softmax

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1 Softmax exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the assignments page on the course website.

This exercise is analogous to the SVM exercise. You will:

- implement a fully-vectorized **loss function** for the Softmax classifier
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** with numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

[1]:
```
import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading extenrnal modules
# see http://stackoverflow.com/questions/1907993/
 ↪autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2
```

[2]:
```
def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test=1000,␣
 ↪num_dev=500):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
    it for the linear classifier. These are the same steps as we used for the
    SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
```

```
    cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'

    # Cleaning up variables to prevent loading data multiple times (which may␣
↪cause memory issue)
    try:
       del X_train, y_train
       del X_test, y_test
       print('Clear previously loaded data.')
    except:
       pass

    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # subsample the data
    mask = list(range(num_training, num_training + num_validation))
    X_val = X_train[mask]
    y_val = y_train[mask]
    mask = list(range(num_training))
    X_train = X_train[mask]
    y_train = y_train[mask]
    mask = list(range(num_test))
    X_test = X_test[mask]
    y_test = y_test[mask]
    mask = np.random.choice(num_training, num_dev, replace=False)
    X_dev = X_train[mask]
    y_dev = y_train[mask]

    # Preprocessing: reshape the image data into rows
    X_train = np.reshape(X_train, (X_train.shape[0], -1))
    X_val = np.reshape(X_val, (X_val.shape[0], -1))
    X_test = np.reshape(X_test, (X_test.shape[0], -1))
    X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

    # Normalize the data: subtract the mean image
    mean_image = np.mean(X_train, axis = 0)
    X_train -= mean_image
    X_val -= mean_image
    X_test -= mean_image
    X_dev -= mean_image

    # add bias dimension and transform into columns
    X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
    X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
    X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
    X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

    return X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev
```

```
# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev =␣
 ↪get_CIFAR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
print('dev data shape: ', X_dev.shape)
print('dev labels shape: ', y_dev.shape)
```

```
Train data shape:  (49000, 3073)
Train labels shape:  (49000,)
Validation data shape:  (1000, 3073)
Validation labels shape:  (1000,)
Test data shape:  (1000, 3073)
Test labels shape:  (1000,)
dev data shape:  (500, 3073)
dev labels shape:  (500,)
```

1.1 Softmax Classifier

Your code for this section will all be written inside `cs231n/classifiers/softmax.py`.

[5]:
```
# First implement the naive softmax loss function with nested loops.
# Open the file cs231n/classifiers/softmax.py and implement the
# softmax_loss_naive function.

from cs231n.classifiers.softmax import softmax_loss_naive
import time

# Generate a random softmax weight matrix and use it to compute the loss.
W = np.random.randn(3073, 10) * 0.0001
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# As a rough sanity check, our loss should be something close to -log(0.1).
print('loss: %f' % loss)
print('sanity check: %f' % (-np.log(0.1)))
```

```
loss: 2.355453
sanity check: 2.302585
```

Inline Question 1

Why do we expect our loss to be close to -log(0.1)? Explain briefly.**

YourAnswer : Because if a classifier randomly predict what the label is, the distribution should be *uniform distribution, then the probability of 10 classes is 0.1, then cross entropy loss is 1*-log(0.1)*

[6]:
```
# Complete the implementation of softmax_loss_naive and implement a (naive)
# version of the gradient that uses nested loops.
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# As we did for the SVM, use numeric gradient checking as a debugging tool.
# The numeric gradient should be close to the analytic gradient.
from cs231n.gradient_check import grad_check_sparse
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)

# similar to SVM case, do another gradient check with regularization
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad, 10)
```

```
numerical: 2.508709 analytic: 2.508708, relative error: 3.799069e-08
numerical: -0.770382 analytic: -0.770382, relative error: 5.984394e-08
numerical: -0.909556 analytic: -0.909556, relative error: 8.764137e-08
numerical: -1.964704 analytic: -1.964704, relative error: 3.195548e-08
numerical: -1.049105 analytic: -1.049105, relative error: 1.669349e-08
numerical: -0.805085 analytic: -0.805085, relative error: 3.318745e-09
numerical: 1.097290 analytic: 1.097290, relative error: 1.022567e-08
numerical: -2.356726 analytic: -2.356726, relative error: 3.885246e-09
numerical: 1.774974 analytic: 1.774974, relative error: 4.130705e-08
numerical: -1.555162 analytic: -1.555162, relative error: 3.767624e-08
numerical: 0.544930 analytic: 0.544930, relative error: 4.331318e-08
numerical: 1.961087 analytic: 1.961087, relative error: 1.934304e-08
numerical: -0.935372 analytic: -0.935372, relative error: 3.841384e-09
numerical: -1.251169 analytic: -1.251169, relative error: 2.636359e-08
numerical: -0.689613 analytic: -0.689613, relative error: 2.852454e-08
numerical: 0.271505 analytic: 0.271505, relative error: 7.968182e-09
numerical: 0.303463 analytic: 0.303463, relative error: 1.281233e-07
numerical: -0.391398 analytic: -0.391398, relative error: 3.125624e-08
numerical: 0.569887 analytic: 0.569887, relative error: 2.238434e-08
numerical: 0.550485 analytic: 0.550485, relative error: 6.345798e-09
```

[8]:
```
# Now that we have a naive implementation of the softmax loss function and its␣
 ↪gradient,
# implement a vectorized version in softmax_loss_vectorized.
# The two versions should compute the same results, but the vectorized version␣
 ↪should be
# much faster.
tic = time.time()
loss_naive, grad_naive = softmax_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('naive loss: %e computed in %fs' % (loss_naive, toc - tic))
```

```
from cs231n.classifiers.softmax import softmax_loss_vectorized
tic = time.time()
loss_vectorized, grad_vectorized = softmax_loss_vectorized(W, X_dev, y_dev, 0.
 ↪000005)
toc = time.time()
print('vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

# As we did for the SVM, we use the Frobenius norm to compare the two versions
# of the gradient.
grad_difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print('Loss difference: %f' % np.abs(loss_naive - loss_vectorized))
print('Gradient difference: %f' % grad_difference)
```

```
naive loss: 2.355453e+00 computed in 0.253335s
vectorized loss: 2.355453e+00 computed in 0.004535s
Loss difference: 0.000000
Gradient difference: 0.000000
```

[10]:

```
# Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of over 0.35 on the validation set.

from cs231n.classifiers.linear_classifier import Softmax
results = {}
best_val = -1
best_softmax = None

################################################################################
# TODO:                                                                        #
# Use the validation set to set the learning rate and regularization strength. #
# This should be identical to the validation that you did for the SVM; save    #
# the best trained softmax classifer in best_softmax.                          #
################################################################################

# Provided as a reference. You may or may not want to change these␣
 ↪hyperparameters
learning_rates = [1e-7, 5e-7]
regularization_strengths = [2.5e4, 5e4]

# *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

for lr in learning_rates:
    for reg in regularization_strengths:
        model = Softmax()
        model.train(X_train, y_train, lr, reg, num_iters=500)
```

```
        y_pre_train, y_pre_val = np.mean(model.predict(X_train) == y_train), np.
 ↪mean(model.predict(X_val) == y_val)
        results[(lr, reg)] = (y_pre_train, y_pre_val)
        if y_pre_val > best_val:
            best_val = y_pre_val
            best_softmax = model

# *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
                lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' %␣
 ↪best_val)
```

```
lr 1.000000e-07 reg 2.500000e+04 train accuracy: 0.306327 val accuracy: 0.331000
lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.307694 val accuracy: 0.325000
lr 5.000000e-07 reg 2.500000e+04 train accuracy: 0.322735 val accuracy: 0.332000
lr 5.000000e-07 reg 5.000000e+04 train accuracy: 0.294286 val accuracy: 0.312000
best validation accuracy achieved during cross-validation: 0.332000
```

[11]:
```
# evaluate on test set
# Evaluate the best softmax on test set
y_test_pred = best_softmax.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('softmax on raw pixels final test set accuracy: %f' % (test_accuracy, ))
```

```
softmax on raw pixels final test set accuracy: 0.339000
```

Inline Question 2 - *True or False*

Suppose the overall training loss is defined as the sum of the per-datapoint loss over all training examples. It is possible to add a new datapoint to a training set that would leave the SVM loss unchanged, but this is not the case with the Softmax classifier loss.

$\color{blue}{\textit{Your Answer :}}$True

$\color{blue}{\textit{Your Explanation :}}$ Because SVM classifier is about its margin and the softmax classifier is about probability. If we add a new datapoint, maybe the loss of this point would be zero when the right label's score of this point is margin Δ larger than than others.
But for softmax, add a new data point will cause a new loss $-y_i log(y_i)$

[12]:
```
# Visualize the learned weights for each class
w = best_softmax.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)

w_min, w_max = np.min(w), np.max(w)
```

```
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse',␣
↪'ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```

[ ]: