Superconducting Hubbard Model Summary

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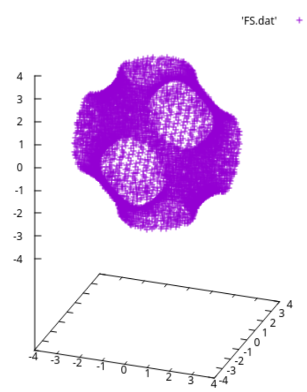
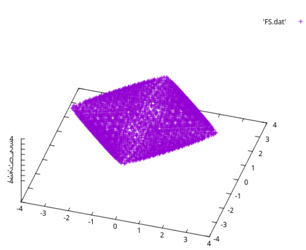
All of the work done here is on the Hubbard Model, in both 2D and 3D. All values of $t = 1$, with U changing for different plots. The value of U and the dimensionality of the model will be listed next to each relevant plot. The single band structure that we will be using follows

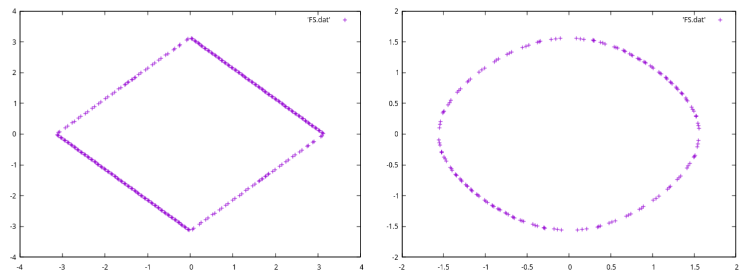
$$\epsilon(k) = -2t(\cos(kx) + \cos(ky) + \cos(kz))$$

1 Fermi Surfaces

Some fermi surfaces found from the above ϵ are shown here, with varying chemical potentials, and in both 2D and 3D. In 3D at $\mu = 0$ there are holes in the surface, in 2D the surface forms a perfect square. In each case, as the chemical potential decreases the surface shrinks and the holes in the 3D surface disappear, just as it should.





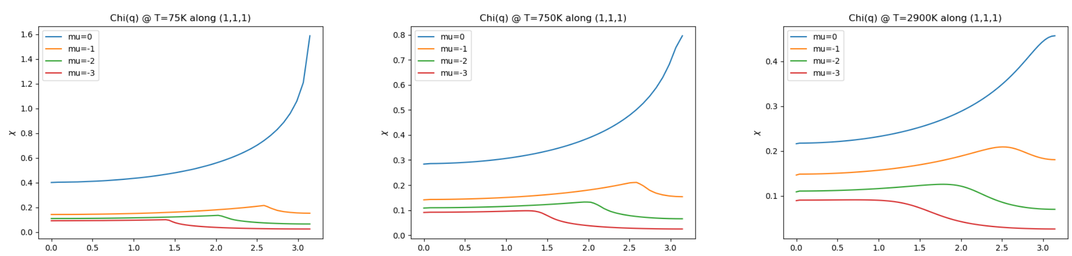


2 $\chi(q, T)$

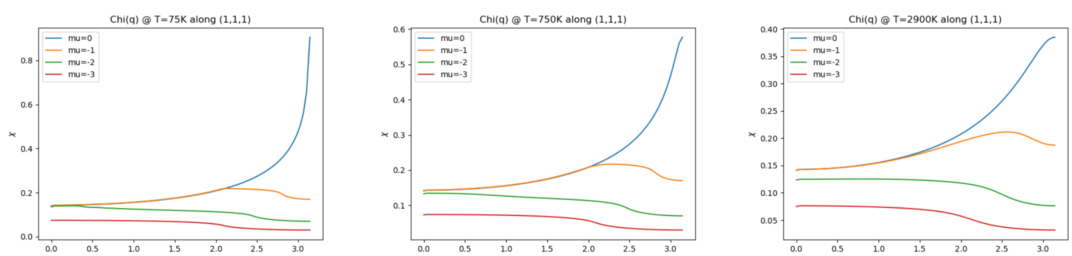
This section all of the susceptibilities for the chemical potentials $\mu = 0, -1, -2, -3$. All of these values are taken inside of the Brillouin Zone for a cube or square, depending on the dimensionality of the model.

In the images below, we can see that at $\mu = 0$, there is a peak at (π, π, π). In 2D there is a peak at (π, π). If we look at the fermi surfaces above, we can see that the most nesting present in the surface, the higher the susceptibility, which is why at $\mu = 0$ the susceptibility is highest.

The following plots are in 2D, with the x-axis corresponding to the magnitude of the x-component of q.

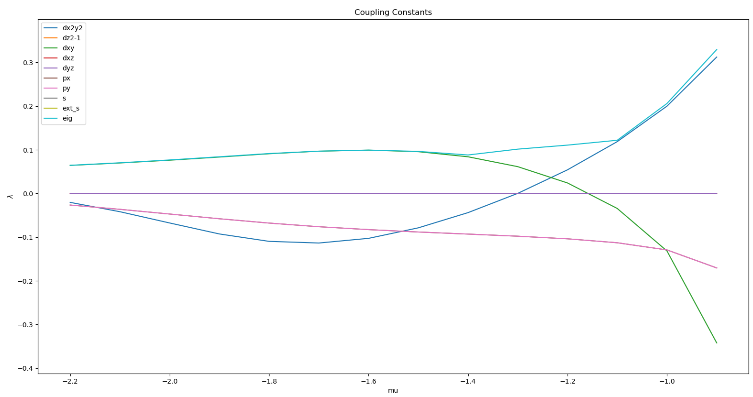


The 3 graphs above show a clear trend, where $\mu = 0$ has the susceptibility peak at (π, π, π), and as the temperature lowers, the susceptibility at $\mu = 0$ increases, and the susceptibility at every other value is unchanged. The graphs below are the same plots but in 3D, and exhibit the same trend.

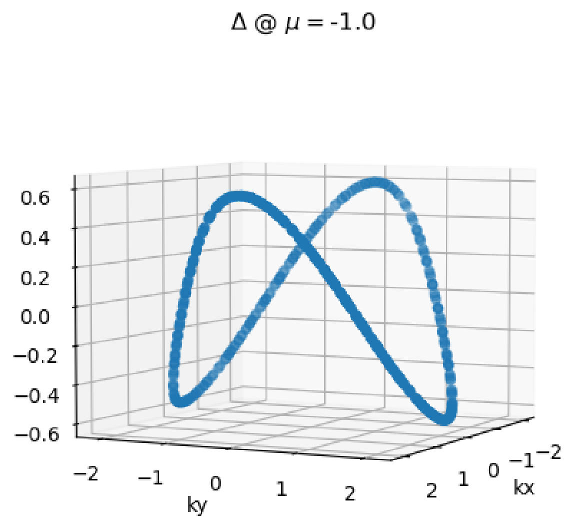


3 Coupling constants as μ varies

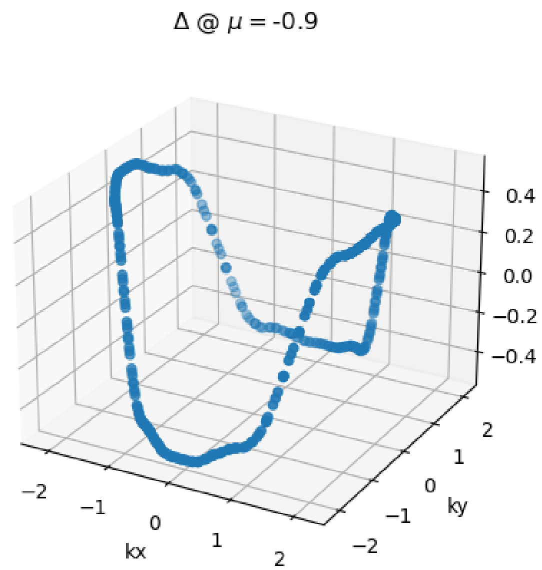
The plot of coupling constants vs chemical potential is shown below, where it can be seen that the d-wave is the second highest, its classical shape following $\cos(k_x) - \cos(k_y)$ just the shape calculated via exact diagonalization (labeled as eig). The difference between their coefficients is extremely small, and the difference in the shape of the gaps is shown in the next section.

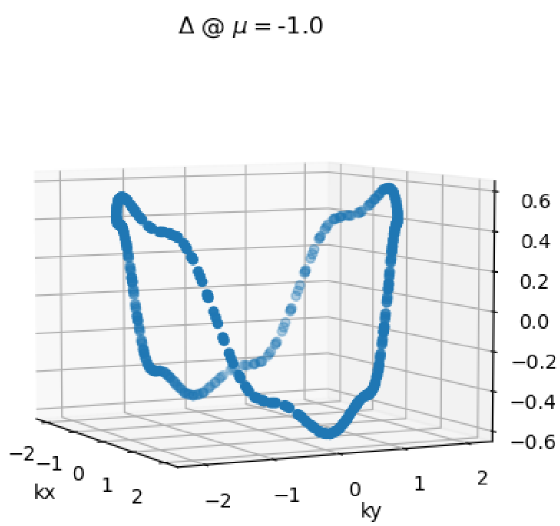


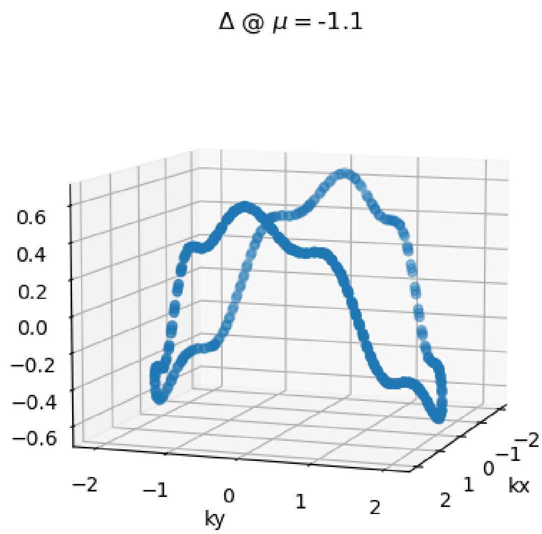
4 Δ shapes as μ varies

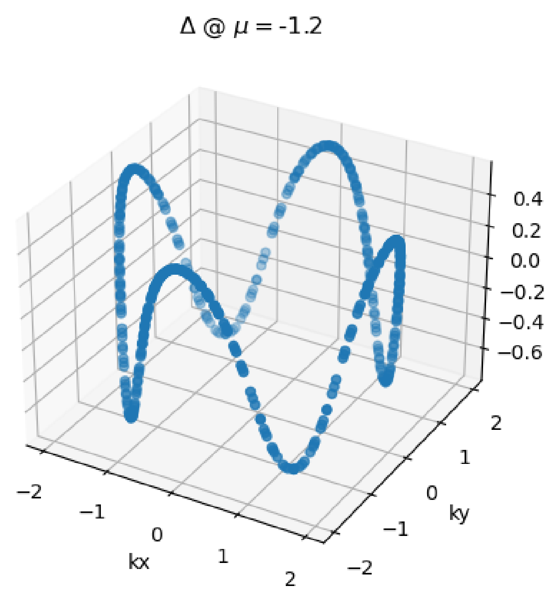


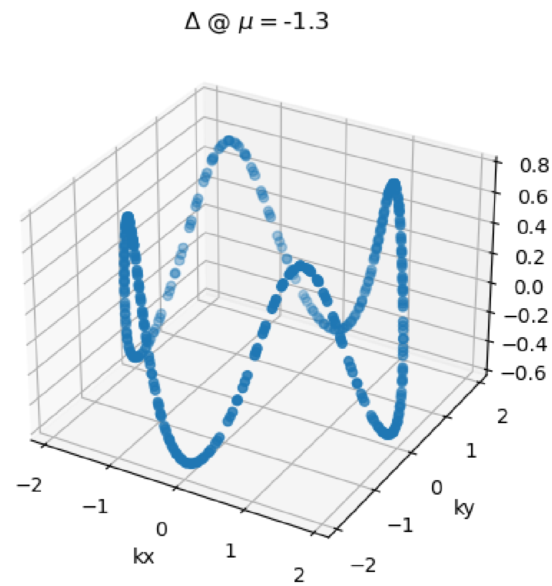
The shape of a classical d-wave $(\cos(x) - \cos(y))$ is shown above for reference. The shape of the energy gap varies quite a bit as the chemical potential is lowered. We can see below the variations from the classic d-wave changing with the chemical potential. It's interesting both how small the variations are when the coupling constants are far away and how large they are when the coupling constants are so close.

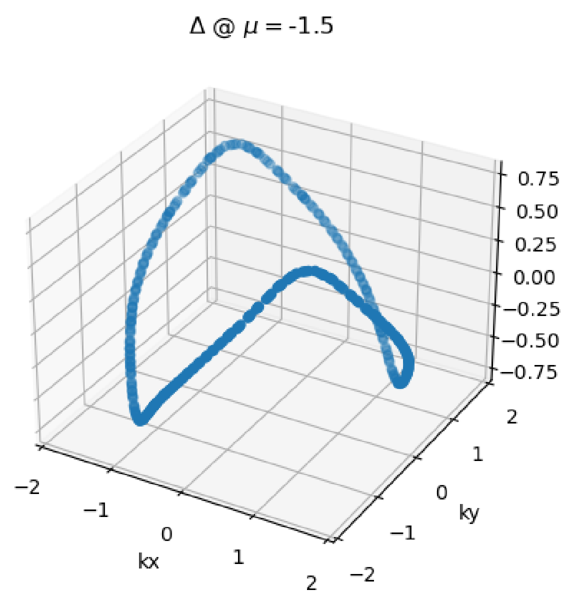

Δ @ μ = -0.9
kx
ky
−2
−1
0
1
2
−2
−1
0
1
2
0.4
0.2
0.0
−0.2
−0.4

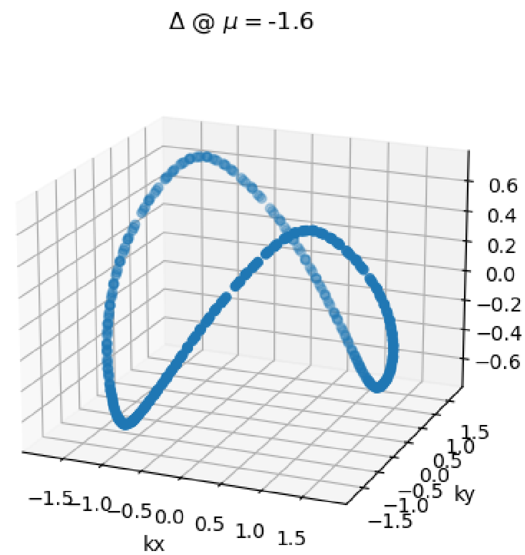

Δ @ μ = -1.0
kx
ky
−2
−1
0
1
2
−2
−1
0
1
2
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6

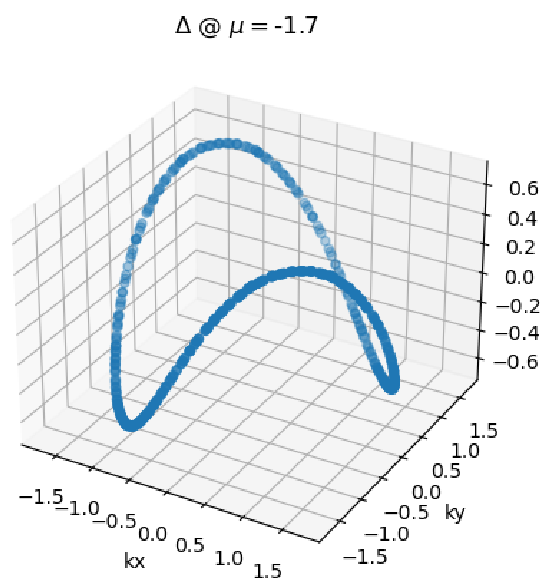

Δ @ μ = -1.1
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−2
−1
0
1
2
ky
kx


Δ @ μ = -1.2
0.4
0.2
0.0
−0.2
−0.4
−0.6
−2
−1
0
1
2
kx
ky


Δ @ $\mu =$ -1.3
kx
ky
−2
−1
0
1
2
−2
−1
0
1
2
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6


Δ @ $\mu =$ -1.5
kx
ky
−2
−1
0
1
2
−2
−1
0
1
2
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75


Δ @ μ = -1.6
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
kx
ky


Δ @ μ = -1.7
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
kx
ky