

习题课

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1 Problem

(1) $\forall M > 0, \exists x_0 > 0, x_0 \rightarrow \infty$ 有 $x > x_0, f(x) > M \iff M \rightarrow \infty \implies f(x) \rightarrow \infty$

(2) $\forall \varepsilon > 0, \exists x_1, x_2 > 0, \text{ , 有 } 0 < x_1 < x < x_2, |f(x_1) - f(x_2)| < \varepsilon$ 来回震荡

2 Problem

$\forall M < 0, \exists N = -\frac{1}{M}, \text{ has } -N < x < 0, \text{ made } f(x) < M$

3 Problem

等价于 $\lim_{x \rightarrow \infty} a^{-x} = 0$

$\forall \varepsilon > 0, \text{ 要找到 } X > 0, \text{ 使得 } x > X \text{ 时, 有}$

$$a^{-x} < \varepsilon$$

\implies

$$x > -\log_a \varepsilon$$

所以取

$$X = \max(-\log_a \varepsilon, 1)$$

则当 $x > X$ 时, 有

$$|a^{-x} - 0| < \varepsilon$$

于是证明了

$$\lim_{x \rightarrow +\infty} a^{-x} = 0$$

4 Problem

$$\begin{aligned} |\sin x - \sin x_0| &= |2 \cos \frac{x+x_0}{2} \sin \frac{x-x_0}{2}| \leq 2 |\sin \frac{x-x_0}{2}| \leq |x - x_0| \\ \forall \varepsilon > 0, \text{ 存在 } \delta = \varepsilon, \text{ 有 } \\ |x - x_0| < \delta, \text{ 进而有 } |\sin x - \sin x_0| < \varepsilon \end{aligned}$$

5 Problem

$$\forall \varepsilon > 0, \exists X = (\sqrt{\frac{7}{\varepsilon}} - 3) > 0, x > |X|, \Rightarrow |f(x) - 2| < \varepsilon$$

6 Problem

$$\text{取 } x_k = \frac{1}{2k\pi}, k = 1, 2, \dots, \text{ hence, } f(x_k) = 2k\pi \rightarrow \infty$$

$$\text{取 } x'_k = \frac{1}{2k\pi + \frac{\pi}{2}}, k = 1, 2, \dots, \text{ hence, } f(x'_k) = 0$$

故 $f(x)$ 在 $x=0$ 的任意领域内无界, 但当 $x \rightarrow \infty$ 时不是无穷大量

7 Problem

$$\lim_{x_0 \rightarrow 1^+} f(x) = 1$$

$$\lim_{x_0 \rightarrow 1^-} f(x) = -1$$

所以该点极限不存在

8 Problem

$$\lim_{x_0 \rightarrow 0^+} f(x) = 1$$

$$\lim_{x_0 \rightarrow 0^-} f(x) = -1$$

所以该点极限不存在

9 Problem

a) \Rightarrow

$$\lim_{x \rightarrow 4} \frac{(x-4)(x-2)}{(x-4)(x-1)} = \frac{2}{3}$$

b) \Rightarrow

$$\lim_{x \rightarrow 0} \left| \frac{x+x^2}{2\sin 2x} \right| \leq \left| \frac{x+x^2}{4x} \right| = \frac{1}{4}$$

c) \Rightarrow

$$f$$

d) \Rightarrow

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x}a}{(\sqrt{x+a} + \sqrt{x})} = \frac{a}{2}$$

10 Problem

a) \Rightarrow

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} \leq \lim_{x \rightarrow a} \frac{x - a}{x - a} = 1$$

b) \Rightarrow

$$\text{let } t = x - \frac{\pi}{2}$$

\Rightarrow

$$\lim_{t \rightarrow 0} \left| \frac{\sin t}{t} \right| \leq \lim_{t \rightarrow 0} \left| \frac{t}{t} \right| = 1$$

c)

$$\text{answer} = 4$$

d)

$$\text{answer} = 1$$

11 Problem

(a)

$$\lim_{n \rightarrow \infty} x_n * \sin \frac{x}{2^n} = \lim_{n \rightarrow \infty} \frac{1}{2^n} \frac{\sin x}{\sin \frac{x}{2^n}} = \lim_{n \rightarrow \infty} \frac{\sin x}{x}$$

(b) 略上课讲