Convert to base 5:

$$\begin{array}{r|l} 5 & 21 \\ \hline 5 & 4 \quad 1 \\ \hline & 0 \quad 4 \end{array} \qquad \begin{array}{r} .4375 \\ \times\ 5 \\ \hline 2.1875 \\ \times\ 5 \\ \hline 0.9375 \\ \times\ 5 \\ \hline 4.6875 \\ \times\ 5 \\ \hline 3.4375 \\ \times\ 5 \\ \hline 2.1875 \end{array} \ \text{*repeats}$$

$$= (41.\overline{2043})_5$$

1.3.3 Base 2^k Conversion

Each of the eight octal digits can be represented by a three-bit binary number. Similarly, each of the 16 hexadecimal digits can be represented by a four-bit binary number. In general, each digit of the base p number system, where p is an integral power k of 2, can be represented by a k-bit binary number.

In converting a base p number to base q, if p and q are both integral powers of 2, the base p number can first be converted to binary, and this in turn can be converted to base q by inspection. This conversion procedure is called the *base 2^k conversion*.

Example 1.13

$$(4\ 2\ A\ 5\ 6\ .\ F\ 1)_{16} = (?)_8$$

$$p = 16 = 2^4, \qquad q = 8 = 2^3$$

Therefore,

$$k_1 = 4, \qquad k_2 = 3$$

Start

$k_1 = 4$ 4   2   A   5   6   .   F   1

Zero included → $\boxed{0}$ 0100 0010 1010 0101 0110 . 1111 0001 $\boxed{0}$ ← Zero included

$k_2 = 3$ 1   0   2   5   1   2   6 . 7   4   2

$$= (1025126.742)_8$$

Example 1.14

$$(AF5.2C)_{16} = (?)_4$$

$A \quad F \quad 5\ .\ 2 \quad C$ base $16 = 2^4$ $k_1 = 4$

1010 1111 0101.0010 1100   base $4 = 2^2$ $\therefore k_2 = 2$

2 2 3 3 1 1 . 0 2 3 0

$$= (223311.0230)_4$$

Example 1.15

$$(567.23)_8 = (?)_{16}$$

5   6   7 . 2   3   base 8 $\therefore k_1 = 3$

Zeros included $\boxed{0}\boxed{0}\boxed{0}$ 101 110 111.010 011 $\boxed{0}\boxed{0}$ base 2

1   7   7 . 4   C   base 16 $\therefore k_2 = 4$

$$= (177.4C)_{16}$$

It is thus possible to represent binary numbers in a very compact form by using octal and hexadecimal systems. The conversion between these