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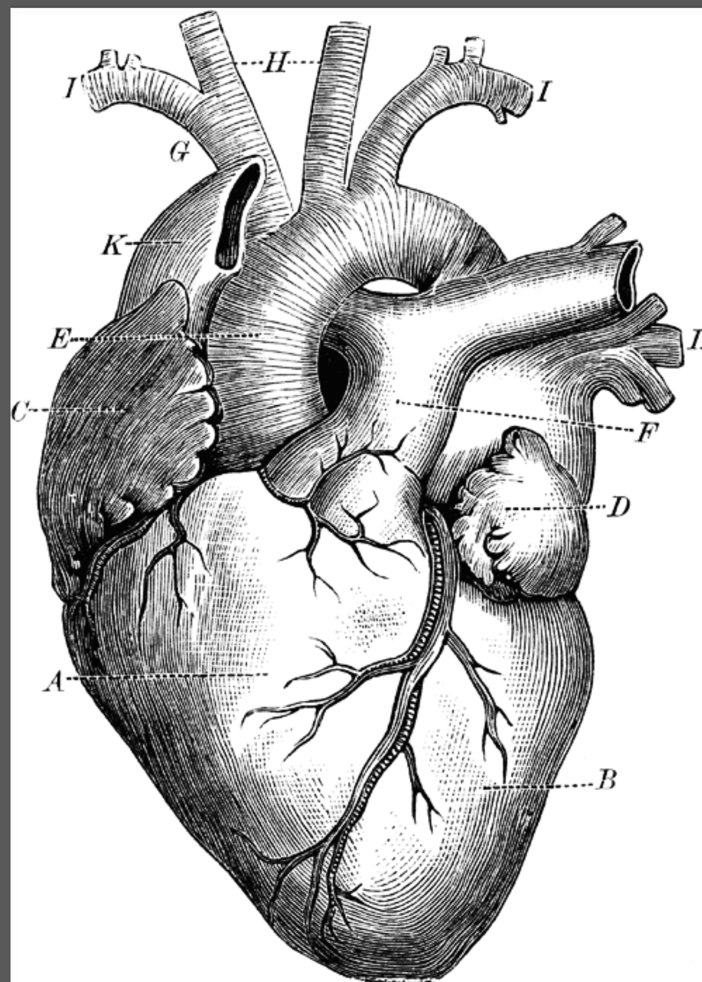
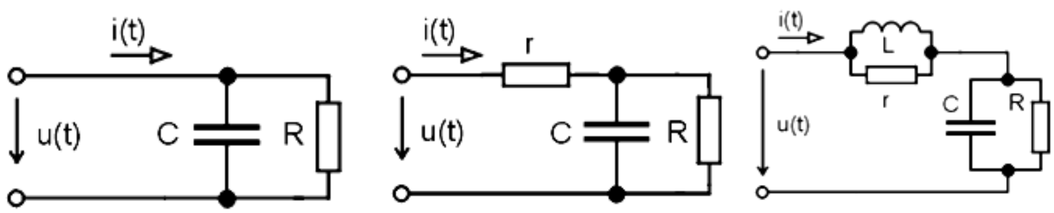


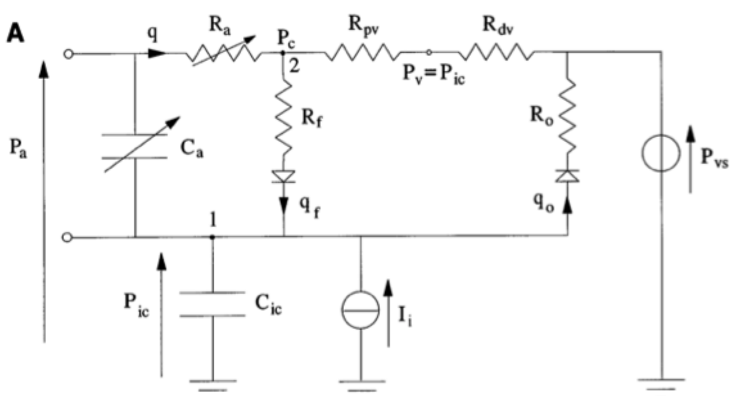
WINDKESSEL MODEL SOLVER

Windkessel Models

Windkessel models are lumped mathematical models used to express the relationship between blood pressure and blood flow rate in the body. Surprisingly, the condition for the blood flowing in blood vessels is mathematically similar to that of an electrical flow in an RC or an RLC circuit. Here, voltage resembles pressure, resistance resembles friction, and capacity accounts for the elasticity of the vessel wall. Various combinations of these circuit elements are possible and each of them represents a specific Windkessel model. Three of the simplest combinations are shown below. They are called Windkessel two-element, three-element and four-element model, respectively.

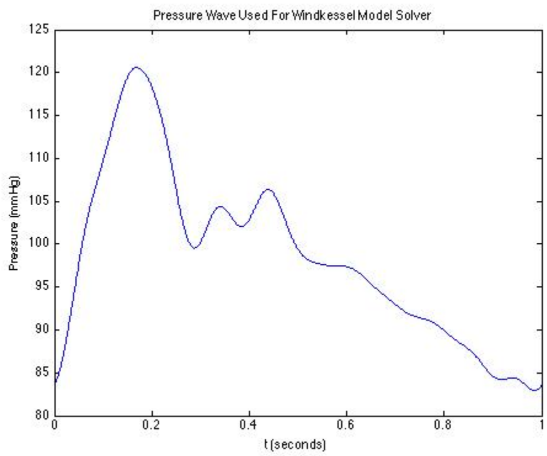


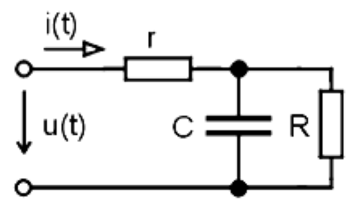
The choice of the Windkessel model we use for modeling depends on the level of accuracy we need. Both very simple and very sophisticated models can be found in literature. For example, while Olufsen has used a three compartment Windkessel model for modeling of the blood flow to the brain during Orthostatic Hypotension – a condition in which the person feels dizzy after a fast getting up from sitting position-, Ursino has used the following Windkessel model for studying cerebral blood flow in aneurysms: [1] [2]



Governing Equations

It is not difficult to obtain governing equations for a specific Windkessel model. A classic circuit theory approach has been shown here to solve the standard three-compartment Windkessel model. It must be noted that we want to solve i, that is blood flow rate, for u, that is, pressure. One should note that the blood flow rate is not steady state; in fact it is a pulsatile flow, due to the pulsatile nature of the pump, i.e. the Heart. The pressure wave of the blood can be measured clinically using non-invasive methods. An analytical expression then can be obtained using Fourier's Theorem. In our MATLAB code, we would use the following pressure wave, which has been provided to us by Auckland Bioengineering Institute:





Now let's continue with solving the three-compartment WK model:

We will write KCL for the specified location above. Let i_1 and i_2 be the currents that leave the node to the resistant and capacitor respectively.

$$i(t) = i_1 + i_2$$

$$We\ know\ that\ i_2 = C\frac{dV(t)}{dt}$$

$$KVL\ implies\ that\ V(t) = u(t) - r.i(t)$$

$$Ohm's\ Law\ then\ implies\ that\ i_1 = \frac{V(t)}{R} = \frac{u(t) - r.i(t)}{R}$$

$$Substitiution\ and\ rearranging\ result\ in\ i(t) = \frac{u(t) - r.i(t)}{R} + C\frac{d\,(u(t) - r.i(t))}{dt}$$

$$So\quad i(t) = \frac{u(t)}{R} - \frac{r}{R}\,i(t) + C\left(\frac{du(t)}{dt} - r\frac{di(t)}{dt}\right)$$

$$and\ finally\quad \left(1 + \frac{r}{R}\right)i(t) + Cr\frac{di(t)}{dt} = \frac{u(t)}{R} + C\frac{du(t)}{dt}$$

For the 2 and 4 compartment models we will have the following equations:

$$i(t) = \frac{u(t)}{R} + C\frac{du(t)}{dt}$$

$$\left(1 + \frac{r}{R}\right)i(t) + \left(rC + \frac{L}{R}\right)\frac{di(t)}{dt} + LC\frac{d^2i(t)}{dt^2} = \frac{u(t)}{R} + C\frac{du(t)}{dt}$$

Windkessel Parameters

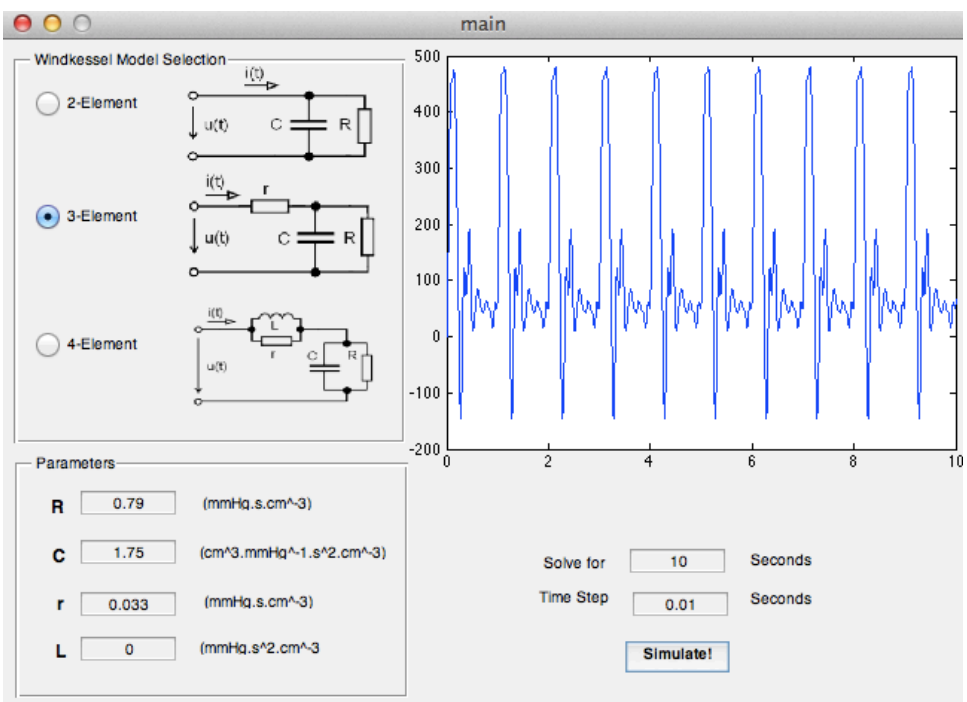
In order to solve a Windkessel model, model parameters, i.e. R, r, C, L, etc., must be known. These parameters are set by comparing the model output with experimental data. The following table gives a set of model parameters found in the literature for 2, 3, and 4-compartment Windkessel models.[3]

Tab. 1: *The values of normal human parameters of the Windkessel models*

	R [mmHg.s.cm^{-3}]	C [cm^3.mmHg^{-1}.s^2.cm^{-3}]	r [mmHg.s.cm^{-3}]	L [mmHg.s^2.cm^{-3}]
2 WM	1	1	-	-
3 WM	1 0.79 0.63	1 1.75 5.16	0.05 0.033 0.03	-
4 WM	1 0.79 0.63	1 1.22 2.53	0.05 0.056 0.045	0.005 0.0051 0.0054

MATLAB GUI

All the user needs to do is to choose model, enter parameters and hit Simulate button. The blood flow rate is in cubic centimeters per seconds. Negative flow rate values imply change of direction of the flow. This is normal and happens at the beginning of heart's diastole phase. The code solves the 2WK model analytically and the 3WK and 4WK models with classic 4 parameter Runge-Kuttah Method. The code has been compiled and tested using MATLAB R2011a under Mac OS X version 10.7.2 (Lion).



References

[1] Mette S. Olufsen Æ April V. Alston Æ Hien T. Tran Æ Johnny T. Ottesen Æ Vera Novak, "Modeling Heart Rate Regulation—Part I: Sit-to-stand Versus Head-up Tilt", Cardiovasc Eng, DOI 10.1007/s10558-007-9050-8

[2] Mauro Ursino and Carlo Alberto Lodi, "A simple mathematical model of the interaction between intracranial pressure and cerebral hemodynamics", *J Appl Physiol* 82:1256-1269, 1997.

[3] Westerhof, N., et. al.: An artificial arterial system for pumping hearts, Journal of Applied Physiology 31 (1971) 776-781.