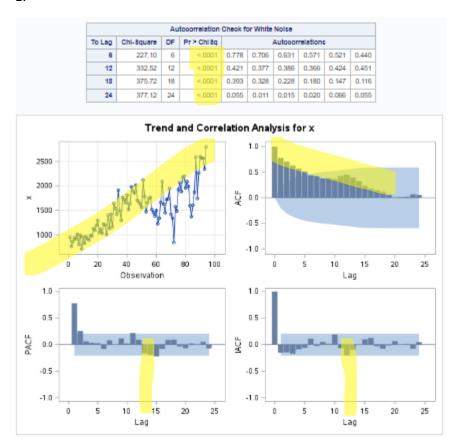
#### **ARIMA Forecasting Variable Temperatures**

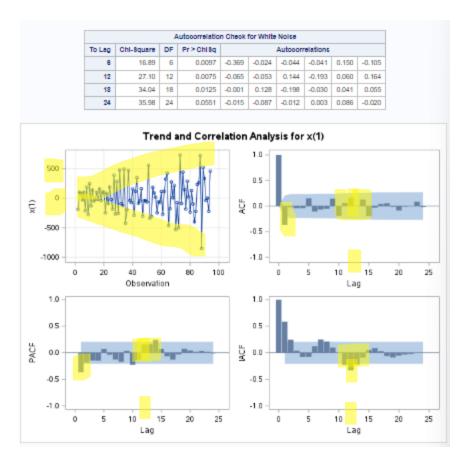
# George Kacoyanis

### STA 4853

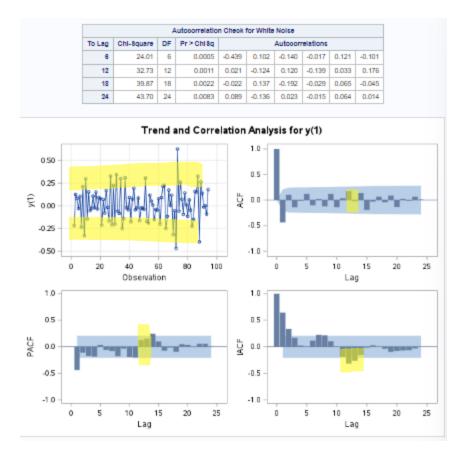
1.



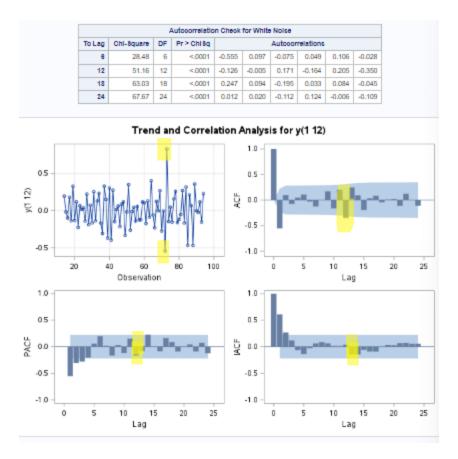
In this series we can see from the ACF that the lags are slowly decreasing to 0 and show no obvious cut offs. Our model also shows that we have an upwards trend. From this we can assume already that our series is not stationary. We start with d=1. We can also see strong positive autocorrelation between our errors in the white noise showing that the lags have strong autocorrelation.



We can see that our ACF has an immediate cutoff to around 0 after the first lag, our series plot no longer has an upward trend either, which can show that our series after differencing we get a stationary model averaging out at around 0. We still see presence of large variance in our series plot, so we can apply a logarithmic transformation to help eliminate some of that variance and make it more normally distributed.



We can see some slight seasonality in the lags where in the IAC and the PACF you can see around lag 12 to have some significant lags. This still leads us to believe that we need seasonal terms. We can assume to use seasonal differencing and the seasonal auto regression since there is autoregression present.

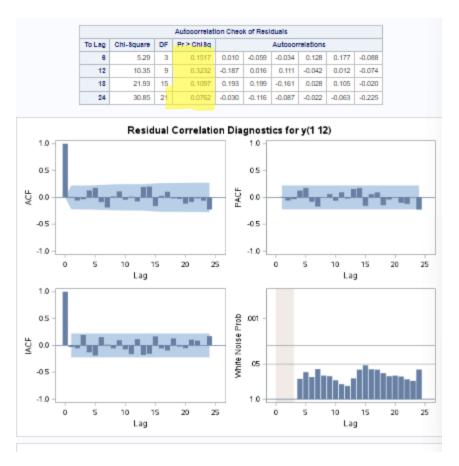


Here we have D=1 and P=1 with AR(1) and MA(1) terms. With this model we can see the Correlation plots all have cut off lags at the beginning. Showing a good fit. We can see the spike in the IACF and PACF at lag 12 disappear and is below the significance.

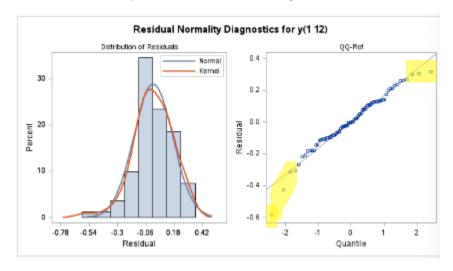
Constant Estimate	0.002318
Variance Estimate	0.028609
8td Error Estimate	0.169143
AIC	-52,2659
8BC	-42,6881
Number of Residuals	81

We have good AIC and SBC values currently which are very small.

Maximum Likelihood Estimation										
Parameter	Estimate	Standard Error	t Value	Approx Pr≥ t	Lag					
MA1,1	0.59031	0.12242	4.82	<.0001	1					
AR1,1	-0.22865	0.13783	-1.66	0.0971	1					
AR1,2	-0.28754	0.11289	-2.55	0.0109	12					



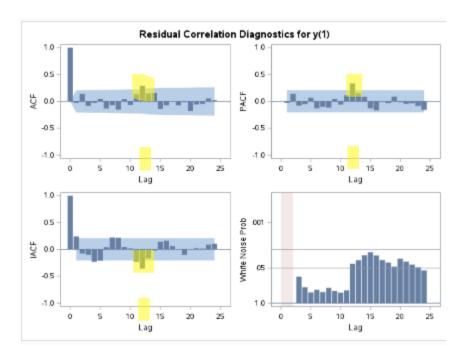
The residual correlation diagnostics plot shows no significant lags and our auto correlation check for residuals shows the p values to be above the significance.



From the histogram of residuals and the Q-Q plot we can see some large outliers, but most of the data fits normal distribution.

From this information, the best model I could find is the  $ARIMA(1,1,1)(1,1,0)_12$ .

(b)



You can see that removing the seasonal differencing and seasonal Auto Regression has made the lags around 12 spike and go past the significance on all the autocorrelation plots. This shows that the seasonal pattern is not being accounted for and that all the data is not being expressed by the model.

Constant Estimate	0.009048
Variance Estimate	0.026388
8td Error Estimate	0.162445
AIC	-70.5522
8BC	-62.9544
Number of Residuals	93

While our AIC and SBC show to be better than with our seasonal terms, we know that the data is not fully expressed by the model from the residual diagnostics.

Maximum Likelihood Estimation										
Parameter	Ectimate	8tandard Error	t Value	Approx Pr >  t	Lag					
MU	0.01146	0.0049351	2.32	0.0202	0					
MA1,1	0.77696	0.10252	7.58	<.0001	1					
AR1,1	0.21040	0.15410	1.37	0.1721	1					

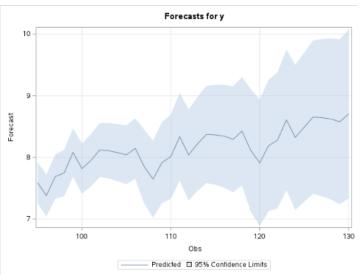
(c).

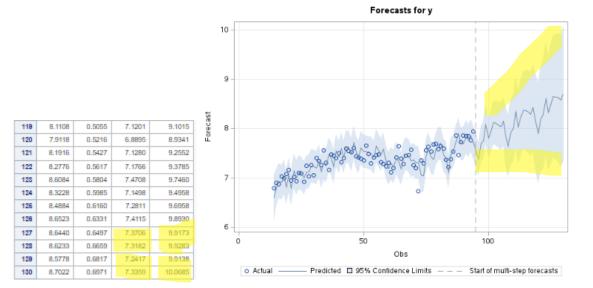
Forecasts with no constant of the next 36 months:

	Fo	recasts for	varlable y				
Obs	Forecast	Std Error		lence Limits			
95	7.5844	0.1681	7.2549	7.9140			
96	7.3726	0.1709	7.0377	7.7075			
97	7.6795	0.1818	7.3233	8.0357			
98	7.7367	0.1898	7.3647	8.1088			
99	8.0663	0.1980	7.6782	8.4544			
100	7.8031	0.2058	7.3998	8.2064			Forecasts for y
101	7.9327	0.2133	7.5147	8.3508		10 -	
102	8.1022	0.2205	7.6700	8.5344			
103	8.0879	0.2275	7.6420	8.5339			
104	8.0555	0.2343	7.5962	8.5149			
105	8.0168	0.2410	7.5446	8.4891		9 -	
106	8.1190	0.2474	7.6341	8.6039			
107	7.8199	0.3035	7.2250	8.4148	ast		
108	7.6130	0.3163	6.9931	8.2329	Forecast		
109	7.8760	0.3316	7.2261	8.5259	Ĭ.		
110	7.9705	0.3461	7.2921	8.6489		8 -	
111	8.2907	0.3600	7.5852	8.9963			
112	7.9910	0.3734	7.2593	8.7228			
113	8.1673	0.3863	7.4103	8.9244			
114	8.3170	0.3987	7.5355	9.0986		7 -	<u> </u>
115	8.3073	0.4109	7.5020	9.1125			100
116	8.2834	0.4226	7.4551	9.1117			100 110 120 130 Obs
117	8.2277	0.4341	7.3770	9.0785			
118	8.3560	0.4452	7.4834	9.2286			——— Predicted ☐ 95% Confidence Limits
						10 -	Forecasts for y
						9 -	
119	8.0350	0.5024	7.0504	9.0196	st		
120	7.8317	0.5184	6.8156	8.8477	Forecast	8 -	
121	8.1064	0.5393	7.0494	9.1635	E.		
122	8.1876	0.5582	7.0935	9.2817			
123	8.5135	0.5768	7.3830	9.6441		7 -	20 A 2000 P
124	8.2230	0.5948	7.0573	9.3887			South o
125	8.3838	0.6122	7.1839	9.5837			711
126	8.5427	0.6291	7.3096	9.7758			
127	8.5296	0.6456	7.2641	9.7950		6 -	
128	8.5040	0.6617	7.2070	9.8010			0 50 100
129	8.4536	0.6774	7.1259	9.7814			Obs
130	8.5732	0.6928	7.2154	9.9310			o Actual — Predicted ☐ 95% Confidence Limits — — Start of multi-step forecasts

Forecasts with constant of the next 36 months:

Forecasts for variable y									
Obs	Forecast	8td Error	96% Confid	lence Limits					
96	7.5901	0.1691	7.2586	7.9216					
96	7.3793	0.1719	7.0424	7.7162					
87	7.6883	0.1828	7.3300	8.0467					
88	7.7474	0.1910	7.3731	8.1216					
88	8.0788	0.1992	7.6884	8.4693					
100	7.8175	0.2070	7.4118	8.2233					
101	7.9490	0.2146	7.5284	8.3696					
102	8.1204	0.2219	7.6856	8.5553					
108	8.1080	0.2289	7.6593	8.5567					
104	8.0775	0.2358	7.6154	8.5396					
106	8.0407	0.2424	7.9655	8.5159					
108	8.1447	0.2489	7.6569	8.6326					
107	7.8516	0.3054	7.2531	8.4501					
108	7.6476	0.3182	7.0240	8.2713					
108	7.9140	0.3336	7.2601	8.5679					
110	8.0119	0.3483	7.3293	8.6944					
111	8.3354	0.3622	7.6255	9.0453					
112	8.0391	0.3757	7.3028	8.7753					
118	8.2187	0.3886	7.4569	8.9804					
114	8.3717	0.4012	7.5854	9.1581					
115	8.3653	0.4134	7.5550	9.1755					
118	8.3447	0.4252	7.5113	9.1782					
117	8.2924	0.4368	7.4364	9.1484					
118	8.4240	0.4480	7.5460	9.3020					





Both forecasts have similar plots and constantly increasing standard errors. But we can see that when we include the constant, our confidence intervals and our forecasts rise and we get greater values for our forecast showing a slightly greater upward trend.

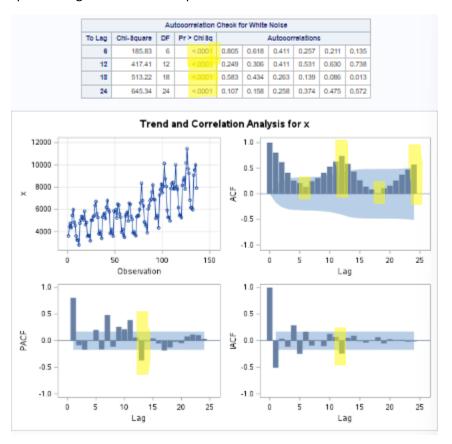
## Forecasts With Constant:

128	8.6523	0.6331	7.4115	9.8930
127	8.6440	0.6497	7.3706	9.9173
128	8.6233	0.6659	7.3182	9.9283
129	8.5778	0.6817	7.2417	9.9138
130	8.7022	0.6971	7.3359	10.0685

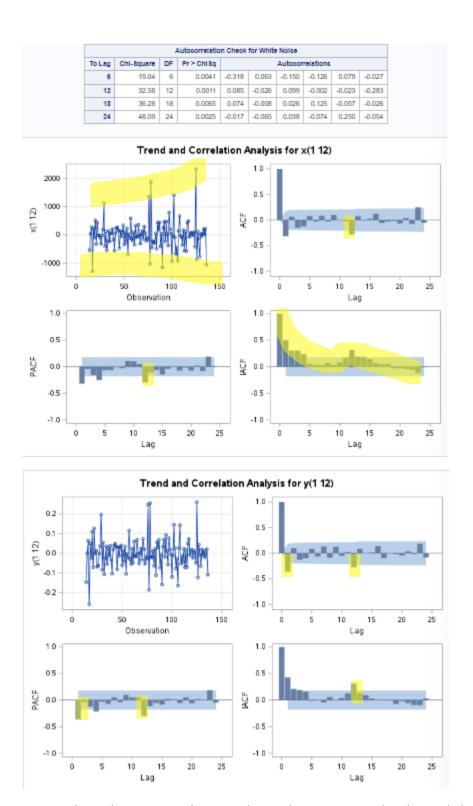
#### **Forecasts Without Constant:**

128	8.5427	0.6291	7.3096	9.7758
127	8.5296	0.6456	7.2641	9.7950
128	8.5040	0.6617	7.2070	9.8010
129	8.4536	0.6774	7.1259	9.7814
130	8.5732	0.6928	7.2154	9.9310

Immediately after running an ARIMA process on the data in BUS.txt we can see the immediate need for differencing and seasonal differencing with the constant acf, obvious patterns in the data series, and lag spikes at lag 12 for all the acf plots.



Looking at the trend and correlation analysis there is a much better acf and pacf. Our IACF shows that there may be some slight over differencing from the constant decrease in the significance of the lags. The series plot has large variance between the values and outliers, which may suggest that a transformation might be needed. The y=log(x) transformation might help with these issues.

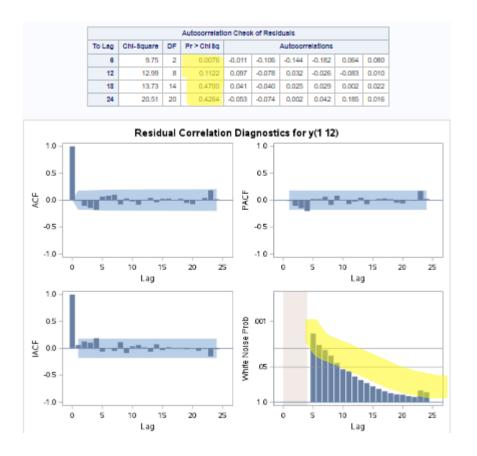


We get about the same graph ACF and PACF, but our series plot shows slightly less variance and helps to make the numbers we are working with slightly smaller. Also the IACF shows to have less evidence of over differencing slightly. From our ACF we can see that there may be evidence of an AR(1) and MA(1) process for seasonal and non-seasonal terms (p=1,q=1,P=1,Q=1).

Maximum Likelihood Estimation									
Parameter	Estimate	Standard Error	t Value	Approx Pr >  t	Lag				
MU	-0.0007189	0.0012061	-0.60	0.5511	0				
MA1,1	0.01150	0.10664	0.11	0.9142	1				
MA1,2	0.98849	1.73198	0.57	0.5682	12				
AR1,1	-0.39926	0.10005	-3.99	<.0001	1				
AR1,2	0.26328	0.12005	2.19	0.0283	12				
	Constant	Estimate	-0.00082						
	Variance 8	Estimate	0.003594						
	8td Error	Estimate	0.059949						
	AIC		-316.009						
	8BC		-301.948						
	Number o	f Residuais	123						

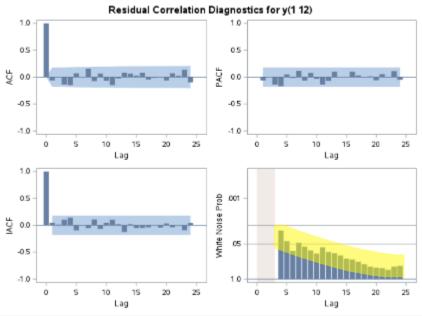
The AIC and SBC are very small which is favorable in a model, but our maximum likelihood estimates show that we might not need a non-seasonal AR(1) term and that there is strong evidence suggesting a seasonal moving average term is necessary. Removing the non-seasonal AR(1) term might be favorable.

With both AR(1) terms:

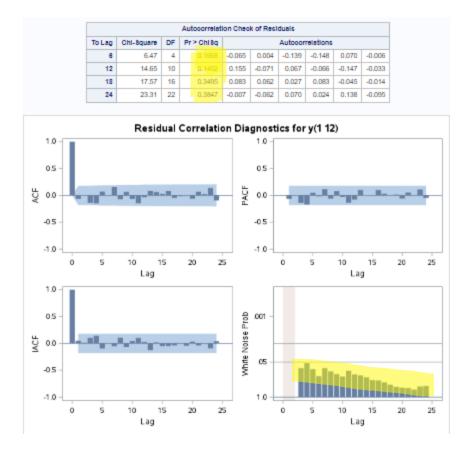


# With only the seasonal AR(1) term:





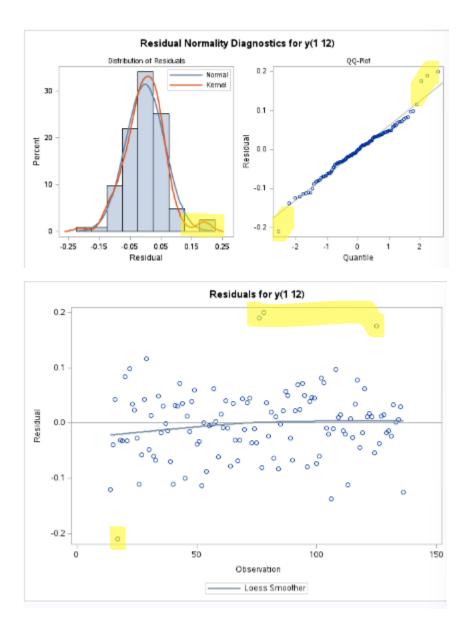
With no AR(1) terms:



When the AR(1) terms are removed, the white noise probability plot improves and shows that the lags are not significantly correlated with each other. We also get improvements in our AIC and SBC which show the model to be favorable as well.

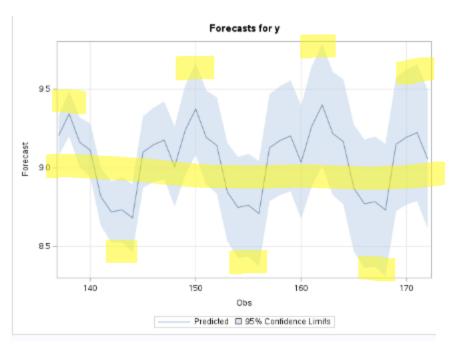
Constant Estimate	-0.00052	
Variance Estimate	0.004091	
8td Error Estimate	0.063962	
AIC	-316.979	
8BC	-308.542	
Number of Residuals	123	

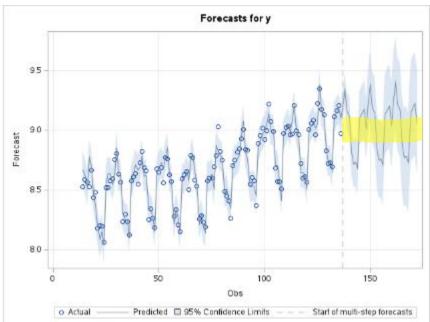
Checking the Residual Normality Diagnostics, it shows the residuals to be distributed normally with some outliers. The residual series plot also shows that while there are some outliers most of the residuals are distributed normally.



Here we can see that we have our best fit model  $ARIMA(0,1,1)(0,1,1)_12$  (b).

Obs	osforecast	osL86	osUBS			
101	8588.30	7569.27	9744.53			
102	9327.68	8221.08	10583.23			
103	8121.44	7158.19	9214.30			
104	8119.91	7157.11	9212.23			
105	6029.67	5314.88	6840.60			
108	6054.87	5337.20	6869.05			
107	5337.05	4704.51	6054.64			
108	4495.82	3963.01	5100.28	123	123 8729.86	123 8729.86 7696.21
109	7798.70	6874.46	8847.21	124	124 8251.44	124 8251.44 7274.45
110	8166.77	7198.90	9264.75			
111	8539.88	7527.81	9688.01	125	126 8527.73	126 8527.73 7518.03
112	7994.64	7047.20	9069.45	128	128 11896.07	128 11896.07 10487.56
113	8796.62	7754.14	9979.25	127	127 9524.95	127 9524.95 8397.23
114	9926.74	8750.38	11261.24	128	128 9108.48	128 9108.48 8030.18
115	8274.03	7293.65	9386.20	129	129 6950.92	128 6950.92 6128.15
1118	7527.19	6635.46	8538.76	130	130 6234.92	130 6234.92 5497.00
117	5682.12	5009.10	6445.57	131	131 6312.07	131 6312.07 5565.11
113	5688.18	5014.55	6452.31	13:2		
119	5592.00	4929.83	6343.11			
120	4934.47	4350.20	6597.22	133	133 9056.24	133 9056.24 7984.67
121	8062.46	7107.83	9145.30	134	134 9478.07	134 9478.07 8356.60
122	8460.08	7458.38	9596.32	135	135 9702.46	135 9702.45 8554.44

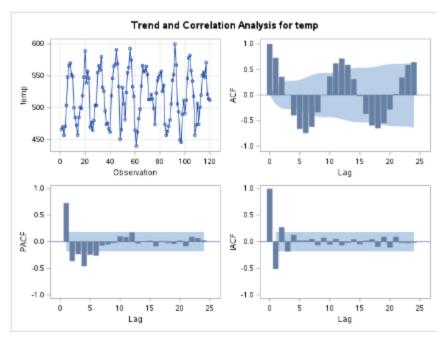


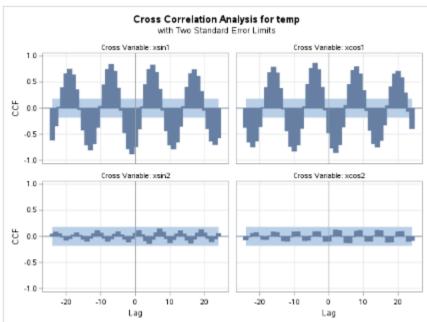


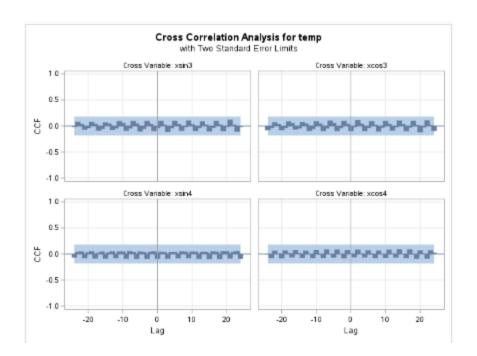
From the forecasts we can see that the repeating pattern continues and our forecasted values somewhat plateau. We have very large spreads in our confidences intervals and the revolving up and down patter also effects the spread of our confidence intervals to be lower or higher depending on what part of the forecasted pattern you are looking at.

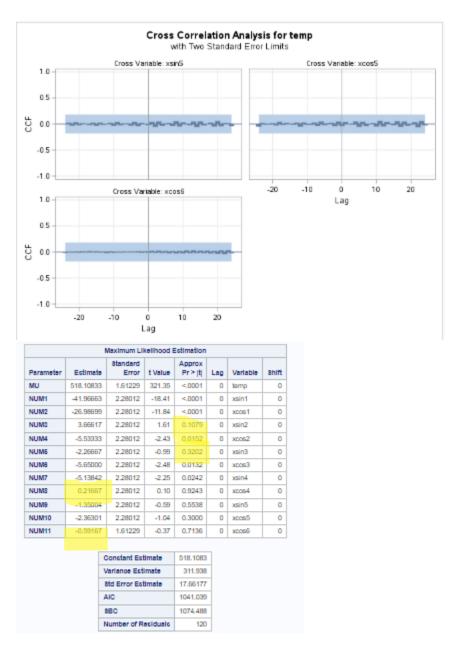
3.

(a)









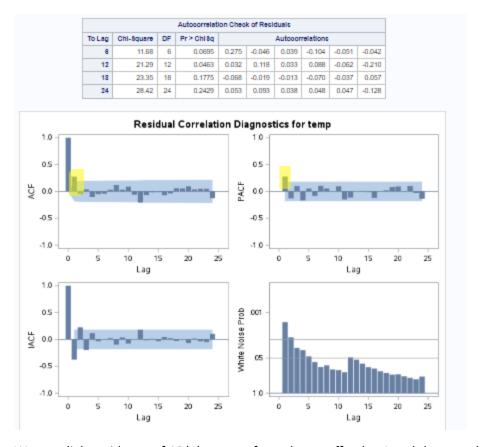
Here it shows that our slope estimates for xcos6 and xcos4 are small and might be not important in the ARIMA process. We start by excluding xcos6 because it's standard error stands our and the pvalue is very high.

	Maximum Likelihood Estimation											
Parameter	Ectimate	Standard Error	t Value	Approx Pr >  t	Lag	Variable	8hirt					
MU	518.10833	1.60115	323.58	<.0001	0	temp	0					
NUM1	-41.96663	2.26437	-18.53	<.0001	0	xsin1	0					
NUM2	-26.98699	2.26437	-11.92	<.0001	0	xpos1	0					
NUMS	3.66617	2.26437	1.62	0.1054	0	xsin2	0					
NUM4	-5.53333	2.26437	-2.44	0.0145	0	xnos2	0					
NUM5	-2.26667	2.26437	-1.00	0.3168	0	xsin3	0					
NUMB	-5.65000	2.26437	-2.50	0.0126	0	жров3	0					
NUM7	-5.13842	2.26437	-2.27	0.0233	0	xsin4	0					
NUMS	0.21667	2.26437	0.10	0.9238	0	xnos4	0					
NUM9	-2.36301	2.26437	-1.04	0.2967	0	xpos/5	0					

Here we see xcos4 to be low and potentially insignificant. We can remove that one and then look at the maximum likelihood extimates again.

Maximum Likelihood Estimation							
Parameter	Estimate	8tandard Error	t Value	Approx Pr >  t	Lag	Variable	Shift
MU	518.10833	1.59399	325.04	<.0001	0	temp	0
NUMI	-41.96663	2.25424	-18.62	<.0001	0	osin1	0
NUM2	-26.98699	2.25424	-11.97	<.0001	0	xpos1	0
NUMS	3.66617	2.25424	1.63	0.1039	0	xsin2	0
NUM4	-5.53333	2.25424	-2.45	0.0141	0	xnos2	0
NUMS	-2.26667	2.25424	-1.01	0.3147	0	osin3	0
NUMB	-5.65000	2.25424	-2.51	0.0122	0	xnos3	0
NUM7	-5.13842	2.25424	-2.28	0.0226	0	osin4	0
NUMS	-2.36301	2.25424	-1.05	0.2945	0	xpos/5	0

Here we see that all estimates are high, but xsin2, xsin3, and xcos5 have very high pvalues well above the significance. We start with xsin3 and xcos5 one at a time because they have the highest pvalues.



We see slight evidence of AR(1) process from the cutoff at lag 1 and there could be a case for a MA(1) process

Maximum Likelihood Estimation							
Parameter	Ectimate	8tandard Error	f Value	Approx Pr >  t	Lag	Variable	Shift
MU	518.10833	1.59407	325.02	<.0001	0	temp	0
NUMI	-41.96663	2.25435	-18.62	<.0001	0	xsin1	0
NUM2	-26.98699	2.25435	-11.97	<.0001	0	xcost	0
NUMS	3,66617	2.25435	1.63	0.1039	0	xsin2	0
NUM4	-5.53333	2.25435	-2.45	0.0141	0	xnos2	0
NUMS	-5.65000	2.25435	-2.51	0.0122	0	xnos3	0
NUMB	-5.13842	2.25435	-2.28	0.0226	0	xsin4	0
NUM7	-2.36301	2.25435	-1.05	0.2945	0	xpos/5	0

	Maximum Likelihood Estimation							
Parameter	Ectimate	8tandard Error	t Value	Approx Pr >  t	Lag	Variable	Shift	
MU	518.10833	1.59476	324.88	<.0001	0	temp	0	
NUMH	-41.96663	2.25534	-18.61	<.0001	0	osin1	0	
NUM2	-26.98699	2.25534	-11.97	<.0001	0	xcos1	0	
NUMS	3.66617	2.25534	1.63	0.1040	0	xsin2	0	
NUM4	-5.53333	2.25534	-2.45	0.0141	0	xnos2	0	
NUMS	-5.65000	2.25534	-2.51	0.0122	0	xnos3	0	
NUM8	-5.13842	2.25534	-2.28	0.0227	0	xsin4	0	

Here we see that our pvalue is not very above the significance but enough to take it out and check the AIC and SBC for the fit.

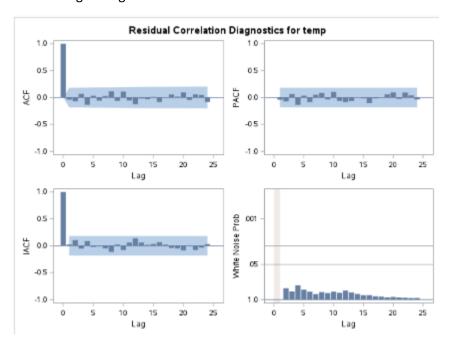
Maximum Likelihood Estimation							
Parameter	Estimate	8tandard Error	t Value	Approx Pr≻ t	Lag	Variable	8hift
MU	518.10833	1.60621	322.57	<.0001	0	temp	0
NUM	-41.96663	2.27153	-18.48	<.0001	0	osin1	0
NUM2	-26.98699	2.27153	-11.88	<.0001	0	xcos1	0
NUMS	-5.53333	2.27153	-2.44	0.0149	0	xxxs2	0
NUM4	-5.65000	2.27153	-2.49	0.0129	0	жоов3	0
NUM5	-5.13842	2.27153	-2.26	0.0237	0	xsin4	0
		Constant Es	timate	518.1083			
		Variance Est	Imate	309.5897			
		8td Error Es	timate	17.59516			
		AIC		1034.62			
		8BC		1051.345			
		Number of Residuals		120			

Here we see that after removing the sines and cosines we can see that these cosines and sines accurately fit the ARIMA model and see improvements in the AIC and SBC.

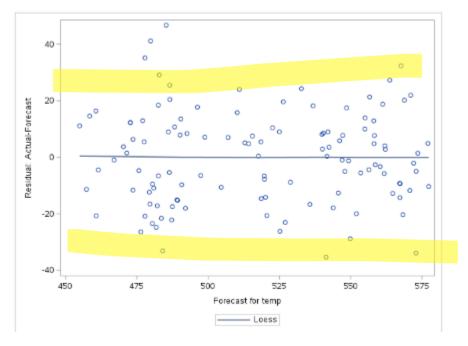
We see that our PACF in our residual correlation diagnostics for temp that we have a cut off at lag 1 from above the significance, so we apply an MA(1) term to the model.

	Maximum Likelihood Estimation							
Parameter	Estimate	Standard Error	t Value	Approx Pr >  t	Lag	Variable	Shift	
MU	518.19807	2.09309	247.58	<.0001	0	temp	0	
MA1,1	-0.36548	0.08884	-4.11	<.0001	1	temp	0	
NUMI	-41.87785	2.88710	-14.51	<.0001	0	xsin1	0	
NUM2	-26.83130	2.87610	-9.33	<.0001	0	xpost 1	0	
NUMS	-5.44357	2.66062	-2.05	0.0408	0	xnos2	0	
NUM4	-5.64730	2.31676	-2.44	0.0148	0	xnos3	0	
NUMS	-4.98457	1.90206	-2.62	0.0088	0	xsin4	0	

When applying the MA(1) term we see that the p-value Is almost 0 showing there to be strong evidence of a moving average.



Here we see that there is little autocorrelation between the lags and there are no lags past the significance. This helps to show that we have a good fit to

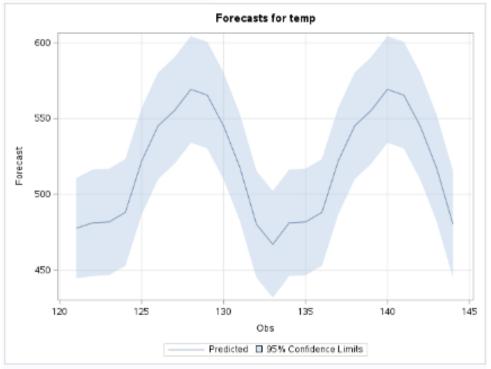


With our residuals forecast we can see that the residual variance remains constant. You can also see this in the Maximum likelihood plots when the STD deviation plateaus to 1 value.

Xt = 518.19807 - 41.87785 sin((2\*pi\*t)/12) - 26.83130 cos((2\*pi\*t)/12) - 5.44357 cos((4\*pi\*t)/12) - 5.64730 cos(6\*pi\*t)/12) - 4.9857 sin((8\*pi\*t)/12) + et - 0.36548 et - 1

# Nonseasonal stationary ARMA process and trend consisting of Cosines and sines

Forecasts for variable temp						
Obs	Forecast	8td Error	95% Confide	ence Limits		
121	477.6385	16.8378	444.6370	510.6399		
122	481.2010	17.9271	446.0646	516.3374		
128	481.7638	17.9271	446.6274	516.9002		
124	488.1042	17.9271	452.9677	523.2406		
126	522.0907	17.9271	486.9543	557.2272		
128	545.2331	17.9271	510.0967	580.3696		
127	555.3350	17.9271	520.1986	590.4715		
128	569.2723	17.9271	534.1358	604.4087		
129	565,5195	17.9271	530.3830	600,6559		
130	545.1020	17.9271	509,9656	580.2385		
131	517.4954	17.9271	482,3589	552.6318		
132	480.2759	17.9271	445.1395	515.4123		
133	466.9840	17.9271	431.8476	502.1205		
134	481.2010	17.9271	446.0646	516.3374		
135	481.7638	17.9271	446.6274	516.9002		
138	488.1042	17.9271	452.9677	523.2406		
137	522.0907	17.9271	486.9543	557.2272		
138	545.2331	17.9271	510.0967	580.3696		
138	555.3350	17.9271	520.1986	590.4715		
140	569.2723	17.9271	534.1358	604.4087		
141	565,5195	17.9271	530.3830	600,6559		
142	545.1020	17.9271	509,9656	580.2385		
148	517.4954	17.9271	482.3589	552.6318		
144	480.2759	17.9271	445.1395	515.4123		



Here we can see the confidence intervals remain constant and the standard errors remain constant through out the forecast. We see a repeating periodic pattern around every 12 months which we also see in the original data.

```
SAS code:
filename repair "/home/u63378438/my_shared_file_links/huffer/repair.txt";
run;
data repair;
infile repair;
input x;
y = log(x);
run;
proc ARIMA data=repair plots=all;
identify var=x(1,12) nlag=24;
 estimate p=1 method=ml;
run;
proc ARIMA data=repair plots=all;
identify var=x(1,12) nlag=24;
estimate p=1 method=ml;
run;
proc ARIMA data=repair plots=all;
identify var=y(1,12) nlag=24;
estimate p=(1,12) q=(1) method=ml noconstant;
forecast lead=36 out=resids;
```

```
run;
proc ARIMA data=repair plots=all;
identify var=y(1,12) nlag=24;
estimate p=(1,12) q=(1) method=ml;
forecast lead=36 out=resids;
run;
filename bus "/home/u63378438/my_shared_file_links/huffer/bus.txt";
run;
filename bus "/home/u63378438/my_shared_file_links/huffer/bus.txt";
run;
data bus;
infile bus;
input x;
y = log(x);
run;
proc arima data=bus plots=all;
identify var=y(1, 12) nlag=24;
estimate q=(1,12) method=ml;
run;
proc arima data=bus plots=all;
identify var=x(1, 12) nlag=24;
estimate method=ml;
run;
```

```
proc arima data=bus plots=all;
identify var=y(1, 12) nlag=24;
estimate q=(1,12) method=ml;
forecast lead=36 out=resids;
run;
data oscale;
keep osforecast osL95 osU95;
set resids;
osforecast=exp(forecast);
osL95=exp(L95);
osU95=exp(U95);
run;
proc print data=oscale(firstobs=101);
run;
filename bus "/home/u63378438/my_shared_file_links/huffer/bus.txt";
run;
data bus;
infile bus;
input x;
y=log(x);
run;
```

```
proc arima data=bus plots=all;
identify var=y(1, 12) nlag=24;
estimate q=(1,12) method=ml;
run;
proc arima data=bus plots=all;
identify var=x(1, 12) nlag=24;
estimate method=ml;
run;
proc arima data=bus plots=all;
identify var=y(1, 12) nlag=24;
estimate q=(1,12) method=ml;
forecast lead=36 out=resids;
run;
data oscale;
keep osforecast osL95 osU95;
set resids;
osforecast=exp(forecast);
osL95=exp(L95);
osU95=exp(U95);
run;
proc print data=oscale(firstobs=101);
run;
```