$$\begin{array}{rcrcl} A \times F = 150 & = & 96 & & \\ 4 \times F = 60 & = & 3C & & \\ E \times F = 210 & = & D2 & & \\ 1 \times F = 15 & = & \underline{0F} & & \\ & & 1C656 & = & P_3 \end{array}$$

Example 1.30: Division

1 A F 3 ÷ E

```
     0 1 E C   <——— Quotient
  E | 1 A F 3
    | 0
    |--------
    | 1 A F 3
    |   E
    |--------
    |   C F 3
    |   C 4
    |--------
    |     B 3
    |     A 8
    |--------
          B    <——— Remainder
```

1.5 Representation of Negative Numbers

The examples shown so far have used only positive numbers. In practice, a digital system must represent both positive and negative numbers. To accommodate the sign of the number, an additional digit, called the sign digit, is included in the representation, along with the magnitude digits. Thus, to represent an n-digit number, we would need $n + 1$ digits. Typically, the sign digit is the MSD. Two popular representation schemes have been used: the *sign-magnitude system* and the *complement system*.

1.5.1 Sign-Magnitude System

In this representation, $n + 1$ digits are used to represent a number, where the MSD is the sign digit and the remaining n digits are magnitude digits. The value of the sign digit is 0 for positive numbers and $r - 1$ for negative numbers, where r is the radix of the number system. Some sample representations follow.

Example 1.31

Here, we assume that five digits are available to represent each number The sign and magnitude portions of the number are separated by a "," for illustration purposes only. The "," is not used in the actual representation.

Number	Representation	
$(-2)_2$	1,0010	All numbers are shown as five-digit numbers.
$(+56)_8$	0,0056	
$(-56)_8$	7,0056	
$(+1F)_{16}$	0,001F	
$(-1F)_{16}$	F,001F	

Sign Magnitude

The sign and magnitude portions are handled separately in arithmetic using sign-magnitude numbers. The magnitude of the result is computed and then the appropriate sign is attached to the result, just as in decimal arithmetic. The sign-magnitude system has been used in such small digital systems as digital meters and typically when the decimal mode of arithmetic is used in digital computers. The decimal (or binary coded decimal) arithmetic mode will be described later in this chapter. Complement number representation is the most prevalent representation mode in modern-day computer systems.

1.5.2 Complement Number System

Consider the subtraction of a number A from a number B. This is equivalent to adding $(-A)$ to B. The complement number system provides a convenient way of representing negative numbers (i.e., complements of positive numbers), thus reducing the subtraction to an addition. Because multiplication and division correspond respectively to repeated addition and subtraction, it is possible to perform the four basic arithmetic operations using only the hardware for addition when the negative numbers are represented in complement form. The two popular complement number systems are radix complement and diminished radix complement.

The *radix complement* of a number $(N)_r$ is defined as

$$\begin{aligned} [N]_r &= r^n - (N)_r \quad && \text{if } (N)_r \neq 0 \\ &= 0 && \text{if } (N)_r = 0 \end{aligned} \tag{1.3}$$