CSE 423 Computer Graphics

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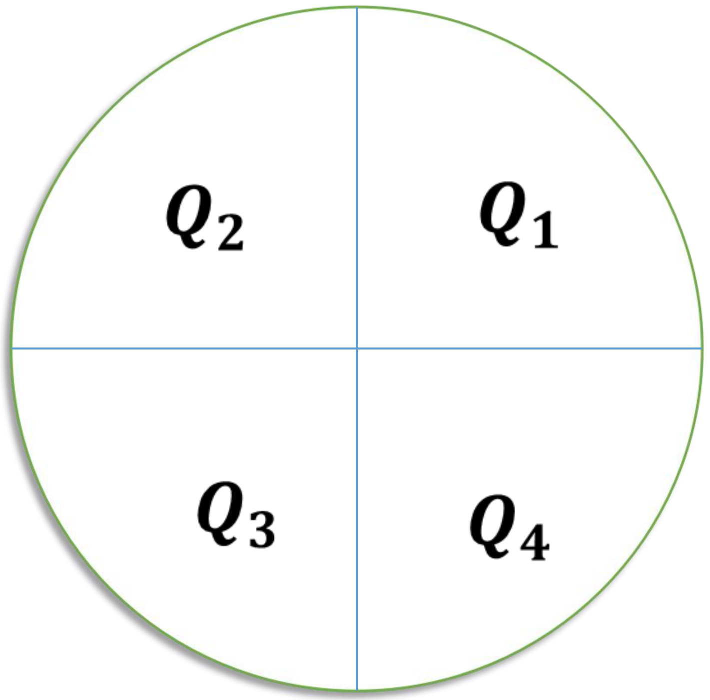
NEUB

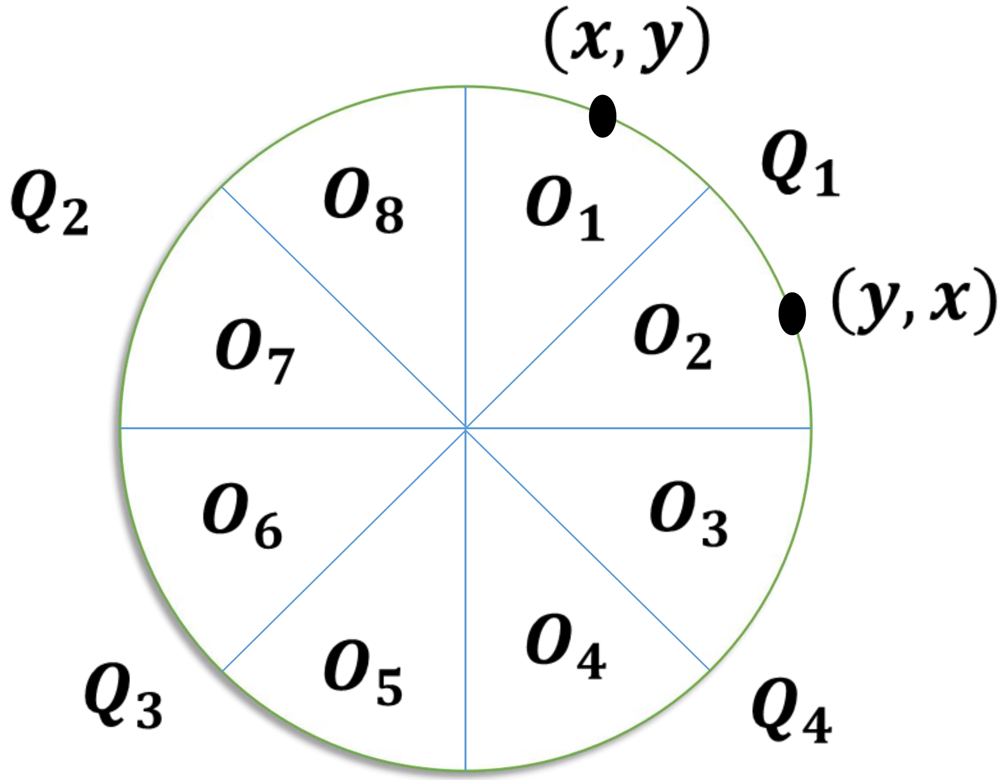
Midpoint Circle Drawing Algorithm concept

- A circle is a set of all points that lie at an **<u>equal distances</u>** from a fixed point called centre.
- If we draw a line from the centre at an equal distance is called Radius (r)
- The position of (x, y) will be interchanged if you change any point with 45^o angle
- Ways to draw a circle
  - Symmetric figure (4 way symmetry and 8 way symmetry)

Symmetric figure

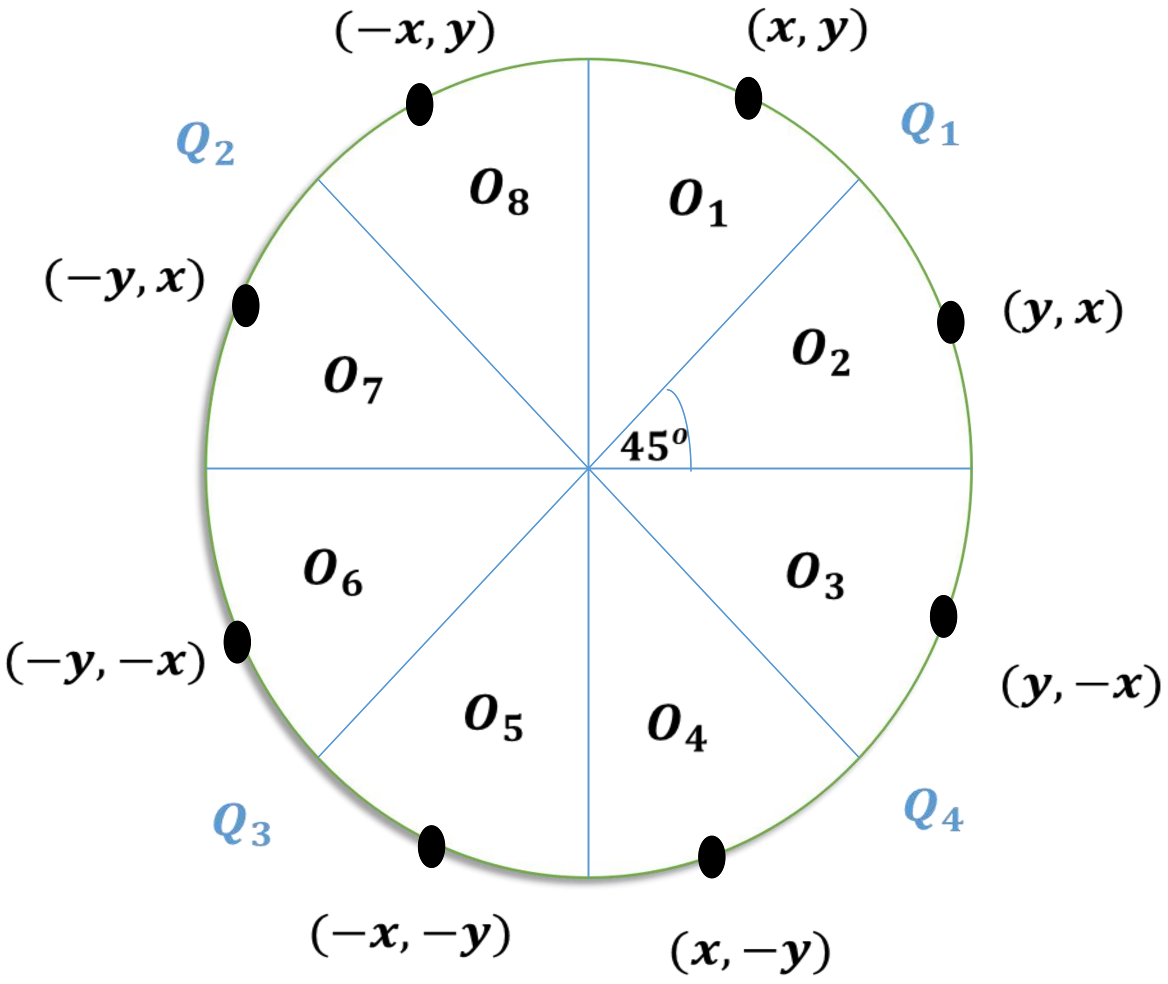


4 way symmetry



8 way symmetry

Midpoint Circle Drawing Algorithm concept



Quadrant

Octant

Circle Equation:

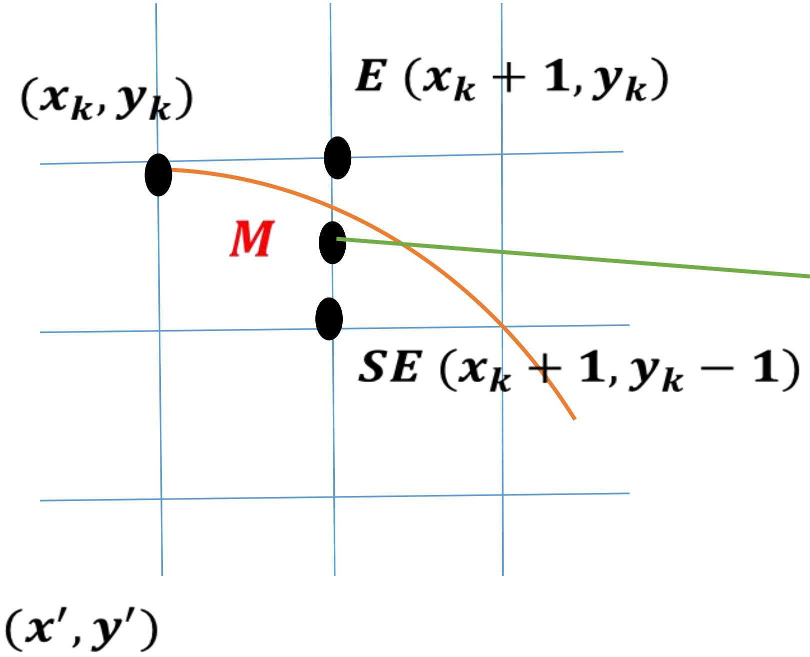
$(x - h)^2 + (y - k)^2 = r^2$

For the axis (x, y) at the point (h, k)

If $h = 0$ and $k = 0$ then

$$x^2 + y^2 = r^2$$

Midpoint Circle Drawing Algorithm evaluation



x axis will be always increment by $x_k + 1$

y axis → y_k

y axis → $y_k - 1$

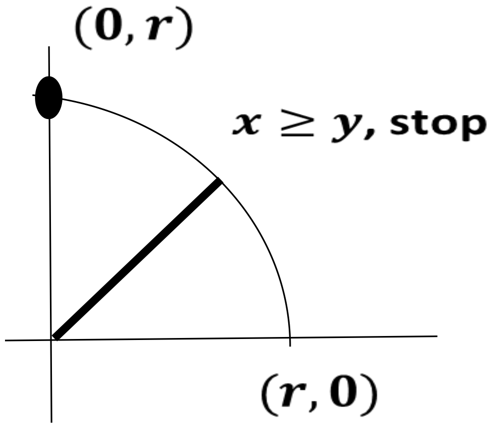
If I put the value of M on the equation of circle then

$M = 0$, lie on the line
$M > 0$, outside the circle boundary, I will pick SE point from the circle boundary
$M < 0$, inside the circle boundary, I will pick E point from the circle boundary

So, if $P_k < 0$, next coordinate $(x_k + 1, y_k)$
Again, if $P_k > 0$, next coordinate $(x_k + 1, y_k - 1)$

Midpoint Circle Drawing Algorithm evaluation

Now, find the initial decision parameter P_0, according to the concept "If we draw a line from the centre at an equal distance is called Radius (r)"



Put that concept into **the equation of circle** with -

$M\left(x_k + 1, y_k - \frac{1}{2}\right)$

In this case,

$$x_k = 0 \text{ and } y_k = r$$

So, the equation of initial decision parameter will be

$$P_0 = (0+1)^2 + \left(r - \frac{1}{2}\right)^2 - r^2$$

Midpoint Circle Drawing Algorithm evaluation

We know the circle equation at the point $(0, 0)$ is $x^2 + y^2 = r^2$

We have got $E(x_k + 1, y_k)$ and $SE\ (x_k + 1, y_k - 1)$

Midpoint: $\left(\frac{x_k+1+x_k+1}{2}, \frac{y_k+y_k-1}{2}\right) = \left(x_k + 1, y_k - \frac{1}{2}\right)$

Put the circle equation at the midpoint in decision parameter P_k

$$P_k = (x_k + 1)^2 + \left(y_k - \frac{1}{2}\right)^2 - r^2$$

Midpoint Circle Drawing Algorithm evaluation

The decision parameter at the $k+1$th position/iteration

$$P_{k+1} = (x_{k+1}+1)^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - r^2$$

Now,

$$P_{k+1} - P_k$$
$$= (x_{k+1}+1)^2 - (x_k+1)^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - \left(y_k - \frac{1}{2}\right)^2 - r^2 + r^2$$

Midpoint Circle Drawing Algorithm evaluation

$$P_{k+1} - P_k$$

$$= (x_{k+1} + 1)^2 - (x_k + 1)^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - \left(y_k - \frac{1}{2}\right)^2 - r^2 + r^2$$

… … …

$$P_{k+1} = P_k + 2x_k + 3 + y_{k+1}^2 - y_{k+1} - y_k^2 + y_k$$

Midpoint Circle Drawing Algorithm evaluation

$$P_{k+1} = P_k + 2x_k + 3 + y_{k+1}^2 - y_{k+1} - y_k^2 + y_k$$

Now, if $P_k < 0$ then $y_{k+1} = y_k$

$$P_{k+1} = P_k + 2x_k + 3 + y_k^2 - y_k - y_k^2 + y_k$$
$$P_{k+1} = P_k + 2x_k + 3$$

if $P_k \geq\ 0$ then $y_{k+1} = y_k - 1$

$$P_{k+1} = P_k + 2x_k + 3 + (y_k - 1)^2 - (y_k - 1) - y_k^2 + y_k$$

$$P_{k+1} = P_k + 2x_k - 2y_k + 5 = P_k + 2(x_k - y_k) + 5$$

Summery

- Initial decision parameter $P_0 = 1 - r$

$\boldsymbol{P_k < 0}$	$\boldsymbol{P_k \geq 0}$
Next coordinate $(x_k + 1, y_k)$ Update the decision parameter by – $P_k = P_k + 2x_k + 3$	Next coordinate $(x_k + 1, y_k - 1)$ Update the decision parameter by – $P_k = P_k + 2(x_k - y_k) + 5$

Solve a problem

Q: Find/Plot the first/second octant of the circle centered at origin, having the radius 7 units.

<u>Solution:</u>

The radius is $r = 7$

Circle is centered at the origin means the point is $(0,0)$

Therefore, the initial point is $(0, 7)$

Find the 1st octant for Q1

Initial decision parameter $P_0 = 1 - r = 1 - 7 = -6$

Iteration	$(\boldsymbol{x_0}, \boldsymbol{y_0})$	$\boldsymbol{P_k}$	$(\boldsymbol{x_k}, \boldsymbol{y_k})$
0	$(0, 7)$	$P_0 = 1 - 7 = -6 < 0$	$(1, 7)$
1	$(1, 7)$	$P_1 = -6 + 2 * 0 + 3 = -3 < 0$	$(2, 7)$
2	$(2, 7)$	$P_2 = -3 + 2 * 1 + 3 = 2 > 0$	$(3, 6)$
3	$(3, 6)$	$P_3 = 2 + 2 * (2 - 7) + 5 = -3 < 0$	$(4, 6)$
4	$(4, 6)$	$P_4 = -3 + 2 * 3 + 3 = 6 > 0$	$(5, 5)$

Find the second octant and complete the Q1

Co-ordinates of 1^{st} octant	**Co-ordinates of 2^{nd} octant**
$(0, 7)$	$(5, 5)$
$(1, 7)$	$(6, 4)$
$(2, 7)$	$(6, 3)$
$(3, 6)$	$(7, 2)$
$(4, 6)$	$(7, 1)$
$(5, 5)$	$(7, 0)$

Assignment

- **Find co-ordinates/octants for rest of the Quadrants (Q2, Q3 and Q4)**