Learning about Initial Play Determines Average Cooperation in Repeated Games: **Online Appendix**

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March 2, 2021

A Additional Depictions of the Data

In the paper we saw that initial round behavior differs between different values of Δ^{RD}, and that the differences increase over time as the participants play more supergames. In figure 1 and 2 we show the corresponding plots but for different memory-1 histories.

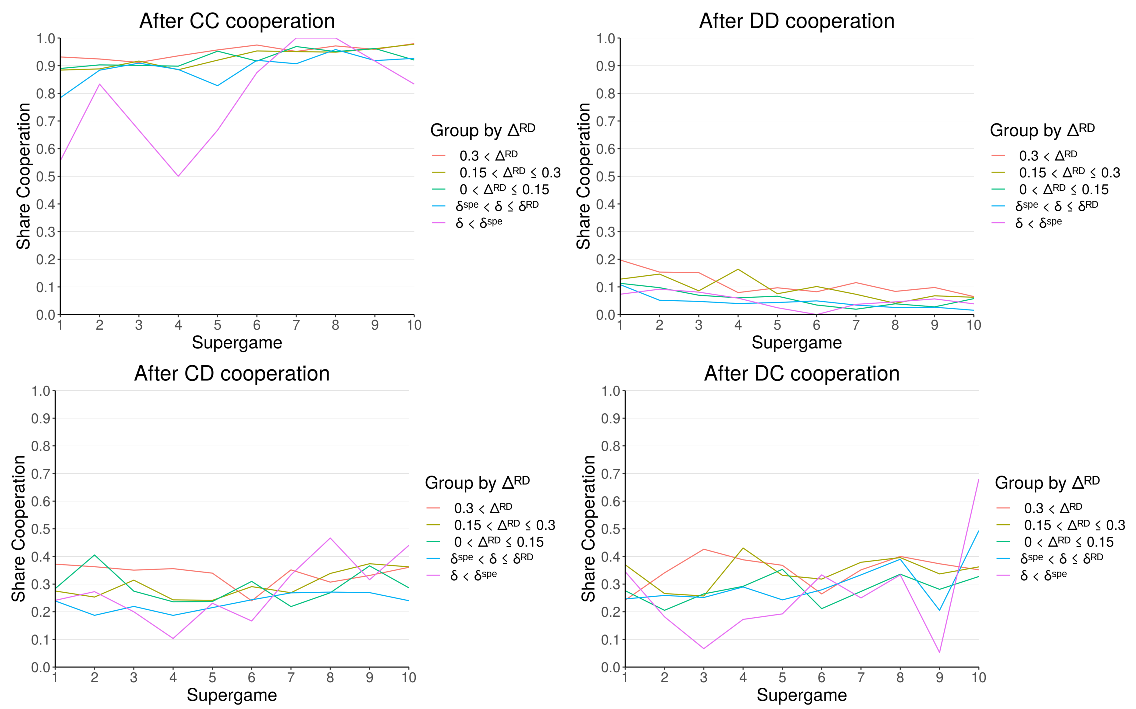

Figure 1: Average cooperation after different memory-1 histories for the first 10 supergames.

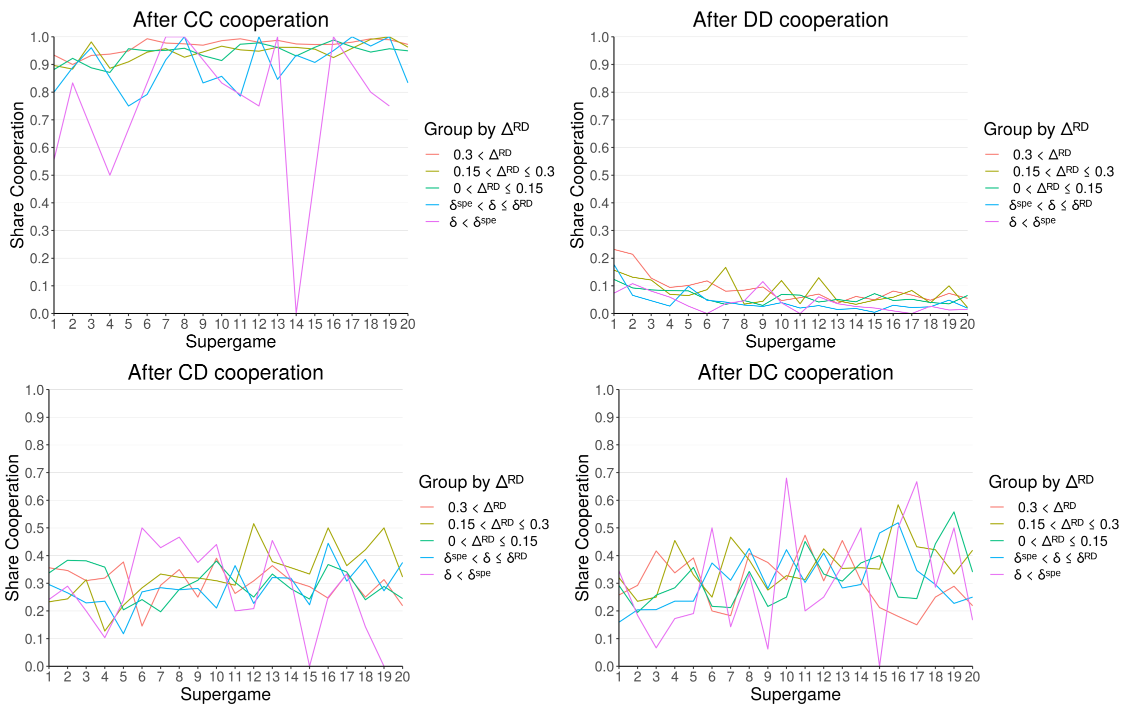

Figure 2: Average cooperation after different memory-1 histories for the first 20 supergames.

Behavior at these memory-1 histories is less variable both between different values of Δ^{RD} and over the course of the experimental sessions. However, in contrast to the initial round, different players face different distributions of the other memory-1 histories, and because these differences are not exogenous there may selection effects. As a consequence, these plots should be interpreted with more caution. Furthermore, some memory-1 histories are uncommon in certain treatments, e.g. there are few CC in games where the average cooperation rate is low.

If the differences in average cooperation between different treatments is driven primarily by the initial round behavior, then average cooperation after the initial round should be should be primarily determined by the outcome of the initial round, and otherwise similar across treatments.

To show this we compare the following three regressions. The outcome variable is the average cooperation by a participant in a supergame in the rounds following the initial round, e.g., if 4 rounds were played in that particular supergame, we calculate the average cooperation by that participant in rounds 2, 3 and 4. The first regression, conditions only on the outcome of the initial round. The second adds game parameters (δ, g, l, and Δ^{RD}), and the last uses only the game parameters and not the initial round.

Table 1: Rest of supergame average cooperation conditional on initial round outcome.

	Dependent variable:		
	y		
	(1)	(2)	(3)
initial = CD	-0.618^{***} (0.005)	-0.602^{***} (0.005)	
initial = DC	-0.626^{***} (0.005)	-0.610^{***} (0.005)	
initial = DD	-0.823^{***} (0.004)	-0.779^{***} (0.005)	
g		0.014^{***} (0.005)	0.006 (0.007)
l		-0.013^{***} (0.003)	-0.036^{***} (0.004)
δ		-0.022 (0.033)	-0.134^{***} (0.046)
Δ^{RD}		0.183^{***} (0.033)	0.875^{***} (0.045)
Constant	0.901^{***} (0.003)	0.873^{***} (0.014)	0.430^{***} (0.019)
Observations	35,726	35,726	35,726
R^2	0.561	0.565	0.172
Adjusted R^2	0.561	0.565	0.172
Note:			*p<0.1; **p<0.05; ***p<0.01

All the game parameters are statistically significant, but they explain almost no extra variance: The difference in R^2 between the model with and without game parameters is less than 0.01. In contrast, removing the outcome of the initial round lowers the R^2 to 0.0172. This is also true for second- round cooperation instead of average cooperation in the rest of the supergame.

Table 2: Second round average cooperation conditional on initial round outcome.

	Dependent variable:		
	second_y		
	(1)	(2)	(3)
initial = CD	-0.674^{***} (0.005)	-0.663^{***} (0.005)	
initial =DC	-0.652^{***} (0.005)	-0.641^{***} (0.005)	
initial =DD	-0.870^{***} (0.005)	-0.837^{***} (0.005)	
g		0.004 (0.006)	-0.004 (0.008)
l		-0.021^{***} (0.004)	-0.047^{***} (0.005)
δ		0.132^{***} (0.037)	0.011 (0.050)
Δ^{RD}		0.038 (0.037)	0.782^{***} (0.050)
Constant	0.948^{***} (0.003)	0.855^{***} (0.016)	0.379^{***} (0.021)
Observations	35,726	35,726	35,726
R^2	0.531	0.534	0.151
Adjusted R^2	0.531	0.534	0.151
Note:			*p<0.1; **p<0.05; ***p<0.01

B The Pure Strategy Belief Learning Model

Here we outline the belief learning model from Dal Bó and Fréchette (2011), and our across-treatment generalization. Individuals are assumed to choose between TFT or AllD at the beginning of each supergame. The decision is made via a logit best reply based on the individuals beliefs about how likely a partner is to play TFT or AllD, and the implied expected payoffs.

The beliefs are tracked by the two values B_{is}^C and B_{is}^D, where i is the individual and s is the supergame. Since only two pure strategies are considered, and they prescribe different actions in the initial round of a supergame, the initial-round actions reveal the partner's strategy. The belief values are updated according to

$$B_{is+1}^a = \theta B_{is}^a + \mathbb{1}\{a_{-i}(s) = a\}$$

where $a_{-i}(s)$ denotes the initial round action taken by the partner of individual i in supergame s, and θ captures recency in the beliefs. Given those two belief values, the belief that the partner will play TFT in supergame s is given by $B_{is}^C/(B_{is}^C + B_{is}^D)$.

Let $u^a(\text{TFT}), u^a(\text{AllD})$ denote the expected payoff from taking action a in the initial round if the partner is playing TFT and AllD respectively. Now given the beliefs and those values, the expected value of each choice is given by

$$U_{is}^a = \frac{B_{is}^C}{B_{is}^C + B_{is}^D} u^a(TFT) + \frac{B_{is}^D}{B_{is}^C + B_{is}^D} u^a(AllD) + \lambda_{is}\epsilon_{is}^a$$

where ϵ_{is}^a follows a type I extreme value distribution $\lambda_{is} = \lambda_i^F + (\phi_i)^s \lambda_i^V$. is a sensitivity parameter. This gives the following probability of subject i playing a in the initial round of supergame s, and thereafter following the according pure strategy,

$$p_{is}^a = \frac{\exp\left(\frac{1}{\lambda_{is}} U_{is}^a\right)}{\exp\left(\frac{1}{\lambda_{is}} U_{is}^C\right) + \exp\left(\frac{1}{\lambda_{is}} U_{is}^D\right)}.$$

B.1 Trembles

Since this model assumes noiseless behavior after the initial round, one possible improvement would be introducing such noise. Therefore, we add a variable ε_i, so that the individual takes the prescribed action with probability $1 - \varepsilon_i$. Otherwise the model remains the same, including using the theoretical values for the value of TFT against TFT etc. We therefore include the model with added trembles as well.

C List of Pure Strategies Considered for Predicting the Next Action

Strategy	Description
Always Cooperate	Alwyas play C.
Always Defect	Always play D.
Tit-for-Tat	Play C unless partner played D the last round.
Tit-for-2-Tats	Play C unless partner played D the last 2 rounds.
Tit-for-3-Tats	Play C unless partner played D the last 3 rounds.
Exploitative Tit-for-Tat	Play D in first round, then play TFT.
2-Tits-for-1-Tat	Play C unless partner played D the last round and punish for 2 rounds.
2-Tits-for-2-Tat	Play C unless partner played D the last 2 rounds and punish for 2 rounds.
Grim	Play C until either player plays D, then defect forever.
Lenient Grim 2	Play C until two consecutive rounds occur in which either player played D, then play D forever.
Lenient Grim 3	Play C until three consecutive rounds occur in which either player played D, then play D forever.

Table 3: List of pure strategies considered for predicting the next action played.

References

Dal Bó, P., and G. R. Fréchette. 2011. "The Evolution of Cooperation in Infinitely Repeated Games: Experimental Evidence." *American Economic Review*, 101: 411–429.