Example 1.1

Decimal point

$$\underbrace{1\,2\,5\,6}_{\text{Integer}}\;.\;\underbrace{9\,3\,2}_{\text{Fraction}}$$

$10^3 10^2 10^1 10^0 \,.\, 10^{-1} 10^{-2} 10^{-3}$ weights

Powers increase by 1 . Powers decrease by 1

This number can also be represented as a polynomial:

$$1 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 9 \times 10^{-1} + 3 \times 10^{-2} + 2 \times 10^{-3}$$

We can thus generalize these two representations to any number system. The general *positional notation* of a number N is

$$N = (a_n \cdots a_3 a_2 a_1 a_0 \,.\, a_{-1} a_{-2} a_{-3} \cdots a_{-m})_r \tag{1.1}$$

where r is the radix of the number systems; a_{-1}, a_0, a_1, a_2, and so on, are digits such that $0 \leq a_i \leq (r - 1)$ for all i; a_n is the most significant digit (MSD), and a_{-m} is the least significant digit (LSD). The *polynomial representation* of the above number is

$$N = \sum_{i=-m}^{n} a_i r^i \tag{1.2}$$

There are $n + 1$ integer digits and m fraction digits in the number shown above.

Consider an integer with n digits. A finite range of values can be represented by this integer. The smallest value in this range is 0 and corresponds to each digit of the n-digit integer being equal to 0. When each digit corresponds in value to $r - 1$, the highest digit in the number system, the n-digit number attains the highest value in the range. This value is equal to $r^n - 1$. Table 1.1 lists the first few numbers in various systems. We will discuss binary, octal, and hexadecimal systems next.

1.2.1 Binary System

In this system, the radix is 2 and the two allowed digits are 0 and 1. *BI*nary digi*T* is abbreviated as BIT. A typical binary number is shown in the positional notation in the following example.

Table 1.1 Number Systems

Decimal ($r = 10$)	Binary ($r = 2$)	Ternary ($r = 3$)	Quaternary ($r = 4$)	Octal ($r = 8$)	Hexadecimal ($r = 16$)
0	0	0	0	0	0
1	1	1	1	1	1
2	10	2	2	2	2
3	11	10	3	3	3
4	100	11	10	4	4
5	101	12	11	5	5
6	110	20	12	6	6
7	111	21	13	7	7
8	1000	22	20	10	8
9	1001	100	21	11	9
10	1010	101	22	12	A
11	1011	102	23	13	B
12	1100	110	30	14	C
13	1101	111	31	15	D
14	1110	112	32	16	E
15	1111	120	33	17	F
16	10000	121	100	20	10
17	10001	122	101	21	11
18	10010	200	102	22	12
19	10011	201	103	23	13
20	10100	202	110	24	14

Example 1.2

$N = (11010.1101)_2$

$2^4 2^3 2^2 2^1 2^0 \,.\, 2^{-1} 2^{-2} 2^{-3} 2^{-4}$ weights

$16\;8\;4\;2\;1 \,.\, \frac{1}{2}\;\frac{1}{4}\;\frac{1}{8}\;\frac{1}{16}$ weights in decimal

Weights double for each move to the left from the binary point.

Weights are halved for each move to the right from the binary point.

In polynomial form, this number is

$$\begin{aligned} N &= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ &\quad + 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4} \\ &= 16 + 8 + 0 + 2 + 0 + \tfrac{1}{2} + \tfrac{1}{4} + 0 + \tfrac{1}{16} \quad \text{(decimal)} \\ &= 26 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{16} \quad \text{(decimal)} \\ &= (26\tfrac{13}{16})_{10} \end{aligned}$$