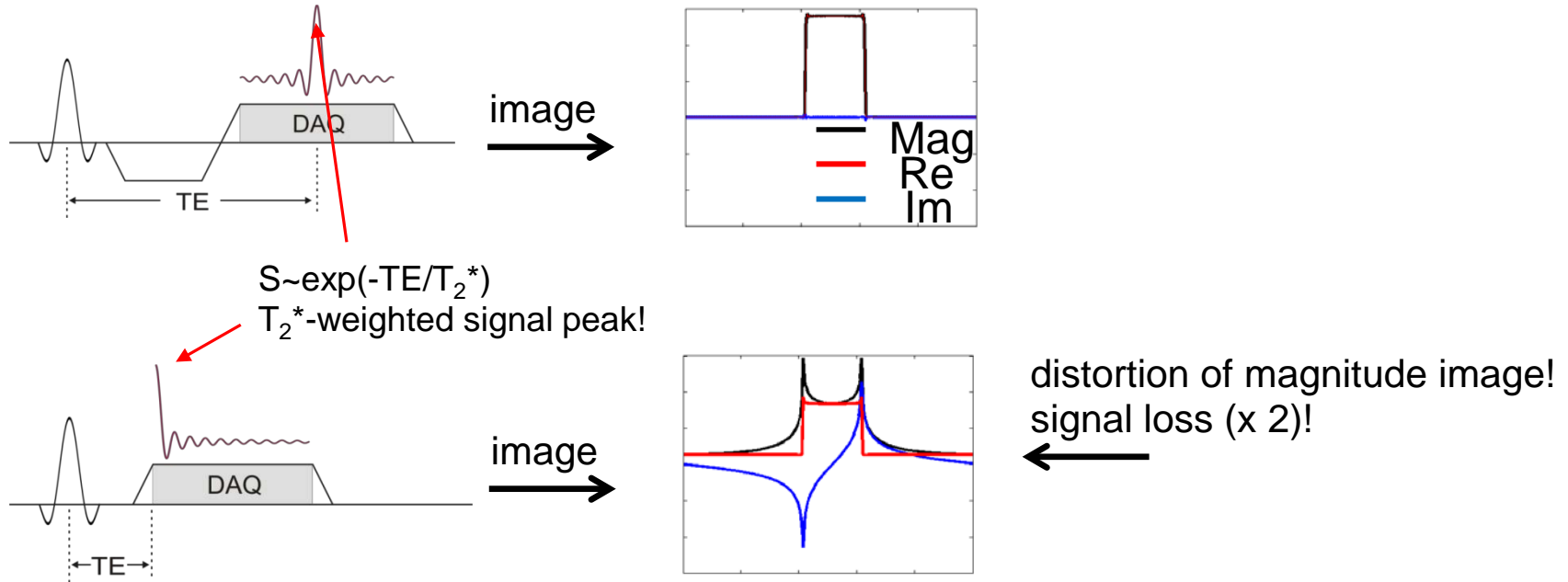# Gradient Echoes, SSFP and steady states

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ISMRM German Chapter Ph.D. student Training, Freiburg 9 Feb 2024

Content

1) The need for an echo

2) Basic GRE imaging

3) SSFP, the SSFP profile, balanced and unbalanced SSFP

4) More echoes of the SSFP family: PSIF and DESS

5) T1-weighted GRE imaging with RF-spoiling

6) The mechanism of RF-spoiling – transverse magentization = 0 (?)

Why Gradient"echo" ? – Fourier transform properties!

image →

$S\sim\exp(-TE/T_2^*)$
T_2^*-weighted signal peak!

Mag
Re
Im

image →

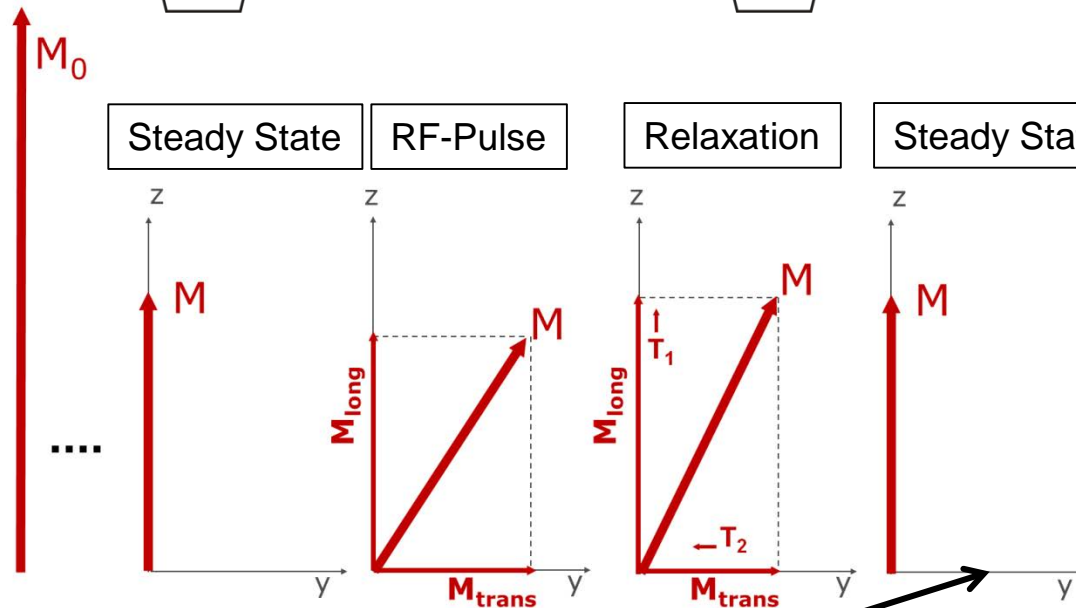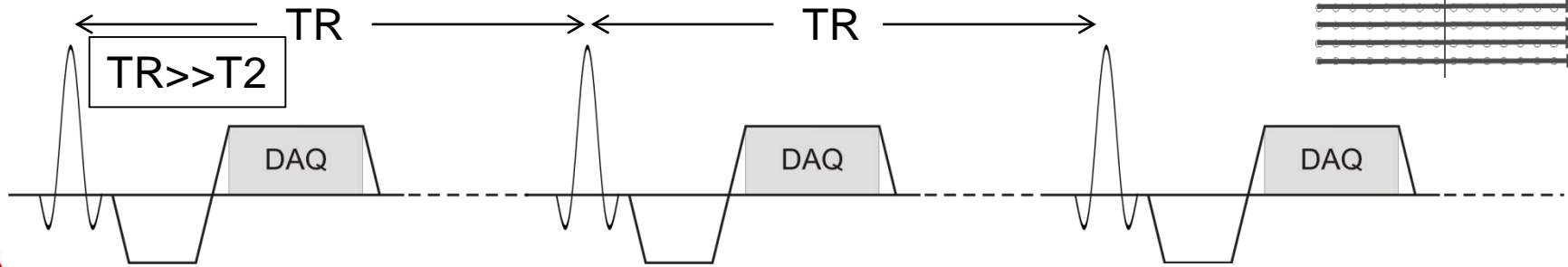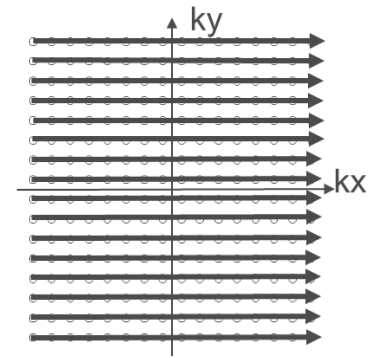distortion of magnitude image!
signal loss (x 2)!

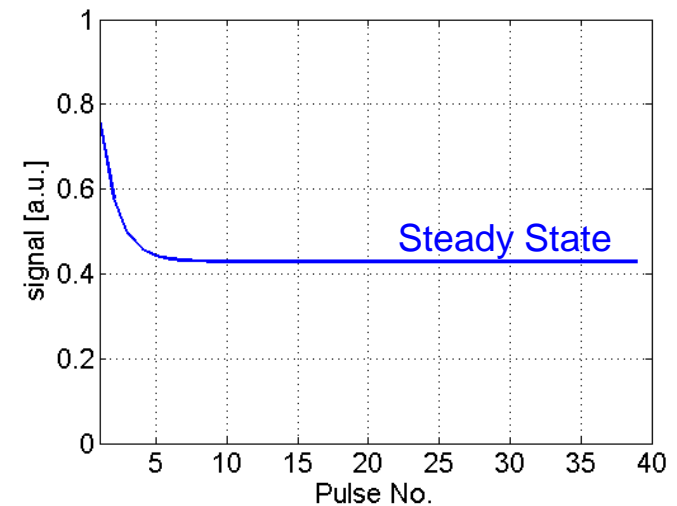Two reasons for creating echos in MRI:
a) Spinecho: refocusing of static offresonances -> T_2 weighting
b) Gradientecho (i.e. applying prephasing gradient): exploiting Fourier transform properties
   ● distortion free magnitude image (no need for cumbersome correction)
   ● Signal gain of factor 2

Simple gradient echo sequence

In order to acquire k-space, a train of gradient echoes is needed:

$$S = M_0 \sin \alpha \frac{1 - e^{-TR/T_1}}{1 - e^{-TR/T_1} \, \cos \alpha}$$

("Ernst-equation")

TR>>T2

TR

TR

DAQ

M_0

M

Steady State | RF-Pulse | Relaxation | Steady State

M_{long}

M_{trans}

T_1

T_2

No transversal magnetisation before the RF-pulse! -> TR>>T2
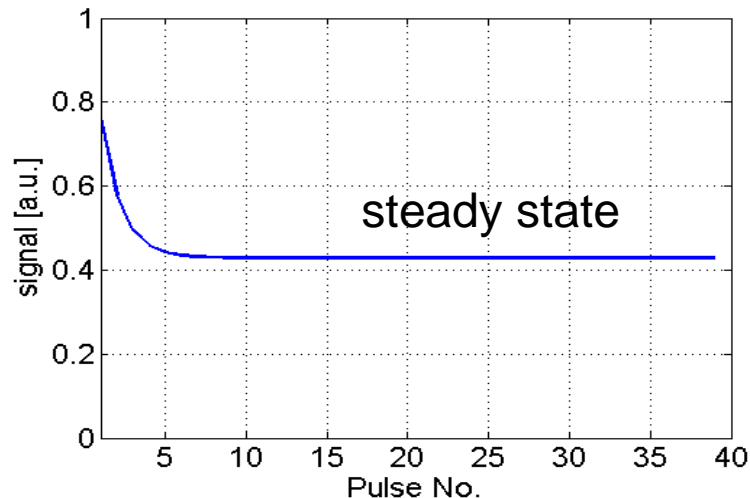
Steady State

Going faster: SSFP

SSFP: <u>S</u>teady <u>S</u>tate <u>F</u>ree <u>P</u>recession

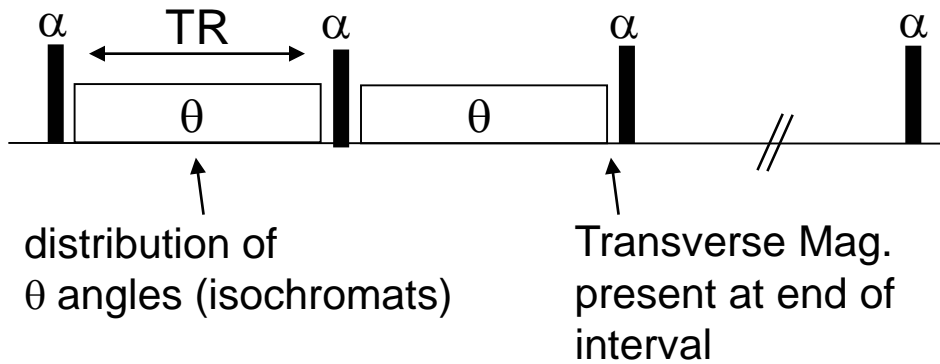<u>Steady State</u>: Magnetisation not returning to equilibrium (M_0) during TR (~TR<3T1)

α TR α α α α

1st pulse 2nd pulse pulse No. *n*
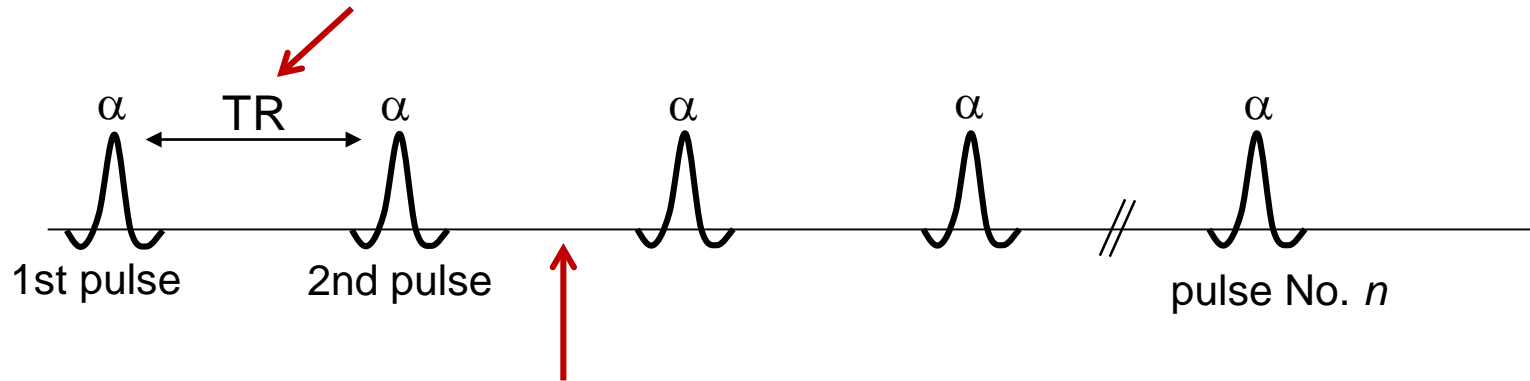
<u>Free Precession</u>: Magnetisation precesses around (local!) B_0 with B_1 switched off
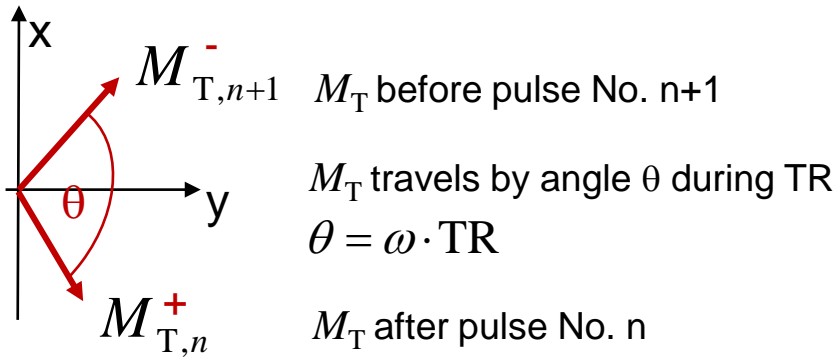(Forced Precession: B_1 switched on)

steady state

From now on: Transversal magnetization at the end of interval is **not** zero, TR<T2 !
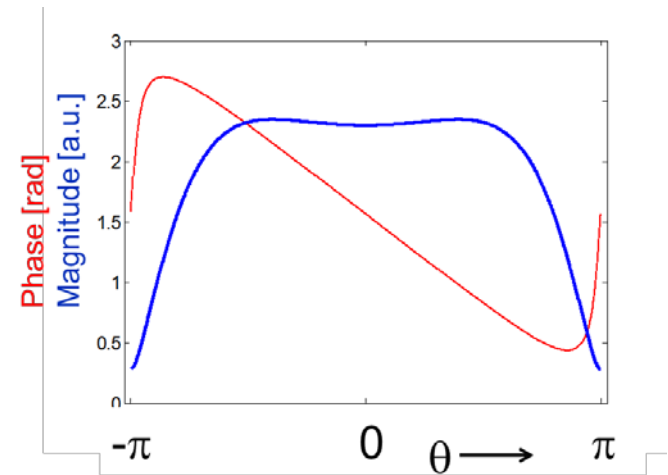Ernst equation is no longer valid

SSFP: RF pulses and dephasing

α   TR   α   α   α

θ   θ

distribution of
θ angles (isochromats)

Transverse Mag.
present at end of
interval

Steady state for

$$M_T^+(\theta) = M_x^+ + iM_y^+$$

Transverse plane:

x

$M_{T,n+1}^-$ M_T before pulse No. n+1

θ   y

$M_{T,n}^+$ M_T after pulse No. n

M_T travels by angle θ during TR

$$\theta = \omega \cdot TR$$

Phase [rad] / Magnitude [a.u.]

−π   0   θ ⟶   π

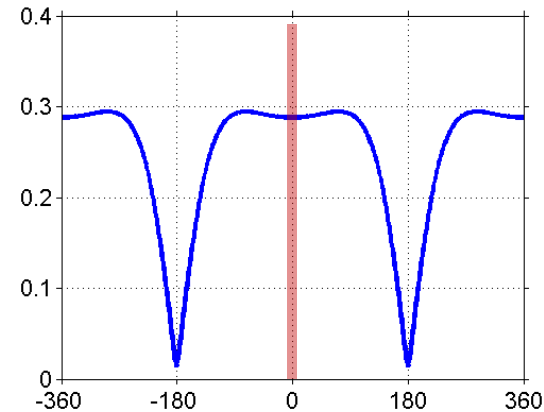α=40°, T_1=0.5s, T_2=0.1s, TR=0.01s

$$M_x^+ = M_0(1-E_1)E_2 \sin\alpha \sin\theta / D$$

$$M_y^+ = M_0(1-E_1)(1-E_2\cos\theta)\sin\alpha / D$$

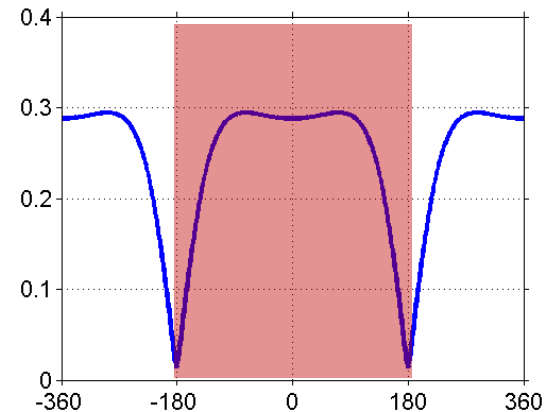$$D = (1-E_1\cos\alpha)(1-E_2\cos\theta) - (E_1-\cos\alpha)(E_2-\cos\theta)E_2$$

$$E_1 = \exp(-TR/T_1), \quad E_2 = \exp(-TR/T_2)$$

Two principle strategies to make a sequence based on SSFP

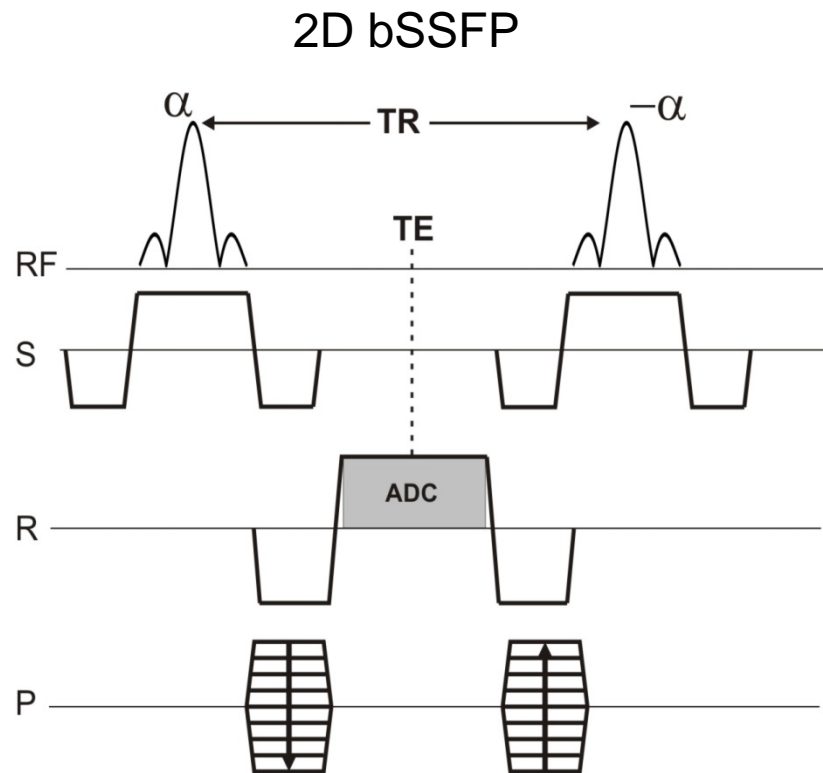1. Selecting a single frequency on the profile: *balanced SSFP*

2. Integration over the profile: *unbalanced SSFP*
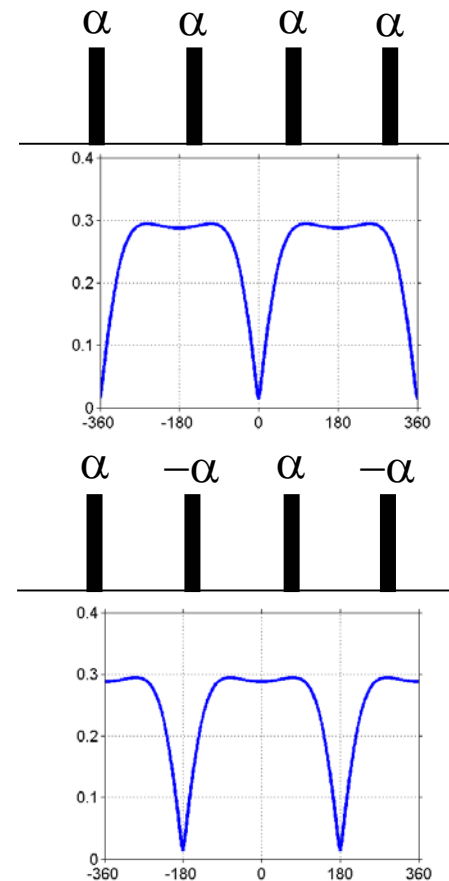
Balanced SSFP sequence diagram

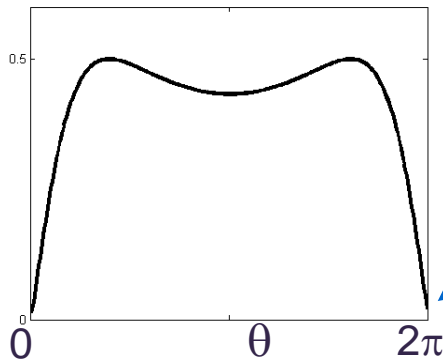For multidimensional k-space acquisition, gradients in all directions are needed:

2D bSSFP

"balanced" gradients!

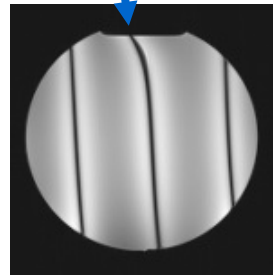### RF-pulses with alternating sign…

…shift the passband to on-resonance

The stripe artefact in bSSFP images

If θ is located in the „stopband", a dark stripe appears in the image
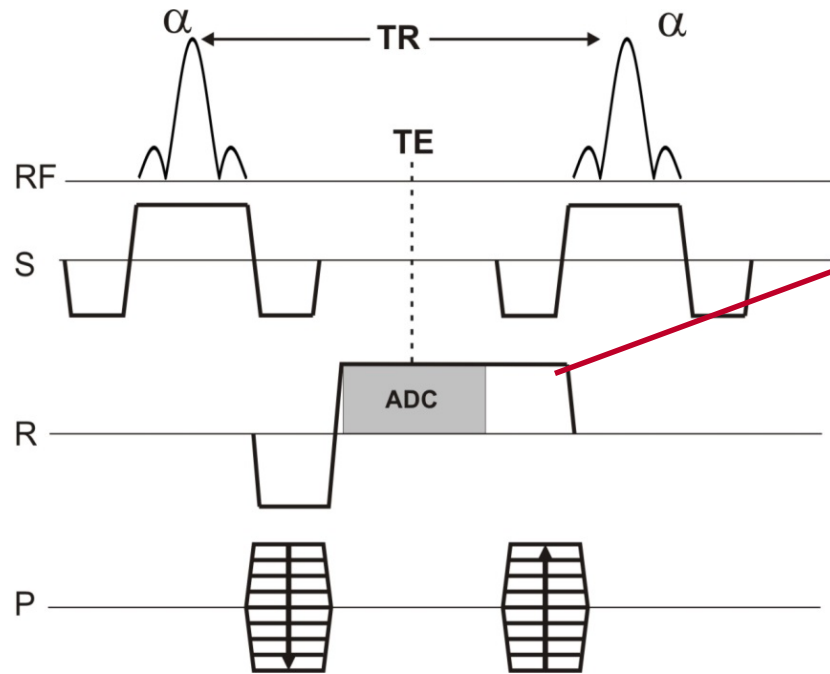
Distance of the stopband from on-resonance (center of passband) in Hz:

$$\Delta f = \frac{1}{2 \cdot TR}$$

Example: $TR = 4$ ms -> $\Delta f = 125$ Hz

Good shim is required to avoid dark stripes! Short TR needed!

Unbalanced SSFP: FISP sequence

The unbalanced area of all gradients must distribute the isochromats such that the voxel signal is the integration over the profile!
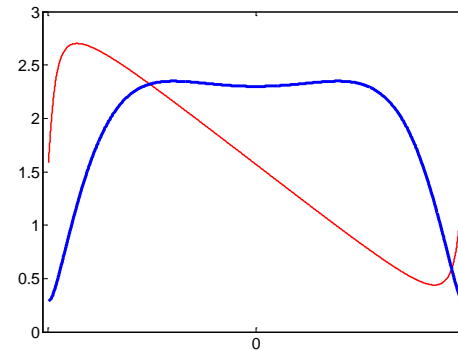
$$\int d\theta = const.$$

-> Need for spoiler gradient(s)!

$$S_{\text{FISP}} = \int_{-\pi}^{\pi} M_{\text{T}}^{+}(\theta)d\theta = \frac{M_0 \sin\alpha \, e^{-TE_{\text{FISP}}/T2}}{1+\cos\alpha}[1 - D'(E_1 - \cos\alpha)]$$

with $D' = \dfrac{\sqrt{1-E_2^2}}{\sqrt{1-E_1^2 E_2^2 - 2E_1(1-E_2^2)\cos\alpha + (E_1^2 - E_2^2)\cos^2\alpha}}$

FISP Signal = „profile squeezed into one voxel"

RO

RO

Courtesy of M.Markl

< 1pixel

Fully Balanced
SSFP

Unbalanced
GRE
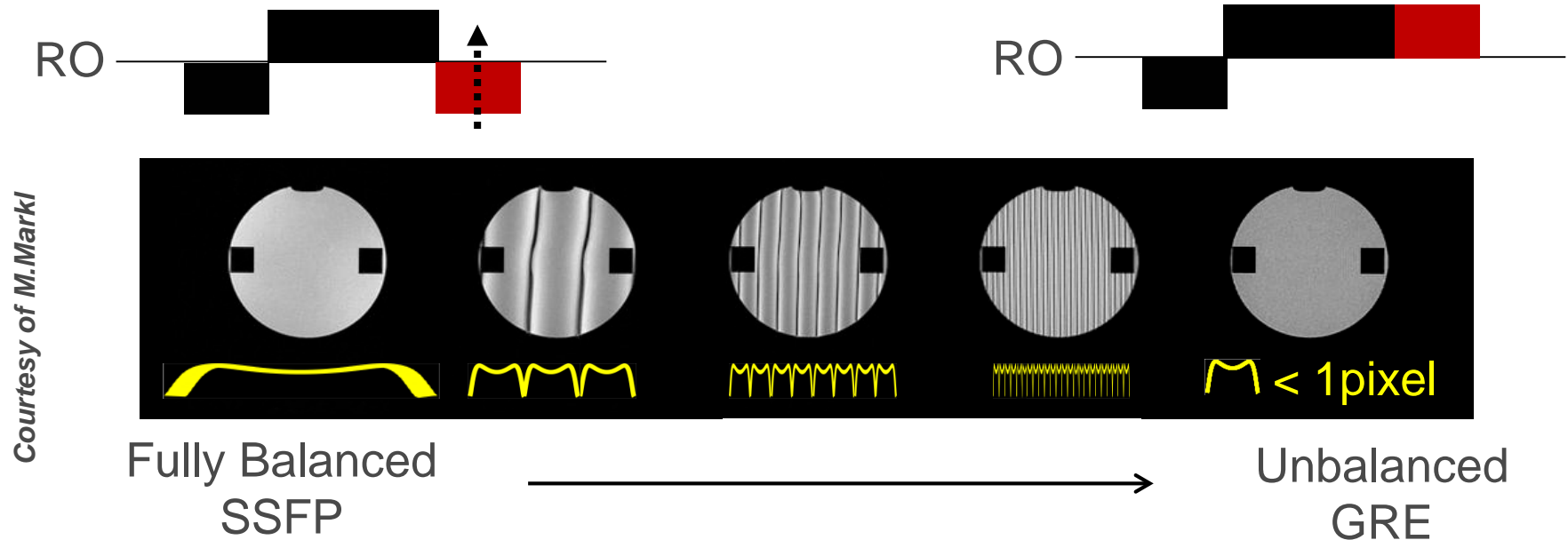
The FISP Signal is the integration of the after-pulse SSFP signal and shows a mixed T_1/T_2-contrast!

More Echos: Stopped pulse experiment 1

$\alpha=40°$, $T_1=0.5s$, $T_2=0.1s$, $TR=0.01s$, $31vox$

FID

Echoes

Terminate sequence in the steady state
(and play long gradient pulse)

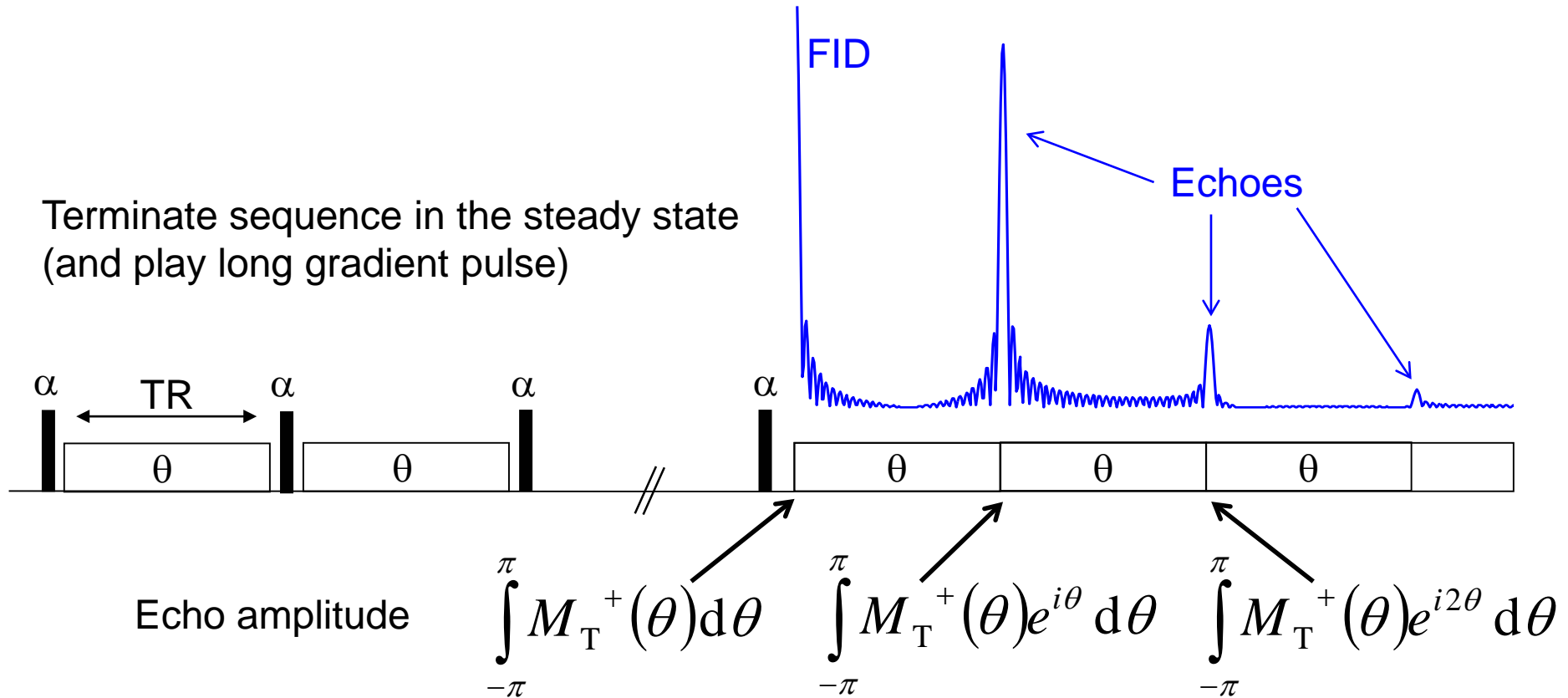α TR α α α

θ θ θ θ θ

Echo amplitude $\displaystyle\int_{-\pi}^{\pi} M_{\mathrm{T}}{}^{+}(\theta)\,\mathrm{d}\theta$ $\displaystyle\int_{-\pi}^{\pi} M_{\mathrm{T}}{}^{+}(\theta)e^{i\theta}\,\mathrm{d}\theta$ $\displaystyle\int_{-\pi}^{\pi} M_{\mathrm{T}}{}^{+}(\theta)e^{i2\theta}\,\mathrm{d}\theta$

$$\Rightarrow \mathrm{Signal(k)}^{+} = \int_{-\pi}^{\pi} M_{\mathrm{T}}{}^{+}(\theta)e^{ik\theta}\,\mathrm{d}\theta, \quad k \geq 0$$

Even more echos: Stopped pulse experiment 2

$\alpha=40°$, $T_1=0.5s$, $T_2=0.1s$, $TR=0.01s$, 31vox

FID

Echoes

Terminate sequence in the steady state
and play long gradient pulse
with inverted amplitude
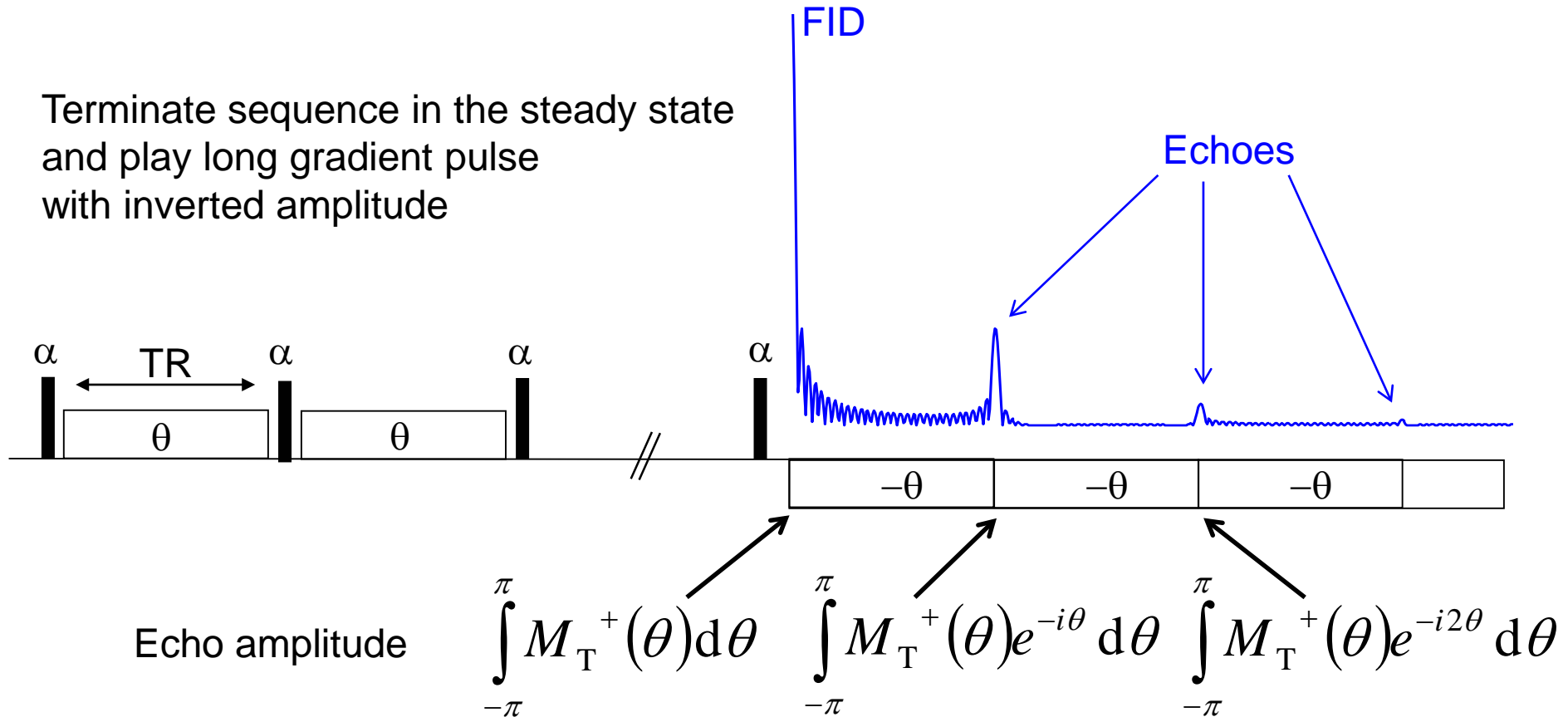
α TR α θ α θ α

$-\theta$ $-\theta$ $-\theta$

Echo amplitude
$$\int_{-\pi}^{\pi} M_{\mathrm{T}}{}^{+}(\theta)\mathrm{d}\theta \quad \int_{-\pi}^{\pi} M_{\mathrm{T}}{}^{+}(\theta)e^{-i\theta}\,\mathrm{d}\theta \quad \int_{-\pi}^{\pi} M_{\mathrm{T}}{}^{+}(\theta)e^{-i2\theta}\,\mathrm{d}\theta$$

$$\Rightarrow \mathrm{Signal(k)}^{+} = \int_{-\pi}^{\pi} M_{\mathrm{T}}{}^{+}(\theta)e^{ik\theta}\,\mathrm{d}\theta, \quad k \le 0$$

Caculation of all echo amplitudes

$$\Rightarrow F_k^+ = \text{Signal}(-k)^+ = \int_{-\pi}^{\pi} M_T^+(\theta)e^{-ik\theta}\,d\theta$$

Fourier transform of $M_T^+(\theta)$!

Butz T. Fourier Transformation for Pedestrians. Berlin Heidelberg: Springer; 2006.
Leupold J. Steady-state free precession signals of arbitrary dephasing order and their sensitivity to T2* Concepts Magn Reson Part A. 2018;e21435
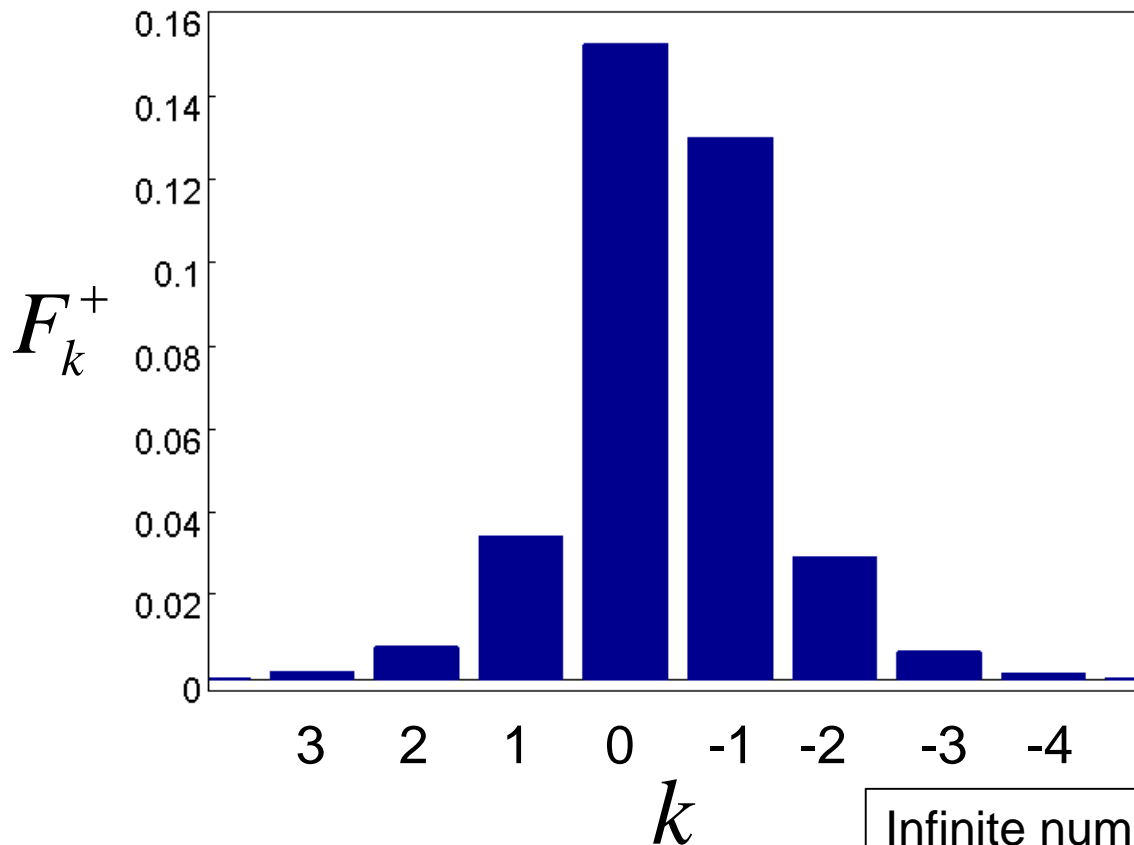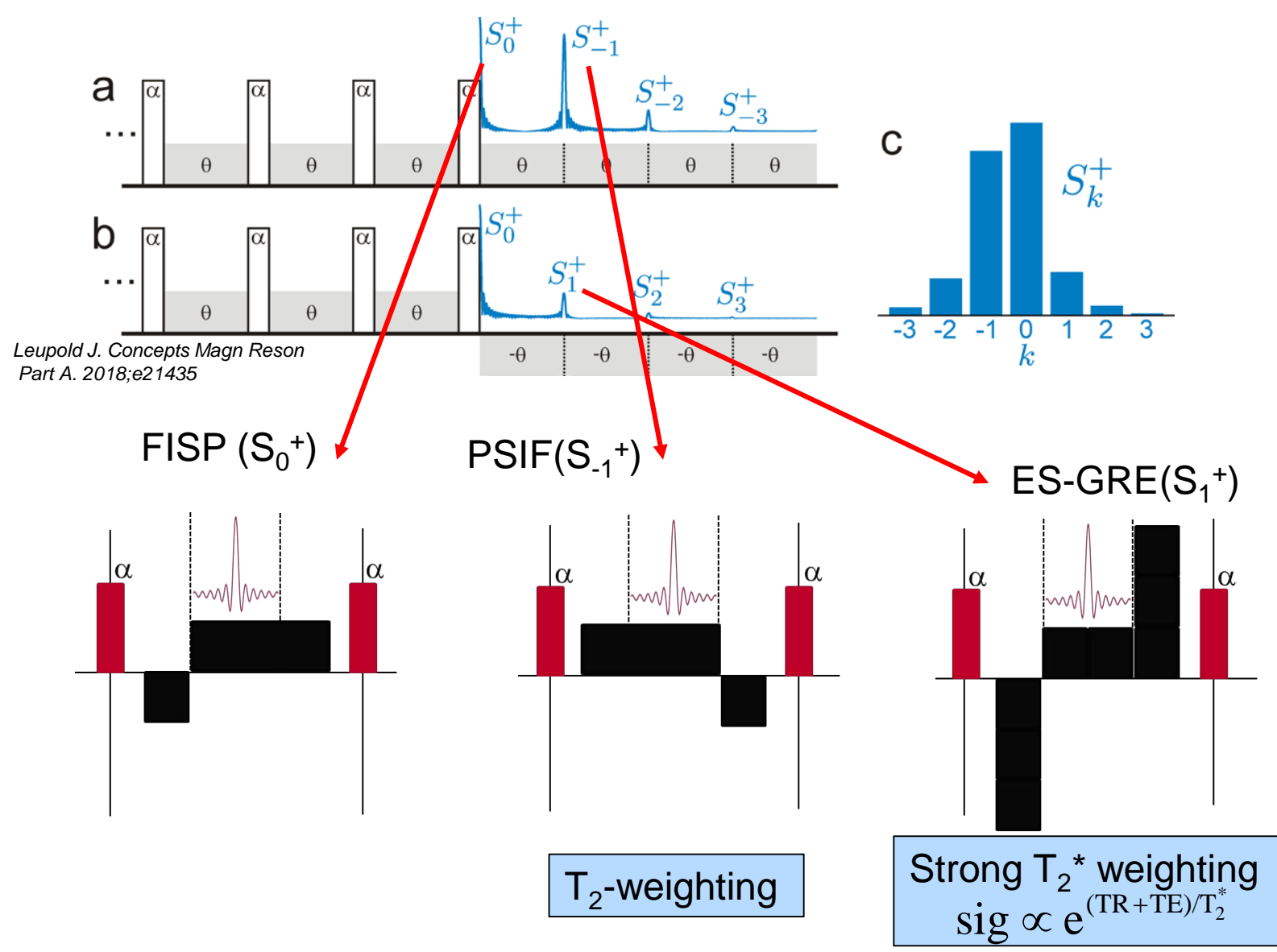

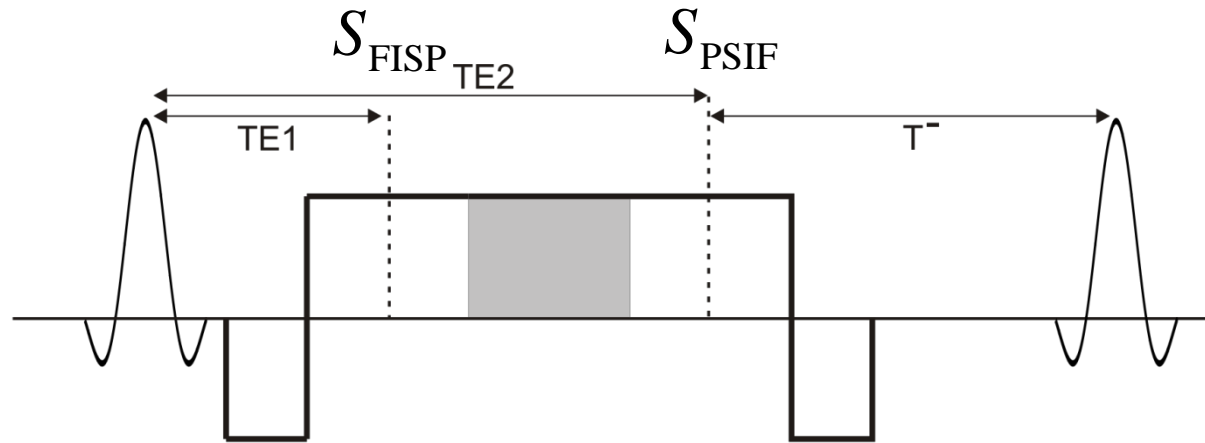Amplitudes of all echoes are given by the Fourier transform of $M_T^+(\theta)$!

$M_T^+(\theta)$

$\alpha=40°$, $T_1=0.5s$, $T_2=0.1s$, $TR=0.01s$
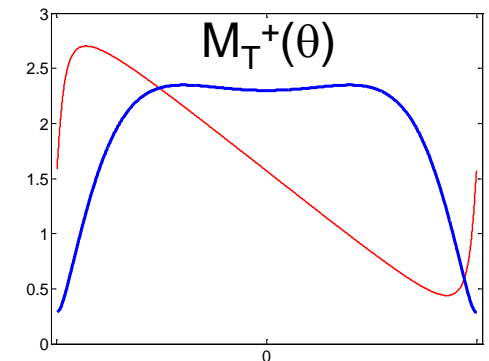
Infinite number of sequences!

How to make imaging SSFP sequences from echoes

Leupold J. Concepts Magn Reson Part A. 2018;e21435

FISP (S_0^+)

PSIF(S_{-1}^+)

ES-GRE(S_1^+)

T_2-weighting

Strong T_2^* weighting
$$\text{sig} \propto e^{(TR+TE)/T_2^*}$$

DESS: Two Signals per interval

$$F_0^+ = \int_{-\pi}^{\pi} M_\mathrm{T}^+(\theta)\,\mathrm{d}\theta$$

$$F_{-1}^+ = \int_{-\pi}^{\pi} M_\mathrm{T}^+(\theta)e^{i\theta}\,\mathrm{d}\theta$$

$$S_\mathrm{FISP} = F_0^+ e^{-TE1/T_2^*}$$

$$S_\mathrm{PSIF} = F_{-1}^+ e^{-TE2/T_2}\, e^{-T^-/T_2'}$$
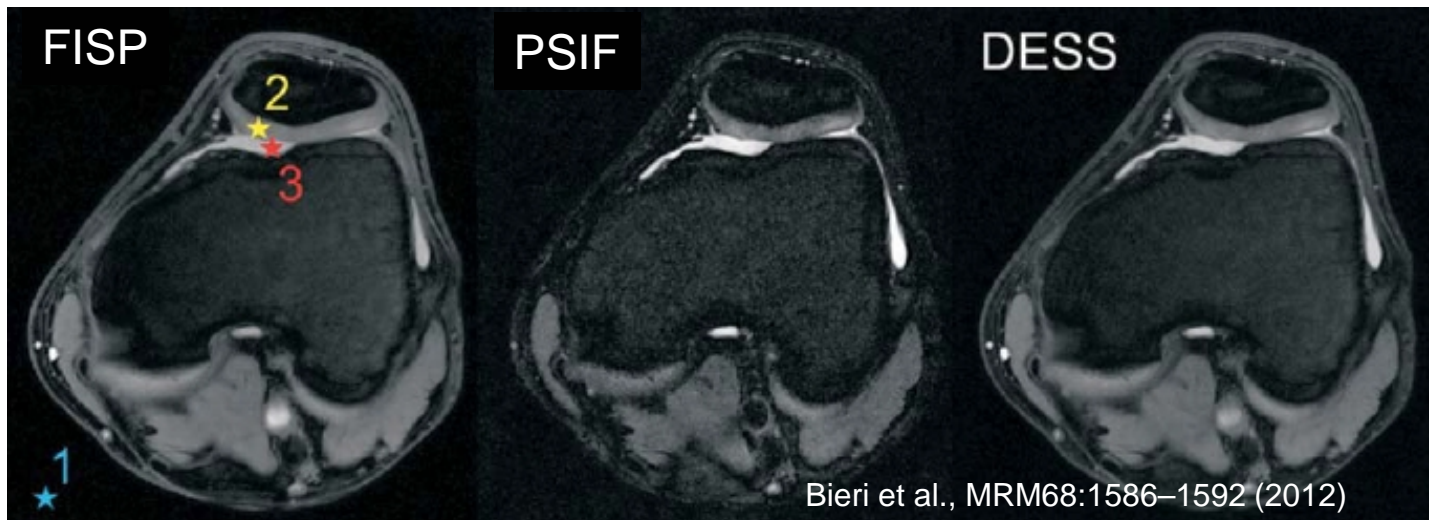

DESS=
|FISP|+|PSIF|

Bieri et al., MRM68:1586–1592 (2012)

Unbalanced SSFP sequences - summary

1) All unbalanced SSFP sequences show some
T_2 weighting (if not TR>>T_2)

2) Most common members of the unbalanced
(non-RF-spoiled) SSFP family:
FISP and DESS

3) Amplitudes of all echoes
(= all sequences) can be calculated from
the M_0^+ profile

T1=1, T2=0.1, TR=0.01, TE=0

Steady state signal [a.u.]

— Trufi
— FISP
— PSIF
— FLASH
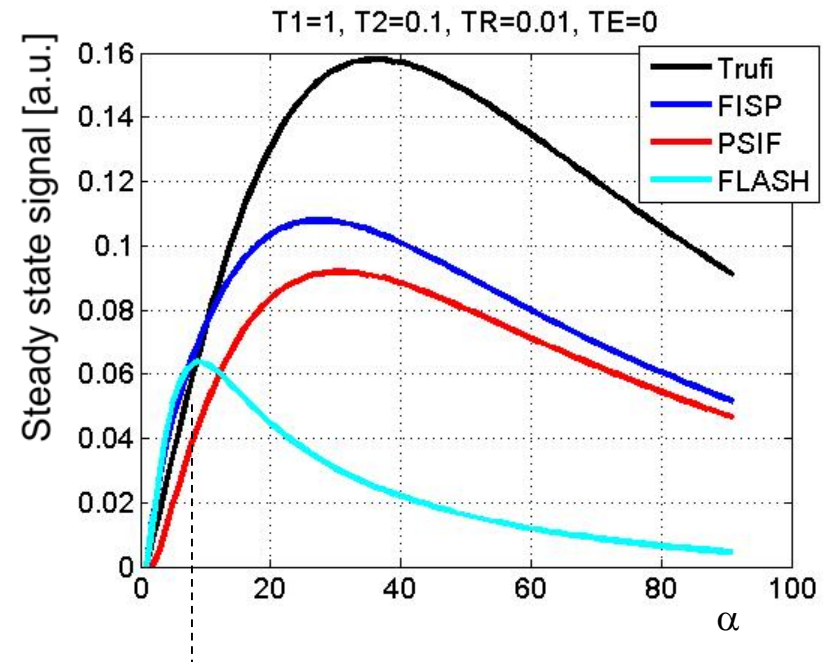
Now: How to achieve T_1-weighting! -> RF-spoiling!
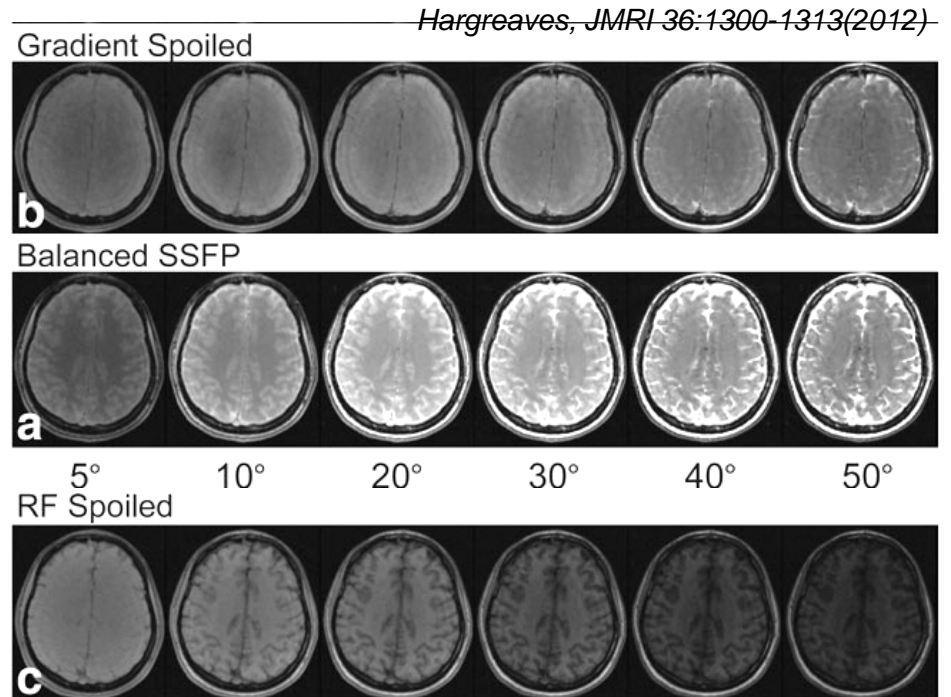
T1-weighting by means of RF-spoiling

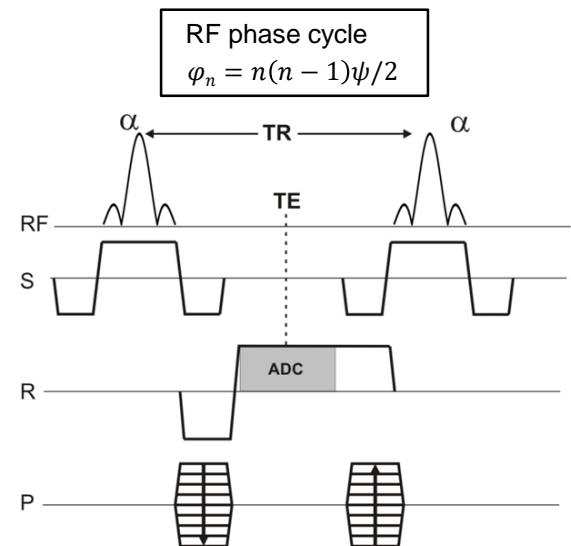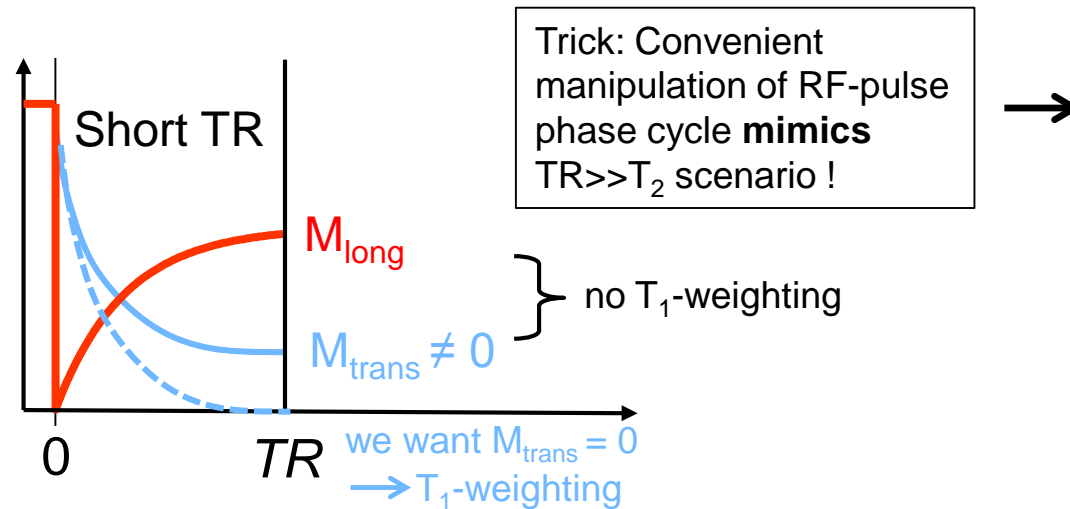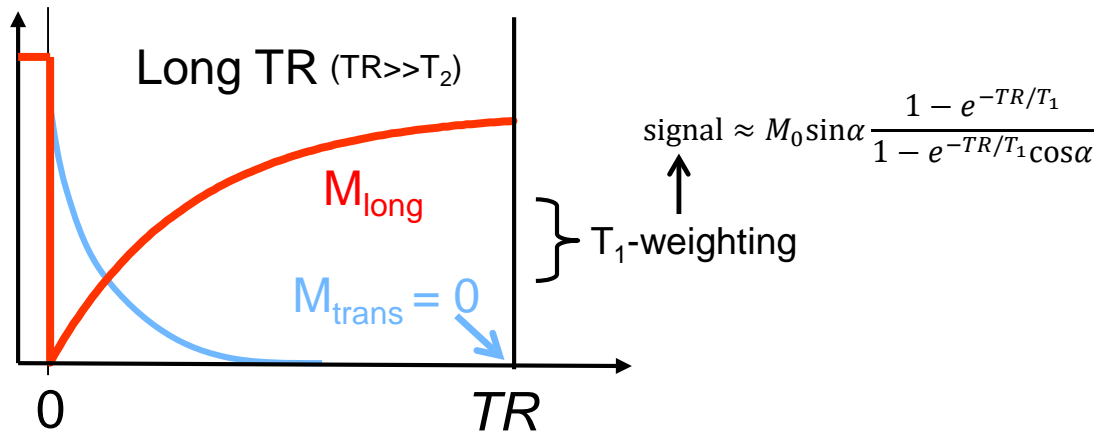Contrasts:

FISP (unbalanced SSFP): mix of T2 and T1

bSSFP: T2/T1 –contrast at on-resonance and optimal α

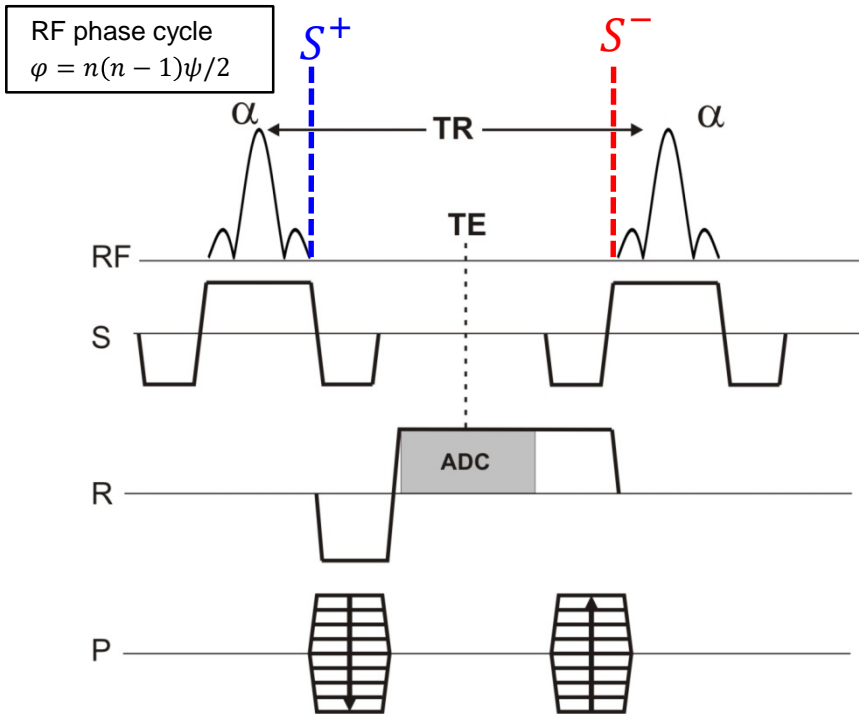Often desired: T1-contrast (e.g. CE MRA)

Obtained by RF-spoiling!

Hargreaves, JMRI 36:1300-1313(2012)

Making T_1 weighted GRE faster: RF - spoiling

Long TR $(TR \gg T_2)$

M_{long}

$M_{trans} = 0$

$$\text{signal} \approx M_0 \sin\alpha \frac{1 - e^{-TR/T_1}}{1 - e^{-TR/T_1}\cos\alpha}$$

T_1-weighting

0   TR

Short TR

M_{long}

$M_{trans} \neq 0$

no T_1-weighting

0   TR   we want $M_{trans} = 0$
→ T_1-weighting

Trick: Convenient manipulation of RF-pulse phase cycle **mimics** $TR \gg T_2$ scenario !

→

RF phase cycle
$\varphi_n = n(n-1)\psi/2$

RF pulse phase
for RF − spoiling:
$$\varphi(n) = n(n-1)\psi/2$$

n ... RF-pulse No.
ψ ... RF-spoil increment (50°,117°...)

RF – spoiling with phase difference increment ψ

RF phase cycle
$\varphi = n(n-1)\psi/2$

Transverse magnetization before and after RF-pulse

$$S^+ = \int\limits_{-\pi}^{\pi} M_T{}^+(\theta)\,d\theta$$

$$S^- = \int\limits_{-\pi}^{\pi} M_T{}^+(\theta)\,e^{i\theta}\,d\theta$$

Vendors' choices:
Siemens: ψ = 50°
GE: ψ = 115.4°
Bruker: ψ = 117°
Philips: ψ = 150°
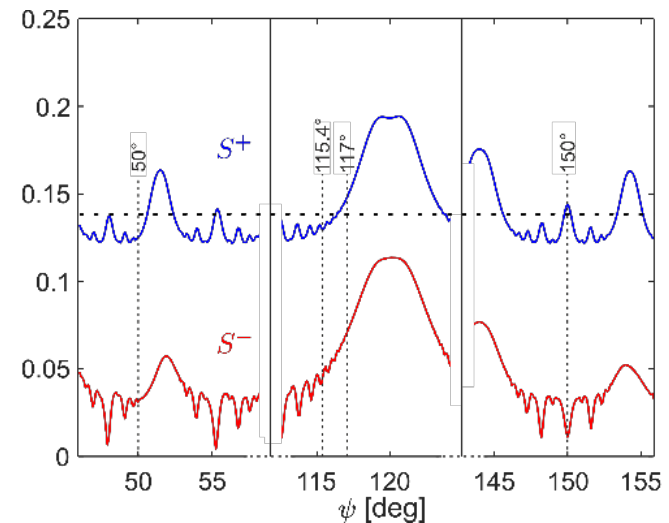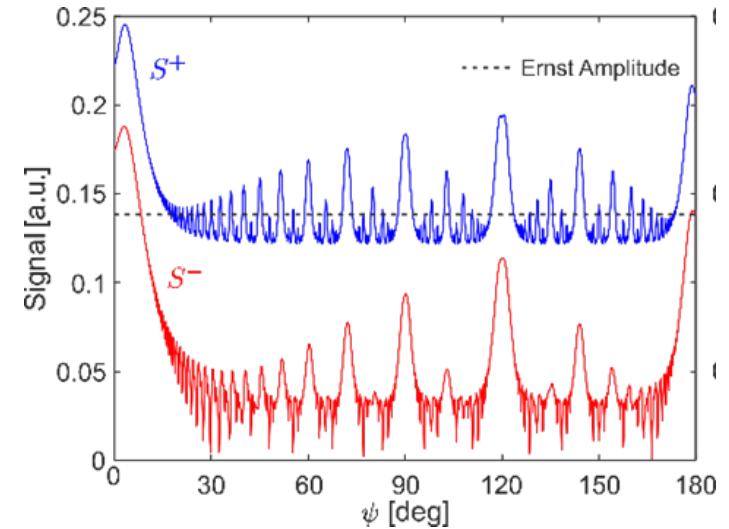
# Paraphrasing RF-spoiling: Is the signal zero at the end of the interval?

Some literature says:

RF spoiling manipulates magnetization such that the |vector addition| of all transverse magnetization components results in a |zero net magnetization at the end| of every sequence interval.

Markl and Leupold, JMRI 35:1274–1289(2012)

$S^- = 0$ at the end of the interval?

… no!
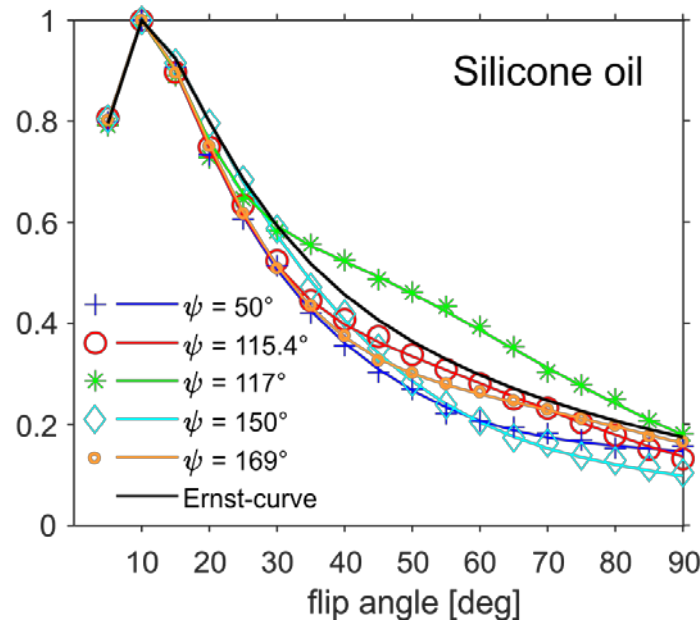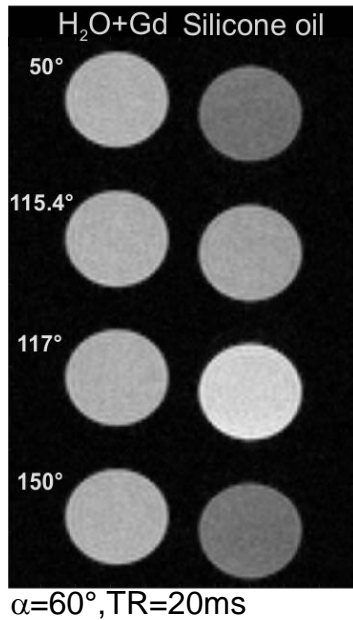
$T_1/TR=T_2/TR=20$, $\alpha=30°$
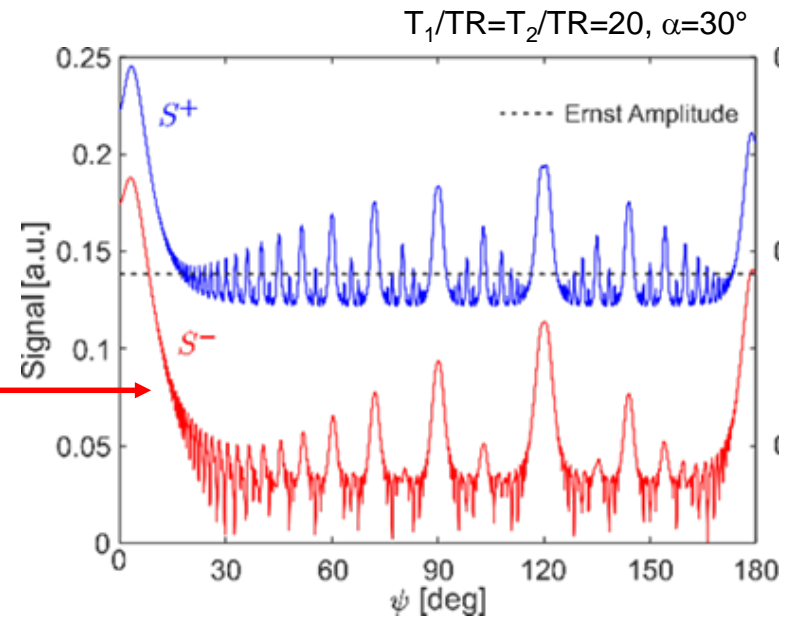
Better:

[RF-Spoiling is] an attempt to restore the contrast properties of long-TR gradient echo techniques.

Denolin et al., MRM 54:937–954 (2005)

Also Better: „RF-spoiling manipulates the 3D magentization vector (per voxel) such that the measured signal obeys approximately the Ernst equation"

Reality of RF-spoiling: Influence of diffusion!

H$_2$O+Gd Silicone oil

50°

115.4°

117°

150°

α=60°,TR=20ms

Silicone oil

- ψ = 50°
- ψ = 115.4°
- ψ = 117°
- ψ = 150°
- ψ = 169°
- Ernst-curve

flip angle [deg]

RF-spoiling works well for:
- water (diffusion!)
- low flip angles (Ernst angle)
- low T2/TR (T2<TR)

(clinical) in-vivo imaging!

T1=540ms;T2=340ms;D=0.002mm^2/s

w/o diffusion

S+
S-
Ernst ampl.

Signal [a.u.]

ψ [deg]

Δx=1mm

S+
S-
Ernst ampl.

Signal [a.u.]

ψ [deg]

Δx=0.5mm

S+
S-
Ernst ampl.

Signal [a.u.]

ψ [deg]