This comprehensive formula and cheat sheet covers the high-yield topics for GATE Engineering Mathematics. It includes key formulas, theorems, and specific "shortcut" methods to solve problems faster.

1. Linear Algebra

Key Formulas

- **Determinant Properties:**
  - $|AB| = |A||B|$
  - $|A^T| = |A|$
  - $|kA| = k^n|A|$ (where A is $n \times n$)
  - $|A^{-1}| = \frac{1}{|A|}$
  - If A is triangular or diagonal, $|A| = \text{product of diagonal elements.}$
- **Rank of Matrix ($\rho(A)$):**
  - $\rho(A) \leq \min(m, n)$ for an $m \times n$ matrix.
  - $\rho(AB) \leq \min(\rho(A), \rho(B))$.
  - **System of Linear Equations ($AX = B$):**
    - **Consistent (Solution exists):** $\rho(A) = \rho(A|B)$
    - **Unique Solution:** $\rho(A) = \rho(A|B) = \text{number of variables } (n)$
    - **Infinite Solutions:** $\rho(A) = \rho(A|B) < n$
    - **Inconsistent (No Solution):** $\rho(A) \neq \rho(A|B)$
- **Eigenvalues (λ) & Eigenvectors (X):**
  - Characteristic Equation: $|A - \lambda I| = 0$
  - Cayley-Hamilton Theorem: Every square matrix satisfies its own characteristic equation (useful for finding A^{-1} or A^k).

🚀 Shortcuts & Tricks

- **Sum of Eigenvalues = Trace(A)** (Sum of main diagonal elements).
- **Product of Eigenvalues = |A|** (Determinant).
- **Eigenvalues of Special Matrices:**
  - **Triangular/Diagonal Matrix:** Eigenvalues are the diagonal elements.
  - **Idempotent Matrix ($A^2 = A$):** Eigenvalues are 0 or 1.
  - **Nilpotent Matrix ($A^k = 0$):** All eigenvalues are 0.
  - **Orthogonal Matrix ($A^T = A^{-1}$):** $|\lambda| = 1$ (Reciprocals are also eigenvalues).

- **Skew-Symmetric Matrix ($A^T = -A$):** Eigenvalues are pure imaginary or 0.
- **Real Symmetric Matrix ($A^T = A$):** Eigenvalues are always real.

- **Inverse Shortcut (2×2):**

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \implies A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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2. Calculus

Key Formulas

- **Limits:**
  - $\lim_{x \to 0} \frac{\sin x}{x} = 1$
  - $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$
  - $\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$
  - $\lim_{x \to \infty} (1 + \frac{1}{x})^x = e$
- **Mean Value Theorems:**
  - **Rolle's Theorem:** If $f(a) = f(b)$, there exists $c \in (a, b)$ such that $f'(c) = 0$.
  - **Lagrange's MVT:** $f'(c) = \frac{f(b) - f(a)}{b - a}$.
- **Vector Calculus:**
  - **Gradient ($\nabla \phi$):** Normal vector to surface $\phi(x, y, z) = c$.
  - **Divergence ($\nabla \cdot F$):** Flux density. If $\nabla \cdot F = 0$, vector is **Solenoidal**.
  - **Curl ($\nabla \times F$):** Circulation. If $\nabla \times F = 0$, vector is **Irrotational** (Conservative field).
  - **Directional Derivative:** $D_u \phi = \nabla \phi \cdot \hat{u}$ (where $\hat{u}$ is unit vector).
- **Integral Theorems:**
  - **Green's Theorem (2D):** $\oint_C (M dx + N dy) = \iint_R (\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}) dx dy$
  - **Stokes' Theorem (Line to Surface):** $\oint_C F \cdot dr = \iint_S (\nabla \times F) \cdot \hat{n} dS$
  - **Gauss Divergence Theorem (Surface to Volume):** $\iint_S F \cdot \hat{n} dS = \iiint_V (\nabla \cdot F) dV$

🚀 Shortcuts & Tricks

- **L'Hospital's Rule:** Apply $\frac{f'(x)}{g'(x)}$ repeatedly for $\frac{0}{0}$ or $\frac{\infty}{\infty}$ forms.
- **Max/Min for Two Variables ($f(x, y)$):**
  1. Find critical points where $f_x = 0$ and $f_y = 0$.

2. Calculate $r = f_{xx}, s = f_{xy}, t = f_{yy}$ at these points.
3. If $rt - s^2 > 0$ and $r > 0 \rightarrow$ **Minima.**
4. If $rt - s^2 > 0$ and $r < 0 \rightarrow$ **Maxima.**
5. If $rt - s^2 < 0 \rightarrow$ **Saddle Point.**

- **Angle of Intersection:** For two surfaces, calculate gradients $\nabla\phi_1$ and $\nabla\phi_2$. The angle θ is:

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

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3. Differential Equations

Key Formulas

- **First Order Linear ($\frac{dy}{dx} + Py = Q$):**
  - Integrating Factor (IF) = $e^{\int P\,dx}$
  - Solution: $y(\text{IF}) = \int Q(\text{IF})dx + C$
- **Exact DE ($M\,dx + N\,dy = 0$):**
  - Condition: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
  - Solution: $\int_{y=\text{const}} M\,dx + \int (\text{terms of } N \text{ free from } x)dy = C$
- **Higher Order Linear ($f(D)y = X$):**
  - General Solution $y = y_c + y_p$ (Complementary Function + Particular Integral).

🚀 Shortcuts (Particular Integral $y_p = \frac{1}{f(D)}X$)

Form of X	Shortcut Formula for y_p	Condition
e^{ax}	$\frac{1}{f(a)}e^{ax}$	$f(a) \neq 0$
$\sin(ax)$ or $\cos(ax)$	Replace D^2 with $-a^2$	$f(-a^2) \neq 0$
x^m	Expand $[f(D)]^{-1}$ using binomial	--
$e^{ax}V(x)$	$e^{ax}\frac{1}{f(D+a)}V(x)$	Shift theorem

Form of X	Shortcut Formula for y_p	Condition
Failure Case ($f(a)=0$):	$x \cdot \frac{1}{f'(a)} e^{ax}$	Multiply by x, diff denominator

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4. Probability & Statistics

Key Formulas

- **Conditional Probability:** $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- **Bayes' Theorem:** $P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum P(B|A_j)P(A_j)}$
- **Random Variables:**
  - **Mean (Expectation)** $E[X]$**:** $\sum xP(x)$ (discrete) or $\int xf(x)dx$ (continuous).
  - **Variance** $Var(X)$**:** $E[X^2] - (E[X])^2$.

Probability Distributions Cheat Sheet

Distribution	PDF / PMF	Mean	Variance
Binomial	${}^nC_x p^x q^{n-x}$	np	npq
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}$	λ	λ
Uniform (a, b)	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential	$\lambda e^{-\lambda x}$ $(x \geq 0)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$	μ	σ^2

🚀 Shortcuts

- **Variance Properties:**
  - $Var(ax + b) = a^2 Var(X)$ (Adding a constant doesn't change variance).
  - $Var(X \pm Y) = Var(X) + Var(Y)$ (If X, Y are independent).
- **Expectation Properties:**
  - $E[ax + b] = aE[X] + b$.
  - $E[XY] = E[X]E[Y]$ (If X, Y are independent).

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5. Complex Variables

Key Formulas

- **Cauchy-Riemann (C-R) Equations:** For $f(z) = u + iv$ to be analytic:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

- **Cauchy's Integral Formula:**

$$\oint_C \frac{f(z)}{(z-a)^n} dz = \frac{2\pi i}{(n-1)!} f^{(n-1)}(a)$$

  (If a is inside C. If outside, integral is 0).

🚀 Shortcuts (Residues)

- **Residue Theorem:** $\oint_C f(z)dz = 2\pi i \times (\text{Sum of Residues inside } C)$.
- **Finding Residue at Simple Pole $z = a$:**

$$\text{Res}(a) = \lim_{z \to a} (z-a) f(z)$$

  Or using L'Hospital's type rule: If $f(z) = \frac{\phi(z)}{\psi(z)}$, then $\text{Res}(a) = \frac{\phi(a)}{\psi'(a)}$.
- **Finding Residue at Pole of Order n:**

$$\text{Res}(a) = \frac{1}{(n-1)!} \lim_{z \to a} \frac{d^{n-1}}{dz^{n-1}} [(z-a)^n f(z)]$$

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6. Numerical Methods

Key Formulas

- **Newton-Raphson Method (Root Finding):**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

  - **Convergence:** Quadratic (Order 2).
- **Trapezoidal Rule:**

$$\int f(x)dx \approx \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})]$$

- **Simpson's 1/3 Rule:** (Number of intervals n must be even)

$$\int f(x)dx \approx \frac{h}{3}[(y_0 + y_n) + 4(\text{odds}) + 2(\text{evens})]$$

🚀 Shortcuts

- **Simpson's 3/8 Rule** is applicable only when n is a multiple of 3.
- **Order of Convergence:**
  - Bisection: 1 (Linear)
  - Secant: 1.62
  - Newton-Raphson: 2 (Quadratic)
- **Error in Integration:** Simpson's rule is exact for polynomials up to degree 3. Trapezoidal is exact for degree 1 (linear).

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7. General GATE Exam Strategy Tips

1. **Virtual Calculator:** Practice using the official GATE virtual calculator. Note that it works differently than physical scientific calculators (e.g., standard order of operations).
2. **Degree vs. Radians:** Always check if the question implies Degrees or Radians, especially in Calculus and Complex Variables (usually Radians).
3. **Check Boundary Conditions:** In differential equations, applying boundary conditions early can eliminate options in multiple-choice questions.
4. **Option Elimination:** For Linear Algebra (Eigenvalues) and Differential Equations, substitute the options back into the problem rather than solving from scratch.