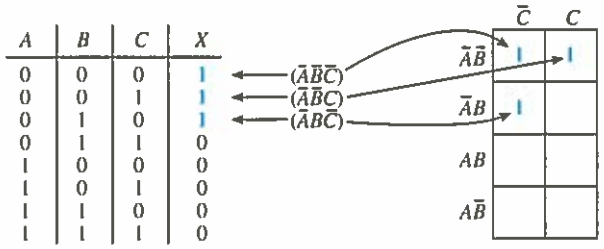
The terms of that SOP expression can be put into a truth table and then transferred to a K-map, as shown in Figure 5–67. Working with the K-map, we now encircle adjacent 1's in groups of two, four, or eight. We end up with two circles of two cells each, as shown in Figure 5–68. The first circle surrounds the two 1's at the top of the K-map, and the second circle surrounds the two 1's in the left column of the K-map.



| A | B | C | X | |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | ← $(\bar{A}\bar{B}\bar{C})$ |
| 0 | 0 | 1 | 1 | ← $(\bar{A}\bar{B}C)$ |
| 0 | 1 | 0 | 1 | ← $(\bar{A}B\bar{C})$ |
| 0 | 1 | 1 | 0 | |
| 1 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 0 | |
| 1 | 1 | 0 | 0 | |
| 1 | 1 | 1 | 0 | |



**Figure 5–67** Truth table and Karnaugh map of $X = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C}$.

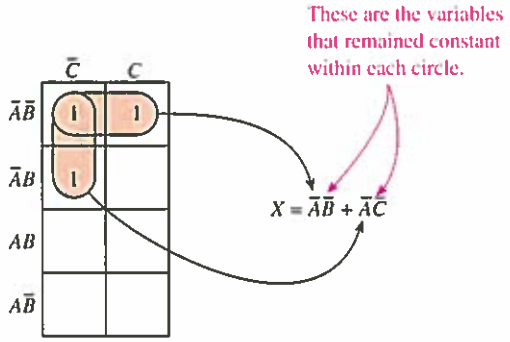




**Figure 5–68** Encircling adjacent cells in a Karnaugh map.

Once the circles have been drawn encompassing all the 1's in the map, the final simplified equation is obtained by determining *which variables remain the same within each circle*. Well, the first circle (across the top) encompasses $\bar{A}\bar{B}\bar{C}$ and $\bar{A}\bar{B}C$. The variables that remain the same within the circle are $\bar{A}\bar{B}$. Therefore, $\bar{A}\bar{B}$ becomes one of the terms in the final SOP equation. The second circle (left column) encompasses $\bar{A}\bar{B}\bar{C}$ and $\bar{A}B\bar{C}$. The variables that remain the same within that circle are $\bar{A}\bar{C}$. Therefore, the second term in the final equation is $\bar{A}\bar{C}$.

Because the final equation is always written in the SOP format, the answer is $X = \bar{A}\bar{B} + \bar{A}\bar{C}$. Actually, the original equation was simple enough that we could have reduced it using standard Boolean algebra. Let's do it just to check our answer:

$$\begin{aligned} X &= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} \\ &= \bar{A}\bar{B}(C + \bar{C}) + \bar{A}B\bar{C} \\ &= \bar{A}\bar{B} + \bar{A}B\bar{C} \\ &= \bar{A}(\bar{B} + B\bar{C}) \\ &= \bar{A}(\bar{B} + \bar{C}) \\ &= \bar{A}\bar{B} + \bar{A}\bar{C} \quad \surd \end{aligned}$$

There are several other points to watch out for when applying the Karnaugh mapping technique. The following examples will be used to illustrate several important points in filling in the map, determining adjacencies, and obtaining the final equation. Work through these examples carefully so that you do not miss any special techniques.