathrm{k}} = \begin{pmatrix} \varepsilon(\mathrm{k})\widehat{\sigma}_0 & \widehat{\Delta}(\mathrm{k}) \\ -\widehat{\Delta}^{*}(-\mathrm{k}) & -\varepsilon(\mathrm{k})\widehat{\sigma}_0 \end{pmatrix}$$

And each ASOC Hamiltonian term looks like this:

$$H_{ASOC} = < k\alpha | \begin{pmatrix} \gamma_3 & \gamma_1 - i\gamma_2 \\ \gamma_1 + i\gamma_2 & -\gamma_3 \end{pmatrix} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\beta} | k\beta >$$

Clearly there will be an energy value for each type of spin pairing, which will modify each of the terms in the $\widehat{\mathscr{E}}_{\mathrm{k}}$ matrix by its respective value. My confusion comes from how this will impact the gap function. If the definition has not changed, it seems like with the 3rd equation defined above that the gap function would be independent, which of coures does not make sense.

Questions

My question is then that since the gap function follows this equation:

$$\Delta_k = \sum_{k'} P_{kk'} \Delta_{k'} \frac{\tanh(\beta E_{k'}/2)}{2E_{k'}}$$

Would the modification to the gap function simply come from changing the E_k term to include the H_{ASOC}? And if so, how does it modify it? Do I simply add the energy for each case to E_k?

Right now I'm using the same P function as before, which doesn't change Δ for each spin. Given that I calculate the energy by finding the eigenvalues of P, I'm not sure how to go about changing one without changing the other. If I'm supposed to change the P function, I'm not sure how to do so to include the H_{ASOC} term.

Issues

Once this is sorted out, should I proceed with determining the correct quantity of each spin combination (singlet, triplet) such that it minimizes the free energy of the system?