Lake Heat Flux Analyzer

Version 1.0 user manual

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Networking Lake Observatories in Europe (https://www.dkit.ie/netlake)

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1. Introduction

Lake Heat Flux Analyzer is a numerical code for calculating the surface energy fluxes in lakes. The program was developed for the rapid analysis of high-frequency instrumented lake buoy data in support of the emerging field of aquatic sensor network science. Some of the available outputs for Lake Heat Flux Analyzer include the surface fluxes of momentum, sensible heat and moisture and their corresponding transfer coefficients, incoming and outgoing long-wave radiation, and evaporation rates. The purpose of this user manual is to describe the physical parameterization and numerical implementation of the Lake Heat Flux Analyzer program. Scientific justification and evaluation of these parameterizations can be found in the referenced scientific papers. This manual includes a description of the data requirements for Lake Heat Flux Analyzer, a demonstration of operating the program and a detailed description of the individual surface energy flux terms

2. Input file formats

Full performance of Lake Heat Flux Analyzer requires various input files, including a water temperature file (extension .wtr), wind data (.wnd), short-wave radiation (.sw), relative humidity (.rh), air temperature (.airT) and configuration file (.lke). Names must be shared among files and the required file format is tab delimited text file format. A list of the input files required for individual outputs can be found in Table 1. All the input files should be located in an identical folder with the user defined name (i.e. lake name).

2.1. Water temperature

The water temperature file is a tab-delimited file with a file extension of (.wtr). The file should contain one header which starts with 'dateTime', followed by the temperature measurements (see example 2.1). Lake Heat Flux Analyzer also accepts .wtr file in the same format as Lake Analyzer (Read et al. 2011) where individual thermistor depths are provided. Lake Heat Flux Analyzer will only use the first temperature column, and assume this is the temperature of the lake surface; all of the other columns will be ignored. The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM].

File Edit Format View Help

dateTime	temp1	temp2	temp3	temp4	temp5	temp6	temp7	temp8	temp9	temp10	temp11	temp12
2009-01-02 00:00	2.8933	3.1461	3.4479	3.4952	3.4862	3.4971	3.5007	3.5614	3.4808	3.5543	3.6139	3.6893
2009-01-02 01:00	2.8214	3.1237	3.4457	3.5297	3.5085	3.51	3.4905	3.5514	3.472	3.5473	3.6232	3.6741
2009-01-02 02:00	2.6804	3.1062	3.5142	3.574	3.5571	3.5104	3.5144	3.5833	3.4906	3.5639	3.6188	3.6591
2009-01-02 03:00	2.5733	3.1246	3.5737	3.5611	3.5198	3.5067	3.5131	3.5474	3.4746	3.555	3.6104	3.6396
2009-01-02 04:00	2.6146	3.1122	3.568	3.5469	3.5078	3.4882	3.5084	3.5473	3.4853	3.5499	3.5825	3.6106
2009-01-02 05:00	2.6431	3.0802	3.5542	3.5572	3.5079	3.4858	3.5409	3.5481	3.4657	3.5441	3.5684	3.6327
2009-01-02 06:00	2.7052	3.0315	3.5386	3.5361	3.487	3.5053	3.5275	3.5413	3.4813	3.538	3.5551	3.6278
2009-01-02 07:00	2.7431	3.0126	3.4441	3.5047	3.5101	3.4992	3.5025	3.5521	3.4726	3.5397	3.5597	3.6026
2009-01-02 08:00	2.6092	3.0843	3.4518	3.548	3.5165	3.4952	3.5268	3.565	3.4296	3.5345	3.5886	3.6041
2009-01-02 09:00	2.3463	3.1615	3.4482	3.5488	3.4935	3.5006	3.5088	3.4915	3.4255	3.5256	3.5939	3.5997
2009-01-02 10:00	2.6042	3.166	3.5732	3.5565	3.4899	3.4865	3.4542	3.4815	3.4179	3.529	3.5985	3.6067
2009-01-02 11:00	2.7695	3.1885	3.6053	3.549	3.4926	3.4587	3.4275	3.4565	3.3755	3.5243	3.566	3.6023
2009-01-02 12:00	2.9931	3.226	3.5527	3.5404	3.477	3.4518	3.4176	3.4264	3.383	3.501	3.5696	3.6218
2009-01-02 13:00	3.2496	3.2977	3.5676	3.529	3.4774	3.4648	3.4323	3.4758	3.425	3.5383	3.5916	3.6417
2009-01-02 14:00	3.4137	3.3564	3.5079	3.5217	3.4493	3.4146	3.4125	3.4505	3.376	3.4808	3.5386	3.5551

Example 2.1. An example temperature file used for Esthwaite Water.

2.2. Wind speed

The wind speed file is a tab-delimited file with a file extension of (.wnd). The file should contain one header which starts with 'dateTime', followed by the wind speed 'wnd' (see example 2.3). The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM], and wind speed data in m s^{-1} should be described.

```
File  Edit  Format  View  Help
dateTime            wnd
2009-01-02 00:00    0.34328
2009-01-02 01:00    0.24373
2009-01-02 02:00    0.2805
2009-01-02 03:00    0.1989
2009-01-02 04:00    0.41765
2009-01-02 05:00    0.58409
2009-01-02 06:00    0.56505
2009-01-02 07:00    0.26356
2009-01-02 08:00    0.86957
2009-01-02 09:00    0.79628
2009-01-02 10:00    0.62732
2009-01-02 11:00    0.88896
2009-01-02 12:00    1.762
2009-01-02 13:00    1.1216
2009-01-02 14:00    2.0852
```

Example 2.2. An example wind speed file used for Esthwaite Water.

2.3. Air temperature

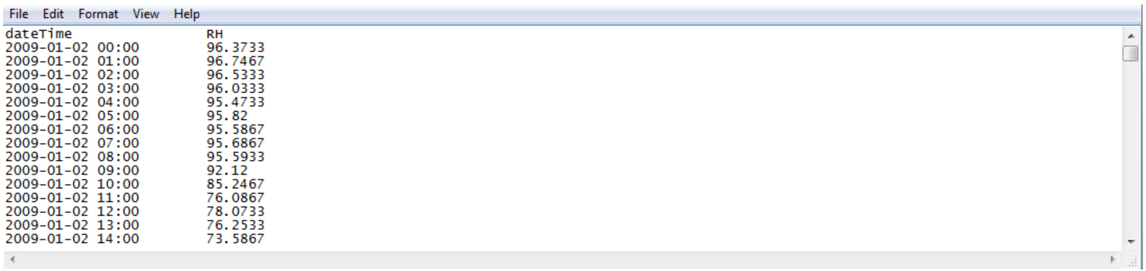
The air temperature file is a tab-delimited file with a file extension of (.airT). The file should contain one header which starts with 'dateTime', followed by the air temperature 'airT' (see example 2.4). The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM], and air temperature data in °C should be described.

```
File  Edit  Format  View  Help
dateTime            airT
2009-01-02 00:00    -1.8672
2009-01-02 01:00    -1.8664
2009-01-02 02:00    -1.4319
2009-01-02 03:00    -1.2977
2009-01-02 04:00    -0.977
2009-01-02 05:00    -0.6893
2009-01-02 06:00    -0.25886
2009-01-02 07:00    0.19604
2009-01-02 08:00    1.0337
2009-01-02 09:00    1.8079
2009-01-02 10:00    2.7698
2009-01-02 11:00    4.2072
2009-01-02 12:00    3.8275
2009-01-02 13:00    4.3101
2009-01-02 14:00    4.3275
```

Example 2.3. An example air temperature file used for Esthwaite Water.

2.4. Relative humidity

The relative humidity file is a tab-delimited file with a file extension of (.rh). The file should contain one header which starts with ‘dateTime’, followed by the relative humidity ‘rh’ (see example 2.5). The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM], and relative humidity data in % should be described.

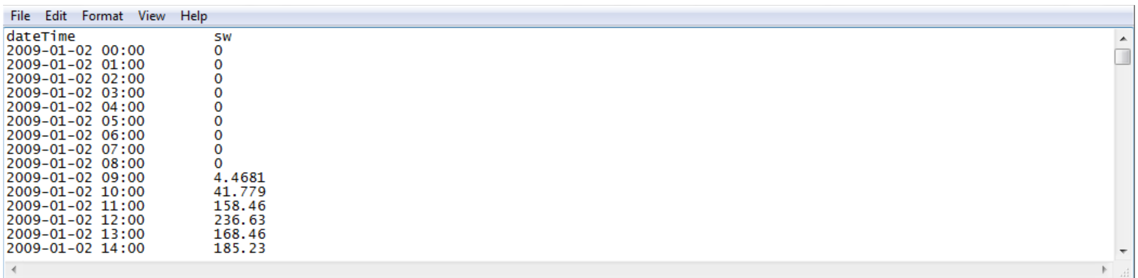


```
dateTime	RH
2009-01-02 00:00	96.3733
2009-01-02 01:00	96.7467
2009-01-02 02:00	96.5333
2009-01-02 03:00	96.0333
2009-01-02 04:00	95.4733
2009-01-02 05:00	95.82
2009-01-02 06:00	95.5867
2009-01-02 07:00	95.6867
2009-01-02 08:00	95.5933
2009-01-02 09:00	92.12
2009-01-02 10:00	85.2467
2009-01-02 11:00	76.0867
2009-01-02 12:00	78.0733
2009-01-02 13:00	76.2533
2009-01-02 14:00	73.5867
```

Example 2.4. An example relative humidity file used for Esthwaite Water.

2.5. Short-wave radiation

The short-wave radiation file is a tab-delimited file with a file extension of (.sw). The file should contain one header which starts from ‘dateTime’, followed by the short-wave radiation ‘sw’ (see example 2.6). The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM], and short-wave radiation data in W m^{-2} should be described.

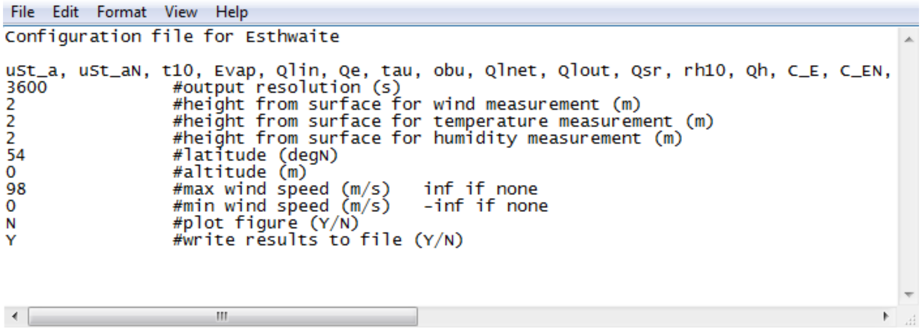


```
dateTime	sw
2009-01-02 00:00	0
2009-01-02 01:00	0
2009-01-02 02:00	0
2009-01-02 03:00	0
2009-01-02 04:00	0
2009-01-02 05:00	0
2009-01-02 06:00	0
2009-01-02 07:00	0
2009-01-02 08:00	0
2009-01-02 09:00	4.4681
2009-01-02 10:00	41.779
2009-01-02 11:00	158.46
2009-01-02 12:00	236.63
2009-01-02 13:00	168.46
2009-01-02 14:00	185.23
```

Example 2.5. An example short-wave radiation file used for Esthwaite Water.

2.6. Configuration file

The configuration file manages the operation of Lake Heat Flux Analyzer with an extension of (.lke). The configuration file is automatically created by Lake Heat Flux Analyzer through the configuration window (see section 3). The user can manually modify the file using the abbreviations shown in Table 1.



```
File  Edit  Format  View  Help
Configuration file for Esthwaite

uSt_a, uSt_aN, t10, Evap, Qlin, Qe, tau, obu, Qlnet, Qlout, Qsr, rh10, Qh, C_E, C_EN,
3600            #output resolution (s)
2               #height from surface for wind measurement (m)
2               #height from surface for temperature measurement (m)
2               #height from surface for humidity measurement (m)
54              #latitude (degN)
0               #altitude (m)
98              #max wind speed (m/s)   inf if none
0               #min wind speed (m/s)   -inf if none
N               #plot figure (Y/N)
Y               #write results to file (Y/N)
```

Example 2.6. An example configuration file used for Esthwaite Water (not all output options are shown).

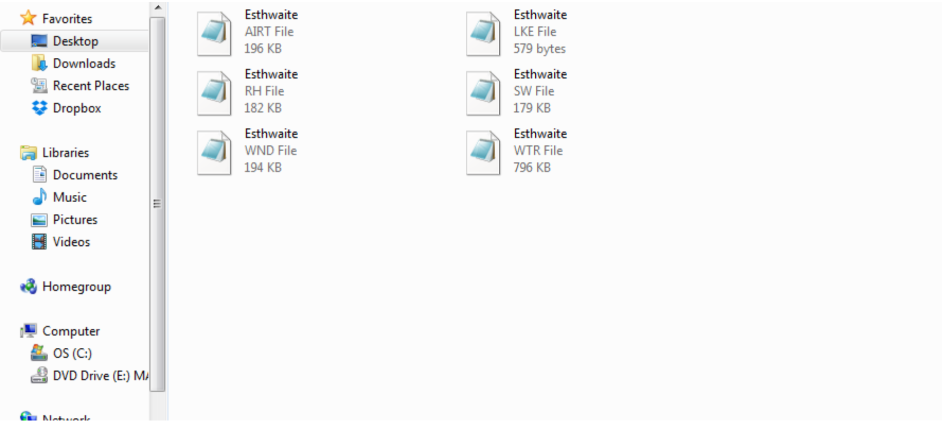
Table 1. List of input files required for the corresponding outputs.

Output	Description	.wtr	.wnd	.airT	.rh	.sw
u_{*a}	Air friction velocity	✓	✓	✓	✓	✗
u_{*aN}	Air friction velocity (neutral)	✗	✓	✗	✗	✗
T_{10}	Air temperature at 10 m	✓	✓	✓	✓	✗
E	Evaporation	✓	✓	✓	✓	✗
Q_{lin}	Incoming long-wave radiation	✓	✗	✓	✓	✓
Q_e	Latent heat flux	✓	✓	✓	✓	✗
τ	Momentum flux	✓	✓	✓	✓	✗
L_w	Monin-Obukhov length scale	✓	✓	✓	✓	✗
Q_{lnet}	Net long-wave radiation	✓	✗	✓	✓	✓
Q_{lout}	Outgoing long-wave radiation	✓	✓	✓	✓	✗
Q_{sr}	Reflected short-wave radiation	✓	✓	✓	✓	✓
rh_{10}	Relative humidity at 10 m	✓	✓	✓	✓	✗
Q_h	Sensible heat flux	✓	✓	✓	✓	✗
C_h	Transfer coefficient for heat	✓	✓	✓	✓	✗
C_{hN}	Neutral transfer coefficient for heat	✗	✓	✗	✗	✗
C_{h10}	Transfer coefficient for heat at 10 m	✓	✓	✓	✓	✗
C_{hN10}	Transfer coefficient for heat at 10 m (neutral)	✗	✓	✗	✗	✗
C_e	Transfer coefficient for humidity	✓	✓	✓	✓	✗
C_{eN}	Transfer coefficient for humidity (neutral)	✗	✓	✗	✗	✗
C_{e10}	Transfer coefficient for humidity at 10 m	✓	✓	✓	✓	✗
C_{eN10}	Transfer coefficient for humidity at 10 m (neutral)	✗	✓	✗	✗	✗
C_d	Transfer coefficient for momentum	✓	✓	✓	✓	✗
C_{dN}	Transfer coefficient for momentum (neutral)	✗	✓	✗	✗	✗
C_{d10}	Transfer coefficient for momentum at 10 m	✓	✓	✓	✓	✗
C_{dN10}	Transfer coefficient for momentum at 10 m (neutral)	✗	✓	✗	✗	✗
u_{10}	Wind speed at 10 m	✓	✓	✓	✓	✗
u_{10N}	Wind speed at 10 m (neutral)	✗	✓	✗	✗	✗

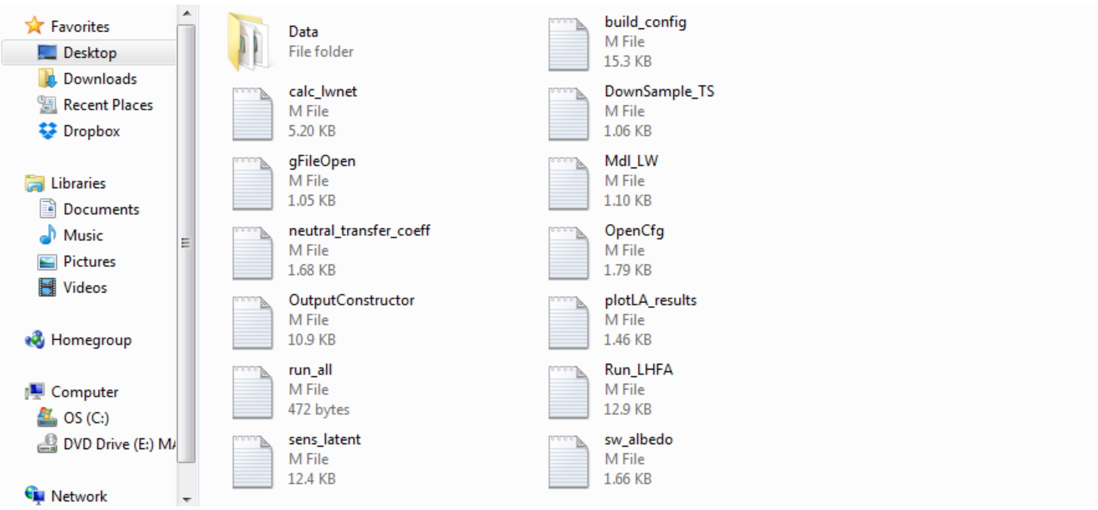
3. Program operation

A step by step guide to Lake Heat Flux Analyzer is provided here.

Step 1 - set up the input files (section 2)

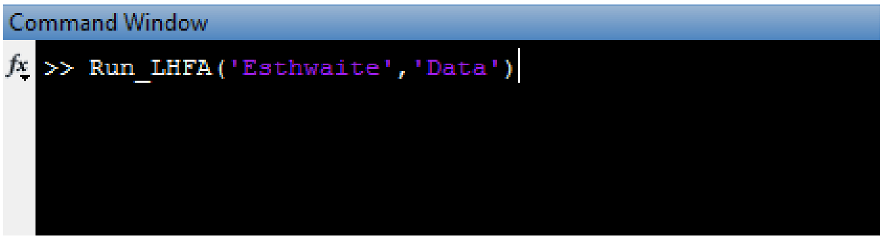


Step 2 - allocate the folder with inputs under the directory of Lake Heat Flux Analyzer

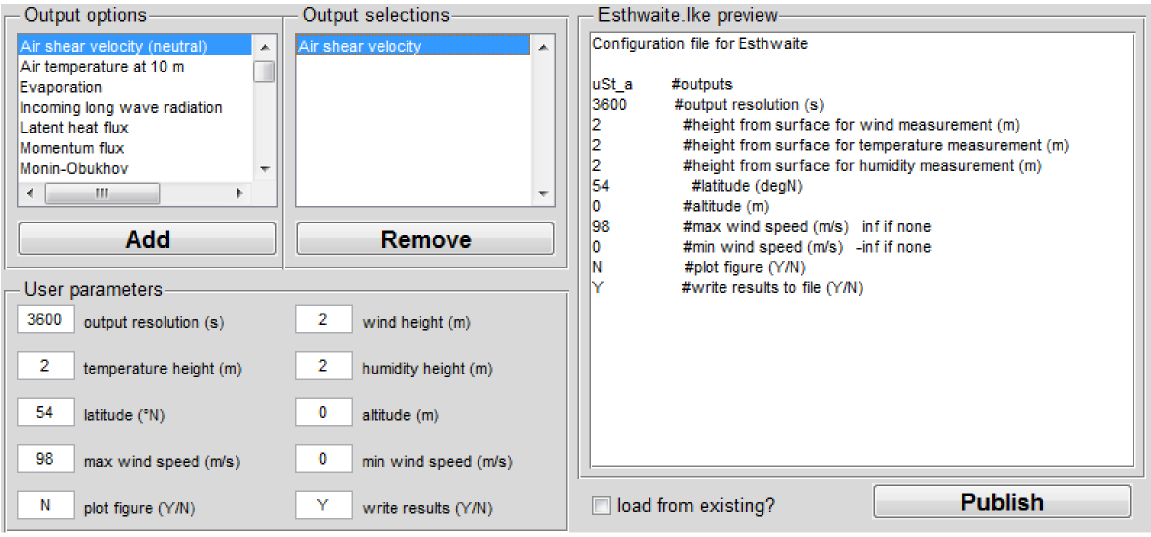


Step 3 - start MATLAB. Set the current directory to the folder where Lake Heat Flux Analyzer is located.

Step 4 - on the command window, type :>> Run_LHFA('LakeName','FolderName'). LakeName is the file name shared in the input files, and the FolderName is the name of the folder that contains the input files. Configuration window will appear.



Step 5 - the configuration window automatically creates the configuration file (.lke). Select the desired outputs and click ‘Add’. Provide specific lake characteristics and then click ‘Publish’ to operate the program.



Step 6 - when the analysis has successfully finished, the folder which the input files are located should contain a new results file. The output file has tab delimited format, thus the files can be viewed by Microsoft Excel or text editors.

Online application (heatfluxanalyzer.gleon.org)

Lake Heat Flux Analyzer has a web application which operates exactly the same way as the MATLAB application. The online application, however, does not show a configuration window, thus the user must prepare the configuration file (.lke) prior to the program operation. Please follow the examples shown in section 2 to create the configuration file. All the input files should have common names. The input files should be zipped into one file and the zip file must share its name with the other input files. Example Esthwaite Water file can be found on the website. Once the input files are prepared, the file name and its location should be chosen by following the instructions on the website.

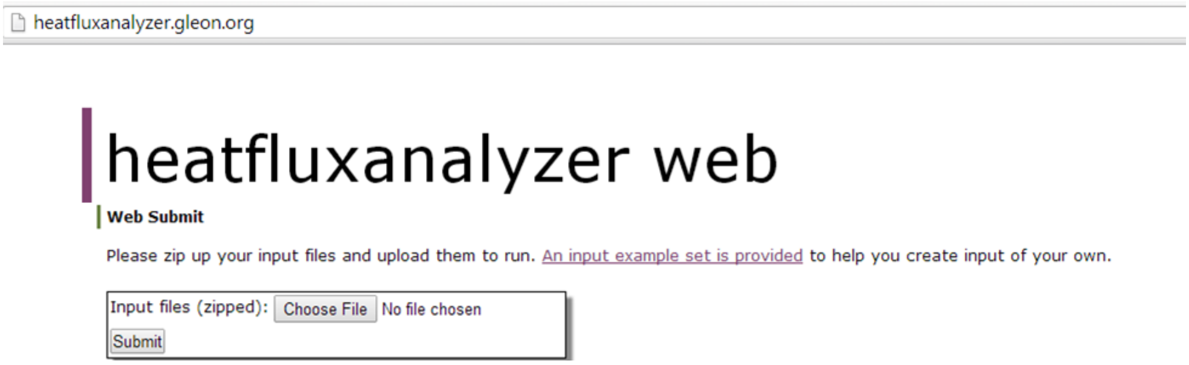


Figure 1. Online Lake Heat Flux Analyzer application (heatfluxanalyzer.gleon.org)

4. Bulk parameterization of surface fluxes

The following section describes the algorithm developed for estimating the surface energy fluxes from lake buoy data. We describe the methods used for calculating the reflected short-wave radiation (Q_{sr}), the sensible (Q_h) and latent (Q_e) heat fluxes, the incoming long-wave (Q_{lin}) and the outgoing long-wave radiation (Q_{lout}), expressed in terms of the total surface heat flux (Q_{tot}) as

$$Q_{tot} = Q_s + Q_{sr} + Q_e + Q_h + Q_{lin} + Q_{lout}, \tag{1}$$

where Q_s is short-wave radiation incident on the lake surface.

4.1. Incident and reflected short-wave radiation

The insolation (direct solar and diffuse sky radiation) reaching the lake surface is a large variable term in the heat budget of a lake and can be measured directly, using relatively inexpensive radiometers. The reflected short-wave radiation, however, is rarely measured by instrumented lake buoys and must, therefore, be estimated from empirical relationships, the most common of which is in terms of the albedo, α,

$$Q_{sr} = \alpha Q_s \ . \tag{2}$$

As albedo is not frequently measured, however, it is common to average the reflectance over time, and to set the albedo as a function of latitude (φ) in suitable tables, from which data are drawn as needed. Various forms of these tables exist and many studies take values directly from them. Here, we calculate α following the methods of Cogley (1979) where α is calculated according to latitude and month of year (Table 2). If no albedo values have been calculated for a specific latitude or time of year, however, α is assumed constant and will be estimated as 7 % following Nunez et al. (1972).

Table 2. Monthly average albedo (%) of open water for 10° latitude belts.

	Month												
φ (°)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Year
80-90	-	30.1	33.3	25.3	16.7	13.3	15.0	22.6	31.7	30.1	-	-	18.1
70-80	30.1	33.7	26.6	17.8	13.8	12.3	13.2	16.3	23.8	32.9	30.1	-	15.7
60-70	34.0	28.1	18.5	12.2	9.9	9.5	9.7	11.3	16.3	25.4	33.6	32.5	12.8
50-60	26.3	19.3	12.7	9.3	8.0	7.7	7.9	8.8	11.4	17.4	24.9	29.4	11.1
40-50	17.8	13.1	9.5	7.7	7.1	7.0	7.0	7.5	8.8	12.0	16.9	19.8	9.4
30-40	12.1	9.7	7.8	6.9	6.6	6.5	6.6	6.8	7.5	9.1	11.6	13.2	8.0
20-30	9.1	7.9	7.0	6.5	6.4	6.4	6.4	6.4	6.8	7.6	8.9	9.7	7.2
10-20	7.6	7.0	6.5	6.3	6.3	6.4	6.3	6.3	6.4	6.9	7.5	7.9	6.7
0-10	6.9	6.5	6.3	6.3	6.5	6.6	6.5	6.3	6.3	6.5	6.8	7.0	6.5

4.2. Net long-wave radiation

The net long-wave heat flux ($Q_{l_{net}}$, W m^{-2}) across the air-water interface is comprised of two main components (i) outgoing long-wave radiation ($Q_{l_{out}}$, W m^{-2}) and (ii) incoming atmospheric long-wave radiation ($Q_{l_{in}}$, W m^{-2}). The bulk formulae may be expressed as

$$Q_{l_{net}} = Q_{l_{out}} - Q_{l_{in}}, \tag{3}$$

where in the absence of direct measurements, we estimate these terms from frequently measured variables. Outgoing long-wave radiation, $Q_{l_{out}}$, which acts to cool the water surface, is estimated by an emission law that is close to a black body:

$$Q_{l_{out}} = \varepsilon_{w} \sigma T_{0k}{}^{4}, \tag{4}$$

where $\varepsilon_w = 0.98$ is the emissivity of water, $\sigma = 5.67e^{-8}$ W m^{-2} K^{-4} is the Stefan-Boltzmann constant, and T_{0k} is the surface water temperature in Kelvin. Incoming long-wave radiation, $Q_{l_{in}}$, which acts to increase the surface water temperature, is estimated from atmospheric emissivity, ε_a, and near-surface air temperature, T_{zk}, as

$$Q_{l_{in}} = \varepsilon_a \sigma T_{zk}{}^4 \, . \tag{5}$$

Following Crawford and Duchon (1999) we estimate atmospheric emissivity as $\varepsilon_a = clf + (1 - clf)\varepsilon_c$, where clf is the cloud cover fraction, estimated by $clf = (I_c - I_m)/I_c$, where I_c is the clear-sky short-wave radiation (W m^{-2}), I_m is the measured short-wave radiation (W m^{-2}), and ε_c is the clear-sky atmospheric emissivity. The clear-sky short-wave radiation is estimated as $I_c = I_o(\cos Z) T_R T_{pg} T_w T_w$ where $I_o = 1370(\bar{r}/r)^2$ is the effective solar constant (W m^{-2}) where $\bar{r}$ and r is the average daily distances between the sun an earth, respectively. The cosine of the solar zenith angle,$\cos Z$, is estimated as $\cos Z = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos H$ where δ is the solar declination (Spencer, 1971) and $H = (\pi/12)(t_{noon} - t)$ is the hour angle where t_{noon} is the local solar noon and t is local solar time. The transmission coefficients, T_i, represent the transmission coefficients for Rayleigh scattering R, absorption by permanent gases pg and water vapour, w, and absorption and scattering by aerosols, a, estimated following Crawford and Duchon (1999).

4.3. Bulk algorithms for momentum, sensible and latent heat fluxes

The air-water fluxes of momentum (τ, N m^{-2}), sensible heat (Q_h, W m^{-2}) and latent heat (Q_e, W m^{-2}) are parameterized by bulk algorithms that relate surface layer data to surface fluxes using formulas based on similarity theory and empirical relationships. The standard procedure involves the calculation of roughness lengths for momentum, heat and moisture (z_o, z_{oh}, z_{oq}) and the corresponding transfer coefficients (C_d, C_h, C_e) from observed wind speed (u), temperature (T) and humidity (q) profiles, via an iteration routine involving a friction velocity term, u_{*a} (m s^{-1}), a scaling temperature term, T_* (K), and a scaling humidity term, q_* (g kg^{-1}). Using the above terms, surface fluxes for momentum, sensible heat and latent heat can be calculated as

$$\tau = C_d\, \rho_z\, u_z^2 = \rho_z {u_{*a}}^2, \tag{6}$$

$$Q_h = \rho_z C_{pa} C_h u_z (T_0 - T_z) = -\rho_z C_{pa} u_{*a} T_*\,, \tag{7}$$

$$Q_e = \rho_z L_v C_e u_z (q_0 - q_z) = -\rho_z L_v u_{*a} q_*\,, \tag{8}$$

where ρ_z is the density of the atmospheric boundary layer (kg m^{-3}); u_z is the wind speed (m s^{-1}) at height z_u (m) above the water surface; C_{pa} is the specific heat of air at constant pressure (J kg^{-1} °C^{-1}); T_0 is the surface water temperature (°C); T_z is air temperature (°C) at height z_t (m) above the water surface; $L_v = 2.501 \times 10^6 - 2370 \times T_0$ is the latent heat of vaporization (J kg^{-1}); $q_0 = \lambda e_{sat}/p + (\lambda - 1)\, e_{sat}$ is the specific humidity at saturation pressure (kg kg^{-1}), where p is the surface air pressure (hPa), λ (= 0.622) is the ratio of the gas constants for dry and moist air; e_{sat} is the saturated vapour pressure (hPa) at T_0, calculated following Bolton (1980) as $e_{sat} = 6.11 \exp[17.27 T_0 / 237.3 + T_0]$; $q_z = \lambda e_z/p + (\lambda - 1)\, e_z$ is the specific humidity of the air (kg kg^{-1}) at height z_q (m) above the water surface, where $e_z = R_h e_s/100$, where R_h is the relative humidity (%) and $e_s = 6.11 \exp[17.27 T_z / 237.3 + T_z]$ is the saturated vapour pressure (hPa) at T_z.

A variety of bulk flux algorithms are presently used in the literature (e.g. Zeng et al. 1998; Fairall et al. 2003; Verburg and Antenucci, 2010). While these algorithms all use equations (6)-(8) to calculate the surface fluxes, they differ in the parameterization of the transfer coefficients, the treatment of free convective conditions and surface layer gustiness. Following Zeng et al. (1998), we calculate these turbulent fluxes according to the Monin-

Obukhov similarity theory applied to the surface of the atmospheric boundary layer. This theory states that wind, temperature and humidity profile gradients depend on unique functions of $\zeta = zL_w{}^{-1}$:

$$\phi_m(\zeta) = \frac{\kappa z_u}{u_{*a}} \frac{\partial u}{\partial z_u}, \tag{9}$$

$$\phi_h(\zeta) = \frac{\kappa z_t}{T_*} \frac{\partial T}{\partial z_t}, \tag{10}$$

$$\phi_e(\zeta) = \frac{\kappa z_q}{q_*} \frac{\partial q}{\partial z_q}, \tag{11}$$

where L_w is the Monin-Obukhov length scale (m), κ is the von Karman constant (= 0.41) and ϕ_m , ϕ_h and ϕ_e are the similarity functions that relate the fluxes of momentum, heat and moisture to the mean profile gradients of wind, temperature, and humidity, respectively. According to Brutsaert (1982), the Monin-Obukhov length scale is a measure of the ratio of the reduction of potential energy due to wind mixing and the growth of atmospheric stratification due to surface fluxes and may be calculated following Monin and Obukhov (1954) as

$$L_w = \frac{-\rho_z u_{*a}^3 T_v}{\kappa g \left(\frac{Q_h}{C_{pa}} + 0.61 \frac{(T_z + 273.16) Q_e}{L_v} \right)}, \tag{12}$$

where $T_v = (T_z + 273.16)[1 + 0.61 q_z]$ is the virtual air temperature (K) and $g = 9.78033\left(1 + (0.0053 \sin^2\varphi - 5.8 \text{ x } 10^{-6} \sin^2\varphi)\right)$ is the gravitational acceleration (m s^{-2}).

Following Panofsky and Dutton (1984), the differential equations for ϕ_m , ϕ_h and ϕ_e can be integrated between the roughness length (z_o, z_{oh}, z_{oq}) and the measurement height, to obtain the wind, temperature and humidity gradients in the atmospheric boundary layer and the corresponding scaling parameters used in calculating the surface fluxes. The roughness length of momentum (z_o) was calculated following Smith (1988) as

$$z_o = \left(\frac{\alpha_1 u_{*a}^2}{g}\right) + \left(\frac{a_2 v_a}{u_{*a}}\right), \tag{13}$$

where α_1 is the Charnock constant (= 0.013), α_2 = 0.11, and v_a is the kinematic viscosity of air (= 1.5 x 10^{-5} m ^{2} s^{-1}). The functional form of Brutsaert (1982) is then used to estimate the roughness length of heat (z_{oh}) and humidity (z_{oq}) as

$$z_{oh} = z_{oq} = z_o \exp(-\mathrm{b}_1 \mathrm{Re}^{0.25} - \mathrm{b}_2)\ , \tag{14}$$

where b_1 = 2.67, b_2 = -2.57, and $Re = u_{*a} z_o / v_a$ is the roughness Reynolds number.

Using the Monin-Obukhov similarity theory, the flux gradient relations for momentum $\left(\phi_m(\zeta)\right)$ are

$$\phi_m(\zeta) = 5 + \zeta \qquad \text{for } \zeta > 1 \text{ (very stable)}\ , \tag{15}$$

$$\phi_m(\zeta) = 1 + 5\zeta \qquad \text{for } 0 \le \zeta \le 1 \text{ (stable)}, \tag{16}$$

$$\phi_m(\zeta) = (1 - 16\zeta)^{-1/4} \qquad \text{for } -1.574 \le \zeta < 0 \text{ (unstable)}, \tag{17}$$

$$\phi_m(\zeta) = \left(0.7\kappa^{2/3}\right)(-\zeta)^{1/3} \qquad \text{for } \zeta < -1.574 \text{ (very unstable)}, \tag{18}$$

and for heat and moisture, where $\phi_e(\zeta) = \phi_h(\zeta)$

$$\phi_h(\zeta) = \phi_e(\zeta) = 5 + \zeta \qquad \text{for } \zeta > 1 \text{ (very stable)}, \tag{19}$$

$$\phi_h(\zeta) = \phi_e(\zeta) = 1 + 5\zeta \qquad \text{for } 0 \le \zeta \le 1 \text{ (stable)}, \tag{20}$$

$$\phi_h(\zeta) = \phi_e(\zeta) = (1 - 16\zeta)^{-1/2} \qquad \text{for } -0.465 \le \zeta < 0 \text{ (unstable)}, \tag{21}$$

$$\phi_h(\zeta) = \phi_e(\zeta) = 0.9\kappa^{4/3}(-\zeta)^{-1/3} \qquad \text{for } \zeta < -0.465 \text{ (very unstable)}, \tag{22}$$

where to ensure continuous functions of $\phi_m(\zeta)$, $\phi_h(\zeta)$ and $\phi_e(\zeta)$, we match the relations for very unstable conditions at $\zeta_m = -1.574$ for $\phi_m(\zeta)$ and $\zeta_h = \zeta_e = -0.465$ for $\phi_h(\zeta) =$

$\phi_e(\zeta)$. The flux gradient relations can be integrated to yield wind profiles, and corresponding friction velocity, u_{*a}, as

$$u_{*a} = \kappa u_z \left[\ln\left(\frac{L_w}{z_o} + 5\right) + [5\ln(\zeta) + \zeta - 1] \right]^{-1}, \tag{23}$$

for very stable conditions $(\zeta > 1)$,

$$u_{*a} = \kappa u_z \left[\ln\left(\frac{z_u}{z_o}\right) + 5\,\zeta \right]^{-1}, \tag{24}$$

for stable conditions $(0 \leq \zeta \leq 1)$,

$$u_{*a} = \kappa u_z \left[\left[\ln\left(\frac{z_u}{z_o}\right) - \Psi_m(\zeta) \right] + \Psi_m\left(\frac{z_o}{L}\right) \right]^{-1}, \tag{25}$$

for unstable conditions ($-1.574 \leq \zeta < 0$), and

$$u_{*a} = \kappa u_z \left[\left[\ln\left(\frac{\zeta_m L_w}{z_o}\right) - \Psi_m(\zeta_m) \right] + 1.14\left[(-\zeta)^{1/3} - (-\zeta)^{1/3}\right] + \Psi_m\left(\frac{z_o}{L_w}\right) \right]^{-1}, \tag{26}$$

for very unstable conditions $(\zeta < -1.574)$ where

$$\Psi_m(\zeta) = 2\ln\left(\frac{1+\chi}{2}\right) + \ln\left(\frac{1+\chi^2}{2}\right) - 2\tan^{-1}\chi + \frac{\pi}{2}, \tag{27}$$

is the stability function for momentum under unstable conditions and

$$\chi = (1 - 16\zeta)^{1/4}. \tag{28}$$

Under unstable conditions $(\zeta < 0)$, u_z is given by

$$u_z = [u_z^2 + (\beta w_{*a})^2]^{0.5}, \tag{29}$$

where w_{*a} is the convective velocity scale

$$w_{*a} = \left(-\frac{g}{T_v} T_{v*} u_{*a} z_i\right)^{1/3}, \tag{30}$$

z_i is the convective boundary layer height (= 1000 m), β = 1, and $T_{v*} = T_*(1 + 0.61q_a/100) + 0.61T_\mathrm{h}$ is the virtual potential temperature scaling parameter and $T_h = (T_z + 273.16)(100/p)^{(287.1/1004.67)}$ is the potential temperature.

The flux gradient relations for heat follow a similar manner to that of wind, but by integrating the flux gradients for heat we obtain

$$T_* = \kappa(T_z - T_0)\left[\left(\ln\left(\frac{L_w}{z_{oh}}\right) + 5\right) + (5\ \ln(\zeta) + \zeta - 1)\right]^{-1}, \tag{31}$$

for very stable conditions $(\zeta > 1)$,

$$T_* = \kappa(T_z - T_0)\left[\ln\left(\frac{z_t}{z_{oh}}\right) + 5\zeta\right]^{-1}, \tag{32}$$

for stable conditions $(\zeta < \zeta_h < -0.465)$,

$$T_* = \kappa(T_z - T_0)\left[\ln\left(\frac{z_t}{z_{oh}}\right) - \Psi_h(\zeta)\right]^{-1}, \tag{33}$$

for unstable conditions $(0 < \zeta < 1)$, and

$$T_* = \kappa(T_z - T_0)\left\{\left[\ln\left(\frac{\zeta_h L_w}{z_{oh}}\right) - \Psi_h(\zeta_h)\right] + 0.8\left[(-\zeta_h)^{-1/3} - (-\zeta)^{-1/3}\right]\right\}^{-1}. \tag{34}$$

for very unstable conditions $(\zeta < \zeta_h = -0.465)$, where the stability function for temperature (Ψ_h) is given by

$$\Psi_h = \Psi_e = 2\ln\left(\frac{1 + \chi^2}{2}\right). \tag{35}$$

The calculation for q_* are the same as those for T_*, except that $[T_z - T_0]$, z_{oh} and z_t are replaced by $[q_z - q_0]$, z_{oq} and z_q, respectively. The scaling terms can then be used to calculate the surface fluxes for momentum, sensible heat and moisture as well as the corresponding transfer coefficients as $C_d = {u_{*a}}^2/{u_z}^2$, $C_h = -u_{*a}T_*/u_z(T_0 - T_z)$ and $C_e = -u_{*a}q_*/u_z(q_0 - q_z)$, respectively. By correcting for measurement height and the stability of the atmospheric boundary, we can also estimate the transfer coefficients at a height of 10 m as $C_{d10} = {u_{*a}}^2/{u_{10}}^2$, $C_{h10} = -u_{*a}T_*/u_{10}(T_0 - T_{10})$, and $C_{e10} = -u_{*a}q_*/u_{10}(q_0 - q_{10})$, respectively, where u_{10} is the wind speed at 10 m, T_{10} is air temperature at 10 m and q_{10} is the specific humidity at 10 m calculated by re-arranging and re-calculating equations 23 to 25 and replacing z_u, z_t and z_q with 10 and using the scaling terms corrected for atmospheric stability and measurement height. Evaporation rates may also be calculated in terms of the latent heat flux as $E = Q_e/\rho_0 L_v$, where ρ_0 is the density of the surface water (kg m^{-3}). We can also calculate the neutral transfer coefficient at a height of 10 m as

$$C_{h10N} = \left[\kappa/\ln\left(\frac{10}{z_o}\right)\right]^2, \tag{36}$$

$$C_{e10N} = C_{e10N} = \kappa^2/\left[\ln\left(\frac{10}{z_{oq}}\right)\ln\left(\frac{10}{z_{oq}}\right)\right], \tag{37}$$

where z_o and z_{oq} are calculated under neutral atmospheric conditions, that is $\zeta = 0$. To calculate wind speed at 10 m and the air friction term, under neutral atmospheric conditions, we follow Amorocho and DeVries (1980).