Lab_11_Hidden_Markov_Model

October 25, 2021

1 LAB 11 : Hidden Markov Model

[5]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
```

Please refer to the following article to understand Hidden Markov Model
Here we will be dealing with 3 major problems :

1. Evaluation Problem
2. Learning Problem
3. Decoding Problem

1. Evaluation Problem : Implementation of Forward and Backward Algorithm

[8]:
```
data = pd.read_csv('/content/data_python.csv') ## Read the data, change the␣
 ↪path accordingly

V = data['Visible'].values

# Transition Probabilities
a = np.array(((0.54, 0.46), (0.49, 0.51)))

# Emission Probabilities
b = np.array(((0.16, 0.26, 0.58), (0.25, 0.28, 0.47)))

# Equal Probabilities for the initial distribution
initial_distribution = ## Write your code here


def forward(V, a, b, initial_distribution):
    alpha = np.zeros((V.shape[0], a.shape[0]))

    ## Write your code here

    return alpha
```

```python
alpha = forward(V, a, b, initial_distribution)


def backward(V, a, b):
    beta = np.zeros((V.shape[0], a.shape[0]))

    ## Write your code here

    return beta


beta = backward(V, a, b)
```

```
alpha :

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```

2. Learning Problem : Implementation of Baum Welch Algorithm

[10]:
```python
def baum_welch(V, a, b, initial_distribution, n_iter=100):

  ## Write your code here

  return (a,b)

data = pd.read_csv('data_python.csv')

V = data['Visible'].values

# Transition Probabilities
a = np.ones((2, 2))
a = a / np.sum(a, axis=1)

# Emission Probabilities
b = np.array(((1, 3, 5), (2, 4, 6)))
b = b / np.sum(b, axis=1).reshape((-1, 1))

# Equal Probabilities for the initial distribution
initial_distribution = np.array((0.5, 0.5))

a,b = baum_welch(V, a, b, initial_distribution, n_iter=100)
```

```
{'a': array([[0.53816345, 0.46183655],
       [0.48664443, 0.51335557]]),
 'b': array([[0.16277513, 0.26258073, 0.57464414],
       [0.2514996 , 0.27780971, 0.47069069]])}
```

3. Decoding Problem : Implementation of Viterbi Algorithm

[11]:
```python
def viterbi(V, a, b, initial_distribution):

  ## Write your code here
```

```
  return result


data = pd.read_csv('data_python.csv')

V = data['Visible'].values

# Transition Probabilities
a = np.ones((2, 2))
a = a / np.sum(a, axis=1)

# Emission Probabilities
b = np.array(((1, 3, 5), (2, 4, 6)))
b = b / np.sum(b, axis=1).reshape((-1, 1))

# Equal Probabilities for the initial distribution
initial_distribution = np.array((0.5, 0.5))

a, b = baum_welch(V, a, b, initial_distribution, n_iter=100)

result = viterbi(V, a, b, initial_distribution)
```

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'A', 'B', 'A', 'A', 'A', 'A', 'A', 'A', 'A', 'A', 'A', 'B', 'A', 'A', 'A', 'A',
'B', 'B', 'B', 'B', 'A', 'B', 'B', 'A', 'A', 'A', 'B', 'B', 'B', 'B', 'A', 'A',
'A', 'A', 'A', 'B', 'A', 'A', 'A', 'A', 'B', 'B', 'A', 'B', 'B', 'B', 'B', 'B',
'B', 'B', 'A', 'B', 'B', 'B', 'A', 'A', 'A', 'A', 'B', 'B', 'B', 'A', 'A', 'A',
'A', 'A', 'A', 'A']
```

4. Use the built-in **hmmlearn** package to fit the data and generate the result using the decoder

[ ]:
```
!pip install hmmlearn
```

[ ]:
```
## Write your code here
```