

# Accepted Manuscript

## A Multi-Objective Model for the Green Capacitated Location-Routing Problem Considering Environmental Impact

Eliana M. Toro, John F. Franco, Mauricio Granada Echeverri, Frederico Gadelha Guimarães

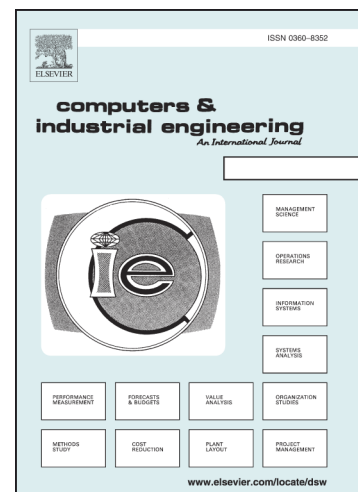
PII: S0360-8352(17)30217-6  
DOI: <http://dx.doi.org/10.1016/j.cie.2017.05.013>  
Reference: CAIE 4744

To appear in: *Computers & Industrial Engineering*

Received Date: 15 December 2015  
Accepted Date: 12 May 2017

Please cite this article as: Toro, E.M., Franco, J.F., Echeverri, M.G., Guimarães, F.G., A Multi-Objective Model for the Green Capacitated Location-Routing Problem Considering Environmental Impact, *Computers & Industrial Engineering* (2017), doi: <http://dx.doi.org/10.1016/j.cie.2017.05.013>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



# A Multi-Objective Model for the Green Capacitated Location-Routing Problem Considering Environmental Impact

Eliana M. Toro<sup>a</sup>, John F. Franco<sup>c</sup>, Mauricio Granada Echeverri<sup>b</sup>, Frederico Gadelha Guimarães<sup>d,1</sup>

<sup>a</sup> Facultad de Ingeniería Industrial, Universidad Tecnológica de Pereira. Pereira, Colombia.

<sup>b</sup> Programa de Ingeniería Eléctrica, Facultad de Ingenierías, Universidad Tecnológica de Pereira. Pereira, Colombia.

<sup>c</sup> Universidade Estadual Paulista Júlio de Mesquita Filho, UNESP, Ilha Solteira, Brazil.

<sup>d</sup> Department of Electrical Engineering, Universidade Federal de Minas Gerais, UFMG, Belo Horizonte, Brazil.

## Abstract

The Capacitated Location-Routing Problem (CLRP) is a strategic-level problem involving the selection of one or many depots from a set of candidate locations and the planning of delivery routes from the selected depots to a set of customers. During the last few years, many logistics and operations research problems have been extended to include greenhouse effect issues and costs related to the environmental impact of industrial and transportation activities. In this paper a new mathematical model for the calculation of greenhouse gas emissions is developed and a new model for the CLRP considering fuel consumption minimization is proposed. This model, named Green CLRP (G-CLRP), is represented by a mixed integer linear problem, which is characterized by incorporating a set of new constraints focused on maintaining the problem connectivity requirements. The model proposed is formulated as a bi-objective problem, considering the minimization of operational costs and the minimization of environmental effects. A sensitivity analysis in instances of different sizes is done to show that the proposed objective functions are indeed conflicting goals. The proposed mathematical model is solved with the classical epsilon constraint technique. The results clearly show that the proposed model is able to generate a set of tradeoff solutions leading to interesting conclusions about the operational costs and the environmental impact. This set of solutions is useful in the decision process because several planning alternatives can be considered at strategic level.

**Keywords:** Capacitated Location-Routing Problem, Green Vehicle Routing Problem, Green logistics, Mixed-Integer Linear Programming, Vehicle Routing Problem.

# A Multi-Objective Model for the Green Capacitated Location-Routing Problem Considering Environmental Impact

Blind Review

## Abstract

The Capacitated Location-Routing Problem (CLRP) is a strategic-level problem involving the selection of one or many depots from a set of candidate locations and the planning of delivery routes from the selected depots to a set of customers. During the last few years, many logistics and operations research problems have been extended to include greenhouse effect issues and costs related to the environmental impact of industrial and transportation activities. In this paper a new mathematical model for the calculation of greenhouse gas emissions is developed and a new model for the CLRP considering fuel consumption minimization is proposed. This model, named Green CLRP (G-CLRP), is represented by a mixed integer linear problem, which is characterized by incorporating a set of new constraints focused on maintaining the problem connectivity requirements. The model proposed is formulated as a bi-objective problem, considering the minimization of operational costs and the minimization of environmental effects. A sensitivity analysis in instances of different sizes is done to show that the proposed objective functions are indeed conflicting goals. The proposed mathematical model is solved with the classical epsilon constraint technique. The results clearly show that the proposed model is able to generate a set of tradeoff solutions leading to interesting conclusions about the operational costs and the environmental impact. This set of solutions is useful in the decision process because several planning alternatives can be considered at strategic level.

**Keywords:** Capacitated Location-Routing Problem, Green Vehicle Routing Problem, Green logistics, Mixed-Integer Linear Programming, Vehicle Routing Problem.

## 1. Introduction

In the last decade, consumers, businesses and governments have increased their attention to the environment. Society in general is becoming increasingly aware and concerned of the environmental impact of human activities and the indiscriminate use of natural resources. Companies are understanding and recognizing the need to assess and reduce the environmental impact of their products and services [1, 2]. In this context, the transportation industry has a significant effect on the planet, because of the large quantity of fuel used in its regular operation and the environmental consequences and greenhouse effects of fuel consumption and pollution. As a consequence, Green Logistics and Green Transportation have emerged in all levels of supply chain management [3], with growing value to researchers and organizations, motivated by the fact that current logistics centered on economic costs without accounting for the negative impacts on the environment is not sustainable in the long term [3].

Approximately 10% of the gross domestic product is devoted to supply chain related activities [4]. Any effort in the optimal management of the supply chain is of great impact on the finances of the organization. The Capacitated Location Routing Problem (CLRP) is an important problem in the strategic level of the supply chain management, dealing with decisions of logistics operations, such as: i) location of factories, warehouses or distribution centers, known as facilities or depots; ii) allocation of customers to each service area; and iii) trans-

portation plans connecting customers, raw materials, manufacturing plants and warehouses. Therefore, the CLRP aims at approaching location and routing decisions together as an integrated problem.

During the last few years, many logistics and operations research problems have been extended to include environmental issues and costs related to the environmental impact of industrial and transportation activities [5, 6, 7, 3]. In this paper we propose a new mathematical model for the CLRP, named Green CLRP (G-CLRP), considering fuel consumption minimization. Briefly, the problem can be stated as follows. Given a set of depots  $I$  and customers  $J$ , the goal of the CLRP is to find the optimal selection of depots and the routes that connect them to the customers. Each facility has a setup or opening cost  $O_i$ . Associated with each edge  $(i, j) \in V$  there is a traveling cost  $c_{ij}$ . Each customer  $j \in J$  has a demand  $d_j$  which must be fulfilled by a single vehicle. A set  $K$  of identical vehicles with capacity  $Q$  is available. Each vehicle, when used by a facility  $i \in I$ , incurs a depot dependent fixed cost  $F_i$  and performs a single route. In the usual CLRP, one objective function is considered in the model: to minimize total operational cost, which includes setup cost of facilities, cost of use of vehicles and cost for transit between a pair of nodes. In the G-CLRP, in addition to the operational costs, a second objective function is included, which considers pollutant emissions generated due to fuel consumption in the routes performed. Given the inherent difficulty of aggregating these objectives into a global criterion, we formu-

late the problem as a bi-objective one. The Pareto-optimal solutions for the problem can be found and analyzed a posteriori by decision-makers or stakeholders. This model corresponds to a mixed integer linear problem formulation and was implemented in AMPL [8] and solved with CPLEX 12.5 (called with the optimality gap option equal to 0%).

The main contributions of this paper are:

- A new mathematical model for the computation of fuel consumption and total emissions is developed based on the forces acting on each vehicle during its operation.
- The proposed G-CLRP extends the CLRP, by considering the environmental impact in terms of fuel consumption minimization.
- The proposed model in this paper corresponds to a mixed integer linear formulation, which is characterized by incorporating a radial constraint that makes it possible to eliminate subtour constraints.
- This paper presents a contribution to the discussion of green VRP, by considering the integrated location of multiple depots and routing of multiple vehicles.

## 2. Overview and Approaches for the CLRP

The CLRP is a special case of the Location Routing Problem (LRP), therefore any methodology proposed for the LRP can be extended to the CLRP, for this one needs to consider facilities and vehicles with limited capacity and customers with deterministic demand. The CLRP is considered an NP-hard problem, due to the combination of the Capacitated VRP (CVRP) and the Capacitated Facility Location problem (CFLP) [9].

LRP have many applications in different economic fronts, such as: localization of central office and routing of army classified documents [10], distribution of documents to cities [11], military logistics planning and operation [12], installation of waste incineration plants and routing and collection of garbage [13], timber supply chain [14], supermarket chains distribution [15], vehicle parts distribution [16], mail distribution [17] and planning assignments in special examinations [18].

LRP have been solved using heuristics and metaheuristics, mathematical programming, and hybrid techniques including *matheuristics*<sup>1</sup>. Berger et al. [19] solved the LRP with standard vehicles and facilities with unlimited capacity, considering the maximum length of each route, using a Branch-and-Price algorithm. Akca et al. [20] described a Branch-and-Price algorithm based on the set partitioning formulation, discussing exact and heuristic variants of this algorithm. Belenguer et al. [21] used a formulation of two indices with two types of binary variables, one of them is associated with the arcs and the other variable indicates whether or not the arc was used twice, because they are considered unique customer routes. Baldacci & Hadjiconstantinou [22] proposed a new integer programming formulation for

the CVRP based on a two-commodity network flow approach and a new Branch-and-Cut exact algorithm for the optimal solution. Contardo et al. [23, 9], based on the formulation in [21], presented new three-index flow formulations and three-index two-commodity flow formulation. The results show that the first formulation type is more efficient than the second one. Contardo et al. [24] proposed a methodology for the CLRP as formulated in [20, 22], where new procedures of inequalities and separations for the flow formulations of the CLRP are presented. The results outperform those of [21], improving the bounds found in the literature, solving to optimality some previously unsolved instances, and improving the upper bounds on some other instances.

In the case of Heuristics, on one hand iterative methods solve both sub-problems simultaneously, feeding back responses obtained from each sub-problem [25]. Hierarchical methods consider the main problem of locating depots and then the routing problem is solved as a subordinate issue [26]. Methods based on grouping of clients or clusters were proposed by [27].

Metaheuristics have also been proposed for the LRP. In [28], a new metaheuristic to solve the LRP with capacitated routes and depots is proposed. The method consists of a GRASP, based on an extended and randomized version of Clarke and Wright algorithm, and a post-optimization using path relinking technique. In [29], a genetic algorithm with management of the population is proposed, denominated Memetic Algorithm with Population Management. The initial population is characterized by a small number of individuals initially generated with the Randomized Extended Clarke and Wright Algorithm (RECWA) and the method of randomized nearest neighbor; both are improved by a local search, and tournament selection is performed. Duhamel et al. [30] proposed an approach called Evolutionary Local Search (GRASP ELS). The initial solutions are constructed using RECWA and are improved using the Local Search as it is proposed in [29]. The solution found is transformed into a giant tour. The result obtained by the giant tour becomes a solution to the LRP, using the division method inspired by [28]. Finally, the solution of the CLRP is improved again by using Local Search. The methodology is tested on several instances, improving some responses achieved by Prins et al. [28, 29].

*Matheuristics* are hybrid strategies that combine elements of heuristic techniques, metaheuristic techniques and mathematical programming techniques. In general, these hybrid strategies involve solving one sub-problem of the general problem by using mathematical programming.

Prins et al. [31] presented a two-phase matheuristic. It combines Lagrangian Relaxation (LR) and Granular Tabu Search (GTS). They improve many results from [27, 28]. Chen & Ting [32] presented a two-stage algorithm to solve the LRP, considering heterogeneous fleet of vehicles, in which first depots are open and the allocation of customers to depots is done, finally, it solves the corresponding VRP at each depot. Results and computation times in several instances are reported in [33, 34, 35, 27].

Lopes et al. [13] presented a sequential route first cluster second method. Four types of clustering methods using six prox-

<sup>1</sup>Combination of mathematical programming and heuristics.

imity measures between clients are tested, these are described in detail in [27]. The tour is improved by using 3-opt based local search. The phase of the allocation problem is solved using a commercial software. Contardo et al. [9] presented a methodology that combines GRASP heuristic method and a column generation method. Cuts proposed by Contardo et al. in [23] are included.

Escobar et al. [36] proposed a two stage matheuristic. In the construction phase, an initial solution is generated from a giant tour obtained with Lin Kernighan Heuristic (LKH), the result of the tour is divided according to the vehicle capacity. The solution obtained is subsequently improved by using the algorithms presented by Groër et al. [37].

Finally, Drexler & Schneider [38] and Prodhon & Prins [39] analyzed the recent literature on the standard LRP and new extensions such as several distribution echelons, multiple objectives or uncertain data. Results of the state-of-the-art metaheuristics are also compared on set of instances from the literature for the classical LRP, the two echelon LRP and the truck and trailer problem.

### 3. Green Issues in Vehicle Routing

Reduction of indirect greenhouse gases emissions, addressed in the vehicle routing problem, represents one of the most common objectives to be optimized. The cost of a route depends on several factors that can be divided in two categories. The first set includes distance, weight, speed, path conditions, percentage of fuel that is generally associated to the unit of distance, and fuel costs. The second set of factors does not have direct relationship on the travel programming, these include tire and vehicle depreciation, maintenance, driver wages, taxes, among others [40, 41].

Comparing the two sets, the first set of factors are directly related to fuel consumption and therefore can be considered as a variable cost or cost of fuel. Also, if other factors remain constant, fuel consumption depends mainly on the distance and load. Entities for environmental impact analysis in the transport sector believe that there is a strong correlation between the gross vehicle weight and distance travelled using a given amount of fuel, see for instance [42].

One of the first publications that consider minimizing the fuel consumption is found in [43], where they define the Energy Minimizing Vehicle Routing Problem (EMVRP) as the CVRP in which the objective function is a product of the total load (including the weight of the empty vehicle) and the length of the arc.

Figliozzi [44] compared different levels of traffic congestion and vehicle speeds, in order to formulate and solve the problem that has been called EVRP (Emissions Vehicle Routing Problem). The problem is an extension of the VRPTW. Greedy heuristic is the solution technique used by the author.

In [5], the authors considered factors such as speed, vehicle load and travel costs. The load and travel speed are factors that can be controlled. They have developed four mathematical formulations for the Pollution Routing Problem (PRP) considering

time windows, speed, load, velocity. Branch-and-Cut is the solution technique chosen, using CPLEX 12.1.

Suzuki [45] developed an approach to the time-constrained, multiple stop, truck-routing problem that minimizes the fuel consumption and pollutants emission to solve the travelling salesman problem with time windows (TSPTW). Their results suggest that the approach may produce up to 6.9% in fuel savings over existing methods. The solution technique used in the study was the compressed annealing.

Xiao et al. [42] defined the Fuel Consumption Vehicle Routing Problem (FCVRP) and proposed Fuel Consumption Rate (FCR) as a load dependent function, adding it to the classical CVRP to extend traditional studies on CVRP with the objective of minimizing fuel consumption. The methodology for solving the problem was based on the simulated annealing algorithm with a hybrid exchange rule to solve it. Their results show that the FCVRP model can reduce fuel consumption by 5% on average compared to the CVRP model.

Erdogan & Miller-Hooks [6] introduced the Green Vehicle Routing Problem (G-VRP). The G-VRP is formulated as a mixed integer linear program. The solution method is based on two construction heuristics and the Modified Clarke and Wright Savings formulation of Bektas & Laporte [5]. They developed two construction heuristics, namely the Modified Clarke and Wright Savings heuristic and the Density-Based Clustering Algorithm, and a customized improvement technique. Results of numerical experiments show good performance of the heuristics. Moreover, problem feasibility depends on customer and station location configurations.

Pradenas et al. [46] formulated a model with emissions of greenhouse gases for the VRPB problem (VRP with Backhauls). Ubeda et al. [47] presented a case study considering environmental criteria based on real estimations. Other approximations that use metaheuristics can be found in [48] and [49]. Demir et al. [7] recently proposed the bi-objective Pollution Routing Problem (PRP), as an extension of the PRP, which consists of routing a number of vehicles to serve a set of customers, and determining their speed on each route segment. Two objective functions related to minimization of fuel consumption and driving time are proposed. Several multi-objective optimization techniques are developed and tested for the problem, finding the trade-off between fuel consumption and driver times.

Küçükoglu et al. [50] presented the G-CVRP optimization model, in which fuel consumption is computed considering the vehicle technical specifications, vehicle load and the distance. Fuel consumption equation is integrated to the model through a regression equation proportional to the distance and vehicle load. The G-CVRP optimization model is validated by various instances with different number of customers. The authors presented a mixed integer programming model, solving it with Gurobi 5.10. Recently, Lin et al. [3] presented an extensive literature review on Green Vehicle Routing Problems.



#### 4. Proposed Model for the Green CLRP

##### 4.1. Nomenclature

The nomenclature for the variables and parameters of the proposed model for the G-CLRP is summarized next.

Sets	
$I$	Set of facilities (depots)
$J$	Set of customers
$V$	Set of nodes $V = I \cup J$
Parameters	
$O_i$	Cost associated with the use of facility $i$
$O_i^{NPV}$	Net present value of the facility cost
$W_i$	Capacity of the facility $i$
$F$	Cost of the vehicle
$F^{NPV}$	Net present value of the cost of the vehicle
$Q$	Maximal weight that the vehicle can carry
$D_j$	Product demand of the customer $j \in J$
$c_{ij}$	Cost of traveling between nodes $i$ and $j$
$d_{ij}$	Distance traveled between nodes $i$ and $j$
$\alpha_{ij}$	Parameter that represents the amount of energy per distance (J/km) required by an unloaded vehicle between nodes $i$ and $j$
$\gamma_{ij}$	Parameter that represents the additional energy per unit of distance and ton of load (J/km-ton) required by a vehicle between nodes $i$ and $j$
$E$	Total emission per unit of energy (kg of $CO_2/J$ )
Variables	
$x_{ij}$	Binary variable indicating the use of the path between nodes $i, j \in V$
$y_i$	Binary variable for the use of a facility $i \in I$
$f_{ij}$	Binary variable that defines if the customer at node $j \in J$ is served by a route that starts at the facility $i \in I$
$z_j$	Binary variable that determines if the customer at node $j \in J$ is the last one served in a route
$a_{ij}$	Binary variable that indicates if a vehicle uses path $j$ to return from the end of its route (at node $j$ ) to a facility (at node $i$ )
$t_{ij}$	Continuous variable indicating the amount of cargo transported between nodes $i$ and $j$

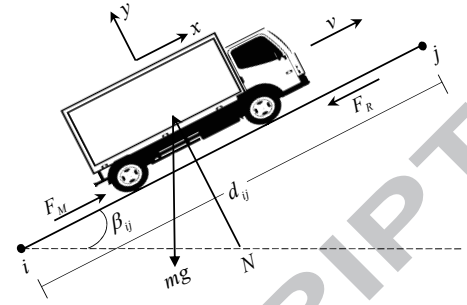


Figure 1: Forces acting on the vehicle.

the inclined plane acting on the vehicle,  $v_{ij}$  is the speed of the vehicle,  $d_{ij}$  is the distance traveled between nodes  $i$  and  $j$ .

The balance of forces is developed as follows (assuming constant speed):

$$\begin{aligned} \sum F_x &= ma_x, & a_x &= 0 \\ \sum F_y &= ma_y, & a_y &= 0 \end{aligned}$$

$$\begin{aligned} \sum F_x &= ma_x \Rightarrow F_M - F_R - mg \sin \beta_{ij} = 0 \\ \sum F_y &= ma_y \Rightarrow N - mg \cos \beta_{ij} = 0 \end{aligned}$$

The force  $\vec{F}_R$  consists of the following components:

$$\vec{F}_R = \vec{F}_{R,\text{tires}} + \vec{F}_{R,\text{wind}} + \vec{F}_{R,\text{internal}} + \frac{mv_{ij}^2}{2d_{ij}} \quad (1)$$

in which  $\vec{F}_{R,\text{tires}}$  represents the force exerted between the wheels without traction and terrain that opposes the movement of the vehicle;  $\vec{F}_{R,\text{wind}}$  is the force exerted by the wind against the movement of the vehicle;  $\vec{F}_{R,\text{internal}}$  represents the equivalent force of the internal forces that oppose the movement of the vehicle; and  $mv_{ij}^2/2d_{ij}$  is the force required by the vehicle to achieve steady state kinetic energy. The mass of the vehicle is given by the mass of the unloaded vehicle  $m_0$  and the load carried between nodes  $i$  and  $j$ :

$$m = m_0 + t_{ij}$$

By definition,  $F_{R,\text{tires}} = Nb$ , with  $b$  a constant depending on the terrain. Thus,

$$F_M = (mg \cos \beta_{ij})b + F_{R,\text{wind}} + F_{R,\text{internal}} + \frac{mv_{ij}^2}{2d_{ij}} + mg \sin \beta_{ij} \quad (2)$$

The work  $U_{ij} = F_M d_{ij}$  from node  $i$  to  $j$  is given by:

$$\begin{aligned} U_{ij} &= \left[ (m_0 + t_{ij})gb \cos \beta_{ij} + F_{R,\text{wind}} + F_{R,\text{internal}} \right. \\ &\quad \left. + \frac{(m_0 + t_{ij})v_{ij}^2}{2d_{ij}} + (m_0 + t_{ij})g \sin \beta_{ij} \right] d_{ij} \end{aligned}$$

##### 4.2. Computation of fuel consumption and total emission

In this section we describe the mathematical model used to compute the fuel consumption of a vehicle between two nodes. The model is developed based on the forces acting on the vehicle as shown in Figure 1.

In Figure 1,  $\beta_{ij}$  is the average inclination of the path between nodes  $i$  and  $j$ ,  $\vec{F}_R$  represents forces opposing to the movement of the vehicle,  $\vec{F}_M$  represents the forces generated by the motor and transmitted to the tires of the vehicle,  $m\vec{g}$  is the weight of the vehicle (mass  $m$  times gravity  $\vec{g}$ ),  $\vec{N}$  is the normal force of

$$U_{ij} = \left[ m_0 g \left( b \cos \beta_{ij} + \sin \beta_{ij} \frac{v_{ij}^2}{2g d_{ij}} \right) + F_{R,wind} + F_{R,internal} \right] d_{ij} + \left[ g \left( b \cos \beta_{ij} + \sin \beta_{ij} \frac{v_{ij}^2}{2g d_{ij}} \right) \right] t_{ij} d_{ij}$$

Assuming constant speed, we can define the constant coefficients:

$$U_{ij} = \alpha_{ij} d_{ij} + \gamma_{ij} t_{ij} d_{ij} \quad (3)$$

The constant  $\alpha_{ij}$  depends on the average inclination of the path between  $i$  and  $j$ , the weight of the unloaded vehicle, the energy to achieve steady state speed, the resistance on the tires, the wind resistance in the path and internal losses of the vehicle. Some of these quantities in turn depend on the speed of the vehicle. The constant  $\gamma_{ij}$  depends on the average inclination of the path between  $i$  and  $j$  and the resistance on the tires.

The work required from node  $i$  to node  $j$  has a component that is related to the unloaded vehicle,  $\alpha_{ij} d_{ij}$ , and another component that is related to the load, i.e.  $\gamma_{ij} t_{ij} d_{ij}$ . If we neglect inclination  $\beta_{ij}$  (or assume the same inclination for all edges) and assume the same speed for all edges, then  $\alpha_{ij} = \alpha$  and  $\gamma_{ij} = \gamma$ , leading to:

$$U_{ij} = \alpha d_{ij} + \gamma t_{ij} d_{ij} \quad (4)$$

The work required for the vehicle to complete one route is given by the sum of work required for each edge. If we associate binary variables to the utilization of the edge  $(i, j)$ , as defined by the variables  $a_{ij}$  and  $x_{ij}$ , we have:

$$\sum_{i,j \in V} U_{ij} = \alpha \left( \sum_{i,j \in V} d_{ij} x_{ij} + \sum_{i,j \in V} d_{ij} a_{ij} \right) + \gamma \left( \sum_{i,j \in V} d_{ij} t_{ij} \right) \quad (5)$$

The amount of fuel required to perform the total work  $\sum_{i,j \in V} U_{ij}$  is obtained with a conversion factor  $E_1$  (gallons/J). The amount of emission per unit of fuel is given by another conversion factor  $E_2$  (kg of CO<sub>2</sub>/gallons). Hence, the total emission is calculated as:

$$E_1 \times E_2 \times \sum_{i,j \in V} U_{ij} = E \times \sum_{i,j \in V} U_{ij} \quad (6)$$

The developed model for computing emission is linear as shown in (5) and defined as an objective in our formulation.

Nonetheless, the model is general. In practical cases, it is possible to calculate the average inclination of the paths  $(i, j)$ , obtaining  $\alpha_{ij}$  and  $\gamma_{ij}$  for each arc. In this case, we would have:

$$\sum_{i,j \in V} U_{ij} = \sum_{i,j \in V} \alpha_{ij} d_{ij} x_{ij} + \sum_{i,j \in V} \alpha_{ij} d_{ij} a_{ij} + \sum_{i,j \in V} \gamma_{ij} d_{ij} t_{ij} \quad (7)$$

which is also a linear objective function.

#### 4.3. Problem formulation

Given a set of nodes  $V = I \cup J$  with a set of depots  $I$  and customers  $J$ , the CLRP defines the optimal selection of facilities and the routes that connect them to the customers. Each facility has an opening cost  $O_i$  and a capacity  $W_i$ . Associated with each edge  $(i, j)$  there is a travelling cost  $c_{ij}$  and a travelling distance  $d_{ij}$ . Each customer  $j \in J$  has a demand  $D_j$  which must be fulfilled by a single vehicle. A set of  $K$  identical vehicles with capacity  $Q$  is available. Each vehicle, when used by a facility  $i \in I$ , incurs a depot dependent fixed cost  $F_i$  and performs a single route. The objective is to determine which depots should be opened and which routes should be constructed to minimize the total cost. Finally, this model was proposed based on the two-index formulation.

The standard CLRP is often criticized because its objective function combines facility locations determined at a strategic level, while vehicle routes are optimized at the operational level. In the facility problem the required cost to build facilities is a function of the output quantity. The incurred transportation cost is a function of shipment size. In both cases, costs are related to quantity to produce, hold in inventory or shipment size to final customers.

Various sorts of assets are frequently leased by business companies, such as offices and buildings, cars and trucks, commercial aircraft, production machinery, industrial equipment, etc. For this reason we propose to put all the values in the same time horizon according to that opening cost and route cost by considering the Net Present Value (NPV) to the lessee, as expressed in (8), see [51]:

$$NPV = V_0 - \sum_{t=0}^N \frac{I_t}{(1+r)^t} \equiv V_0 - I \quad (8)$$

with

$$I_t = L_t(1 - T) + D_t T \quad (9)$$

$$r = r_B(1 - T) \quad (10)$$

In these equations,  $V_0$  is the current value (cost) of the leased asset,  $L_t$  is the lease rental payment at time  $t$ ,  $D_t$  is the depreciation expense at time  $t$ ,  $T$  is the lessee's effective corporate tax rate,  $r_B$  is the before tax cost of borrowing, and  $N$  is the life, i.e. maturity of the lease.

In this way,  $O_i^{NPV}$  represents the NPV of the leasing cost associated to the use of a facility  $i \in I$  and  $F^{NPV}$  is the NPV of the leasing cost associated to the use of a vehicle, assuming  $F_k = F$ ,  $\forall k \in K$ .

The G-CLRP model is proposed with two objective functions to be minimized:

$$\Psi_1 = \sum_{i \in I} O_i^{NPV} y_i + \sum_{i \in I, j \in J} F^{NPV} a_{ij} + \sum_{i,j \in V} c_{ij} x_{ij} + \sum_{i \in I, j \in J} c_{ij} a_{ij} \quad (11)$$

and

$$\Psi_2 = \alpha E \left( \sum_{i,j \in V} d_{ij} x_{ij} + \sum_{i,j \in J} d_{ij} a_{ij} \right) + \gamma E \sum_{i,j \in V} d_{ij} t_{ij} \quad (12)$$

The first objective represents the operational cost, in which the first sum is the setup cost of facilities, using the NPV, the second sum is the associate cost to open the route, and the last two summations compute the routing cost. The second objective models the fuel consumption and the total emission associated to this fuel consumption.

The complete model is presented below.

$$\min \Psi_1, \Psi_2 \quad (13)$$

subject to:

$$\sum_{i \in V} x_{ij} = 1, \quad \forall j \in J \quad (14)$$

$$\sum_{k \in J} x_{jk} + \sum_{i \in I} a_{ij} = \sum_{i \in V} x_{ij}, \quad \forall j \in J \quad (15)$$

$$\sum_{j \in J} x_{ij} = \sum_{j \in J} a_{ij}, \quad \forall i \in I \quad (16)$$

$$x_{ij} + x_{ji} \leq 1, \quad \forall i, j \in V \quad (17)$$

$$\sum_{i \in V, i \neq j} t_{ij} = \sum_{k \in V, k \neq j} t_{jk} + D_j, \quad \forall j \in J \quad (18)$$

$$\sum_{i, j \in V} x_{ij} = |J|, \quad (19)$$

$$\sum_{i \in I} f_{ij} \leq 1, \quad \forall j \in J \quad (20)$$

$$t_{ij} \leq Qx_{ij}, \quad \forall i, j \in V \quad (21)$$

$$\sum_{j \in J} t_{ij} \leq W_i y_i, \quad \forall i \in I \quad (22)$$

$$\sum_{k \in V} x_{jk} = 1 - z_j, \quad \forall j \in J \quad (23)$$

$$1 + a_{ij} \geq f_{ij} + z_j, \quad \forall i \in I, \forall j \in J \quad (24)$$

$$-(1 - x_{ju} - x_{uj}) \leq f_{ij} - f_{iu}, \quad \forall i \in I, \forall j, u \in V \quad (25)$$

$$f_{ij} - f_{iu} \leq (1 - x_{ju} - x_{uj}), \quad \forall i \in I, \forall j, u \in V \quad (26)$$

$$f_{ij} \geq x_{ij}, \quad \forall i \in I, j \in J \quad (27)$$

$$\sum_{i \in I} y_i \geq \frac{\sum_j D_j}{\sum_i W_i}, \quad \forall i \in I \quad (28)$$

$$\sum_{j \in J} x_{ij} \leq W_i / Q \quad (29)$$

$$\sum_{i \in I, j \in J} x_{ij} \geq \sum_{j \in J} D_j / Q \quad (30)$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in V \quad (31)$$

$$y_i \in \{0, 1\} \quad \forall i \in I \quad (32)$$

$$f_{ij} \in \{0, 1\} \quad \forall i \in I, \forall j \in V \quad (33)$$

$$z_j \in \{0, 1\} \quad \forall j \in J \quad (34)$$

$$a_{ij} \in \{0, 1\} \quad \forall i \in I, \forall j \in J \quad (35)$$

$$t_{ij} \in \mathbb{R} \quad \forall i, j \in V \quad (36)$$

The model for the second objective function is developed in the previous section. If we assume constant inclination and constant speed,  $\alpha$  and  $\gamma$  are constants and the same for all edges. Nonetheless, if one considers distinct values for inclination and

speed in each edge,  $\alpha_{ij}$  and  $\gamma_{ij}$  should be calculated for each path. In any case, the model for the emission due to fuel consumption is linear.

The expression for the fuel consumption is formed by two parts. The first part, multiplied by  $\alpha$ , corresponds to the amount of energy required considering the unloaded vehicle, where  $x_{ij}$  are the active arcs and  $a_{ij}$  are the return arcs to the facility. The second term, multiplied by  $\gamma$ , corresponds to the amount of extra energy required considering the load carried on that arc.  $\alpha$  is a parameter representing how much energy an unloaded vehicle spends in crossing the arc and is given in J/km.  $\gamma$  is a parameter representing the additional energy (per unit of load) that a loaded vehicle spends in that arc and is given in J/km-ton. Finally, both terms are multiplied by  $E$ , which is the total emission per unit of energy, giving the total emission associated with the solution.

The G-CLRP model is formulated as a multi-objective mixed integer linear programming model, completely defined by (13)-(36).

Equation (14) establishes that the number of arcs arriving to a customer node must be 1, that is, every customer node is visited by a route. Equation (15) represents that the sum of the arcs of output of a demand is equal to the sum of the input arcs: may be a normal arc  $x$  or an arc back to a depot  $a$ . Equation (16) ensures that for a facility  $i$ , the number of output arcs  $x$  must be equal to the number of arcs of arrival  $a$ . To avoid duplication of the arcs, the orientation of an arc is defined, i.e., if the direction is from  $i$  to  $j$ , then Equation (17) is formulated. The flow balance for a node  $j$  is defined by (18), in terms of the incoming and outgoing flows and the demand at node  $j$ . The load transported by a vehicle through the arc  $(i, j)$  is represented by the positive flow variable  $t_{ij}$ .

The number of active arcs needed to connect all customer nodes is established by (19). Thus, it is assured that the routes are radials and do not have cycles. Equation (20) ensures that the demand for a route is connected to a facility. Equation (21) limits flow routes according to the capacity of the vehicles.

Equation (22) limits flows leaving a deposit according to the ability and decision to build the facility. Equation (23) identifies the terminal nodes of the routes when no output arc is demanded for that node. Constraint (24) ensures that if  $j$  is a terminal node, then the viewing constraint requires that there is a return arc. Constraints (25) and (26) ensure that the active arcs are connected to the same facility to form the route.

If the arc between facility  $i$  and demand  $j$  is active, then it is ensured that the node  $j$  is connected to the facility  $i$  through Equation (27). By the Equation (28), a lower limit to the number of deposits that must be constructed according to the sum of the demands and the capacity of the facility is determined. Using equation (29) the number of routes that can leave a deposit is restricted according with the facility capacity and vehicle capacity. Constraint (30) ensures that the number of routes is sufficient to attend all clients demand. Constraints (31)-(35) define the binary nature of the variables  $x_{ij}$ ,  $y_i$ ,  $f_{ij}$ ,  $z_j$  and  $a_{ij}$ . Finally, constraint (36) defines  $t_{ij}$  as continuous variable.

The new mathematical model proposed for the CLRP contains a second objective related to the emission cost, and relies



on radial constraints to ensure the routes are radial and have no cycles. The mathematical model is inspired by the radial power distribution networks [52], and their similarities with transportation distribution networks. A radial sub-graph corresponds to a minimum spanning tree if the sub-graph found is connected and has a number of arcs that is equal to the number of demand nodes. The model presented guarantees a minimum spanning tree through constraints (18)-(19). Constraints (14)-(16) ensure that the sub-graph found is composed of Hamiltonian routes, such that each demand node (client) must have a degree smaller or equal to two. Given the radially constraint, the routes are open and therefore the additional variables  $z_j$  and  $a_{ij}$  are used to close the routes with a return arc from the last node in the route back to the facility. Therefore, the proposed mathematical model ensures that feasible solutions must be connected and without cycles (thus avoiding the generation of sub-tours). In the case of problems with multiple depots, the reasoning remains valid. The only difference is that forests formed by Hamiltonian paths must be generated. There are no mathematical models in the literature for the CLRP based on this approach.

#### 4.4. Multi-objective optimization

Multi-objective optimization is the computational process of simultaneously optimizing two or more conflicting objectives subject to a set of constraint functions. For non-trivial multi-objective problems, there is not a single solution that simultaneously optimizes all objectives. Instead, there is set of solutions for which, when attempting to improve an objective, other objectives get worse. These solutions are called Pareto optimal or Pareto efficient solutions. Finding a representative set of such solutions, and quantifying the trade-offs in satisfying the different objectives, is the goal of setting up and solving a MOO problem. In multi-objective optimization, the main focus is on producing trade-off solutions representing the best possible compromises among different (possibly conflicting) objectives.

In order to employ a suitable concept of optimality, the Pareto-optimality is defined:

**Definition 4.1.** Let  $\Omega$  be a non-empty set of feasible solutions and  $\Psi(\cdot)$  be a vector of objective functions. A feasible solution  $x^* \in \Omega$  is called a Pareto optimal solution of the multi-objective optimization problem if and only if there does not exist any  $x \in \Omega$  such that  $\Psi(x) < \Psi(x^*)$ . The relation  $(\cdot < \cdot)$  denotes that each coordinate of the first argument is less than or equal to the corresponding coordinate of the second argument, and at least one coordinate of the first argument is strictly smaller than the corresponding coordinate of the second argument.

**Definition 4.2.** The image of the Pareto optimal set in the objective space is called the Pareto front.

In this paper, we approach the proposed multi-objective problem using an *a posteriori* methodology, in which the optimization returns some Pareto-optimal solutions, leaving the decision-making process to a post-optimization stage [53].

In order to solve the multi-objective problem, one can adopt parameterized scalar problems, the solution of which is a Pareto-optimal solution. For further discussion on multi-objective methods, we refer to [53, 54, 55]. In the  $\epsilon$ -constraint method, one objective is selected to be optimized, while the others are converted into inequality constraints by imposing upper bounds  $\epsilon$ .

$$\min_x \Psi_1(x), \quad (37)$$

$$\Psi_k(x) \leq \epsilon_k \quad k = 2, \dots, m \quad (38)$$

$$x \in \Omega$$

Cohon & Marks [56] show that the  $\epsilon$ -constraint method can be derived from the Kuhn-Tucker conditions for optimality for a MOO problem. A systematic variation of the parameters  $\epsilon_k$  can yield Pareto optimal solutions [53]. If it exists, a solution to the  $\epsilon$ -constraint formulation is weakly Pareto optimal, as shown in [54]. Moreover, if the solution is unique, then it is Pareto-optimal.

For more than two objectives, the  $\epsilon$ -constraint formulation can lead to infeasibility problems, for some combinations of values of  $\epsilon_k$ . Nevertheless, for two objectives, as is the case in our formulation, the method can yield Pareto optimal solutions with a systematic variation of  $\epsilon$ . Another advantages of using the  $\epsilon$ -constraint method in our problem are: (i) unlike other methods such as distance-to-goal approaches and penalty-based boundary intersection, the  $\epsilon$ -constraint formulation preserves the linearity of the original problem; (ii) only one constraint is added to model, since one of the objectives becomes an additional constraint, and one can choose which function is more convenient as an objective in practice; (iii) it is not sensitive to the shape of the Pareto front such as the classical weighted sum method.

In order to generate points on the Pareto front, first we optimize each objective individually with the original constraints of the model and neglecting the other objective. This yields the minimum and maximum values of each objective that contain the Pareto front. Intermediate points on the front are obtained with discrete steps, varying  $\epsilon$  within the minimum and maximum range.

The generation of the Pareto front is independent of which objective is chosen to be minimized. If we select  $\Psi_1$  as objective and convert  $\Psi_2$  into a constraint or vice-versa, the result is practically the same, the Pareto front is obtained regardless of the selected objective. Nonetheless, we have observed in our experiments that when the emission ( $\Psi_2$ ) is used as objective and the operational costs ( $\Psi_1$ ) are posed as a constraint, the model is solved much faster than when the converse is done, allowing the solution of larger instances.

## 5. Computational results

### 5.1. Configuration and parameters

In this study eight test scenarios were used. The first two scenarios are proposed by us and are used to show in detail the

main features of the results. One with 20 and another with 30 clients, and both with 5 depots, named G-CLRP 20-5 and G-CLRP 30-5, respectively. The numbers from 1 to 5 identify facilities and from 6 on identify customers to serve. The data is available online<sup>2</sup>. A homogeneous fleet of vehicles with capacity of 20 tons is considered. The remaining six test scenarios correspond to instances from the literature on the capacitated location-routing problem, as presented in [31], which include 20, 50 and 100 customers and from 5 to 10 facilities.

The calculation of consumption of the vehicle was taken from the report of the University of Michigan Transportation Research Institute [57], in which it is established that the average fuel consumption of a vehicle with these characteristics is 1 gallon per 15.81 km travelled. This value was used as a reference for calculating vehicle consumption at full load, which was estimated at 12 km per gallon.

As for the amount of emissions per gallon of gasoline, we consider 8.70645kg of  $CO_2$  per gallon, this information is related to the fuel consumption guide [58]. In this report  $CO_2$  emissions vary according to the type of fuel used and engine characteristics such as size, type, vehicle brand and optimum cruising speed. The cost of emissions is calculated based on quoted prices presented in [59]. Based on this, we consider the value of 0.009 USD per kg of  $CO_2$ . To quantify the price of a gallon of gasoline we consulted information available online<sup>3</sup>. This information is set according to the territory where the case study is contemplated. In our case, this parameter was set as 6.92 USD per gallon.

The mathematical model was solved under CPLEX 12.5 [60] on a computer Intel Core i7-4770 3.4 GHz, 16 GB of RAM and written in AMPL: A Modeling Language for Mathematical Programming [8].

The method used to generate the Pareto front is the  $\epsilon$ -constraint method, using  $\Psi_2$  as objective and  $\Psi_1$  as a constraint. In order to generate points on the Pareto front, first we optimize each objective individually with the original constraints of the model and neglecting the other objective. Assume:

$$s_1 = \arg \min_{x \in \Omega} \Psi_1(x) \quad (39)$$

$$s_2 = \arg \min_{x \in \Omega} \Psi_2(x) \quad (40)$$

This yields the minimum and maximum values of each objective that contain the Pareto front, in other words, the Pareto front is within the range  $[\Psi_1(s_1), \Psi_1(s_2)]$  in the first objective and the range  $[\Psi_2(s_2), \Psi_2(s_1)]$  in the second objective. Intermediate points on the front are obtained with discrete steps, varying  $\epsilon$  within the minimum and maximum range. More specifically, the  $i$ th point of the Pareto front can be obtained with:

$$\epsilon = \Psi_1(s_1) + \left( \frac{\Psi_1(s_2) - \Psi_1(s_1)}{N - 1} \right) \times i \quad (41)$$

where  $N$  is the number of discrete steps (number of desired points in the Pareto front).

## 5.2. Instances G-CLRP 20-5 and G-CLRP 30-5

Further detail about the solutions on the Pareto front for each case are shown in Table 1 and Table 2. Three solutions will be analyzed in each case: (i) the solution corresponding to the minimal value of  $\Psi_1$  (operational costs); (ii) the solution corresponding to the minimal value of  $\Psi_2$  (environmental impact); and (iii) the solution corresponding to the min max criterion, usually an intermediate point on the Pareto front.

It is intuitive that more fuel will be consumed if more vehicles are used. However, in the context of CLRP considering fuel consumption minimization, generally using more vehicles does not necessarily imply lengthening the travel distance. By observing Tables 1 and 2, for the instances considered, it is interesting to notice that by increasing the number of vehicles the fuel consumption and hence the total emission can be reduced. Using few vehicles at full capacity does not necessarily imply in fuel economy, whereas using more vehicles, not fully loaded, can translate into reduced fuel cost and less environmental impact. In the long term, by having more facilities and more routes, the fuel economy can balance out the initial setup costs. This same behavior can be observed in the results obtained on the instances of Prins et al. [31].

It is important to remark that in the results reported in Tables 1 and 2, each line of these tables corresponds to the solution of an  $\epsilon$ -constraint formulation, varying the value of  $\epsilon$  in order to produce different point of the Pareto front. In the  $\epsilon$ -constraint method, the generation of the Pareto front in a bi-objective problem is theoretically independent of which objective function is chosen as main objective (and which objective becomes a constraint). Nonetheless, in our computational experiments, we have observed that when emission ( $\Psi_2$ ) is used as main objective and the operational cost ( $\Psi_1$ ) is posed as a constraint, the model is solved much faster than when the opposite is done. In these results, we minimize  $\Psi_2$  while constraining the operational cost by the value of  $\epsilon$ , increasing  $\epsilon$  according to (41). In the first lines of these tables, the value of  $\epsilon$  is smaller, therefore the  $\epsilon$ -constraint is tighter, making the problem harder to solve. As the value of  $\epsilon$  increases, the  $\epsilon$ -constraint becomes less tight and less constraining to the model, which is why it is faster to solve.

Figure 2a shows the solution for CLRP 20-5 corresponding to the minimal value of  $\Psi_1$ , Figure 2b shows the solution corresponding to the minimal value of  $\Psi_2$ , and Figure 2c shows the min max solution.

The solution in Figure 2a presents one depot and five routes. The selected depot is located in a region that happens to be a load center, from which the largest possible number of customers are served, according to their capacity, keeping a relationship between distance and amount of load served. This optimal solution for  $\Psi_1$  reduces the cost of opening depots, cost of vehicles and freight. In this case the direction of travelling does not affect the result.

The solution in Figure 2b corresponds to the minimization of  $\Psi_2$  (environmental issues). In this scenario three depots are selected and six routes are identified. Notice that the distances travelled by vehicles are shorter compared to the previous scenario. When attending customers, priority is given to higher

<sup>2</sup><http://blog.utp.edu.co/problemasderuteo/2015/03/11/gclrp-instances/>

<sup>3</sup>[http://es.globalpetrolprices.com/gasoline\\_prices/](http://es.globalpetrolprices.com/gasoline_prices/)

Table 1: Computational results for the G-CLRP 20-5

No. of routes	No. of depots	$\Psi_1$ (USD)	$\Psi_2$ (kg CO <sub>2</sub> )	Cost of routes (USD)	Cost of depots (USD)	Cost of vehicles (USD)	Fuel cost (USD)	Fuel (gallon)	Run time (second)	
5	1	4764	180.6	2881	1383	500	79.8	20.4	10	
5	1	4764	177.2	2881	1383	500	79.8	20.4	16	
5	2	5222	174.2	2826	1896	500	78.5	20.0	14	
5	2	5639	166.3	2690	2449	500	74.9	19.1	13	
5	2	5727	153.5	2498	2729	500	69.1	17.6	13	
5	3	6185	150.5	2443	3242	500	67.8	17.3	9	
5	3	6602	142.6	2307	3795	500	64.3	16.4	7	
6	3	6719	142.3	2324	3795	600	64.1	16.3	3	
6	4	7232	142.3	2324	4308	600	64.1	16.3	3	
6	5	7790	142.3	2324	4866	600	64.1	16.3	5	

Table 2: Computational results for the G-CLRP 30-5

No. of routes	No. of depots	$\Psi_1$ (USD)	$\Psi_2$ (kg CO <sub>2</sub> )	Cost of routes (USD)	Cost of depots (USD)	Cost of vehicles (USD)	Fuel cost (USD)	Fuel (gallon)	Run time (second)	
7	2	6106	216.5	3510	1896	700	97.6	24.9	31318	
7	2	6331	198.6	3219	2412	700	89.5	22.8	1475	
7	3	6807	196.5	3182	2925	700	88.5	22.6	2309	
8	3	6920	195.5	3195	2925	800	88.1	22.5	1903	
7	3	7071	192.2	3129	3242	700	86.6	22.1	431	
7	3	7352	175.9	2857	3795	700	79.3	20.2	108	
8	3	7465	175.6	2870	3795	800	79.1	20.2	74	
7	4	7825	173.5	2817	4308	700	78.2	19.9	41	
8	4	7938	172.5	2830	4308	800	77.7	19.8	40	
8	4	8496	172.5	2830	4866	800	77.7	19.8	46	

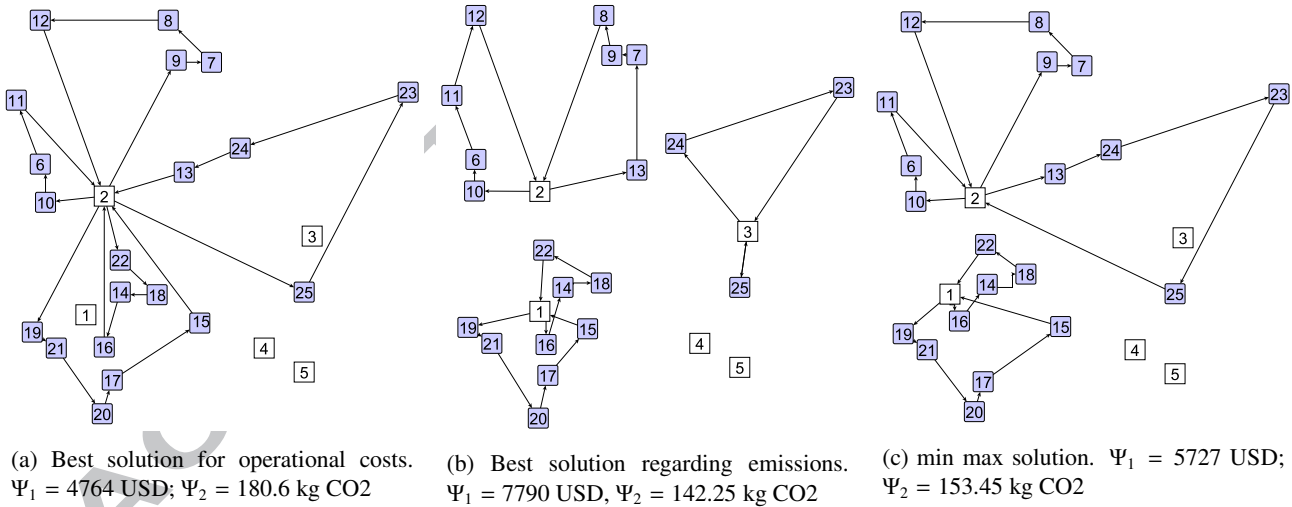


Figure 2: Three solutions of optimal Pareto front: G-CLRP 20-5.

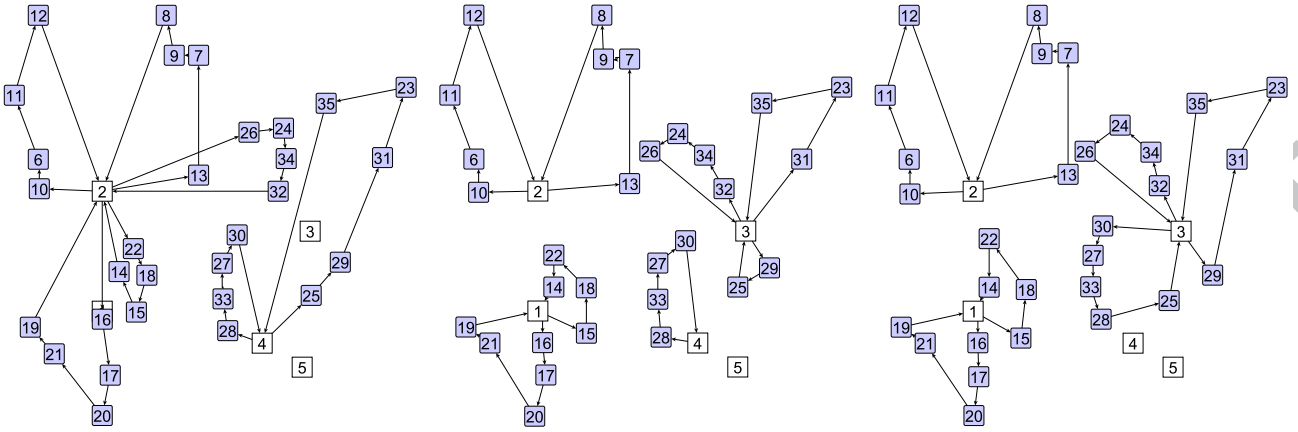
demand and/or closest customer, since making larger deliveries in the shortest time possible decreases the fuel consumption and consequently emission of greenhouse gases. In this case the direction of travel of the vehicle is important, influencing the impact of emissions.

In the third scenario, Figure 2c, the min max solution combines operational costs and environmental impact. Under these new conditions two depots are opened and five routes are selected. There is a clear balance between operational costs and emissions by fuel consumption. The consideration of reducing

emissions gives priority to larger deliveries as soon as possible, also identifying short routes and taking into account the direction of the vehicle. The operational cost influences the result in terms of number of depots and number of routes used.

Figure 3a shows the solution for CLRP 30-5 corresponding to the minimal value of  $\Psi_1$ , Figure 3b shows the solution corresponding to the minimal value of  $\Psi_2$ , and Figure 3c shows the min max solution.

Solution in Figure 3a is the optimal solution for the G-CLRP 30-5 considering operational costs ( $\Psi_1$ ). The solution presents



(a) Best solution for operational costs.  $\Psi_1 = 6106$  USD;  $\Psi_2 = 216.5$  kg CO<sub>2</sub>  
 (b) Best solution regarding emissions.  $\Psi_1 = 8496$  USD,  $\Psi_2 = 172.46$  kg CO<sub>2</sub>  
 (c) min max solution.  $\Psi_1 = 7352$  USD;  $\Psi_2 = 175.94$  kg CO<sub>2</sub>

Figure 3: Three solutions of optimal Pareto front: G-CLRP 30-5.

two depots and seven routes. As in the previous case, the optimal solution here serves the customers using the smallest possible number of depots and vehicles. The solution is characterized by few long routes.

In Figure 3b, the solution that minimizes emissions ( $\Psi_2$ ) presents four depots and eight routes. The routes are short, with priority given to the customers with higher demand, in order to reduce the weight of the loaded vehicle, hence reducing fuel required by the vehicles.

In Figure 3c it is possible to see the min max solution for the G-CLRP 30-5, with three depots and seven routes. As before, it represents a tradeoff between operational costs and environmental impact as modeled in this paper. This solution tries to balance the number of depots used and the amount of fuel consumed.

### 5.3. Instances from the literature

In this subsection, we have used six instances from Prins et al. [31] in order to verify the model in additional instances and to see if the behavior observed with the previous instances can be confirmed. In other words, we want to verify if, in the context of CLRP considering fuel consumption minimization, by increasing the number of vehicles the fuel consumption and hence total emission can be reduced indeed.

The results in the first five instances in Table 3 show the Pareto-optimal fronts obtained. The solution corresponding to the minimization of  $\Psi_1$  matches those solutions represented as BKS in [61]. By minimizing  $\Psi_1$ , the solution is again characterized by small number of depots and vehicles, hence reducing initial operational costs. The routes of the vehicles tend to be longer leading to higher fuel consumption. Contrastingly, by minimizing  $\Psi_2$ , the environmental impact is greatly reduced at the expense of higher initial costs. In this case, more vehicles are used and allocated to fewer costumers in shorter routes.

The last instance in Table 3 corresponds to a large size instance (instance 100\_10\_1), which is harder to solve with mathematical programming methods. Nevertheless, the proposed

model presented good scalability up to instances with 50 customer nodes and 5 depots. In that instance with 100 customers and 10 depots, we were able to obtain solutions with GAP inferior to 4%. Although the obtained front is an approximation of the true Pareto-optimal front, it is possible to identify the same behavior noticed before. The overall trend is also observed here in this large size instance, that is, using more vehicles in shorter routes can help reducing the cost of emissions.

It is important to highlight that the solutions obtained with the relaxation of  $\Psi_1$  correspond to minimizing the Pollution-Routing Problem (PRP), considering the parameters presented in the beginning of this section. Therefore, these results can be considered as a reference for future studies involving the single objective PRP.

The results also show that the min-max criterion for selecting one solution from the Pareto front could be a good compromise between investment cost and environmental impact in the long term, but of course the availability of a multi-objective approach for the G-CLRP allows for the use of more sophisticated multi-criteria decision-making methods, tailored for the specific application.

## 6. Conclusions and future work

This paper proposed a multi-objective model for the G-CLRP considering fuel consumption. The model proposed is a bi-objective problem, considering the minimization of operational costs and the minimization of environmental effects. The computation of the fuel emission is developed from a mathematical model that could in principle be applied to more realistic scenarios, if one considers distinct inclinations and speeds for each edge ( $i, j$ ).

The use of the radial constraint in the G-CLRP model has shown to be an alternative way to deal with the problem of removing sub tours. The proposed mathematical model can be a reference for approaching larger instances, when other strategies for the solution should be employed, such as hybrid tech-

Table 3: Computational results for instances used by Prins et al. [31]

Instance	No. of routes	No. of depots	$\Psi_1$ (USD)	$\Psi_2$ (kg CO <sub>2</sub> )	Cost of routes (USD)	Cost of depots (USD)	Cost of vehicles (USD)	Fuel cost (USD)	Fuel (gallon)	Run time (second)	Optimal Pareto front
20_5_1a	6	5	72054	151960	22094	43960	6000	120869	17466	4022	
	6	4	61261	152290	22142	33119	6000	121132	17505	9451	
	5	3	55987	164785	23959	27028	5000	131070	18941	15210	
	5	3	54793	166750	24244	25549	5000	132633	19166	1279	
20_5_1b	3	3	74946	133032	19342	52604	3000	105814	15291	11	
	3	2	44614	139772	20322	21292	3000	111175	16066	10	
	3	2	39104	141736	20607	15497	3000	112735	16291	9	
20_5_2a	6	5	71321	130170	18926	46395	6000	103538	14962	69	
	6	4	57562	132172	19217	32345	6000	105130	15192	173	
	5	4	56734	133355	19389	32345	5000	106071	15328	213	
	5	3	48908	135578	19712	24196	5000	107839	15584	341	
50_5_1	13	4	113150	352616	51268	35186	13000	280472	40530	7236	
	12	4	98777	354839	51591	35186	12000	282240	40786	15604	
	12	3	90111	362256	52669	25442	12000	288139	41639	8935	
50_5_2bbis	6	5	65097	184467	26820	32277	6000	146725	21203	80234	
	6	4	59219	185120	26915	26304	6000	147245	21278	17647	
	6	3	51822	186110	27059	18763	6000	148033	21392	44087	
100_10_1	28	10	627547	463555	67398	532149	28000	368713	53282	26312	
	28	9	580375	468322	68091	484284	28000	372505	53830	18138	
	28	8	525578	477004	69353	428225	28000	379410	54828	22184	
	27	7	469871	484337	70419	372452	27000	385243	55671	22181	
	26	3	289755	748414	108813	154942	26000	595290	86025	40000	

niques including set partitioning and heuristics and matheuristics<sup>4</sup>. The results presented here might be considered as a reference point for future work, since there are not results for the G-CLRP in the literature.

The multi-objective approach to this problem is entirely appropriate, since the tradeoff between economical aspects and environmental aspects can be selected from the Pareto front using a decision-making process. Perhaps the most interesting conclusion from this study is that using more vehicles can lead to large fuel economy in the long term and hence less emission. From the point of view of environmental impact, more vehicles performing shorter routes and serving as soon as possible those customers with higher demand seems to be the preferred strategy.

In future work we should explore other methodologies for the calculation of fuel consumption and should consider aspects such as the slope of the road and vehicle speed in different

edges.

## References

- [1] S. E. Daniel, D. C. Diakoulaki, C. P. Pappis, Operations research and environmental planning, *European Journal of Operational Research* 102 (2) (1997) 248–263.
- [2] J. Q. Frota Neto, G. Walther, J. Bloemhof, J. A. E. E. van Nunen, T. Spengler, A methodology for assessing eco-efficiency in logistics networks, *European Journal of Operational Research* 193 (3) (2009) 670–682.
- [3] C. Lin, K. L. Choy, G. T. S. Ho, S. H. Chung, H. Y. Lam, Survey of green vehicle routing problem: Past and future trends, *Experts Systems with Applications* 41 (4) (2014) 1118–1138.
- [4] D. Simchi-Levi, P. Kaminsky, E. Simchi-Levi, Designing and managing the supply chain: concepts, strategies and case studies, McGraw-Hill/Irwin, New York, 1999.
- [5] T. Bektas, G. Laporte, The pollution-routing problem, *Transportation Research Part B* 45 (8) (2011) 1232–1250.
- [6] S. Erdogan, E. Miller-Hooks, A green vehicle routing problem, *Transportation Research Part E: Logistics and Transportation Review* 48 (1) (2012) 100–114.
- [7] E. Demir, T. Bektas, G. Laporte, The bi-objective pollution-routing problem, *European Journal of Operational Research* 223 (3) (2014) 464–478.

<sup>4</sup>Integration of metaheuristics and mathematical programming.



- [8] R. Fourer, D. M. Gay, B. W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*, 2nd Edition, Brooks/Cole-Thomson, 2002.
- [9] C. Contardo, J.-F. Cordeau, B. Gendron, A computational comparison of flow formulations for the capacitated location-routing problem, *Discrete Optimization* 10 (4) (2013) 263–295.
- [10] Y. Chan, S. F. Baker, The multiple depot, multiple traveling salesmen facility location problem: Vehicle range, service frequency, and heuristic implementations, *Mathematical and Computer Modelling* 41 (8-9).
- [11] C. K. Y. Lin, R. C. W. Kwok, Multi-objective metaheuristics for a location-routing problem with multiple use of vehicles on real data and simulated data, *European Journal of Operational Research* 175 (3) (2006) 1833–1849.
- [12] R. E. Burks, J. T. Moore, J. W. Barnes, J. E. Bell, Solving the theater distribution problem with tabu search, *Military Operations Research* 15 (4) (2010) 5–26.
- [13] R. B. Lopes, S. Barreto, C. Ferreira, B. S. Santos, A decision-support tool for a capacitated location-routing problem, *Decision Support Systems* 46 (1) (2008) 366–375.
- [14] Y. Marinakis, M. Marinaki, A bilevel genetic algorithm for a real life location routing problem, *International Journal of Logistics Research and Applications* 11 (1) (2008) 49–65.
- [15] D. Ambrosino, A. Sciomachen, M. G. Scutell, A heuristic based on multi-exchange techniques for a regional set assignment location-routing problem, *Computers & Operations Research* 36 (2) (2009) 442–460.
- [16] P. Schittekat, K. Sorensen, OR practice – supporting 3PL decisions in the automotive industry by generating diverse solutions to a large-scale location-routing problem, *Operations Research* 57 (5) (2009) 1058–1067.
- [17] S. Cetiner, C. Sepil, H. Sural, Hubbing and routing in postal delivery systems, *Annals of Operations Research* 181 (1) (2010) 109–124.
- [18] J. Ahn, O. D. Weck, Y. Geng, D. Klabjan, Column generation based heuristics for a generalized location routing problem with profits arising in space exploration, *European Journal of Operational Research* 223 (2) (2012) 47–59.
- [19] R. T. Berger, C. R. Coullard, M. S. Daskin, Location-routing problems with distance constraints, *Transportation Science* 41 (1) (2007) 29–43.
- [20] Z. Akca, R. T. Berger, T. K. Ralphs, A branch-and-price algorithm for combined location and routing problems under capacity restrictions, in: J. W. Chinneck, B. Kristjansson, M. J. Saltzman (Eds.), *Operations Research and Cyber-infrastructure*, Vol. 47 of *Operations Research/Computer Science Interfaces Series*, Springer, 2009, pp. 309–330.
- [21] J.-M. Belenguer, E. Benavent, C. Prins, C. Prodhon, R. W. Calvo, A branch and cut algorithm for the capacitated location routing problem, *Computers & Operations Research* 38 (6) (2011) 931–941.
- [22] M. A. Baldacci, R. Hadjiconstantinou, An exact algorithm for the capacitated vehicle routing problem based on a two-commodity network flow formulation, *Operations Research* 52 (2004) 723–738.
- [23] C. Contardo, J.-F. Cordeau, B. Gendron, A computational comparison of flow formulations for the capacitated location-routing problem, Tech. rep., CIRRELT (2011).
- [24] C. Contardo, J.-F. Cordeau, B. Gendron, An exact algorithm based on cut-and-column generation for the capacitated location-routing problem, *INFORMS Journal on Computing* 26 (2013) 88–102.
- [25] S. Salhi, G. Rand, The effect of ignoring routes when locating depots, *European Journal of Operational Research* 39 (2) (1989) 150–156.
- [26] M. Albareda-Sambola, J. Díaz, E. Fernández, A compact model and tight bounds for a combined location-routing problem, *Computers & Operations Research* 32 (3) (2005) 407–428.
- [27] S. Barreto, C. Ferreira, J. Paixao, B. Santos, Using clustering analysis in a capacitated location-routing problem, *European Journal of Operational Research* 19 (3) (2007) 968–977.
- [28] C. Prins, C. Prodhon, R. W. Calvo, Solving the capacitated location-routing problem by a grasp complemented by a learning process and path relinking, *4OR A Quarterly Journal of Operations Research* 4 (3) (2006) 221–238.
- [29] C. Prins, C. Prodhon, R. W. Calvo, A memetic algorithm with population management (MA—PM) for the capacitated location-routing problem, in: J. Gottlieb, G. R. Raidl (Eds.), *Proceedings of the 6th European Conference on Evolutionary Computation in Combinatorial Optimization, EvoCOP 2006*, Vol. 3906 of *Lecture Notes in Computer Science*, Springer, Budapest, Hungary, 2006, pp. 183–194.
- [30] C. Duhamel, P. Lacomme, C. Prins, C. Prodhon, A GRASP x ELS approach for the capacitated location-routing problem, *Computers & Operations Research* 37 (11) (2010) 1912–1923.
- [31] C. Prins, C. Prodhon, A. Ruiz, P. Soriano, R. W. Calvo, Solving the capacitated location-routing problem by a cooperative lagrangean relaxation-granular tabu search heuristic, *Transportation Science* 41 (4) (2007) 470–483.
- [32] C. Chen, C. Ting, A hybrid lagrangian heuristic/simulated annealing algorithm for the multi-depot location routing problem, *Journal of Heuristics* 6 (2007) 137–150.
- [33] J. Perl, M. S. Daskin, A warehouse location-routing methodology, *Journal of Business Logistics* 5 (1) (1984) 92–111.
- [34] P. H. Hansen, B. O. B. Hegedahl, S. Hjortkjær, A heuristic solution to the warehouse location-routing problem, *European Journal of Operational Research* 76 (1) (1994) 111–127.
- [35] X. Wang, X. Sun, Y. Fang, A two-phase hybrid heuristic search approach to the location-routing problem, in: *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*, Vol. 4, IEEE, 2005, pp. 3338–3343.
- [36] J. W. Escobar, R. Linfati, P. Toth, A two-phase hybrid heuristic algorithm for the capacitated location-routing problem, *Computers & Operations Research* 40 (1) (2013) 70–79.
- [37] C. Groër, B. Golden, E. Wasil, A library of local search heuristics for the vehicle routing problem, *Mathematical Programming Computation* 2 (2) (2010) 79–101.
- [38] M. Drexl, M. Schneider, A survey of location-routing problems, Tech. rep., Gutenberg School of Management and Economics, Johannes Gutenberg University, IM-2013-03 (2013).  
URL <http://logistik.bwl.uni-mainz.de/Dateien/IM-2013-03.pdf>
- [39] C. Prodhon, C. Prins, A survey of recent research on location-routing problems, *European Journal of Operational Research* 238 (1) (2014) 1–17.
- [40] A. Palmer, The development of an integrated routing and carbon dioxide emissions model for goods vehicles, Ph.D. thesis, School of Management, Cranfield University (November 2007).
- [41] K. Boriboonsomsin, A. Vu, M. Barth, Eco-driving: pilot evaluation of driving behavior changes among U.S. drivers, Tech. rep., University of California Transportation Center, uCTC-FR-2010-20 (2010).  
URL <http://www.uctc.net/research/papers/UCTC-FR-2010-20.pdf>
- [42] Y. Xiao, Q. Zhao, I. Kaku, Y. Xu, Development of a fuel consumption optimization model for the capacitated vehicle routing problem, *Computers & Operations Research* 39 (7) (2012) 1419–1431.
- [43] I. Kara, B. Y. Kara, M. K. Yetis, Energy minimizing vehicle routing problem, in: A. Dress, Y. Xu, B. Zhu (Eds.), *Proceedings of the First International Conference on Combinatorial Optimization and Applications, COCOA 2007*, Vol. 4616 of *Lecture Notes in Computer Science*, Springer, Xi'an, China, 2007, pp. 62–71.
- [44] M. A. Figliozzi, An iterative route construction and improvement algorithm for the vehicle routing problem with soft time windows, *Transportation Research Part C: Emerging Technologies* 18 (5) (2010) 668–679.
- [45] Y. Suzuki, A new truck-routing approach for reducing fuel consumption and pollutants emission, *Transportation Research Part D: Transport and Environment* 16 (1) (2011) 73–77.
- [46] L. Pradenas, B. Oportus, V. Parada, Mitigation of greenhouse gas emissions in vehicle routing problems with backhauling, *Expert Systems with Applications* 40 (8) (2013) 2985–2991.
- [47] S. Ubeda, F. J. Arcelus, J. Faulin, Green logistics at Eroski: A case study, *International Journal of Production Economics* 131 (1) (2011) 44–51.
- [48] E. Demir, T. Bektaş, G. Laporte, An adaptive large neighborhood search heuristic for the pollution-routing problem, *European Journal of Operational Research* 223 (2) (2012) 346–359.
- [49] J. Jemai, M. Zekri, K. Mellouli, An NSGA-II algorithm for the green vehicle routing problem, in: J.-K. Hao, M. Middendorf (Eds.), *Proceedings of the 12th European Conference on Evolutionary Computation in Combinatorial Optimization, EvoCOP 2012*, Vol. 7245 of *Lecture Notes in Computer Science*, Springer, Málaga, Spain, 2012, pp. 37–48.
- [50] İlker Küçükoğlu, S. Ene, A. Aksoy, N. Öztürk, A green capacitated vehicle routing problem with fuel consumption optimization model, *Internat-*

- tional Journal of Computational Engineering Research 3 (7) (2013) 16–23.
- [51] L. Trigeorgis, Real Options: Managerial Flexibility and Strategy in Resource Allocation, The MIT Press, 1996.
  - [52] M. Lavorato, J. F. Franco, M. J. Rider, R. Romero, Imposing radiality constraints in distribution system optimization problems, IEEE Transactions on Power Systems 27 (1) (2012) 172–180.
  - [53] T. Marler, J. S. Arora, Multi-Objective Optimization: Concepts and Methods for Engineering, VDM Verlag Dr. Müller, 2009.
  - [54] K. Miettinen, Nonlinear Multiobjective Optimization, Kluwer Academic Publishers, Boston, 1999.
  - [55] M. Ehrgott, X. Gandibleux, Multiobjective combinatorial optimization – theory, methodology, and applications, in: X. Gandibleux (Ed.), Multiple Criteria Optimization: State of the Art Annotated Bibliographic Surveys, International Series in Operations Research & Management Science, Springer, 2002, pp. 369–444.
  - [56] J. L. Cohon, D. H. Marks, A review and evaluation of multiobjective programming techniques, Water Resources Research 11 (2) (1975) 208–220.
  - [57] Transportation Research Institute, Large drop in fuel economy in september (October 2014).  
URL <http://www.umtri.umich.edu/what-were-doing/news/large-drop-fuel-economy-september>
  - [58] 2015 fuel consumption guide (May 2015).  
URL <http://www.nrcan.gc.ca/energy/efficiency/transportation/cars-light-trucks/buying/7487>
  - [59] SENDECO2, The European bourse for European Unit Allowances (EUA) and Carbon Credits (CER's) (September 2014).  
URL <http://www.sendeco2.com/>
  - [60] CPLEX optimization subroutine library guide and reference, version 11.0 (2008).
  - [61] J. W. Escobar, R. Linfati, M. G. Baldoquin, P. Toth, A granular variable tabu neighborhood search for the capacitated location-routing problem, Transportation Research Part B: Methodological 67 (2014) 344–356.

**Highlights**

The Capacitated Location-Routing Problem considering environmental impact is proposed.

A new model for computing greenhouse gas emissions in vehicle routing is proposed.

The Green CLRP is formulated as a bi-objective mixed integer linear programming.

Using more vehicles can lead to large fuel economy in the long term and hence less emission.

More vehicles in shorter routes and prioritizing high demand clients lead to less emission.