MATH564: Pursuit of Happiness

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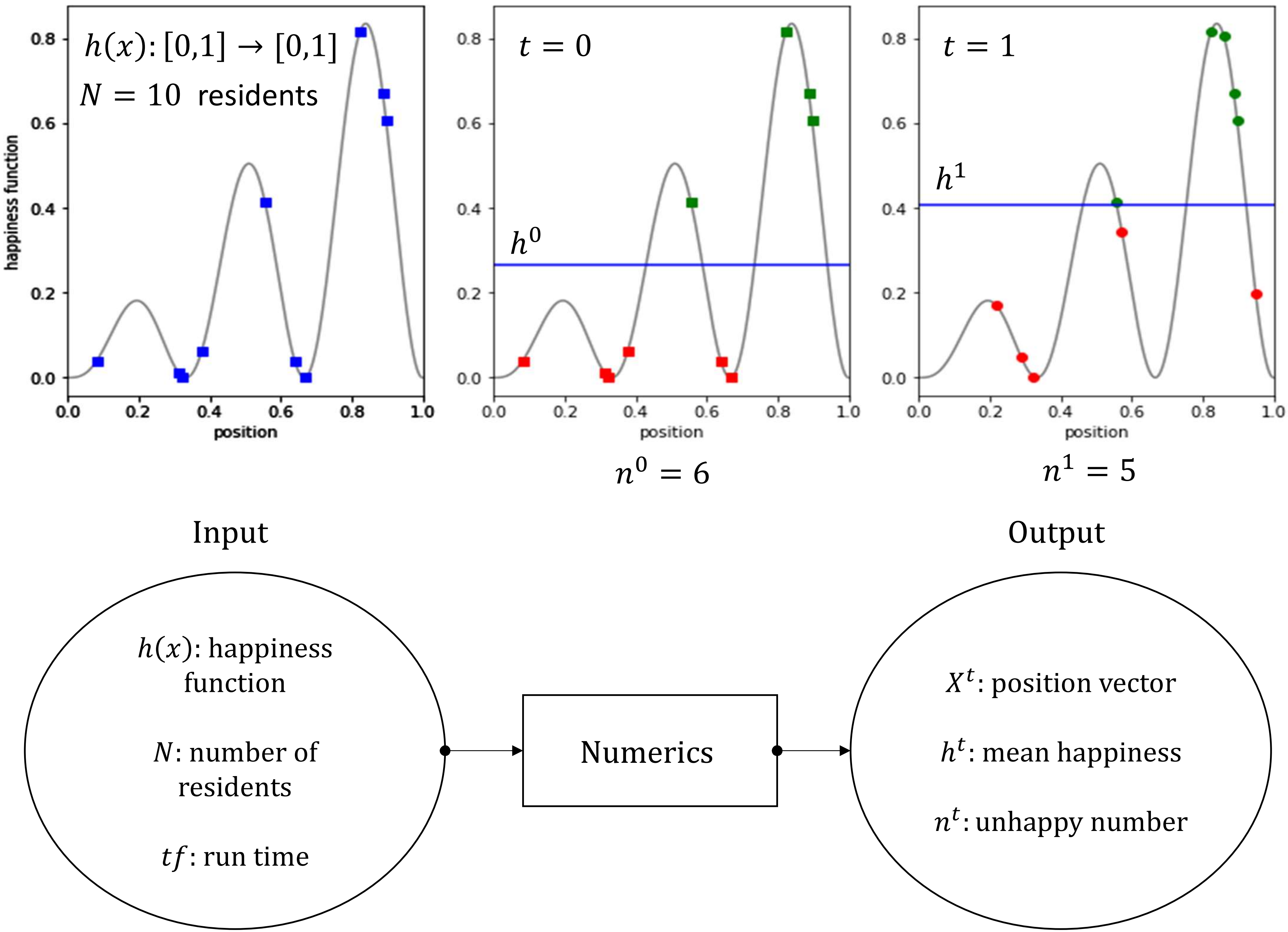
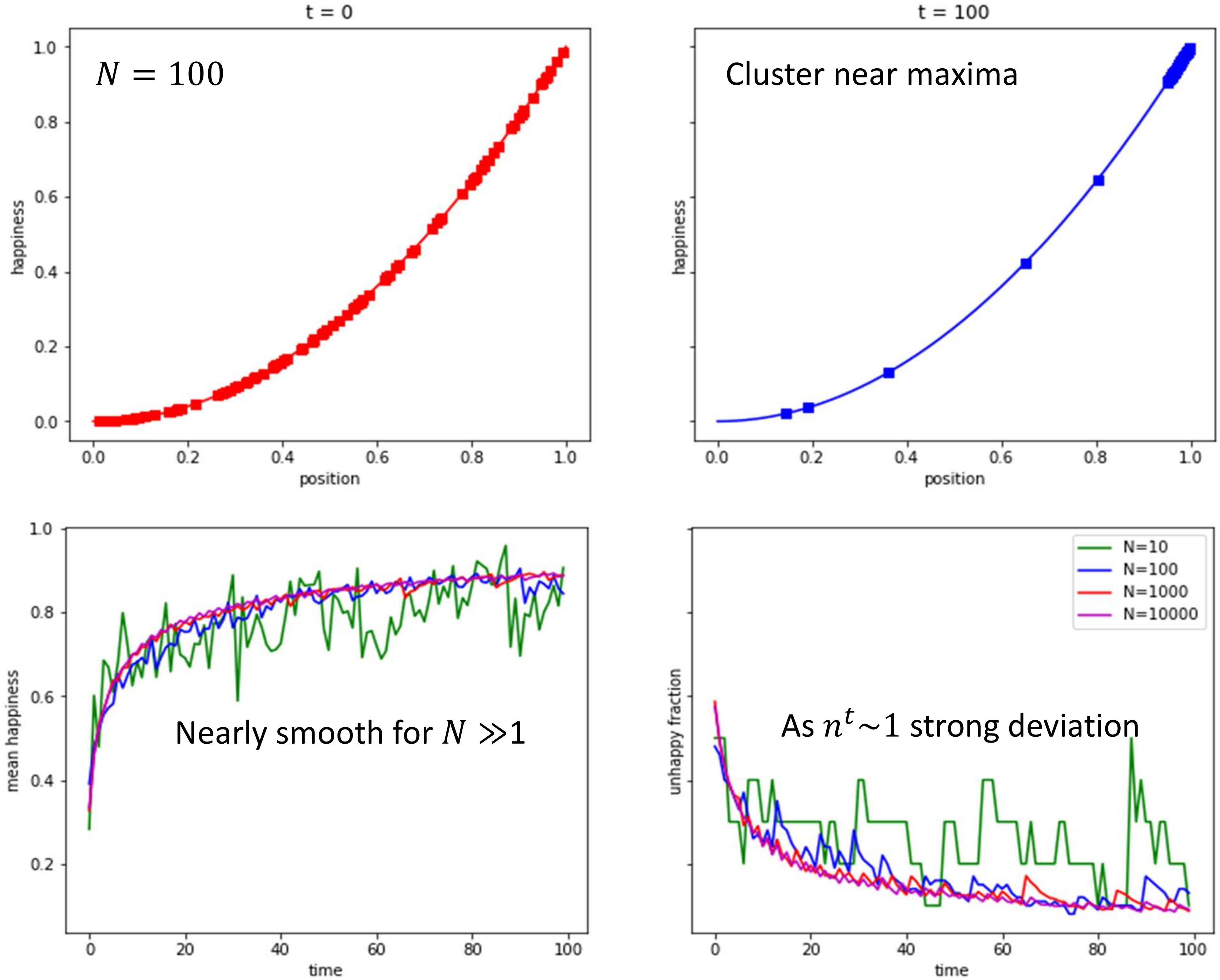


h(x): [0,1] → [0,1]
N = 10 residents
happiness function
position
t = 0
h^0
n^0 = 6
t = 1
h^1
n^1 = 5
Input
Output
h(x): happiness function
N: number of residents
tf: run time
Numerics
X^t: position vector
h^t: mean happiness
n^t: unhappy number



What happens as $N \to \infty$?

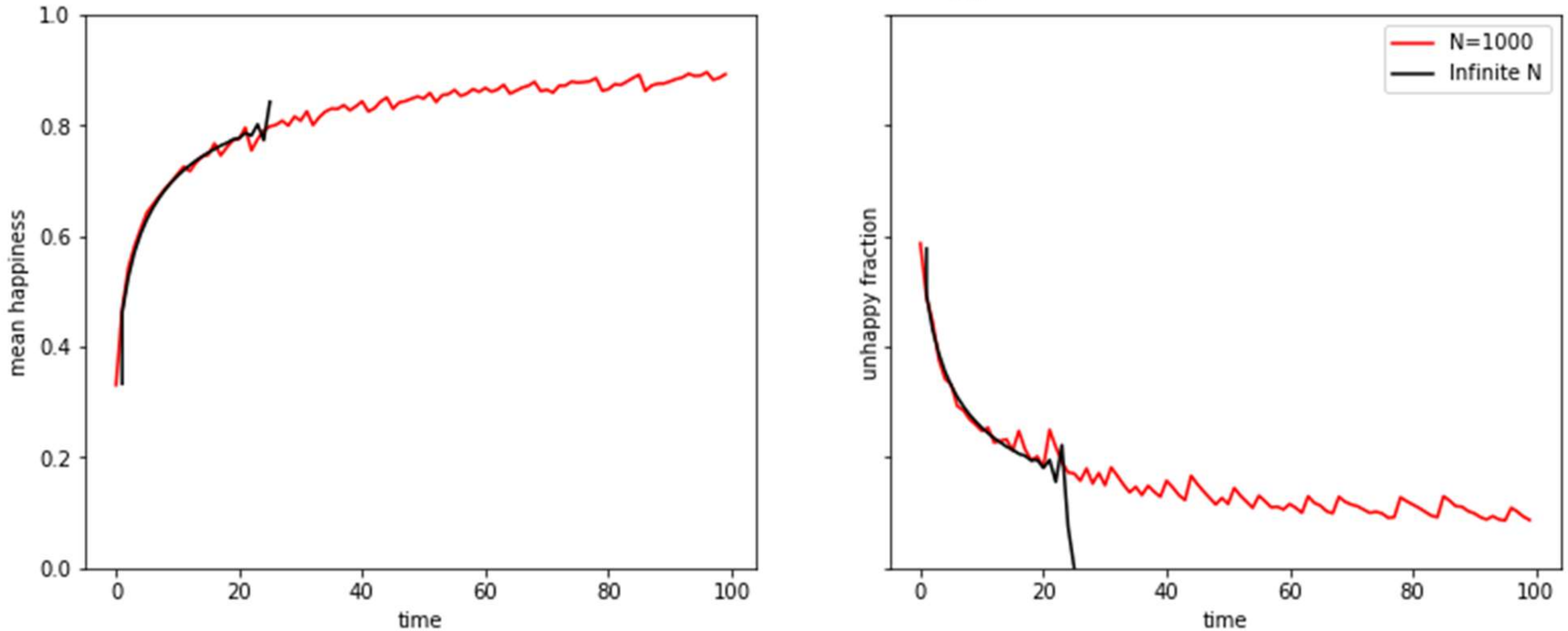
$$\chi_k \in [a, b] : k \leq M$$

$$\text{Ergodic Theorem :} \quad \lim_{M\to\infty} \frac{1}{M} \sum_{k=1}^{M} h(\chi_k) = \frac{1}{b-a} \int_a^b h(z)dz$$

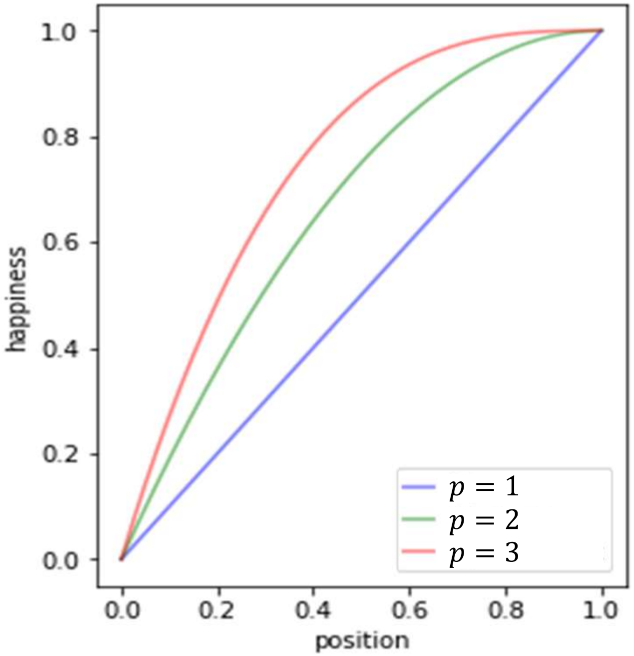
$$\exists\, g(h(x)) = x \;\; \text{and} \;\; s^t = \frac{1}{N} \sum_{i=0}^{t} n^i$$

$$s^t = s^{t-1} g(h^{t-1}) + 1 \;\; \text{and} \;\; h^t = s^t \int_0^1 h(z)dz - s^{t-1} \int_0^{g(h^{t-1})} h(z)dz$$

$$\text{Initial Condition :} \quad s^0 = N \;\; \text{and} \;\; h^0 = \int_0^1 h(z)dz.$$



Numerical error diverges at t ≈ 20 , hard problem … but



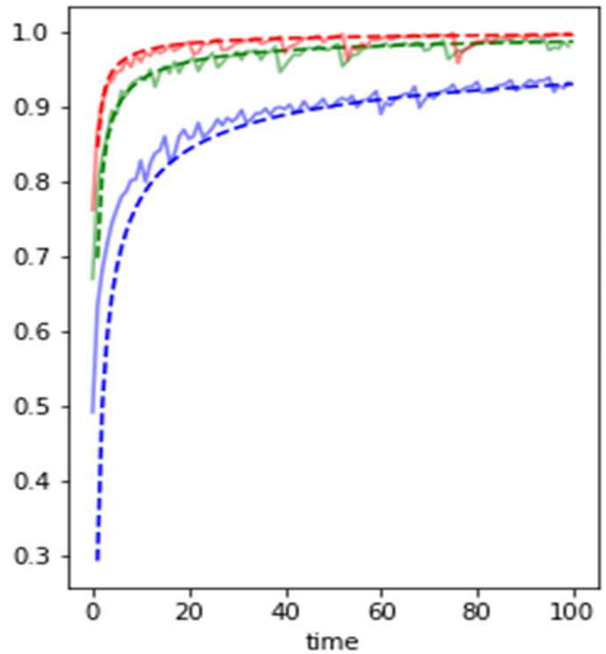
$$h(x) = 1 - (1 - x)^p$$

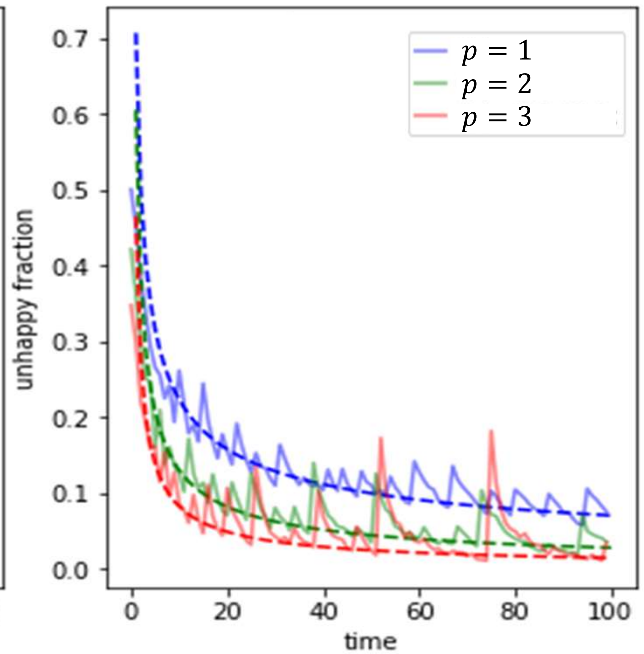
Long time Asymptotics $t \to \infty$

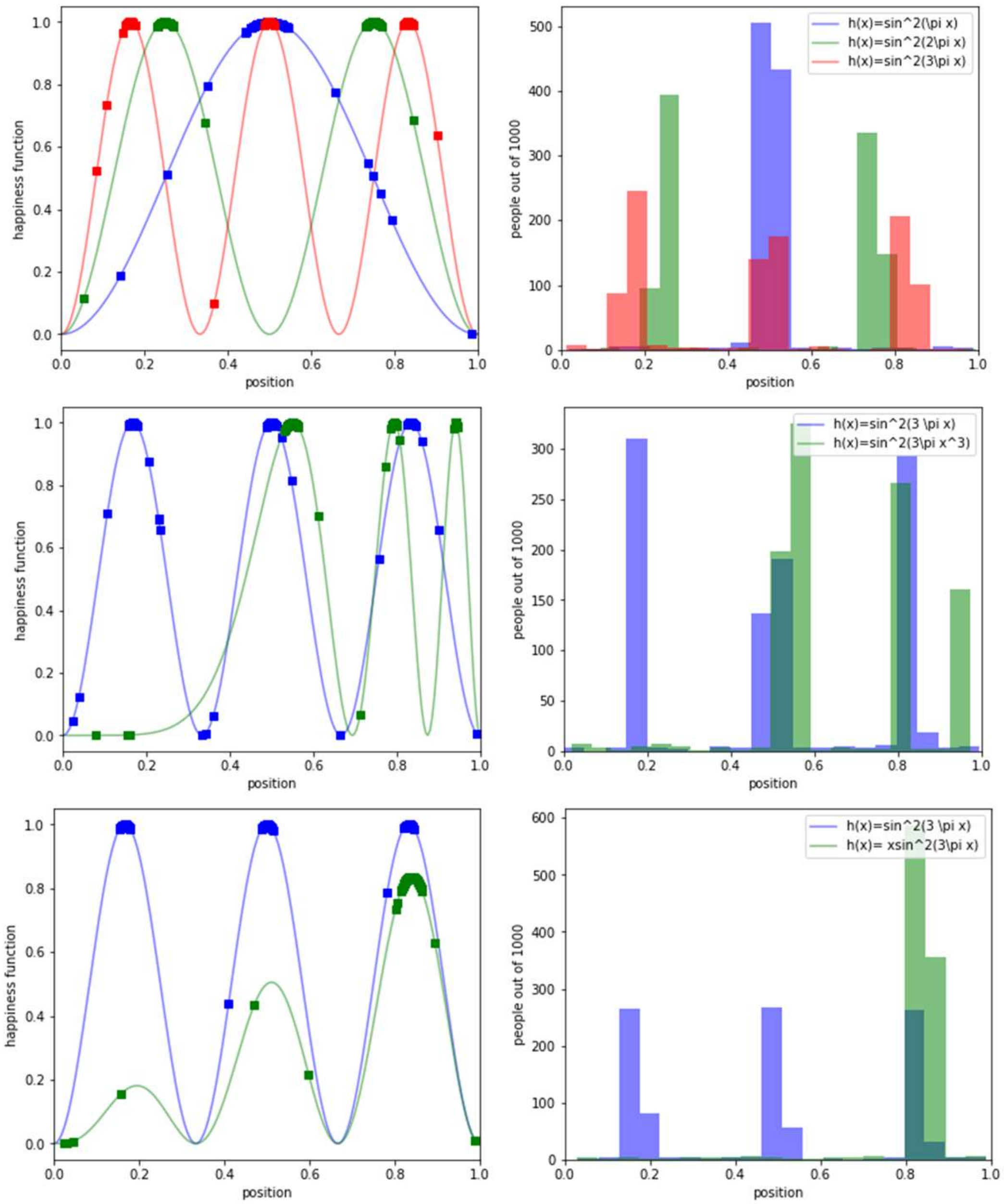
$$h^t = 1 - (p(p+1))^{\beta} t^{\beta} + \mathcal{O}(t^{2\beta})$$

$$n^t = N p^{\beta+1} (p+1)^{\beta} t^{\beta} + \mathcal{O}(t^{2\beta}) \text{ where } \beta = \frac{-p}{p+1}$$

Hypothesis: For any general function with maxima valued h_M at x_M and $h_M - h \sim |x_M - x|^p$ the approach with time follows power law with exponent $\frac{-p}{p+1}$







Symmetric Maxima:
Equal proportion at
each peak

Equivalued maxima:
In inverse proportion to
peak width

Unique Global Maxima:
Cluster at global max

Conclusion: The pursuit of happiness is a maximum search algorithm
Which exhibits power-law convergence with exponent in (-1,0)