6-11章习题

习题6.2、6.5 中均考察了Z变换的性质

6.2 (a) $\mathcal{Z}\{-\alpha\mu[n]\} = \dfrac{-\alpha}{1-z^{-1}}$, ROC: $|z|>1$

$\mathcal{Z}\{-\alpha\mu[-n]\} = \dfrac{-\alpha}{1-z}$, ROC: $|z|<1$

$X_a(z) = \mathcal{Z}\{-\alpha\mu[-n-1]\} = \dfrac{-\alpha\, z}{1-z} = \dfrac{-\alpha}{z^{-1}(1-z)} = \dfrac{\alpha}{1-z^{-1}}$, ROC: $|z|<1$

(b) $\mathcal{Z}\left\{-\left(\dfrac{1}{\alpha}\right)^n \mu[n]\right\} = \dfrac{-1}{1-\alpha^{-1}z^{-1}}$, ROC: $|z| > \dfrac{1}{|\alpha|}$

$\mathcal{Z}\left\{-\left(\dfrac{1}{\alpha}\right)^{-n} \mu[-n]\right\} = \mathcal{Z}\{-\alpha^n\mu[-n]\} = \dfrac{-1}{1-\alpha^{-1}z}$, ROC: $|z|<|\alpha|$

$$\mathrm{x}_b[\mathrm{z}] = \alpha^{-1} Z\left\{-\alpha^{n+1}\mu[-n-1]\right\} = \alpha^{-1} \bullet \frac{-z}{1-\alpha^{-1}z} = \frac{1}{1-\alpha z^{-1}}, |\mathrm{z}|<|\alpha|$$

Property	Sequence	z -Transform	ROC
	$g[n]$ $h[n]$	$G(z)$ $H(z)$	$\mathcal{R}_g$ $\mathcal{R}_h$
Conjugation	$g^*[n]$	$G^*(z^*)$	$\mathcal{R}_g$
Time-reversal	$g[-n]$	$G(1/z)$	$1/\mathcal{R}_g$
Linearity	$\alpha g[n] + \beta h[n]$	$\alpha G(z) + \beta H(z)$	Includes $\mathcal{R}_g \cap \mathcal{R}_h$
Time-shifting	$g[n - n_o]$	$z^{-n_o} G(z)$	$\mathcal{R}_g$, except possibly the point $z = 0$ or ∞
Multiplication by an exponential sequence	$\alpha^n g[n]$	$G(z/\alpha)$	$\lvert\alpha\rvert \mathcal{R}_g$
Differentiation of $G(z)$	$n g[n]$	$-z \dfrac{dG(z)}{dz}$	$\mathcal{R}_g$, except possibly the point $z = 0$ or ∞
Convolution	$g[n] \circledast h[n]$	$G(z)H(z)$	Includes $\mathcal{R}_g \cap \mathcal{R}_h$
Modulation	$g[n]h[n]$	$\frac{1}{2\pi j} \oint_C G(v)H(z/v)v^{-1}\, dv$	Includes $\mathcal{R}_g \mathcal{R}_h$
Parseval's relation		$\sum_{n=-\infty}^{\infty} g[n]h^*[n] = \frac{1}{2\pi j} \oint_C G(v)H^*(1/v^*)v^{-1}\, dv$	

Note: If $\mathcal{R}_g$ denotes the region $R_{g-} < |z| < R_{g+}$ and $\mathcal{R}_h$ denotes the region $R_{h-} < |z| < R_{h+}$, then $1/\mathcal{R}_g$ denotes the region $1/R_{g+} < |z| < 1/R_{g-}$ and $\mathcal{R}_g \mathcal{R}_h$ denotes the region

6.5 **(a)** $\mathcal{Z}\{\delta[n]\} = \sum_{n=-\infty}^{\infty} \delta[n] z^{-n} = \delta[0] = 1$, which converges everywhere in the z-plane.

(b) $x[n] = \alpha^n \mu[n]$. From Table 6.1, $\mathcal{Z}\{x[n]\} = X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \dfrac{1}{1-\alpha z^{-1}}, |z| > |\alpha|$.

Let $g[n] = nx[n]$. Then, $\mathcal{Z}\{g[n]\} = G(z) = \sum_{n=-\infty}^{\infty} nx[n] z^{-n}$. Now,

$\dfrac{dX(z)}{dz} = -\sum_{n=-\infty}^{\infty} ng[n] z^{-n-1}$. Hence, $z\dfrac{dX(z)}{dz} = -\sum_{n=-\infty}^{\infty} nx[n] z^{-n} = -G(z)$, or,

$$G(z) = -z\frac{dX(z)}{dz} = \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}, |z| > |\alpha|.$$

6.7 $\mathcal{Z}\{(0.6)^n\mu[n]\} = \frac{1}{1-0.6z^{-1}}, |z|>0.6;\ \mathcal{Z}\{(-0.8)^n\mu[n]\} = \frac{1}{1+0.8z^{-1}}, |z|>0.8;$

$\mathcal{Z}\{-(0.6)^n\mu[-n-1]\} = \frac{1}{1-0.6z^{-1}}, |z|<0.6;\ \mathcal{Z}\{(-0.8)^n\mu[-n-1]\} = -\frac{1}{1+0.8z^{-1}}, |z|<0.8;$

(a) $\mathcal{Z}\{x_1[n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}} = \frac{2+0.2z^{-1}}{(1-0.6z^{-1})(1+0.8z^{-1})}, |z|>0.8.$

$$(b)\ Z\{x_2[n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}} = \frac{2+0.2z^{-1}}{(1-0.6z^{-1})(1+0.8z^{-1})}, 0.6<|z|<0.8$$

$$(c)\ Z\{x_2[n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}} = \frac{2+0.2z^{-1}}{(1-0.6z^{-1})(1+0.8z^{-1})}, |z|<0.6$$

$$(d)\ Z\{x_2[n]\} = \frac{1}{1-0.6z^{-1}} + \frac{1}{1+0.8z^{-1}} = \frac{2+0.2z^{-1}}{(1-0.6z^{-1})(1+0.8z^{-1})}, |z|<0.6 \text{ and } |z|>0.8,$$

$$\text{ROC} \quad \textit{not} \quad \textit{exist}$$

本题考查了右边序列、左边序列和双边序列的收敛域形状，说明不同的序列有可能获得相同的Z变换表达式，必须结合收敛域才能唯一地确定一个序列。

$n \geq -1$ 含有左边部分的右边序列

$$6.8(\mathrm{a-i}) \quad x_1[n] = 0.2^n \mu[n+1] \qquad \mathrm{ROC}: 0.2 < |z| < \infty$$

$$6.8(\mathrm{a-iv}) \quad x_4[n] = (-0.5)^n \mu[-n-3] \qquad \mathrm{ROC}: |z| < 0.5$$

$n \leq -3$ 纯左边序列

6.13 **(a)** $X_a(z) = \dfrac{7+3.6z^{-1}}{1+0.9z^{-1}+0.18z^{-2}} = \dfrac{2}{1+0.6z^{-1}} + \dfrac{5}{1+0.3z^{-1}}$.

$$\textit{left sided} \quad |z| < 0.3, \mathrm{x_a[n]} = -2(-0.6)^n u[-\mathrm{n}-1] - 5(-0.3)^n u[-\mathrm{n}-1]$$

$$\textit{two sided} \quad 0.3 < |z| < 0.6, \mathrm{x}_b\mathrm{[n]} = -2(-0.6)^n u[-\mathrm{n}-1] + 5(-0.3)^n u[\mathrm{n}]$$

$$\textit{right sided} \quad |z| > 0.6, \mathrm{x_c[n]} = 2(-0.6)^n u[\mathrm{n}] + 5(-0.3)^n u[\mathrm{n}]$$

6.16(a) $X_a(z) = Z\{\mu[\mathrm{n}+2] - \mu[\mathrm{n}-3]\} = \dfrac{z^2}{1-z^{-1}} - \dfrac{z^{-3}}{1-z^{-1}}$

$$= \sum_{n=-2}^{2} z^{-n} = z^2 + z + 1 + z^{-1} + z^{-2}$$

$$ROC: (0 \quad \infty)$$

$$X_a(z)\big|_{z=e^{j\omega}} = \frac{e^{2j\omega}}{1-e^{-j\omega}} - \frac{e^{-3j\omega}}{1-e^{-j\omega}}$$

(b) $x_b[n] = \alpha^n \mu[n-1] - \alpha^n \mu[n-4], |\alpha| < 1$. From Table 6.1,

$x_b[z] = \dfrac{\alpha z^{-1}}{1-\alpha z^{-1}} - \dfrac{\alpha^4 z^{-4}}{1-\alpha z^{-1}}$. The ROC is exterior to the circle at $|z| = |\alpha| < 1$. Hence, the ROC includes the unit circle. On the unit circle,

$$X_b(z)\Big|_{z=e^{j\omega}} = X_b(e^{j\omega}) = \frac{\alpha e^{-j\omega}}{1-\alpha e^{-j\omega}} - \frac{\alpha^4 e^{-4j\omega}}{1-\alpha e^{-j\omega}}$$

(c) $x_c[n] = 2n\alpha^n \mu[n], |\alpha| < 1$. From Table 6.1, $x_c[z] = \dfrac{2\alpha z^{-1}}{\left(1-\alpha z^{-1}\right)^2}$. The ROC is exterior to the circle at $|z| = |\alpha| < 1$. Hence, the ROC includes the unit circle. On the unit circle, $x_c[e^{j\omega}] = \dfrac{2\alpha e^{-j\omega}}{\left(1-\alpha e^{-j\omega}\right)^2}$

6.44 **(a)** A partial-fraction expansion of $H(z)$ in z^{-1} using the M-file `residuez` yields

$$H(z) = -5 + \frac{4.0909}{1+0.4z^{-1}} + \frac{0.9091}{1-0.15z^{-1}}.$$ Hence, from Table 6.1 we have

$$h[n] = -5\delta[n] + 4.0909(-0.4)^n \mu[n] + 0.9091(0.15)^n \mu[n].$$ causal

(b) $x[n] = 2.1(0.4)^n \mu[n] + 0.3(-0.3)^n \mu[n]$. Its z–transform is thus given by

$$X(z) = \frac{2.1}{1-0.4z^{-1}} + \frac{0.3}{1+0.3z^{-1}} = \frac{2.4+0.51z^{-1}}{(1-0.4z^{-1})(1+0.3z^{-1})}, |z| > 0.4.$$ The z–transform of the output $y[n]$ is then given by

$$Y(z) = H(z)X(z) = \left[\frac{2.4+0.51z^{-1}}{(1-0.4z^{-1})(1+0.3z^{-1})}\right] \cdot \left[\frac{-1.5z^{-1}+0.3z^{-2}}{1+0.25z^{-1}-0.06z^{-2}}\right].$$

A partial-fraction expansion of $Y(z)$ in z^{-1} using the M-file `residuez` yields

$$Y(z) = \frac{9.2045}{1+0.4z^{-1}} - \frac{3.15}{1-0.4z^{-1}} - \frac{5}{1+0.3z^{-1}} - \frac{1.0545}{1-0.15z^{-1}}, |z| > 0.4.$$ Hence, from Table 6.1 we have $y[n] = \left(9.2045(-0.4)^n - 3.15(0.4)^n - 5(-0.3)^n - 1.0545(0.15)^n\right)\mu[n]$.

6.81 **(a)** The frequency response exists if the ROC contains the unit circle. Since $H(z)$ has poles at -0.3, 0.6, and -5, a two-sided sequence corresponding to an ROC of $0.6 < |z| < 5$ would allow the existence of the frequency response.

(b) The system can be stable if the ROC is $0.6 < |z| < 5$. However, it cannot be both stable and causal because this ROC corresponds to a two-sided sequence.

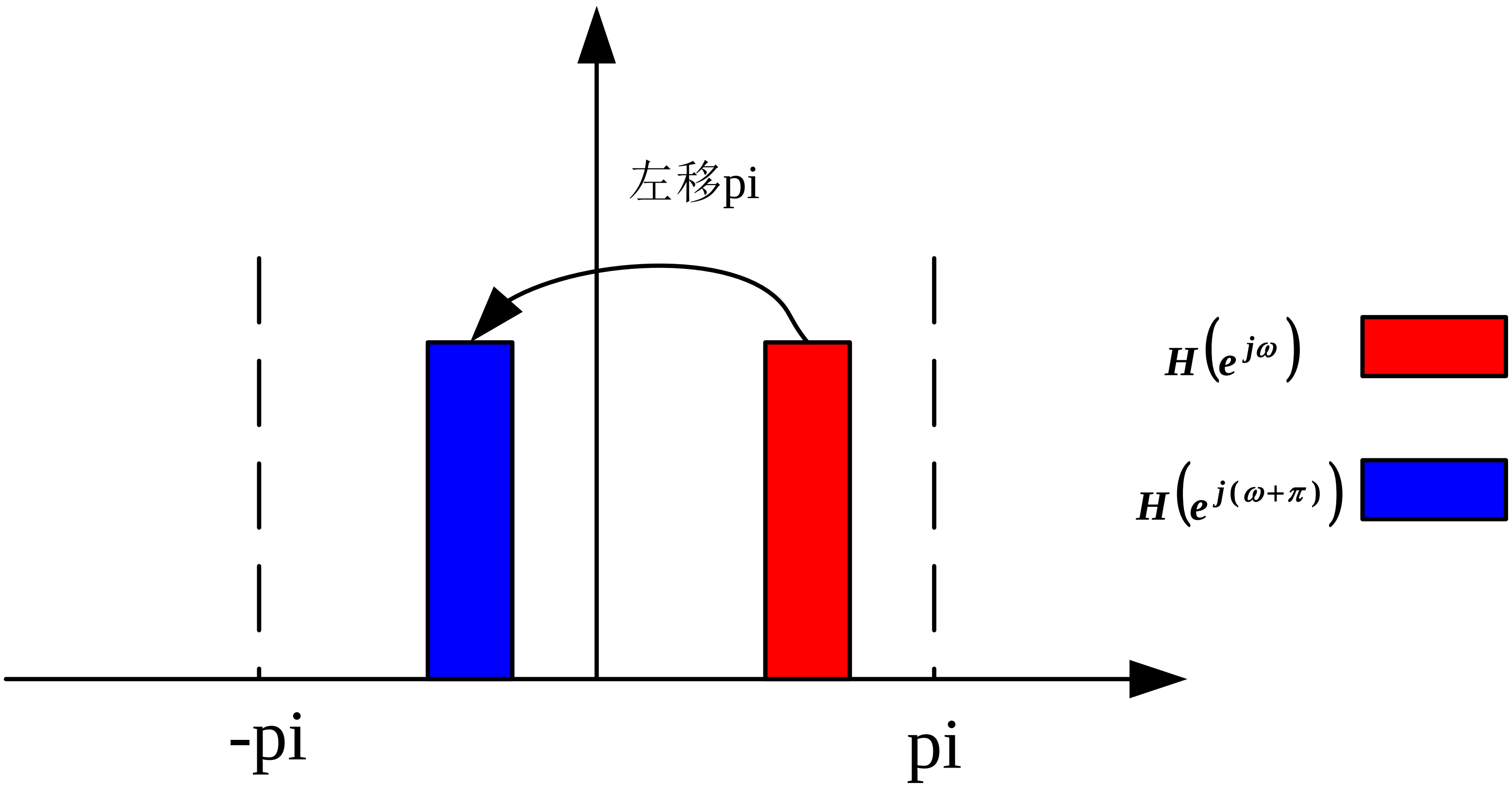
(c) $h[n] = A(-0.3)^n \mu[n] + B(0.6)^n \mu[n] + C(-5)^n \mu[-n-1]$

7.7 $H(e^{j\omega}) = \begin{cases} 1, & \omega_{p1} \le |\omega| \le \omega_{p2}, \\ 0, & 0 \le |\omega| \le \omega_{s1}, \\ 0, & \omega_{s2} \le |\omega| < \pi. \end{cases}$ $G(e^{j\omega}) = H(e^{j(\pi-\omega)})$, This implies that the frequency response of $H(-z)$ is a shifted version of the frequency response of $H(z)$, shifted by π radians. Therefore,

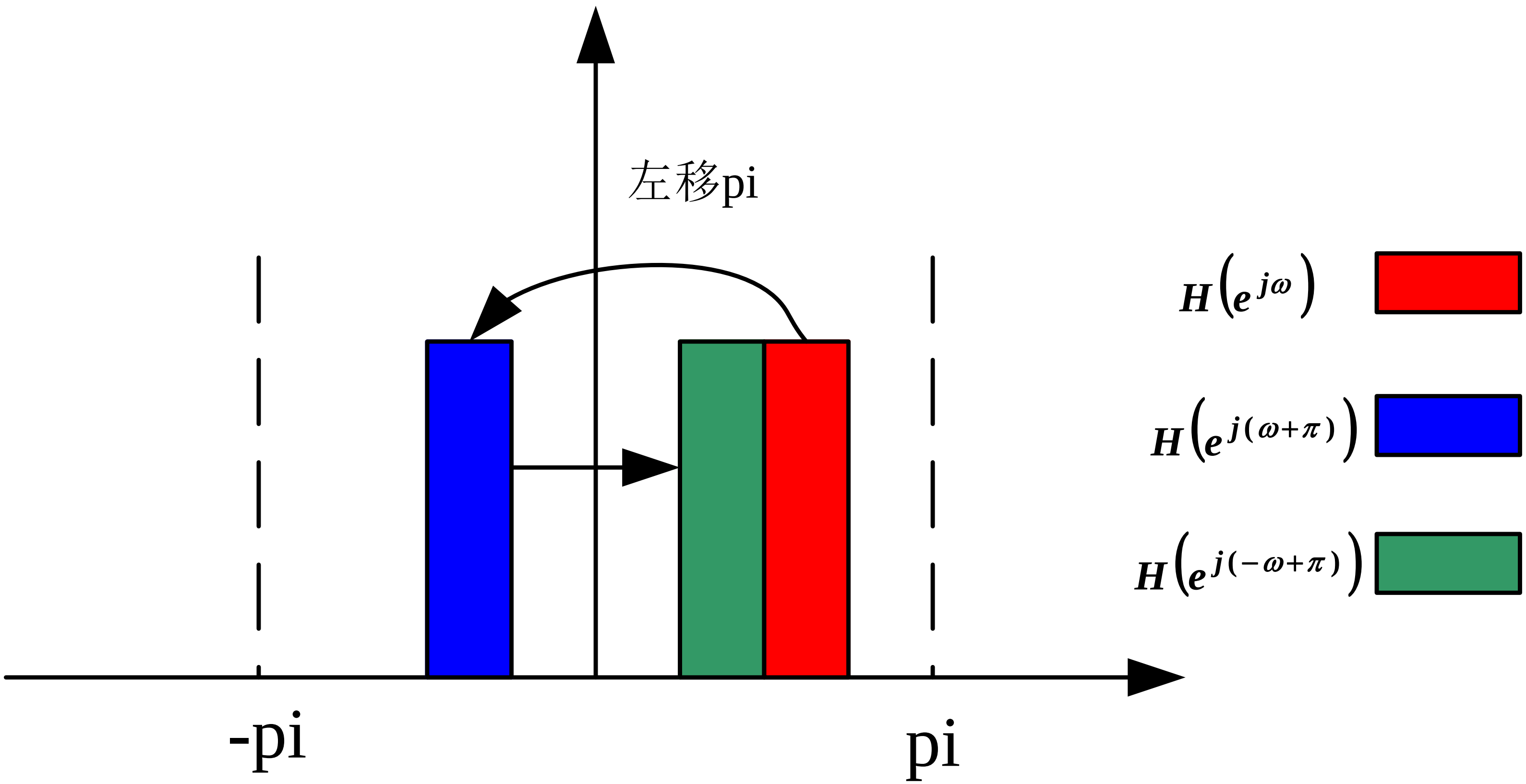
$$G(e^{j\omega}) = H(e^{j(\pi-\omega)}) = \begin{cases} 1, & \pi-\omega_{p2} \le |\omega| \le \pi-\omega_{p1}, \\ 0, & 0 \le |\omega| \le \pi-\omega_{s2}, \\ 0, & \pi-\omega_{s1} \le |\omega| < \pi. \end{cases}$$

Hence, $H(-z)$ is also a bandpass filter with passband edges at $\pi-\omega_{p2}$ and $\pi-\omega_{p1}$, and stopband edges at $\pi-\omega_{s2}$ and $\pi-\omega_{s1}$ with $\pi-\omega_{s2} < \pi-\omega_{p1} < \pi-\omega_{p1} < \pi-\omega_{s1}$.

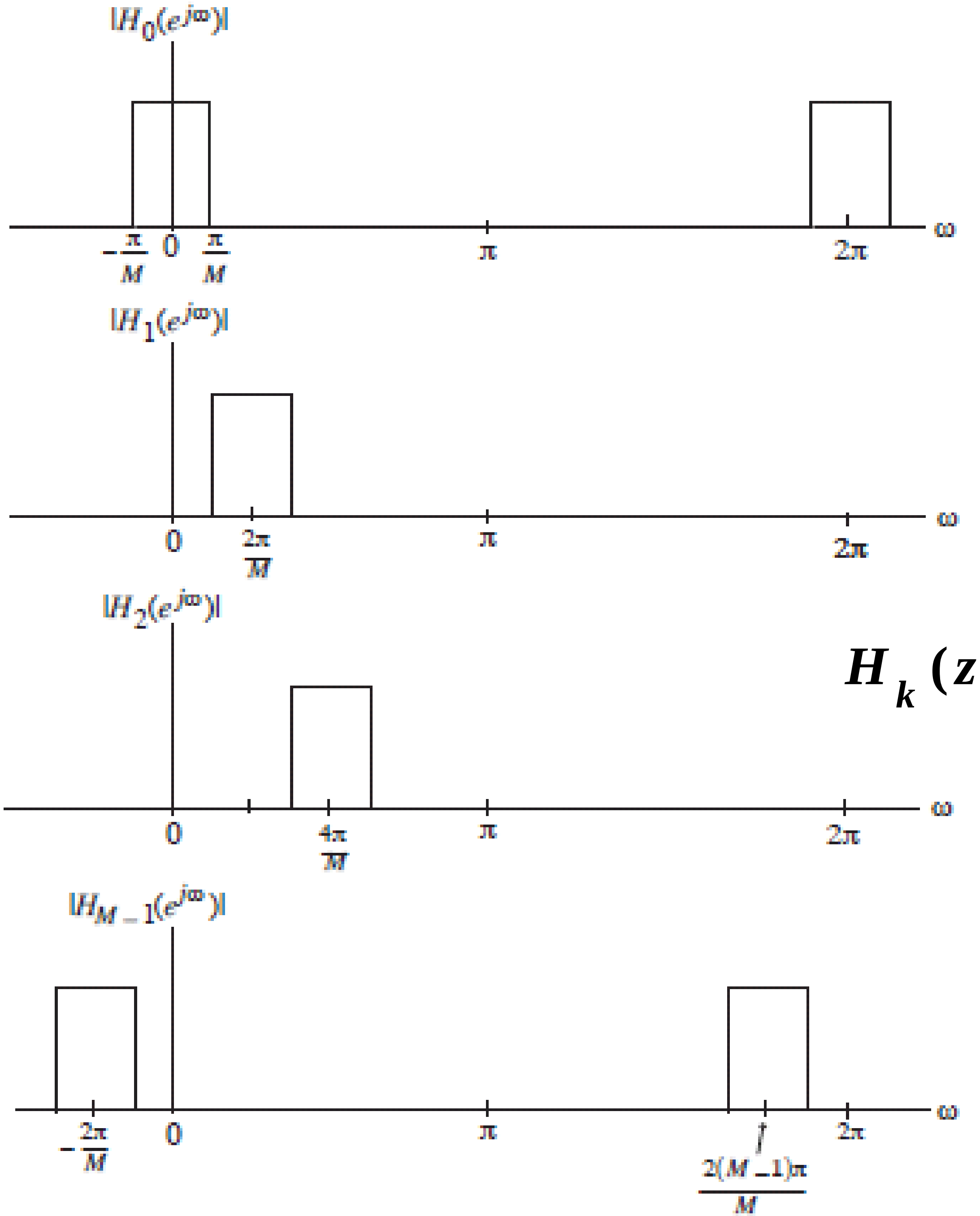
$$H(-z)\Big|_{z=e^{j\omega}} = H\left(e^{j(\pi+\omega)}\right)$$



$$G\left(e^{j\omega}\right)=H\left(e^{j(\pi-\omega)}\right)$$



7.8



$$H_k(z) = H(ze^{-j2\pi k/M})$$

7.18 Now, $\mathcal{A}_M(z) = \pm \dfrac{z^{-M} D_M^*(1/z^*)}{D_M(z)}$, where $D_M(z) = 1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_M z^{-M}$.

$$\left|\mathcal{A}_M(z)\right|^2 = \mathcal{A}_M(z)\mathcal{A}_M^*(1/z^*) = \frac{D_M^*(1/z^*)}{D_M(z)} \cdot \frac{D_M(z)}{D_M^*(1/z^*)} = 1.$$

7.39 (a) The specified zeros of a real coefficient Type 1 FIR transfer function $H_1(z)$ are at $z_1 = 1$, $z_2 = -0.6$, and $z_3 = -1 + j$. Since a Type 1 FIR transfer function can have no zeros or an even number zeros at $z = 1$, $H_1(z)$ must have another zero at $z_4 = 1$. Also mirror-image symmetry of the zeros imply that the other zeros are at

$z_5 = 1/z_2 = -1.6667$, $z_6 = z_5^* = -1 - j$, $z_7 = 1/z_6 = -0.5 - j0.5$,

$z_8 = z_7^* = -0.5 + j0.5$.

(b) Therefore the transfer function is of the form

$$H_1(z) = (z - z_1)(z - z_4)(z - z_2)(z - z_5)(z - z_3)(z - z_6)(z - z_7)(z - z_8)$$

$$= 1.0 + 3.2667z^{-1} + 2.7667z^{-2} - 3.1333z^{-3} - 7.8z^{-4} - 3.1333z^{-5} + 2.7667z^{-6} + 3.2667z^{-7} + z^{-8}.$$

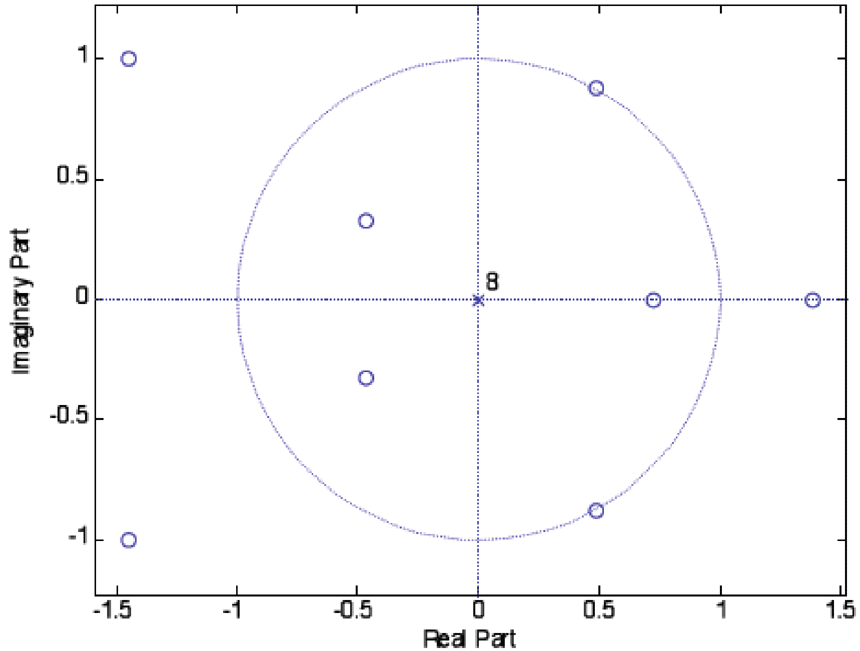
7.45 **(a)** Type 1: $\{h[n]\} = \{2,\ \ 1.5,\ \ -3.2,\ \ -5.2,\ \ 6.4,\ \ -5.2,\ \ -3.2,\ \ 1.5,\ \ 2\}$. Hence,

$$H(z) = 2 + 1.5z^{-1} - 3.2z^{-2} - 5.2^{-3} + 6.4z^{-4} - 5.2z^{-5} - 3.2z^{-6} + 1.5z^{-7} + 2z^{-8}.$$

The zero plot obtained using the M-file `zplane` is shown below:

偶对称，

奇数长度



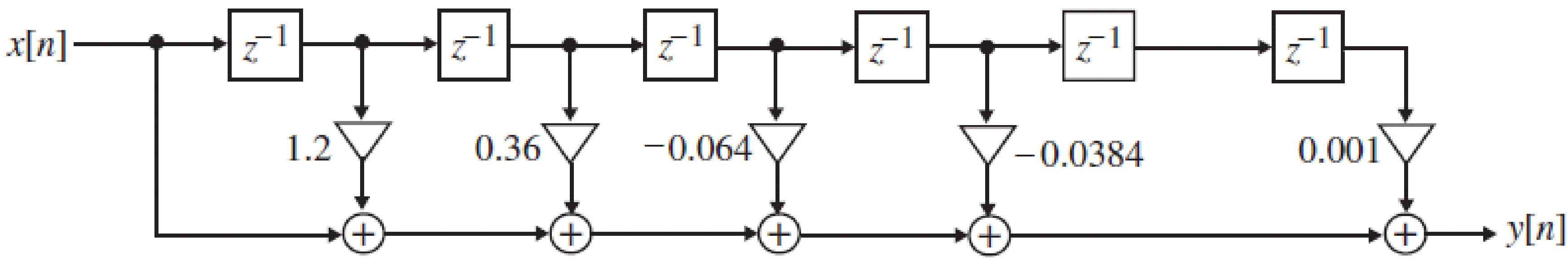
(b) Type 2: $\{h[n]\} = \{2,\ \ 1.5,\ \ -3.2,\ \ -5.2,\ \ 6.4,\ \ 6.4,\ \ -5.2,\ \ -3.2,\ \ 1.5,\ \ 2\}$.

Hence,

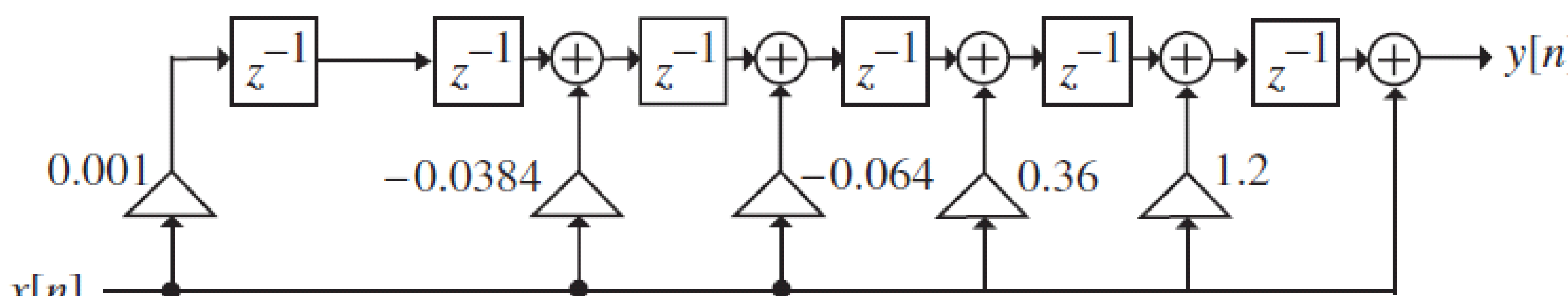
$$H(z) = 2 + 1.5z^{-1} - 3.2z^{-2} - 5.2^{-3} + 6.4z^{-4} + 6.4z^{-5} - 5.2z^{-6} - 3.2z^{-7} + 1.5z^{-8} + 2z^{-9}.$$

7.55 $H_0(z) = \frac{1}{2}(1 + z^{-1})$. Thus, $\left|H_0(e^{j\omega})\right| = \cos(\omega/2)$. Now, $G(z) = \left(H_0(z)\right)^M$. Hence, $\left|G(e^{j\omega})\right|^2 = \left|H_0(e^{j\omega})\right|^{2M} = \left(\cos(\omega/2)\right)^{2M}$. The 3–dB cutoff frequency ω_c of $G(z)$ is thus given by $\left(\cos(\omega_c/2)\right)^{2M} = \frac{1}{2}$. Hence, $\omega_c = 2\cos^{-1}(2^{-1/2M})$.

8.13 **(a)** $H(z) = (1+0.4z^{-1})^4(1-0.2z^{-1})^2$
$= 1+1.2z^{-1}+0.36z^{-2}-0.64z^{-3}-0.0384z^{-4}+0.001z^{-6}$. A direct form realization of $H(z)$ is shown below:



The transposed form of the above structure yields another direct form realization as indicated below:



(b) A realization in the form of cascade of six first-order sections is shown below:

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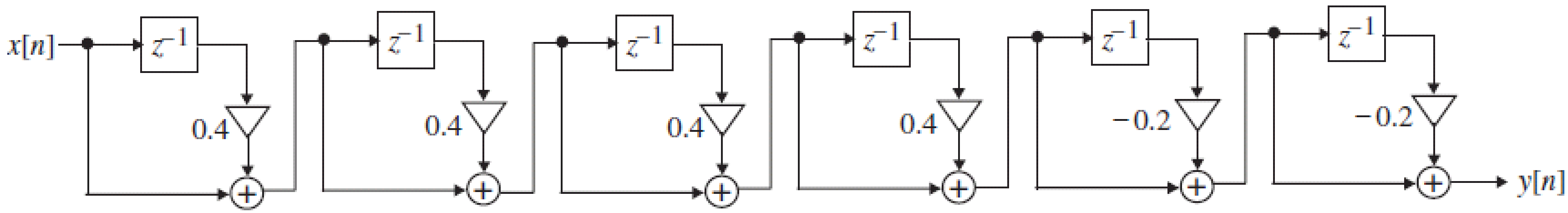
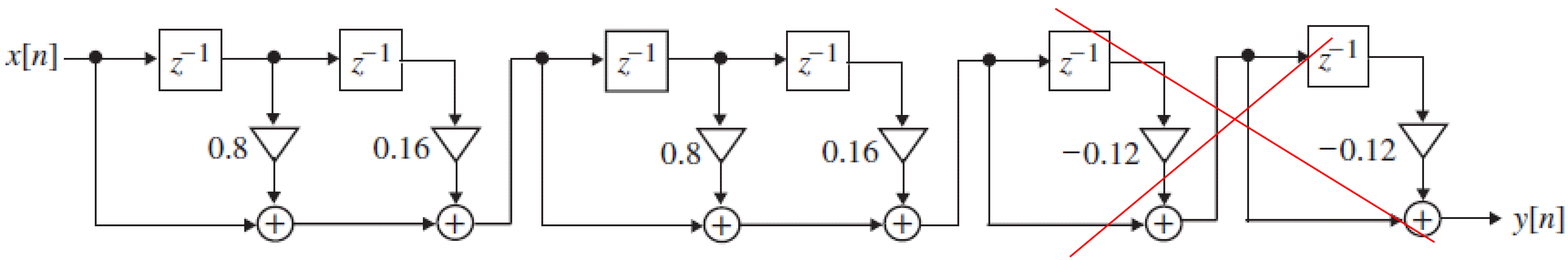
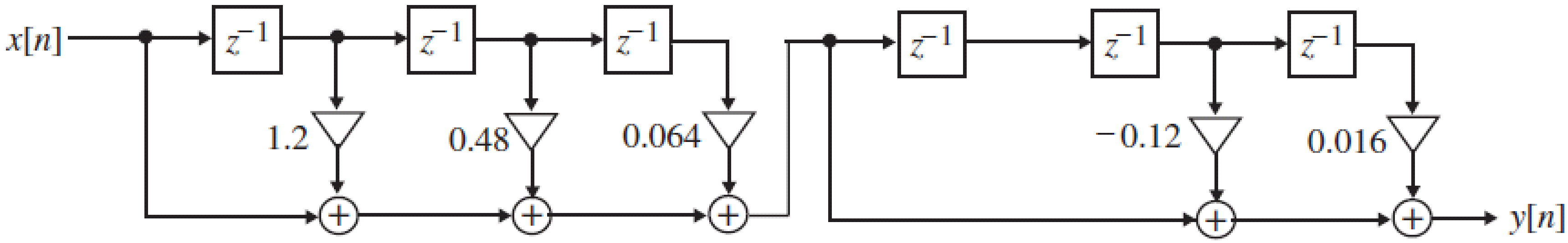


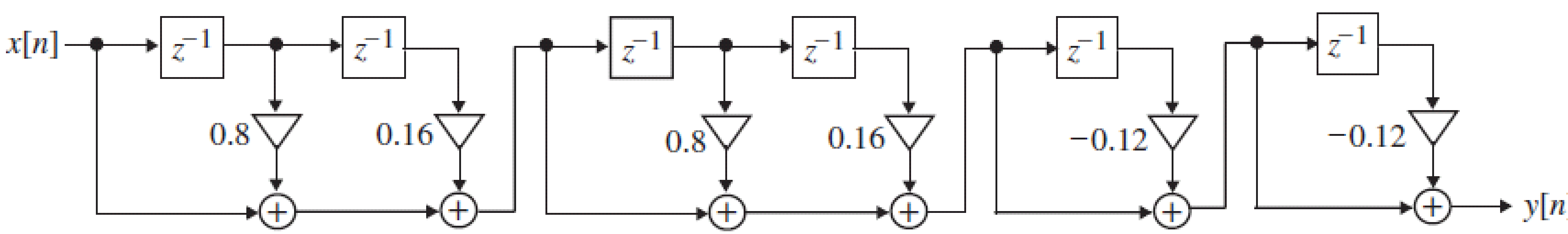
(c) A realization in the form of cascade of three second-order sections is shown below:



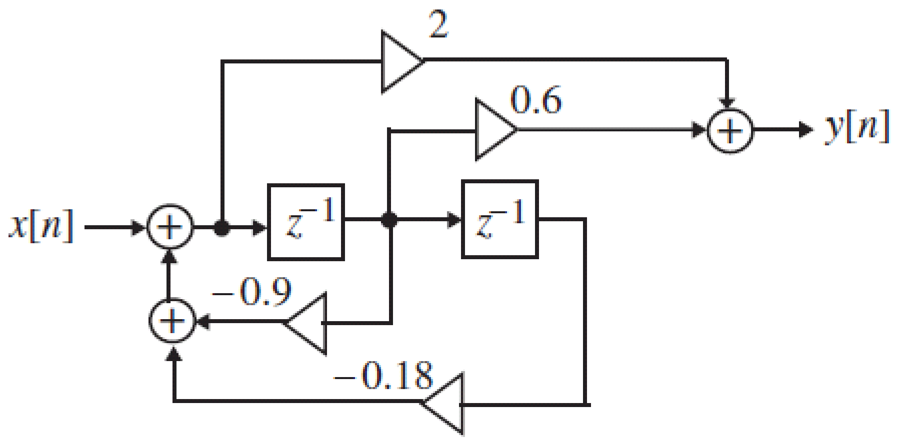
(d) A realization in the form of cascade of two third-order sections is shown below:



(e) A realization in the form of cascade of two first-order sections and two second-order sections is shown below:



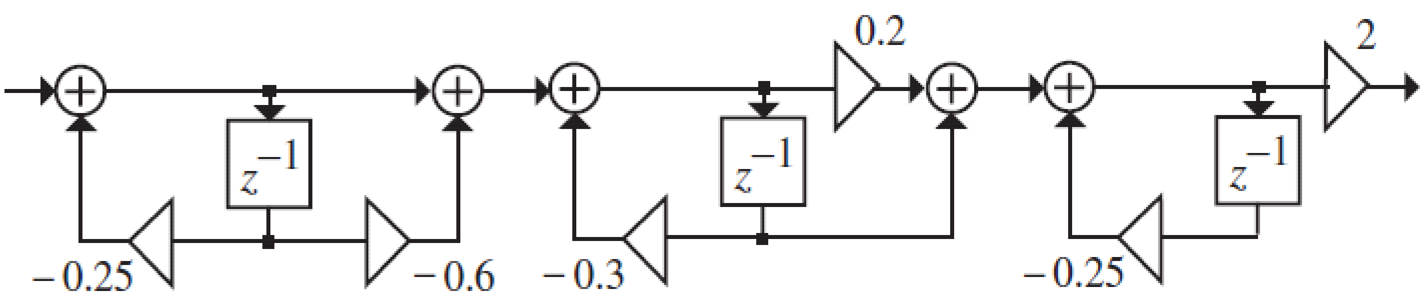
8.24 **(a)** A direct form II realization of $H_1(z)$ is shown below:



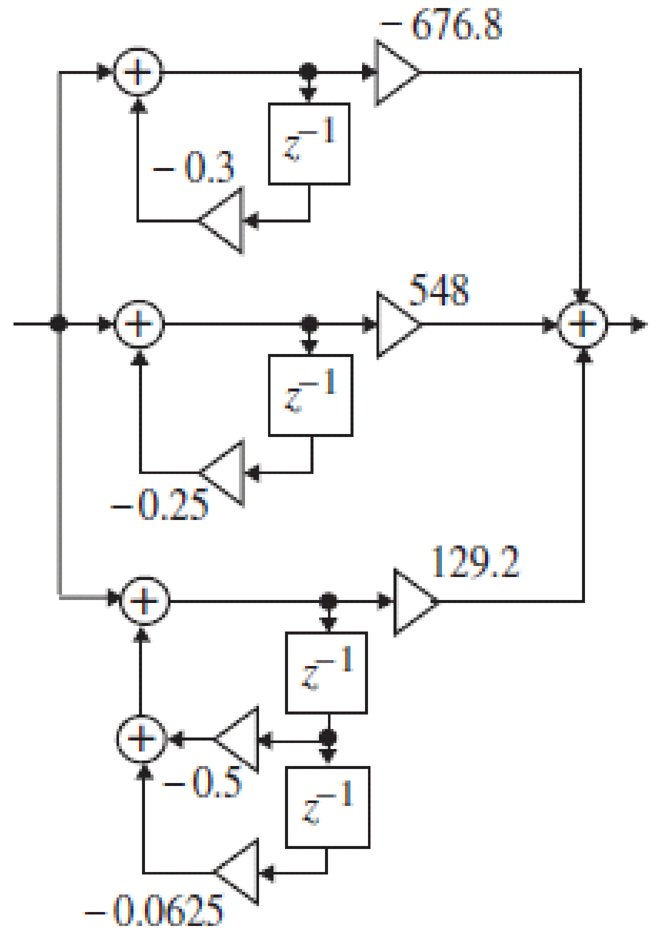
8.28 **(a)** $H(z) = \frac{1-0.6z^{-1}}{1+0.25z^{-1}} \cdot \frac{0.2+z^{-1}}{1+0.3z^{-1}} \cdot \frac{2}{1+0.25z^{-1}} = \frac{0.4+1.76z^{-1}-1.2z^{-2}}{1+0.8z^{-1}+0.2125z^{-2}+0.0187z^{-3}}.$

(b)
$y[n] = 0.4x[n]+1.76x[n-1]-1.2x[n-2]-0.8y[n-1]-0.2125y[n-2]-0.0187y[n-3].$

(c) A cascade realization of $H(z)$ is shown below:



(d) A partial-fraction expansion of $H(z)$ in z^{-1} obtained using `residuez` is given by
$$H(z) = \frac{548}{1+0.25z^{-1}} + \frac{129.2}{(1+0.25z^{-1})^2} + \frac{-676.8}{1+0.3z^{-1}}.$$ The Parallel Form I realization based on this expansion is shown on the next page.



- Parallel realizations are obtained by making use of the partial fraction expansion of the transfer function

Parallel form I:

$$H(z) = \gamma_0 + \sum_k \left(\frac{\gamma_{0k} + \gamma_{1k} z^{-1}}{1 + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

Parallel form II:

$$H(z) = \delta_0 + \sum_k \left(\frac{\delta_{1k} z^{-1} + \delta_{2k} z^{-2}}{1 + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

9.9 **(a)** $$H_a(s) = \frac{2(s+2)}{(s+3)(s^2+4s+5)} = \frac{-1}{s+3} + \frac{0.5-0.5j}{(s+2-j)} + \frac{0.5+0.5j}{(s+2+j)}$$

$$= \frac{-1}{s+3} + \frac{s+3}{(s+2)^2+1^2} = \frac{-1}{s+3} + \frac{s+2}{(s+2)^2+1^2} + \frac{1}{(s+2)^2+1^2}.$$

Using Eq (9.71), we get

$$G_a(z) = \frac{-1}{1-e^{-3T}z^{-1}} + \frac{1-z^{-1}e^{-2T}\cos(T)}{1-2z^{-1}e^{-2T}\cos(T)+e^{-4T}z^{-2}} + \frac{z^{-1}e^{-2T}\sin(T)}{1-2z^{-1}e^{-2T}\cos(T)+e^{-4T}z^{-2}}.$$

Since $T = 0.25$, we get

$$G_a(z) = \frac{-1}{1-0.4724z^{-1}} + \frac{1-0.4376z^{-1}}{1-1.1754z^{-1}+0.3679z^{-2}}..$$

9.11 (a) $H_a(s) = G_a(z)\Big|_{z=4\left(\frac{1+s}{1-s}\right)} = \dfrac{4\left(5s^2+18s+9\right)}{75s^2+154s+91}.$

(b) $H_b(s) = G_b(z)\Big|_{z=4\left(\frac{1+s}{1-s}\right)} = \dfrac{105s^3+385s^2+467s+195}{(13s+11)\left(27s^2+46s+23\right)}.$

$$\frac{5+s}{5-s}$$

补充作业题：Using the bilinear transformation and a lowpass analog Butterworth filter

$$H(s)=\frac{1}{s^2+\sqrt{2}s+1}$$

Design a second-order lowpass digital filter with 3-dB cutoff frequency 3kHz and operating at a sampling rate of 12kHz.

(a) Determine the system function of the desired lowpass digital filter.

(b) Draw the canonical realization form of the designed filter.

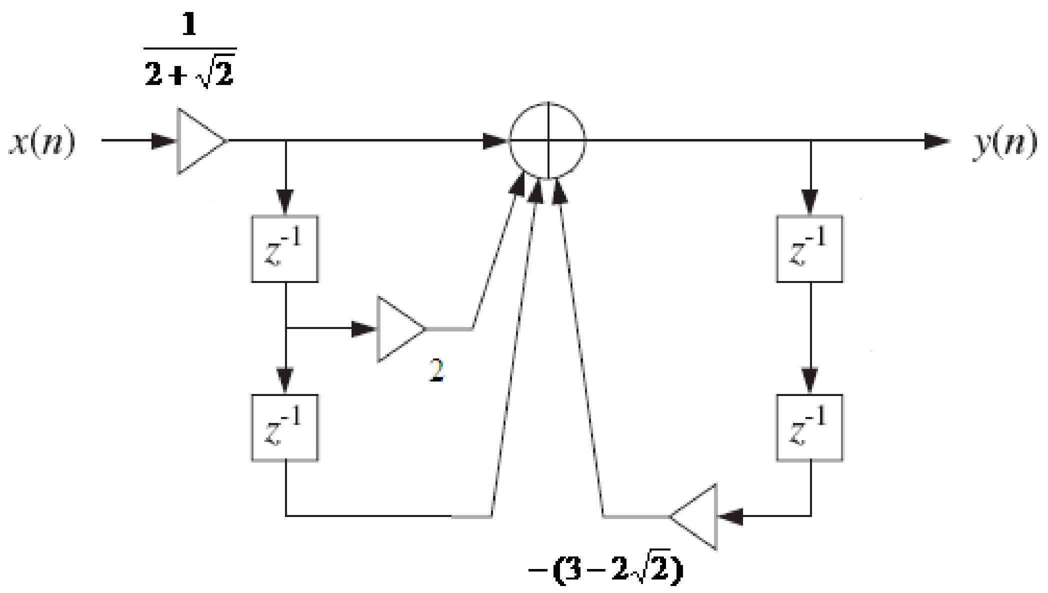
$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2\pi \times 3}{12} = 0.5\pi$$

$$\Omega_c = \tan(\frac{\omega_c}{2}) = \tan(0.25\pi) = 1$$

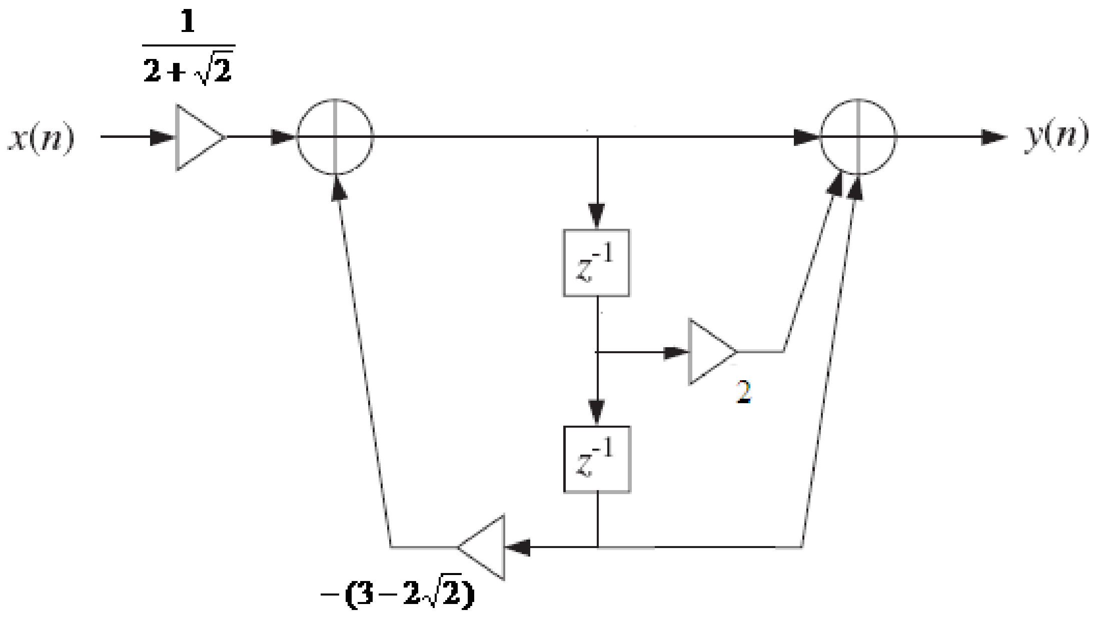
$$H_a(s) = H(\frac{s}{\Omega_c})$$

$$H(z) = H_a(s)|_{\frac{1-z^{-1}}{1+z^{-1}}} = \frac{1}{(\frac{1-z^{-1}}{1+z^{-1}})^2 + \sqrt{2}\frac{1-z^{-1}}{1+z^{-1}} + 1}$$

$$= \frac{1+2z^{-1}+z^{-2}}{(2+\sqrt{2})+(2-\sqrt{2})z^{-2}} = \frac{1}{2+\sqrt{2}} \cdot \frac{1+2z^{-1}+z^{-2}}{1+(3-2\sqrt{2})z^{-2}}$$



$\frac{1}{2+\sqrt{2}}$
$x(n)$
$y(n)$
z^{-1}
z^{-1}
2
z^{-1}
z^{-1}
$-(3-2\sqrt{2})$



$\frac{1}{2+\sqrt{2}}$
$x(n)$
$y(n)$
z^{-1}
2
z^{-1}
$-(3-2\sqrt{2})$

10.14 From Eq. (10.9): $\Phi_R = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H_t\left(e^{j\omega}\right)-H_d\left(e^{j\omega}\right)\right|^2 d\omega$,,

where $H_t\left(e^{j\omega}\right)=\sum_{n=-M}^{M}h_t[n]e^{-j\omega n}$.

Using Parseval's relation, we can write: $\Phi_R = \sum_{n=-\infty}^{\infty}\left|h_t[n]-h_d[n]\right|^2$

$$= \sum_{n=-M}^{M}\left|h_t[n]-h_d[n]\right|^2 + \sum_{n=-\infty}^{-M-1}h_d^2[n] + \sum_{n=M+1}^{\infty}h_d^2[n].$$

Therefore: $\Phi_{Haan} = \sum_{n=-\infty}^{\infty}\left|h_d[n]\cdot w_{Hann}[n]-h_d[n]\right|^2$

$$= \sum_{n=-M}^{M}\left|h_d[n]\left(\frac{1}{2}+\frac{1}{2}\cos\left(\frac{2\pi n}{2M+1}\right)\right)-h_d[n]\right|^2 + \sum_{n=-\infty}^{-M-1}h_d^2[n] + \sum_{n=M+1}^{\infty}h_d^2[n]$$

Hence: $\Phi_{Excess} = \Phi_R - \Phi_{Haan} = \sum_{n=-M}^{M} \left| h_d[n] \cdot w_R[n] - h_d[n] \right|^2$

$$- \sum_{n=-M}^{M} \left| h_d[n] \cdot \left(\frac{1}{2} + \frac{1}{2} \cos\left(\frac{2\pi n}{2M+1} \right) \right) - h_d[n] \right|^2$$

$$= - \sum_{n=-M}^{M} \left| \frac{h_d[n]}{2} \cos\left(\frac{2\pi n}{2M+1} \right) - \frac{h_d[n]}{2} \right|^2 = \frac{1}{2}(1+2M) \left| \cos\left(\frac{2\pi M}{2M+1} \right) - 1 \right|^2 .$$

10.16 The responses of an ideal lowpass filter with a cutoff frequency $\omega_c = \pi/2$ are:

$$h_{HB}[n] = \frac{\sin(\omega_c n)}{\pi n} = \frac{\sin(\pi n/2)}{\pi n}, \qquad H_{HB}(e^{j\omega}) = \begin{cases} 1, & -\pi/2 < \omega < \pi/2, \\ 0, & \text{otherwise.} \end{cases}$$

The frequency responses of the Hilbert Transformer (from Eq. (10.24)) and the ideal discrete-time Differentiator (from Eq. (10.26)) are as follows:

$$H_{HT}(e^{j\omega}) = \begin{cases} j, & -\pi < \omega < 0, \\ -j, & 0 < \omega < \pi. \end{cases} \qquad H_{DIF}(e^{j\omega}) = j\omega.$$

In order to get derive the impulse response of the Hilbert Transformer, we note that:

$$\boxed{H_{HT}(e^{j\omega}) = jH_{HB}(e^{j(\omega-\pi/2)}) - jH_{HB}(e^{j(\omega+\pi/2)}).}$$

Therefore, using the properties of the DTFT, we get:

$$h_{HT}[n] = jh_{HB}[n]e^{jn(\pi/2)} - jh_{HB}[n]e^{-jn(\pi/2)}$$

$$= 2h_{HB}[n]\left(\frac{1}{2j}e^{jn(\pi/2)} - \frac{1}{2j}e^{-jn(\pi/2)}\right) = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi}{2}\right).$$

In order to derive the impulse response of the Differentiator, we note that:

$$\boxed{H_{DIF}(e^{j\omega}) = j\omega H_{HB}(e^{j(\omega-\pi/2)}) + j\omega H_{HB}(e^{j(\omega+\pi/2)})}$$

Therefore, using the properties of the DTFT, and in particular: $\frac{\partial}{\partial n}x[n] \Leftrightarrow j\omega X\left(e^{j\omega}\right)$

We get (by use of the double angle formula):

$$h_{DIF}[n] = \frac{\partial}{\partial n}\left(h_{HB}[n]e^{jn(\pi/2)}\right) + \frac{\partial}{\partial n}\left(h_{HB}[n]e^{-jn(\pi/2)}\right)$$

$$= 2\frac{\partial}{\partial n}\left(h_{HB}[n]\left[\frac{1}{2}e^{jn(\pi/2)} + \frac{1}{2}e^{-jn(\pi/2)}\right]\right) = 2\frac{\partial}{\partial n}\left(h_{HB}[n]\cos\left(\frac{\pi n}{2}\right)\right)$$

$$= \frac{\partial}{\partial n}\left(\frac{2\sin(\pi n/2)\cos(\pi n/2)}{\pi n}\right) = \frac{\partial}{\partial n}\left(\frac{\sin(\pi n)}{\pi n}\right) = \frac{\cos(\pi n)}{\pi n} - \frac{\sin(\pi n)}{\pi^2 n^2}.$$

10.17 For each problem, the codes used to generate the plots are given below, assuming that ω_c is the average of the stopband and passband frequencies:

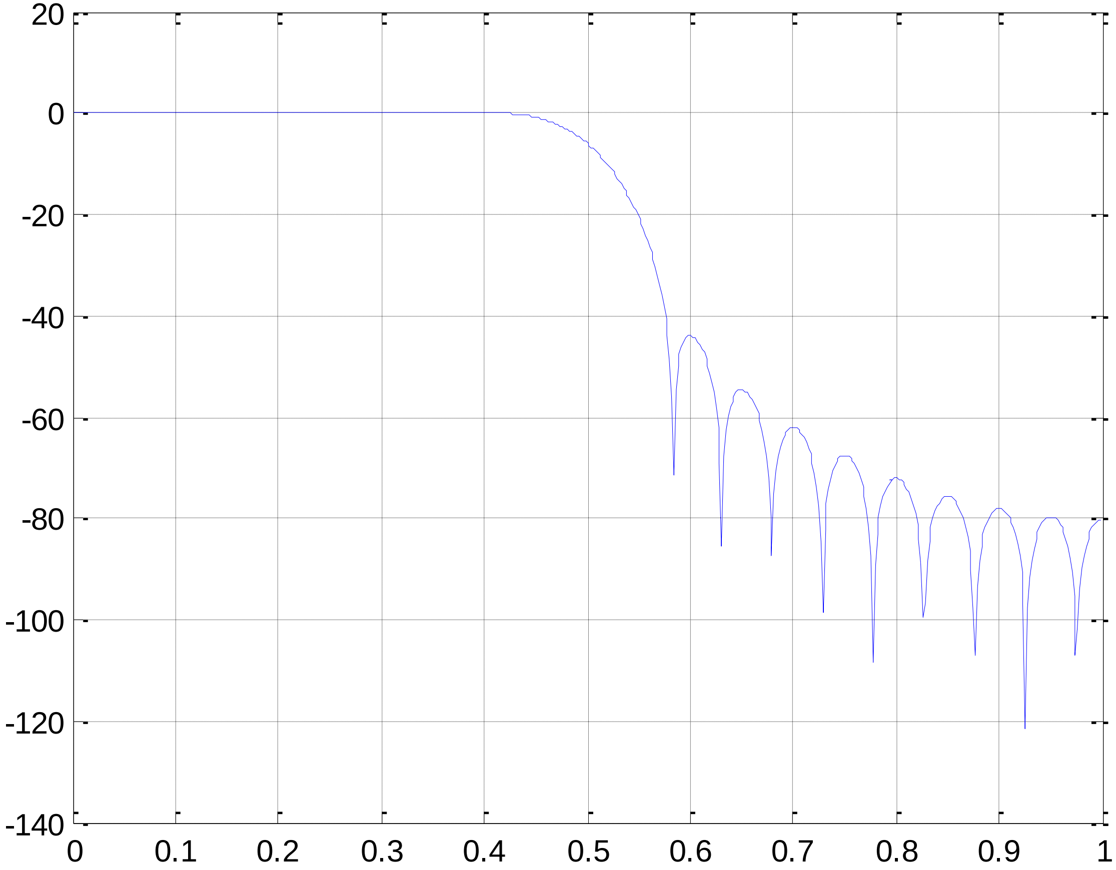
```
fRange = -M:M;
idealLPF = (wc/pi)*sinc((wc/pi)*fRange);
fNum = idealLPF.*hann(L)';
[h,w] = freqz(fNum,1,512);
plot(w/pi,20*log10(abs(h)));grid;
xlabel('\omega/\pi'); ylabel('Gain, dB');
```

(a) Given: $\omega_p = 0.42\pi,\ \omega_s = 0.58\pi,\ \delta_p = 0.002,\ \delta_s = 0.008.$

Thus: $\Delta\omega = 0.16\pi, \alpha_s = -20\log_{10}\delta_s = 41.93$ dB.

From Table 10.2, we see that for fixed-window functions, we can achieve the minimum stopband attenuation by using Hann, Hamming, or Blackman windows. Hann will have the lowest filter length:

Since $M = \dfrac{3.11\pi}{0.16\pi} = 19.43,\ N_{Haan} = \lceil 2M+1 \rceil = 40.$


20
0
-20
-40
-60
-80
-100
-120
-140
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

11.12 The k–th sample of an N–point DFT is given by $X[k]=\sum_{n=0}^{N-1}x[n]W_N^{nk}$. Thus the computation of $X[k]$ requires N complex multiplications and $N-1$ complex additions. Now, each complex multiplication, in turn, requires 4 real multiplicatio and 2 real additions. Likewise, each complex addition requires 2 real additions. A a result, the N complex multiplications needed to compute $X[k]$ require a total of $4N$ real multiplications and a total of $2N-2$ real additions. Therefore, each sample of the N–point DFT involves $4N$ real multiplications and $4N-2$ real additions. Hence, the computation of all DFT samples thus requires $4N^2$ real multiplications and $(4N-2)N$ real additions.

$$(A+Bj)\times(C+Dj)=(A\times C-B\times D)+(B\times C+A\times D)j$$

2次实数加法　　　4次实数乘法

$$(A+Bj)+(C+Dj)=(A+C)+(B+D)j$$

2次实数加法

21 Direct computation of M samples of an N-point DFT requires M^2 multiplications, whereas, the Radix-2 FFT algorithm requires $\frac{N}{2}\log_2 N$ multiplications. In order for a N-point radix-2 FFT algorithm to be computationally more efficient than a direct computation of M samples of an N-point DFT, the following inequality must hold: $M > \left\lfloor \sqrt{\frac{N}{2}\log_2 N} \right\rfloor$.

a) $N = 32, M = 9$, b) $N = 64, M = 14$, c) $N = 128, M = 22$.

对长度为N的序列作N点DFT，计算一个x(k)，需要N次复数乘法；计算M个x(k)，需要MN次复数乘法；

$$MN > \frac{N}{2}\log_2 N$$

1.32 **(a)** # of zero-valued samples to be added is 256 – 197 = 59.

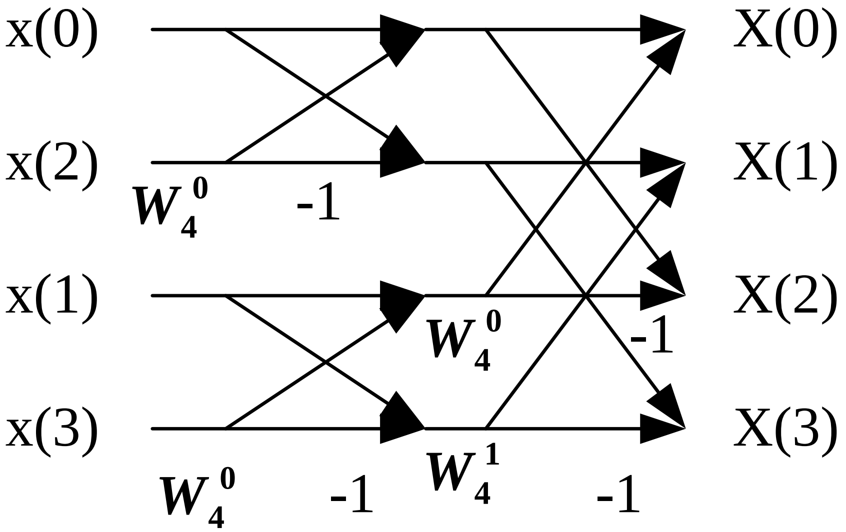
(b) Direct computation of a 256-point DFT of a length-197 sequence requires $(197)^2 = 38809$ complex multiplications and $196 \times 197 = 38612$ complex additions.

(c) A 256-point Cooley-Tukey type FFT algorithm requires $128 \times \log_2(256) = 1024$ complex multiplications and $256 \times \log_2(256) = 2048$ complex additions.

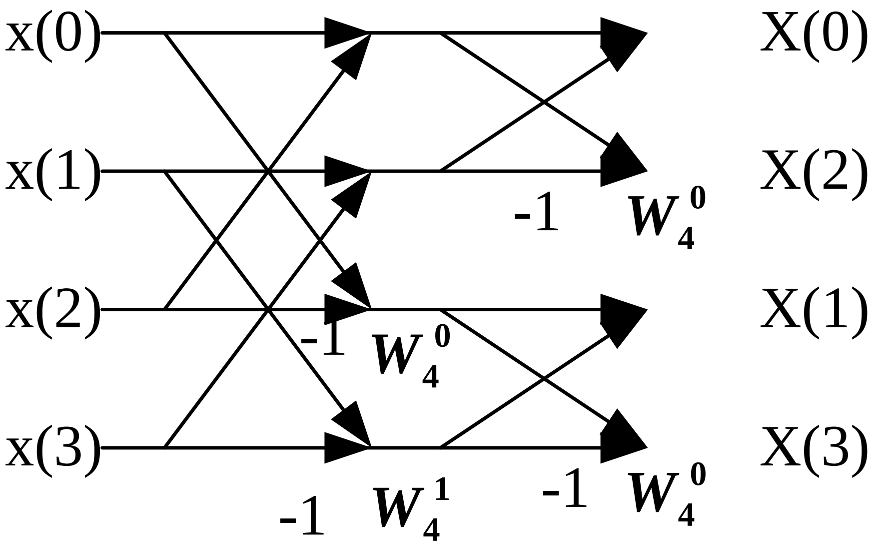
256*197

256*196

- **Plot the butterfly flow-graph for 4 point DIT and DIF FFT algorithm**



DIT



DIF

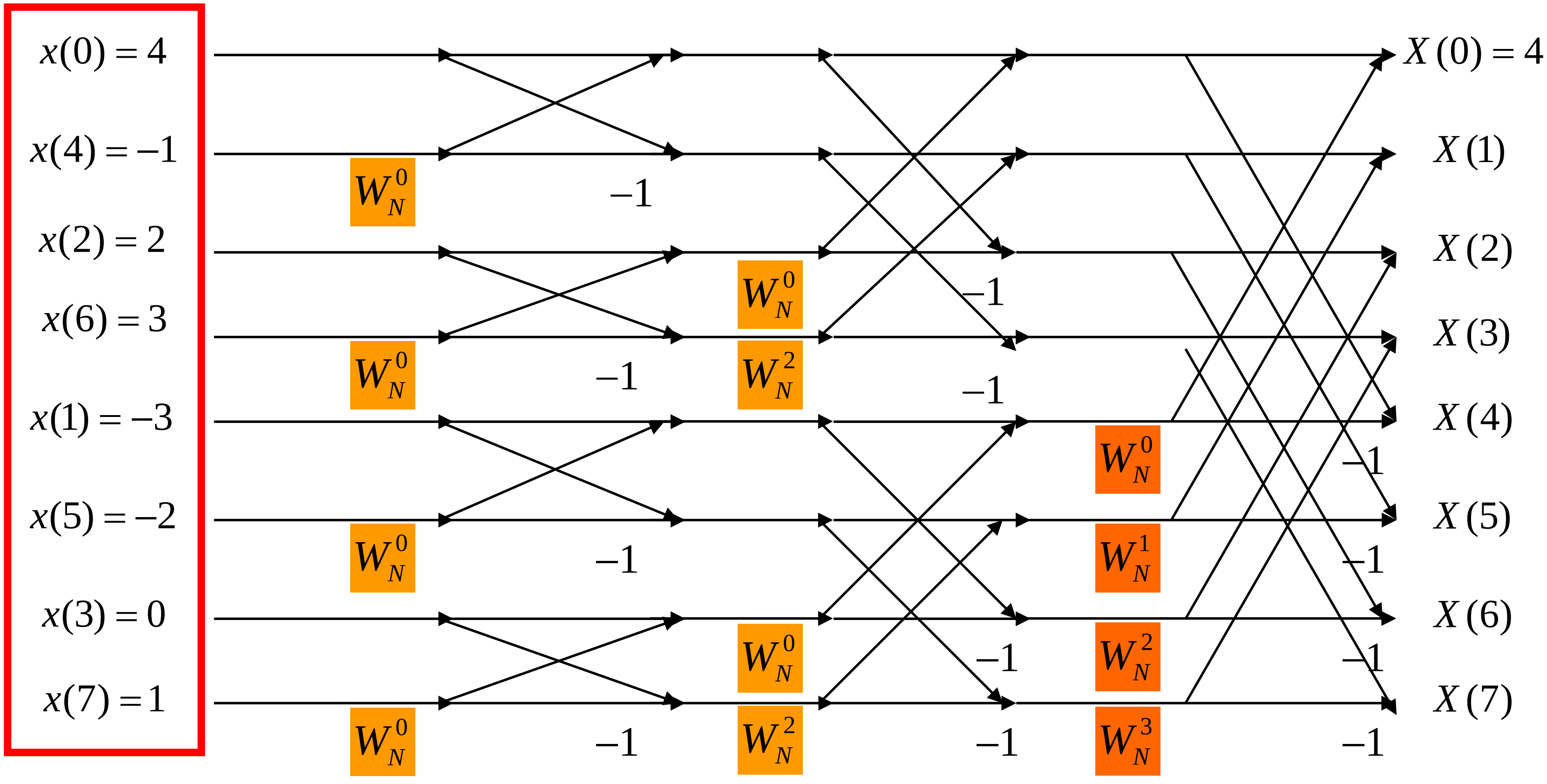
- ***Supplementary Problem:***

Using the FFT algorithm, compute the 8-point DFT of the 8-point signal x = [4, −3, 2, 0, −1, −2, 3, 1].

X=[4 5+2.4142i -2+6i 5+0.4142i 12 5-0.4142i -2-6i 5- 2.4142i]

DIT Algorithms

$$\mathbf{x} = [4, -3, 2, 0, -1, -2, 3, 1]$$



DIF Algorithms