Name: zhaoyu

StudentId: 2023232115

Exercise 1.8

The probability that the number of red balls is less than or equal to 1 is:

$$P = 0.1^{10} + C_{10}^{1} \times 0.1^{9} \simeq 9 \times 10^{-9}$$

Exercise 1.9

$$cause \quad u = 0.9$$

Hoeffding Inequality: $\text{P(|u - v|} > \epsilon) \leq 2e^{-2\epsilon^2 N}$

thus : $$\text{P(v} \leq 0.1) = P(0.9 - v \geq 0.8)$$
$$= \text{P (u - v} \geq 0.8)$$
$$\leq P(|u - v| \geq 0.8)$$
$$\leq 2e^{-2\times 0.8^2 \times 10}$$

Problem 1.10

(a)

It is known from the question , $E_{off} = \frac{1}{M}\sum_{m=1}^{M}[(N+m)round2]$

We know that there are $\lfloor\frac{N}{2}\rfloor$ positive integers divisible by 2 in 1 to N . thus $E_{off} = \frac{1}{M}(\lfloor\frac{N+M}{2}\rfloor - \lfloor\frac{N}{2}\rfloor)$

(b)

If there is no error on the training set D, the value on $(x_{N+1} \ldots, x_{N+M})$ is arbitrary, and there are two values for each point, so there are a total of 2^M fs that can be fitted. ANSWER IS 2^M.

(C)

There are a total of M points, and there are k points that are different from the objective function, so there are C_M^k types.

(d)

$$E_f[E_{off}(h,f)] = \sum_{k=0}^{M} \frac{k}{M}\frac{C_M^k}{2^M}$$

$$= \frac{\sum_{k=0}^{M} kC_M^k}{M2^M}$$

$$= \frac{\sum_{k=0}^{M} MC_{M-1}^{k-1}}{M2^M}$$

$$= \frac{2^{M-1}}{2^M} = \frac{1}{2}$$

(e)

it is easy to get both of them equal to p, thus

$$E_f[E_{off}(A_1(D),f)] = E_f[E_{off}(A_2(D),f)] = \sum_{k=0}^{M} \frac{k}{M}p$$

Problem 1.12

(a)

The derivative of h can be obtained, $E'_{in}(h) = 2\sum_{n=1}^{N}(h - y_n)$

$$E''_{in}(h) = 2N \geq 0$$

so $E_{in}(h)$ takes the minimum value of $E'_{in}(h) = 0$ and solves it,

$$h = h_{mean} = \frac{1}{N}\sum_{n=1}^{N} y_n$$

(b)

Construct a distribution such that $P(y = y_i) = \frac{1}{N}(i = 1, 2, \ldots N)$ thus ,

$$F(h) = \frac{1}{N}E_{in}(h) = \frac{1}{N}\sum_{n=1}^{N} |h - y_n|$$

obtain a minimum value when $h = y_{med}$.

(c)

Suppose y_N is preturbed to $y_N + \epsilon$, where $\epsilon \to \infty$,By definition $h_{mean} \to \infty$, y_{med} is the median. It doesn't change much, because the relative order of the elements only changes by y_N.