Code Steps

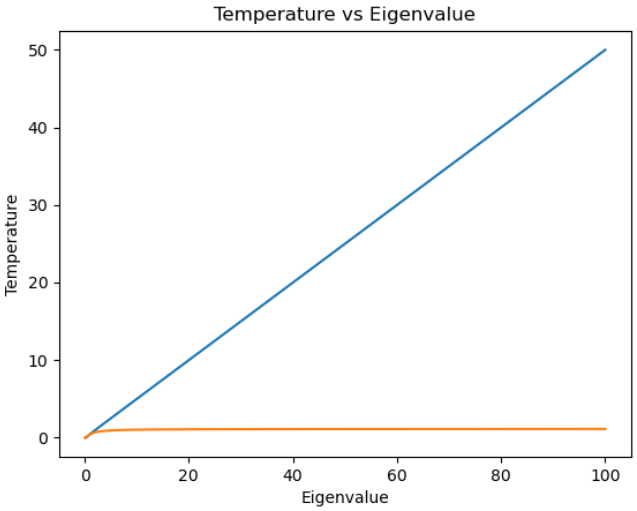
1. Define $P, N = 1, \omega_c = 1$
2. $f(T_c)PN\psi = \epsilon\psi$. Find eigenvalues of P (denoted by p) and set $\epsilon = 1$.
3. For a given eigenvalue of P, solve for $f_p = \frac{\epsilon}{pN} = \frac{1}{p}$. Choose the lowest positive eigenvalue and solve for f_p.
4. $f(T_c) = \int_{-\omega_c}^{\omega_c} \frac{tanh(\epsilon/(2T_c))}{2\epsilon} d\epsilon$. Using numerical integration to find the value of f for a given T_c, and then vary T_c to find the zeros of $f(T_c) - f_p = 0$
5. T_c is now found. We compare it to the approximate equation given by $T_c = 1.134\omega_c e^{-1/(PN)}$. We use p for the value of P.

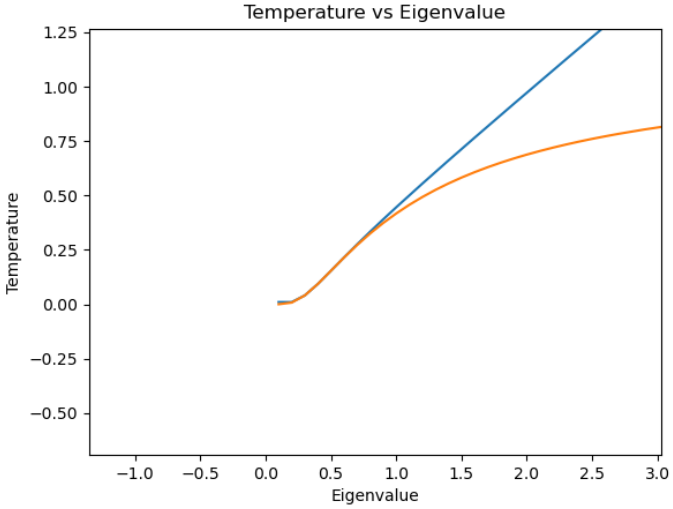
Tests

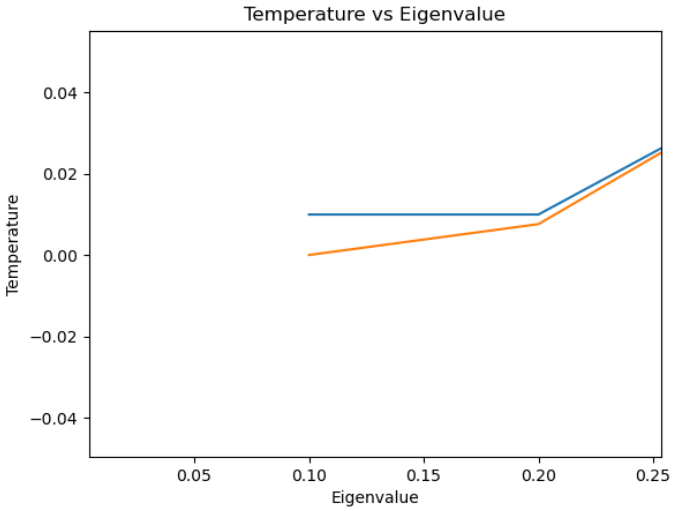
1. Behavior makes sense, since the temperature equation agrees at low temperature values.
2. Integral is compared against wolfram alpha for various values and found to be exact

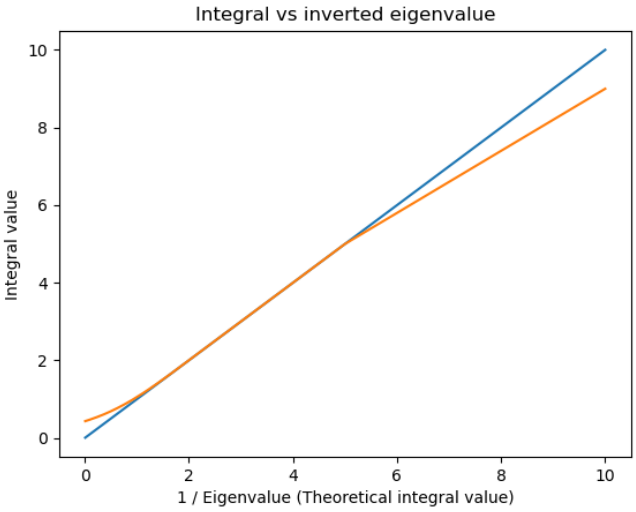
Plots

1. The approximate formula (orange) begins to diverge when $T_c \sim 0.5$, which makes sense since the condition for the approximation is $T_c << \omega_c$.
2. We use the integral plot to illustrate how accurate the value of $f(T_c)$ is. Orange is using T_c from the approximate formula, and blue is exactly calculated, which is why it is linear. The integrals remain close because at high values of T_c, where the values diverge, the integral becomes very small. Even though the integral values diverge, the difference appears small compared to the integral values at lower T_c's.
3. There is one failure in the temperature vs eigenvalue plot at extremely low temperatures, where the two lines diverge. This is not due to an inaccuracy in the approximation, but rather in the integration limit for python's code, where the accuracy decreases and the temperature cannot be accurately measured, which is why it flatlines. This can be seen when running the code and AccuracyWarning's are given.


Temperature vs Eigenvalue
Temperature
Eigenvalue
0
10
20
30
40
50
0
20
40
60
80
100


Temperature vs Eigenvalue
Temperature
Eigenvalue
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0


Temperature vs Eigenvalue
Temperature
Eigenvalue
0.04
0.02
0.00
−0.02
−0.04
0.05
0.10
0.15
0.20
0.25


Integral vs inverted eigenvalue
Integral value
1 / Eigenvalue (Theoretical integral value)
0
2
4
6
8
10