Learning about Initial Play Determines Average Cooperation in Repeated Games: Online Appendix

Drew Fudenberg[1] and Gustav Karreskog[2]

[1]Department of Economics, MIT

[2]Department of Economics, Stockholm School of Economics

October 21, 2020

A In-sample Regressions

Here we present in-sample regressions for the time-path and average cooperation prediction tasks.

Table 1: In-Sample OLS for Average Cooperation

	(1)	(2)	(3)
Δ^{RD}	0.919^{***} (0.072)		0.207 (0.483)
δ		0.700^{***} (0.108)	-0.174 (0.338)
rd			0.008 (0.096)
g		-0.102^{***} (0.018)	-0.013 (0.038)
l		-0.090^{***} (0.017)	-0.035 (0.024)
N rounds			0.001 (0.001)
N supergames			-0.003 (0.003)
$rd \times \Delta^{RD}$			0.636^{*} (0.378)
$rd \times$ N rounds			0.0002 (0.001)
$rd \times$ N supergames			0.001 (0.003)
Constant	0.297^{***} (0.016)	0.116 (0.079)	0.395^{*} (0.222)
Observations	100	100	100
R^2	0.621	0.597	0.688
Adjusted R^2	0.618	0.585	0.653
MSE	0.0178	0.0189	0.0147
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 2: In-Sample OLS for the Time Path of Cooperation.

	(1)	(2)	(3)
Δ^{RD}	0.936^{***} (0.012)		0.010 (0.064)
δ		0.768^{***} (0.016)	0.445^{***} (0.043)
rd			0.013 (0.011)
g		-0.111^{***} (0.003)	-0.024^{***} (0.006)
l		-0.083^{***} (0.003)	-0.044^{***} (0.004)
round			0.003^{*} (0.002)
initial			0.098^{***} (0.005)
supergame			-0.0002 (0.0002)
Length of prev Supergame - E(length)			0.006^{***} (0.0003)
$rd \times \Delta^{RD}$			0.744^{***} (0.050)
$rd \times round$			-0.002 (0.002)
$rd \times supergame$			0.005^{***} (0.0003)
Constant	0.309^{***} (0.003)	0.078^{***} (0.012)	-0.024 (0.024)
Observations	8,809	8,809	8,809
R^2	0.423	0.406	0.503
Adjusted R^2	0.423	0.406	0.503
MSE	0.0388	0.0399	0.0333

Note: *p<0.1; **p<0.05; ***p<0.01

B Second round cooperation

Table 3: Second round average cooperation conditional on initial round outcome.

	(1)	(2)	(3)
initial = CD	-0.661^{***} (0.006)	-0.647^{***} (0.007)	
initial = DC	-0.635^{***} (0.006)	-0.620^{***} (0.007)	
initial = DD	-0.866^{***} (0.005)	-0.830^{***} (0.006)	
g		0.010^{*} (0.006)	0.004 (0.008)
l		-0.016^{***} (0.004)	-0.044^{***} (0.005)
δ		0.069 (0.042)	0.037 (0.055)
Δ^{RD}		0.109^{***} (0.041)	0.860^{***} (0.054)
Constant	0.943^{***} (0.004)	0.874^{***} (0.019)	0.342^{***} (0.025)
Observations	25,574	25,574	25,574
R^2	0.509	0.513	0.151
Adjusted R^2	0.509	0.512	0.151
Note:			*p<0.1; **p<0.05; ***p<0.01

C Initial round learning with cross validated mixed strategies

In the main specification of the initial round learning model, we let the behavior at non-initial histories be fixed at the averages in the whole data set. This means that the cross-validations we perform on this model are not true out of sample estimations, since these averages come from the whole data-set. We do this since we want to emphasize the point that these values do not change much between treatments and can thus be set to constant values, and the natural choice for those constants are the averages in the data.

Here we instead let this averages be given by the averages in the respective training sets, to get true cross-validate estimates of the out of sample MSE and parameters. In table 4 we see that the MSE are basically the same and in table 5 that the parameters

estimates are also the same.

Model	Time path	Average Cooperation
Main specification	0.0338 (0.0001)	0.0166 (0.0001)
Averages taken from training data	0.0340 (0.0001)	0.0168 (0.0001)

Table 4: Comparison of the initial round learning model with non-initial play taken as the average of the full data set or the training data.

Variable	Mixed strategies given by average on	
	Full data	Training data
α	-0.38 (0.05)	-0.38 (0.05)
β	3.09 (0.32)	3.07 (0.30)
ρ	0.92 (0.03)	0.92 (0.03)
$\beta_{reinforce}$	0.13 (0.03)	0.13 (0.03)

Table 5: Average and standard deviations, in parentheses, for the parameter estimates for the main specification and the true cross validation with averages taken on the training data.

D Full table for one step ahead predictions

We here report variants of the models used to predict the next action taken. In particular, we consider a version of the initial- round learning model where behavior at subsequent memory-1 histories depend on Δ^{RD}. So while learning in this model only happens in the initial round, the mixed strategy played afterwards varies across treatments. This captures a lot of the difference between the initial round learning model and the full learning model, but that extending learning to all memory-1 histories give a slight additional improvement.

Table 6: Out of sample one-step ahead predictive performance comparisons.

Model	N	AllD	Loss	Accuracy	Relative Accuracy
Naive			0.439	84.1%	
Pure			0.335	87.0%	18.6%
Mixed	1		0.362	83.6%	-2.6%
	2		0.313	87.2%	19.4%
	3		0.3	87.2%	19.6%
Initial round learning	1		0.328	87.2%	19.8%
	1	Yes	0.309	87.5%	21.4%
	3		0.306	87.9%	24.1%
	3	Yes	0.297	87.9%	24.0%
Initial round learning with flexible mixed	1		0.326	87.2%	19.7%
	2		0.298	88.0%	24.7%
	3		0.287	88.1%	25.1%
Full learning	1		0.323	87.8%	23.2%
	2		0.295	88.1%	25.6%
	3		0.285	88.3%	26.7%
GBT:next action			0.242	90.0%	38.6%

References