As we can see from the polynomial expansion and summation shown here, the positions containing a 0 do not contribute to the sum. To convert a binary number into decimal, we can simply accumulate the weights corresponding to each nonzero bit of the number.

Each bit can take either of the two values: 0 or 1. With two bits, we can derive 2^2, or 4, combinations: 00, 01, 10, and 11. The decimal values of these combinations (binary numbers) are 0, 1, 2, and 3, respectively. Similarly, with three bits we can derive 2^3, or 8, combinations ranging in value from 000 (0 in decimal) to 111 (7 in decimal). In general, with n bits it is possible to generate 2^n combinations of 0s and 1s, and these combinations when viewed as binary numbers range in value from 0 to $(2^n - 1)$. Table 1.2 shows some binary numbers for various values of n. The 2^n combinations possible for any n are obtained by starting with n 0s and counting in binary until the number with n 1s is reached. A more mechanical method of generating these combinations is described herein.

The first combination has n 0s and the last has n 1s. As we can see from Table 1.2, the value of the least significant bit (LSB)—i.e., bit position 0—alternates in value between 0 and 1 every row, as we move from row to row. Similarly, the value of the bit in position 1 alternates every two rows (i.e., two 0s followed by two 1s). In general, the value of the bit in position i alternates every 2^i rows starting from 0s. This observation can be utilized in generating all the 2^n combinations.

Table 1.2 Binary Numbers

$n = 2$	$n = 3$	$n = 4$	
1 0	2 1 0	3 2 1 0	← Bit position
00	000	0000	
01	001	0001	
10	010	0010	
11	011	0011	
	100	0100	
	101	0101	
	110	0110	
	111	0111	
		1000	
		1001	
		1010	
		1011	
		1100	
		1101	
		1110	
		1111	

1.2.2 Octal System

In this system, $r = 8$, and the allowed digits are 0, 1, 2, 3, 4, 5, 6, and 7. A typical number is shown in positional notation in the following example.

Example 1.3

$$
\begin{aligned}
N &= (4\,5\,2\,6\,.\,2\,3)_8 \\
&= 8^3 8^2 8^1 8^0 . 8^{-1} 8^{-2} \quad \text{weights} \\
&= 4 \times 8^3 + 5 \times 8^2 + 2 \times 8^1 + 6 \times 8^0 + 2 \times 8^{-1} \\
&\quad + 3 \times 8^{-2} \quad \text{polynomial form} \\
&= 2048 + 320 + 16 + 6 + \tfrac{2}{8} + \tfrac{3}{64} \quad \text{(decimal)} \\
&= (2390\tfrac{19}{64})_{10}
\end{aligned}
$$

1.2.3 Hexadecimal System

In this system, $r = 16$, and the allowed digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. Digits A through F correspond to decimal values 10 through 15, respectively. A typical number is shown in the following example.

Example 1.4

$$
\begin{aligned}
N &= (A\,1\,F\,.\,1\,C)_{16} \\
&\quad 16^2 16^1 16^0 . 16^{-1} 16^{-2} \quad \text{weights} \\
&= A \times 16^2 + 1 \times 16^1 + F \times 16^0 + 1 \times 16^{-1} \\
&\quad + C \times 16^{-2} \quad \text{polynomial form} \\
&= 10 \times 16^2 + 1 \times 16^1 + 15 \times 16^0 + 1 \times 16^{-1} \\
&\quad + 12 \times 16^{-2} \quad \text{(decimal)} \\
&= (2591\tfrac{28}{256})_{10}
\end{aligned}
$$

1.3 Conversion

To convert numbers from a nondecimal system to decimal, we simply expand the given number as a polynomial and evaluate the polynomial using decimal arithmetic, as shown in Examples 1.1 through 1.4. When a decimal number is converted to any other system, the integer and fraction portions of the number are handled separately. The *radix divide technique* is used to convert the integer portion, and the *radix multiply technique* is used for the fraction portion.