Hw 1

1. Proof the following. If A is an $n \times p$ matrix and B a $p \times q$ matrix, then the product $C = AB$ has the following properties:

a) Every column of C is a linear combination of columns of A.

b) Every row of C is a linear combination of rows of B.

2. Show directly that if A is an $n \times p$ matrix and B is $p \times n$, then

$$(I_n - AB)^{-1} = I_n + A(I_p - BA)^{-1}B$$

provided the inverses exist. (This verifies a simplified version of the Woodbury binomial inverse theorem.)

3. Verify that the Moore-Penrose inverse A^+ of a symmetric matrix A is symmetric. (Thus, $A^{+'}$ is also a Moore-Penrose inverse of A.)

4. Use the fact that $r(AB) \leq \min(r(A), r(B))$, show that

$$r(A) = r(PA), r(A) = r(AQ)$$

if P and Q are invertible matrices.