

Bungee Jumping

The dynamics of a person bungee jumping are governed by the equation below

$$m\ddot{x} = F_{gravity} + F_{cord} + F_{drag} = -mg + b(x) - \frac{1}{2}\rho C_D A |\dot{x}| \dot{x}$$

where m is the body mass, g is gravity, $b(x)$ is the spring force of the elastic cord, and the last term represents air resistance. The $||$ operator above denotes the absolute value operation.

Prior to jumping, the person is located at a height x_0 above the ground and is tied to an elastic bungee cord with spring constant k . The length of the unstretched cord is L . Once the bungee cord begins stretching (at $x < x_0 - L$), it provides a restoring spring force which can be characterized as

$$b(x) = \begin{cases} k((x_0 - L) - x), & \text{if } x < x_0 - L \\ 0, & \text{if } x \geq x_0 - L \end{cases}$$

So, the overall equation is

$$m\ddot{x} = -mg + b(x) - \frac{1}{2}\rho C_D A |\dot{x}| \dot{x}$$

Create a model of the system dynamics and simulate using the parameters and initial conditions below. Plot the position of the person over time.

$m = 90$	$A = 0.5$
$g = 9.81$	$L = 20$
$k = 25$	$x_0 = 80$
$\rho = 1.2$	$\dot{x}_0 = 0$
$C_D = 0.6$	Stop Time = 50

Outputs

x – Position

States

x – Position

\dot{x} – Velocity

Parameters

m – Mass (90 kg)

g – Gravity constant (9.81 m/s²)

k – Spring constant of cord (25 N/m)

ρ – Air density (1.2 kg/m³)

C_D – Drag coefficient (0.6)

A – Frontal surface area of person (0.5 m²)

L – Length of unstretched bungee cord (20 m)

