## Statistics for Biology and Health

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# Regression Methods in Biostatistics

Linear, Logistic, Survival, and Repeated Measures Models

Second edition



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For Rupert & Jean; Kay & Minerva; Caroline, Erik & Hugo; and J.R.

### **Preface**

In the second edition of Regression Methods in Biostatistics: Linear, Logistic, Survival, and Repeated Measures Models, we have substantially revised and expanded the core chapters of the first edition, and added two new chapters. The first of these, Chap. 9, on strengthening causal inference, introduces potential outcomes, average causal effects, and two primary methods for estimating these effects, what we call potential outcomes estimation and inverse probability weighting. It also covers propensity scores in detail, then more briefly discusses time-dependent exposures, controlled and natural direct effects, instrumental variables, and principal stratification. The second, Chap. 11, on missing data, explains why this is a problem, classifies missingness by mechanism, and discusses the shortcomings of some simple approaches. Its focus is on three primary approaches for dealing with missing data: maximum likelihood estimation, multiple imputation, and inverse weighting, and lays out in detail when each of these approaches is most appropriate.

Among the core chapters of the first edition, Chap. 5, on logistic regression, has substantial new sections on models for ordinal and multinomial outcomes, as well as exact logistic regression. Chapter 6, on survival analysis, has an in-depth new section on competing risks, as well as new coverage of interval censoring and left truncation. Chapter 7, on repeated measures analysis, introduces recently developed methods for distinguishing between- and within-cluster effects, and for estimating the effects of fixed and time-dependent covariates (TDCs) on change. Chapter 8, on generalized linear models, adds coverage of negative binomial as well as zeroinflated and zero-truncated models for counts. Chapters 4-8 all now cover restricted cubic splines, take a new approach to mediation, and provide methods for sample size, power, and detectable effect calculation. Chapter 10, on predictor selection, has expanded coverage of developing and assessing models for prediction, as well as a new section on directed acyclic graphs. Our summary in Chap. 13 includes a new discussion of multiple comparisons and updated coverage of software packages. All Stata examples have been updated. As before, Stata, SAS, and Excel datasets and Stata do-files for most examples are provided on the website for the book, http://www.biostat.ucsf.edu/vgsm. We also posted implementations of analyses for time-dependent exposures too complicated for inclusion in the text.

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At UCSF, we have used the first edition for a two-quarter course on regression methods for clinical researchers and epidemiologists, the first quarter covering linear and logistic models and predictor selection, and the second covering survival and repeated measures analysis. The new chapter on strengthening causal inference is the basis of new quarter-long course, and the new missing data chapter will play an important role in a more advanced quarter-long course next year. The new breadth of coverage of the second edition should make it more widely useful in year-long biostatistics courses for students like ours, MPH students, and for masters-level courses in biostatistics.

Finally, we gratefully acknowledge the very important contributions made by Professors Joseph Hogan of Brown University, Michael Hudgens of the University of North Carolina, Barbara McKnight of the University of Washington, and Maya Peterson of the University of California, Berkeley, who generously provided detailed, insightful reviews of the two new chapters. Any remaining errors and shortcomings are of course entirely ours.

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### **Preface to the First Edition**

The primary biostatistical tools in modern medical research are single-outcome, multiple-predictor methods: multiple linear regression for continuous outcomes, logistic regression for binary outcomes, and the Cox proportional hazards model for time-to-event outcomes. More recently, generalized linear models (GLMs) and regression methods for repeated outcomes have come into widespread use in the medical research literature. Applying these methods and interpreting the results require some introduction. However, introductory statistics courses have no time to spend on such topics and hence they are often relegated to a third or fourth course in a sequence. Books tend to have either very brief coverage or to be treatments of a single topic and more theoretical than the typical researcher wants or needs.

Our goal in writing this book was to provide an accessible introduction to multipredictor methods, emphasizing their proper use and interpretation. We feel strongly that this can only be accomplished by illustrating the techniques using a variety of real data sets. We have incorporated as little theory as feasible. Further, we have tried to keep the book relatively short and to the point. Our hope in doing so is that the important issues and similarities between the methods, rather than their differences, will come through. We hope this book will be attractive to medical researchers needing familiarity with these methods and to students studying statistics who would like to see them applied to real data. The methods we describe are, of course, the same as those used in a variety of fields, so non-medical readers will find this book useful if they can extrapolate from the predominantly medical examples.

A prerequisite for the book is a good first course in statistics or biostatistics or an understanding of the basic tools: paired and independent samples t-tests, simple linear regression and one-way analysis of variance (ANOVA), contingency tables and  $\chi^2$  (chi-square) analyses, Kaplan–Meier curves, and the logrank test.

We also think it is important for researchers to know how to interpret the output of a modern statistical package. Accordingly, we illustrate a number of the analyses with output from the Stata statistics package. There are a number of other packages that can perform these analyses, but we have chosen this one because of its accessibility and widespread use in biostatistics and epidemiology.

We begin the book with a chapter introducing our viewpoint and style of presentation and the big picture as to the use of multipredictor methods. Chapter 2 presents descriptive numerical and graphical techniques for multipredictor settings and emphasizes choice of technique based on the nature of the variables. Chapter 3 briefly reviews the statistical methods we consider prerequisites for the book.

We then make the transition in Chap. 4 to multipredictor regression methods, beginning with the linear regression model. This chapter also covers confounding, mediation, interaction, and model checking in the most detail. In Chap. 5, we turn to binary outcomes and the logistic model, noting the similarities to the linear model. Ties to simpler, contingency table methods are also noted. Chapter 6 covers survival outcomes, giving clear indications as to why such techniques are necessary, but again emphasizing similarities in model building and interpretation with the previous chapters. Chapter 7 looks at the accommodation of correlated data in both linear and logistic models. Chapter 8 extends Chap. 5, giving an overview of GLMs.

In the second edition, new sections of Chaps. 4–8 deal with pooled and exact logistic regression (Chap. 5), competing risks (Chap. 6), and time-varying predictors and separating between and within cluster information (Chap. 7). Chapters 4–8, also now conclude with short sections on calculating sample size, power, and minimum detectable effects.

The next three chapters, two of them new in the second edition, cover broader issues. Chapter 9 looks more closely at making causal inferences, using the models discussed in Chaps. 4–8, as well as alternatives including propensity scores and instrumental variables. Chapter 10 deals with predictor selection, with expanded treatment of methods for prediction problems. Chapter 11 considers missing data and methods for dealing with it, including maximum likelihood models, multiple imputation, and complete case analysis, the problematic default.

Finally, Chap. 12 is a brief introduction to the analysis of complex surveys. The text closes with a summary, Chap. 13, attempting to put each of the previous chapters in context. Too often it is hard to see the forest for the trees of each of the individual methods. Our goal in this final chapter is to provide guidance as to how to choose among the methods presented in the book and also to realize when they will not suffice and other techniques need to be considered.

San Francisco, CA, USA

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