ShanghaiTech University

CS240 Algorithm Design and Analysis

Fall 2023

Problem Set 3

Due: 23:59, Dec. 12, 2023

1. Submit your solutions to Gradescope (www.gradescope.com).
2. In "Account Settings" of Gradescope, set your FULL NAME to your Chinese name and enter your STUDENT ID correctly.
3. If you want to submit a handwritten version, scan it clearly. Camscanner is recommended.
4. When submitting your homework, match each of your solution to the corresponding problem number.

Problem 1:

NAE-4-SAT: A clause in an instance of 4-SAT is a disjunction with four literals i.e. $(x_1 \vee x_2 \vee x_3 \vee x_4)$. A satisfied assignment for a collection of clauses is called *not all equal* (NAE) if the literals in each clause are not all equal to each other. In other words, at least one false, and at least one true. Prove NAE-4-SAT is NP-complete, using a reduction from 3-SAT.

Problem 2:

NAE-3-SAT: Prove NAE-3-SAT is NP-complete.

Problem 3:

4-SAT: Prove 4-SAT is NP-complete.

Problem 4:

Feedback Vertex Set problem Let $G = (V, E)$ be a directed graph. A set $F \subseteq V$ is a *feedback vertex set* if every cycle of G contains at least one vertex from F. The *Feedback Vertex Set problem* asks whether G has a feedback vertex set with at most K vertices. Show that Feedback Vertex Set is NP-complete. (Hint: use Vertex Cover problem for reduction.)

Vertex Cover Problem

Given an undirected graph $G' = (V', E')$ and a number K', the Vertex Cover problem asks if there is a vertex cover of size at most K', i.e., a subset of vertices $V'' \subseteq V'$ such that every edge in E' is incident to at least one vertex in V''.

Problem 5:

STINGY SAT is the following problem: given a set of clauses (each a disjunction of literals) and an integer k, find a satisfying assignment in which at most k variables a true, if such an assignment exists. Prove that STINGY SAT is NP-complete.

Problem 6:

Given a tree T, a set of terminal vertices, and an integer k, is there a set of at most k edges which, when removed from T, separates every pair of terminal vertices? Prove that this problem is NP-complete.