A	B	C	X
0	0	0	1 ← ($\bar A\bar B\bar C$)
0	0	1	1 ← ($\bar A\bar B C$)
0	1	0	1 ← ($\bar A B\bar C$)
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

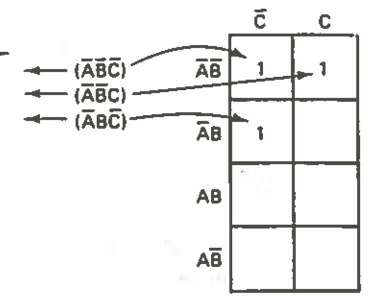

Figure 3–37 Truth table and Karnaugh map of $X = \bar A\bar B\bar C + \bar A\bar B C + \bar A B\bar C$.

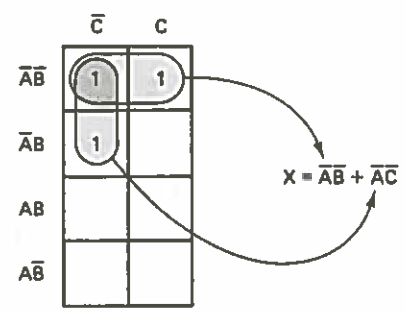

Figure 3–38 Encircling adjacent cells in a Karnaugh map.

circle. Well, the first circle (across the top) encompasses $\bar A\bar B\bar C$ and $\bar A\bar B C$. The variables that remain the same within the circle are $\bar A\bar B$. Therefore, $\bar A\bar B$ becomes one of the terms in the final SOP equation. The second circle (left column) encompasses $\bar A\bar B\bar C$ and $\bar A B\bar C$. The variables that remain the same within that circle are $\bar A\bar C$. Therefore, the second term in the final equation is $\bar A\bar C$.

Since the final equation is always written in the SOP format, the answer is $X = \bar A\bar B + \bar A\bar C$. Actually, the original equation was simple enough that we could have reduced it using standard Boolean algebra. Let's do it just to check our answer:

$$\begin{aligned} X &= \bar A\bar B C + \bar A\bar B\bar C + \bar A B\bar C \\ &= \bar A\bar B(C + \bar C) + \bar A B\bar C \\ &= \bar A\bar B + \bar A B\bar C \\ &= \bar A(\bar B + B\bar C) \\ &= \bar A(\bar B + \bar C) \\ &= \bar A\bar B + \bar A\bar C \;\checkmark \end{aligned}$$

There are several other points to watch out for when applying the Karnaugh mapping technique. The following examples will be used to illustrate several important points in filling in the map, determining adjacencies, and obtaining the final equation. Work through these examples carefully so that you do not miss any special techniques.

EXAMPLE 3–14

Simplify the following SOP equation using the Karnaugh mapping technique:

$$X = \bar A B + \bar A\bar B\bar C + AB\bar C + A\bar B\bar C$$

Solution:

1. Construct an eight-cell K-map (Figure 3–39) and fill in a 1 in each cell that corresponds to a term in the original equation. (Note that $\bar A B$ has no C variable in it. Therefore, $\bar A B$ is satisfied whether C is HIGH or LOW, so $\bar A B$ will fill in two cells: $\bar A BC + \bar A B\bar C$.)

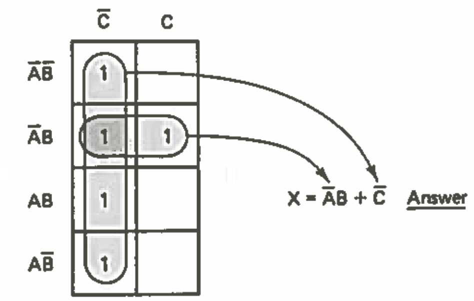

Figure 3–39 Karnaugh map and final equation for Example 3–14.

2. Encircle adjacent cells in the largest group of two or four or eight.
3. Identify the variables that remain the same within each circle and write the final simplified SOP equation by ORing them together.

EXAMPLE 3–15

Simplify the following equation using the Karnaugh mapping procedure:

$$X = \bar A B\bar C D + A\bar B\bar C D + \bar A\bar B\bar C D + AB\bar C D + AB\bar C\bar D + ABCD$$

Solution:

Since there are four different variables in the equation, we need a 16-cell map ($2^4 = 16$), as shown in Figure 3–40.

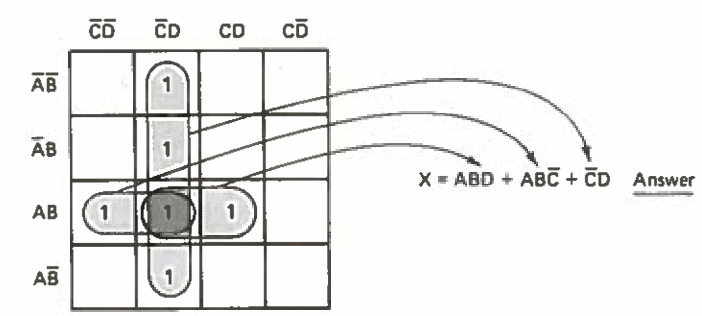

Figure 3–40 Solution to Example 3–15.

EXAMPLE 3–16

Simplify the following equation using the Karnaugh mapping procedure:

$$X = B\bar C\bar D + \bar A B\bar C D + AB\bar C D + \bar A BCD + ABCD$$

Solution:

Note in Figure 3–41 that the $B\bar C\bar D$ term in the original equation fills in *two* cells: $AB\bar C\bar D + \bar A B\bar C\bar D$. Also note in Figure 3–41 that we could have encircled four cells, then two cells, but that would not have given us the simplest final equation. By encircling four cells, then four cells, we will be sure to get the simplest final equation. (Always encircle the largest number of cells possible, even if some of the cells have already been encircled in another group.)