Solving Method

$$\vec{d}(k) = -k_B T \sum_{k'} V_{kk'}((G_+G_+ + G_-G_-)\vec{d}(k') + 2G_-G_-(\hat{g}_{k'} \cdot \vec{d}(k')) - \vec{d}(k')))$$

$$\vec{d}(k) = -\sum_{k'} V_{kk'}(a\vec{d}(k') + b(\hat{g}_{k'}(\hat{g}_{k'} \cdot \vec{d}(k')) - \vec{d}(k'))), a(\epsilon_k, T) = G_+G_+ + G_-G_-, b(\epsilon_k, T) = 2G_-G_-$$

$$\vec{d}(k) = -\sum_{k'} V_{kk'}(a(d_x\vec{(k')} + d_y(k') + d_z(k')) + b((g_x d_x + g_y d_y + g_z d_z)(\hat{g}_{k'} - \vec{d}(k'))))$$

$$d_i(k) = -\sum_{k'} V_{kk'}(a d_i(k') + b(d_x g_x + d_y g_y + d_z g_z)(g_i - d_i(k'))$$

$$\begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} = \sum_{k'} V_{kk'} \begin{pmatrix} (a + b(g_x^2 - 1))d_x + bg_x(g_y d_y + g_z d_z) \\ (a + b(g_y^2 - 1))d_y + bg_y(g_x d_x + g_z d_z) \\ (a + b(g_z^2 - 1))d_z + bg_z(g_y d_y + g_x d_x) \end{pmatrix}$$

$$\begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} = \sum_{k'} V_{kk'} Q(\epsilon_{k'}, T) \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix}, \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} = Q \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} = \begin{pmatrix} (a + b(g_x^2 - 1))d_x + bg_x(g_y d_y + g_z d_z) \\ (a + b(g_y^2 - 1))d_y + bg_y(g_x d_x + g_z d_z) \\ (a + b(g_z^2 - 1))d_z + bg_z(g_y d_y + g_x d_x) \end{pmatrix}$$

$$Q = \begin{pmatrix} a + b(g_x^2 - 1) & bg_x g_y & bg_x g_z \\ bg_x g_y & a + b(g_y^2 - 1) & bg_y g_z \\ bg_y g_z & bg_x g_z & a + b(g_z^2 - 1) \end{pmatrix}$$

This is significant, because it means that $\vec{d}$ at a given value of k' has the same direction regardless of $V(k, k')$, since it is a constant at that value of k'. Therefore, $\vec{d}(k')$ is an eigenvector of Q.

Results

$$g = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \to d = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \to d = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$g = \begin{pmatrix} 0 & 1 & 2 \end{pmatrix} \to d = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \to d = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

d perfectly follows g

Checks

1. Confirm that critical temperature optimization code gives the same result for both integration and matrix diagonalization
2. Confirm that the same result is given for singlet states and triplet states as $g \to 0$

Check Results

Check #1 is confirmed, though for radial cases, good accuracy requires a very large matrix.

Issues

Next steps

Extract negative sign from eigenstate and put it on eigenvalue for all states, that way we can see s-wave, p-wave, d-waves, etc without skipping any
Confirm check #2