Also, all four corners are adjacent to each other because the K-map can be wrapped around in both the vertical *and* horizontal directions. Encircling the four corners results in $\overline{B}\,\overline{D}$. The final equation is

$$X = \overline{A} + \overline{B}\,\overline{D}$$

EXAMPLE 3–20

Simplify the following equation using the Karnaugh mapping procedure:

$$X = \overline{A}\,\overline{B}\,\overline{D} + A\overline{C}\,\overline{D} + \overline{A}B\overline{C} + AB\overline{C}D + A\overline{B}\,\overline{C}\,\overline{D}$$

Solution:

Encircling the four corners forms $\overline{B}\,\overline{D}$, as shown in Figure 3–45. The other group of four forms $B\overline{C}$. You may be tempted to encircle the $\overline{C}\,\overline{D}$ group of four as shown by the dotted line, but that would be *redundant* because each of those 1s is already contained within an existing circle. Therefore, the final equation is

$$X = \overline{B}\,\overline{D} + B\overline{C}$$

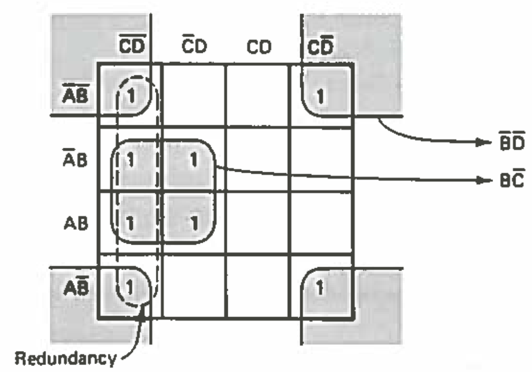

Figure 3–45 Solution to Example 3–20.

3–6 SYSTEM DESIGN APPLICATIONS

Let's summarize the entire chapter now by working through two complete design problems. The following examples illustrate practical applications of a K-map to ensure that when we implement the circuit we will have the simplest possible solution.

Note: The construction of digital circuits with higher complexity than those of these examples will be more practically suited for implementation using what is called *programmable logic devices,* which are discussed in Chapter 8 and Appendix K.

SYSTEM DESIGN 3–1

Design a circuit that can be built using logic gates that will output a HIGH (1) whenever the 4-bit BCD input is an odd number from 0 to 9.

Solution:

First, build a truth table (Table 3–3) to identify which BCD codes from 0 to 9 produce odd numbers. (Use the variable A to represent the 2^0 BCD input, B for 2^1, C for 2^2, and D for 2^3.) Next, reduce that equation into its simplest form by using a Karnaugh map, as shown in Figure 3–46a. Finally, using basic logic gates, the circuit can be constructed, as shown in Figure 3–46b.

TABLE 3–3
Truth Table Used to Determine the Equation for Odd Numbers[a] from 0 to 9

D	C	B	A	DEC	
0	0	0	0	0	
0	0	0	1	1	$\leftarrow A\overline{B}\,\overline{C}\,\overline{D}$
0	0	1	0	2	
0	0	1	1	3	$\leftarrow AB\overline{C}\,\overline{D}$
0	1	0	0	4	
0	1	0	1	5	$\leftarrow A\overline{B}C\overline{D}$
0	1	1	0	6	
0	1	1	1	7	$\leftarrow ABC\overline{D}$
1	0	0	0	8	
1	0	0	1	9	$\leftarrow A\overline{B}\,\overline{C}D$

[a]Odd number = $A\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + A\overline{B}C\overline{D} + ABC\overline{D} + A\overline{B}\,\overline{C}D$.

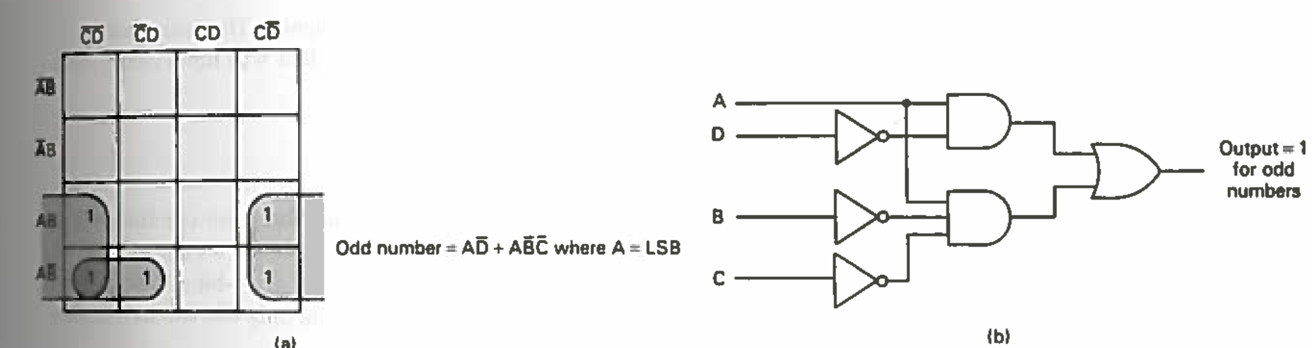

Figure 3–46 (a) Simplified equation derived from a Karnaugh map; (b) logic circuit for the "odd-number decoder."

SYSTEM DESIGN 3–2

A chemical plant needs an alarm system developed to warn of critical conditions in one of its chemical tanks. The tank has four HIGH/LOW (1/0) switches, monitoring temperature (T), pressure (P), fluid level (L), and weight (W). Design a system that will activate an alarm when any of the following conditions arise:

1. A high fluid level with a high temperature and a high pressure
2. A low fluid level with a high temperature and a high weight
3. A low fluid level with a low temperature and a high pressure
4. A low fluid level with a low weight and a high temperature

Solution:

First, write in Boolean equation form, the conditions that will activate the alarm:

$$\text{Alarm} = LTP + \overline{L}TW + \overline{L}\,\overline{T}P + \overline{L}\,\overline{W}T$$

Next, factor the equation into its simplest form by using a Karnaugh map, as shown in Figure 3–47a. Finally, the logic circuit can be constructed, as shown in Figure 3–47b.