算设

Hw 1 Feb 24, Spring 2022

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1 标题左对齐

第一个 section 使用 `\firstsection`，避免切换到新的一页；后续使用 `\section`，会自动从新的一页开始 section。

1.1 子标题也左对齐

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1.1.1 子子标题也左对齐

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1.2 展示带圈字符

① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩ ⑪ ⑫ ⑬ ⑭ ⑮ ⑯ ⑰ ⑱ ⑲ ⑳ , ㊎

2 问题列表

整个文档使用同一个问题计数器，当然，也可以使用 \setcounte{ProblemCounter}{1} 重设计数。

2.1 双列问题列表

Problem 1

Prove: $2n + \Theta\left(n^2\right) = \Theta\left(n^2\right)$

Solution:

$\because$ 根据 Θ 定义

$\therefore \exists n_0, c_1, c_2,\ s.t.\ \forall n > n_0,\ c_1 n^2 \leqslant \Theta(n^2) \leqslant c_2 n^2$

$$\therefore c_1 + 2/n \leqslant \frac{2n + \Theta(n^2)}{n^2} \leqslant c_2 + 2/n$$

$$\therefore c_1 \leqslant \frac{2n + \Theta(n^2)}{n^2} \leqslant c_2 + 3$$

$$\therefore c_1 n^2 \leqslant 2n + \Theta(n^2) \leqslant (c_2 + 3) n^2$$

$$\therefore 2n + \Theta(n^2) = \Theta(n^2)$$

Problem 2

Prove: $\Theta(g(n)) \cap o(g(n)) = \varnothing$

Solution:

$\forall f \in \Theta(g),\ \exists c_2, n_0,\ s.t.\ \forall n > n_0,\ g \leqslant c_2 f$

若 $f \in o(g)$, 则 $\forall c,\ \exists n_1,\ s.t.\ \forall n > n_1,\ cf < g$

这与$f \in \Theta(g)$矛盾

$\therefore \Theta(g(n)) \cap o(g(n)) = \varnothing$

在这里使用 *\columnbreak* 强制换栏

Problem 3

Prove: $\Theta(g(n)) \cup o(g(n)) \neq O(g(n))$

Ref to book.

Solution:

直观理解三个符号:

$\Theta(g) \iff f/g$ 有界, 且下确界>0

$o(g) \iff f/g \to 0$

$O(g) \iff f/g \leqslant const$

$\therefore$ 设 $g = n^2,\ f = (1 + (-1)^n) n^2$

此时, $f = O(g)$, 但是$f \neq \Theta(g)$且$f \neq o(g)$

Problem 4

Prove: $\max(f(n), g(n)) = \Theta(f(n) + g(n))$

Solution:

$\because f, g \geqslant 0$

$\therefore max(f,\ g) \leqslant f + g \leqslant 2max(f,\ g)$

$\therefore c_1 = 1,\ c_2 = 2$

$\therefore \max(f(n), g(n)) = \Theta(f(n) + g(n))$

2.2 单列问题列表

Problem 1 **Relative asymptotic growths** CLRS, P61, 3-2

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω, or Θ of B. Assume that $k \geq 1, \epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

Solution:

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ϵ	Y	Y	N	N	N
b.	n^k	c^n	Y	Y	N	N	N
c.	$\sqrt{n}$	$n^{\sin n}$	N	N	N	N	N
d.	2^n	$2^{n/2}$	N	N	Y	Y	N
e.	$n^{\lg c}$	$c^{\lg n}$	Y	N	Y	N	Y
f.	$\lg(n!)$	$\lg(n^n)$	Y	N	Y	N	Y

Problem 2 **Ordering by asymptotic growth rates** CLRS, P61, 3-3

a. Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, \ldots, g_{30}$ of the functions satisfying $g_1 = \Omega(g_2), g_2 = \Omega(g_3), \ldots, g_{29} = \Omega(g_{30})$. Partition your list into equivalence classes such that functions $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$.

$\lg(\lg^* n)$	$2^{\lg^* n}$	$(\sqrt{2})^{\lg n}$	n^2	$n!$	$(\lg n)!$
$(\frac{3}{2})^n$	n^3	$\lg^2 n$	$\lg(n!)$	2^{2^n}	$n^{1/\lg n}$
$\ln\ln n$	$\lg^* n$	$n \cdot 2^n$	$n^{\lg\lg n}$	$\ln n$	1
$2^{\lg n}$	$(\lg n)^{\lg n}$	e^n	$4^{\lg n}$	$(n+1)!$	$\sqrt{\lg n}$
$\lg^*(\lg n)$	$2^{\sqrt{2}\lg n}$	n	2^n	$n \lg n$	$2^{2^{n+1}}$

b. Give an example of a single nonnegative function $f(n)$ such that for all functions $g_i(n)$ in part (a), $f(n)$ is neither $O(g_i(n))$ nor $\Omega(g_i(n))$

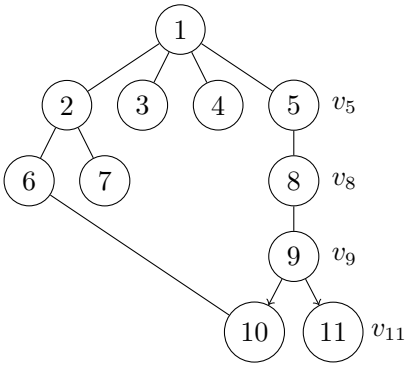
Solution:

a. $a > b$ 表示 $a = \Omega(b)$, $a = b$ 表示 $a = \Theta(b)$。顺序为: $2^{2^{n+1}} > 2^{2^n} > (n+1)! > n! > e^n > n \cdot 2^n > 2^n > (3/2)^n > (\lg n)^{\lg n} = n^{\lg\lg n} > (\lg n)! > n^3 > n^2 = 4^{\lg n} > n \ln n = \lg n! > n = 2^{\lg n} > (\sqrt{2})^{\lg n} > 2^{\sqrt{2\lg n}} > \lg^2 n > \ln n > \sqrt{\lg n} > \ln\ln n > 2^{\ln^* n} > \lg^* n = \lg^*(\lg n) > \lg(\lg^* n) > n^{1/\lg n} = 1$

b. $f(n) = (1 + (-1)^n)\, n$

3 绘制图、树

如果要绘制图、树，使用 LuaLaTeX 编译，否则使用 XeLaTeX 编译，要快一些。



更多例子可见 https://texample.net/

4 伪代码示意

Algorithm 1 Subproblem Optimize

```
if r − l ⩽ 30 then
    sort p.pointlist by a specific sort algorithm
    calculate minimal distance brutely here
    return minimal distance
else if l ⩾ r − 1 then
    return INF
```

引用代码使用 *Algorithm 1*。如果要允许代码块换行，使用 breakablealgorithm 代替 algorithm。

5 代码渲染

这里展示文档内代码渲染功能，支持内插代码和从文件读取代码。

Example Code 1

```cpp
// This is an example of code block.
// Language: C++
#include <iostream>
using namespace std;
int main ()
{
        cout << "Hello World!" << endl;
        return 0;
}
```

Load from file

```python
"""
一个无聊的生成四则运算表达式的小程序
"""
from random import random, randint

toStrFunc = 0

class Node:
    _randMax = 100
    _numtype = 'num'
    _nodetype = 'node'

    type = None
    num = 0
    lson = None
    rson = None
    op = ''

    def __init__(self) -> None:
        self.type = self._numtype
        self.num = int(random() * self._randMax)
        self.op = '+-*/'[randint(0, 3)]

    def __str__(self) -> str:
        if self.type == self._numtype:
            return str(self.num)
        elif self.type == self._nodetype:
            global toStrFunc
            if toStrFunc == 1:
                return f'({str(self.lson)}{self.op}{self.rson})'

            exp = ''
```

```
            if self.lson.priority() < self.priority(): exp = f'({self.lson}){self.op}'
            else: exp = f'{self.lson}{self.op}'
            if self.rson.priority() < self.priority(): exp = f'{exp}({self.rson})'
            else: exp = f'{exp}{self.rson}'
            return exp
        else:
            return 'NULL NODE!!!'

    def expand(self):
        if self.type == self._numtype:
            self.type = self._nodetype
            self.lson = Node()
            self.rson = Node()
        return self

    def priority(self):
        if self.type == self._numtype: return 1000
        else:
            if self.op in '+-': return 10
            elif self.op in '*/': return 20


def dfs(h: Node, k: int):
    while k > 0:
        k -= 1
        h.expand()
        h = [h.lson, h.rson][randint(0, 1)]


for i in range(10):
    h = Node()
    dfs(h, 5)
    print(h)
```