Lake Heat Flux Analyzer

user manual

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Contributors

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Preface

Lake Heat Flux Analyzer is a numerical code for calculating the surface energy fluxes in lakes. The program was developed for the rapid analysis of high-frequency instrumented lake buoy data in support of the emerging field of aquatic sensor network science. The purpose of this user manual is to describe the physical parameterization and numerical implementation of the Lake Heat Flux Analyzer program. Scientific justification and evaluation of these parameterizations can be found in the referenced scientific papers. This manual is split into three chapters:

Chapter 1 includes a description of the basic structure of Lake Heat Flux Analyzer and how to format the input files.

Chapter 2 goes into the details of the operation of the program as well as introducing the on-line application.

Chapter 3 explains the various terms of the surface energy fluxes and how these fluxes are calculated within the Lake Heat Flux Analyzer program.

Source code for Lake Heat Flux Analyzer is available at https://github.com/GLEON/HeatFluxAnalyzer and the on-line application can be found at http://heatfluxanalyzer.gleon.org/.

Contents

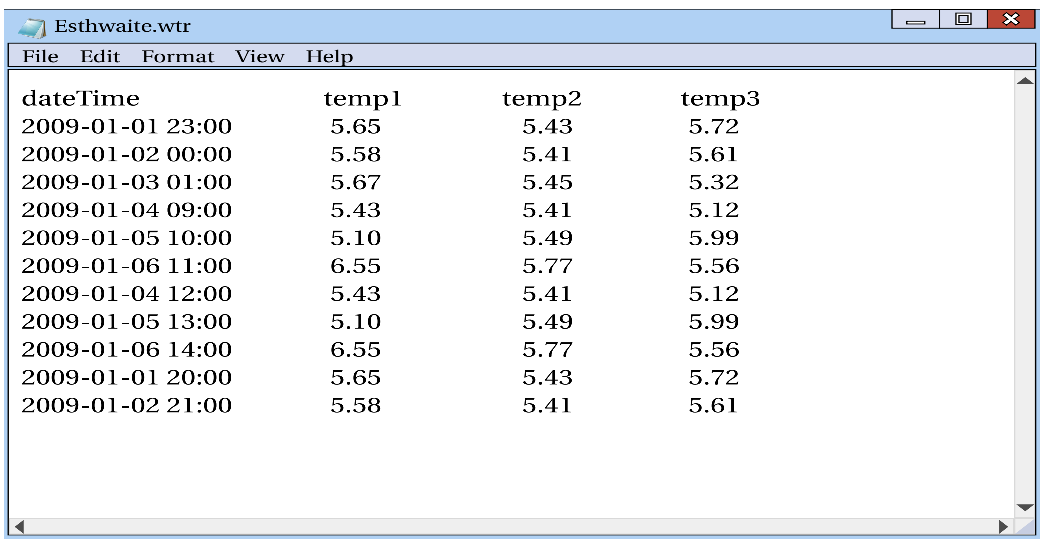
Chapter 1

Input file formats

Full performance of Lake Heat Flux Analyzer requires various input files, including a water temperature file (extension .wtr), wind data (.wnd), short-wave radiation (.sw), relative humidity (.rh), air temperature (.airT) and configuration file (.hfx). The program can also accept net long-wave radiation data (.lwnet), incoming long-wave radiation data (.lw) and photosynthetically active radiation data (.par). Names must be shared among files and the required text file format is tab-delimited. A list of the input files required for individual outputs can be found in Woolway et al (2014). All the input files should be located in an identical folder with the user defined name (i.e. lake name).

1.1 Water temperature

The water temperature file is a tab-delimited file with a file extension of (.wtr). The file should contain one header which starts with ‘dateTime’, followed by the temperature measurements. Lake Heat Flux Analyzer also accepts .wtr file in the same format as Lake Analyzer (Read et al., 2011) where individual thermistor depths are provided. Lake Heat Flux Analyzer will only use the first temperature column, and assume this is the temperature of the lake surface; all of the other columns will be ignored. The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM].

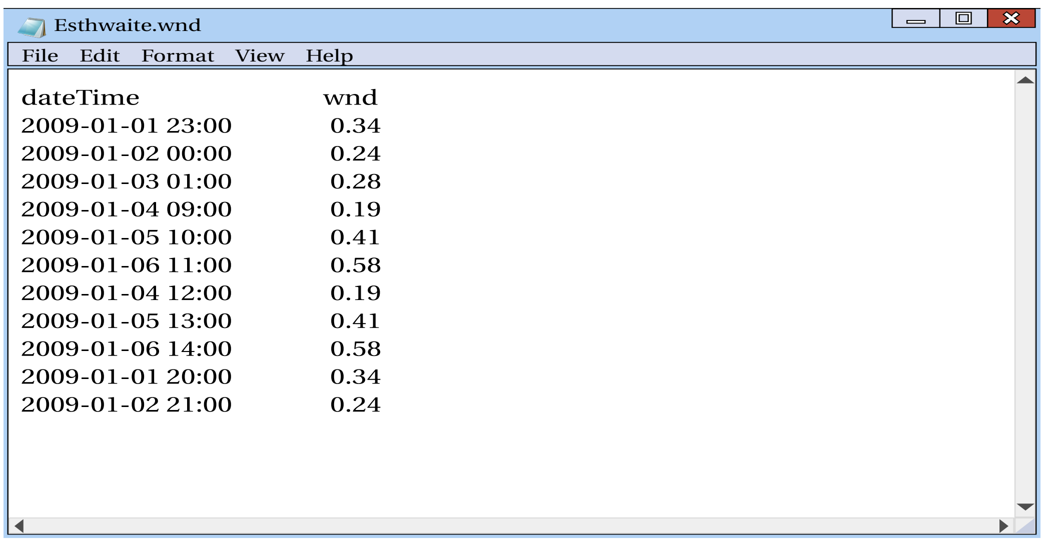

Esthwaite.wtr
File Edit Format View Help

dateTime	temp1	temp2	temp3
2009-01-01 23:00	5.65	5.43	5.72
2009-01-02 00:00	5.58	5.41	5.61
2009-01-03 01:00	5.67	5.45	5.32
2009-01-04 09:00	5.43	5.41	5.12
2009-01-05 10:00	5.10	5.49	5.99
2009-01-06 11:00	6.55	5.77	5.56
2009-01-04 12:00	5.43	5.41	5.12
2009-01-05 13:00	5.10	5.49	5.99
2009-01-06 14:00	6.55	5.77	5.56
2009-01-01 20:00	5.65	5.43	5.72
2009-01-02 21:00	5.58	5.41	5.61

An example temperature file used for Esthwaite Water.

1.2 Wind speed

The wind speed file is a tab-delimited file with a file extension of (.wnd). The file should contain one header which starts with ‘dateTime’, followed by the wind speed, ‘wnd’. The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM], and wind speed data in m s^{-1} should be described.



Esthwaite.wnd

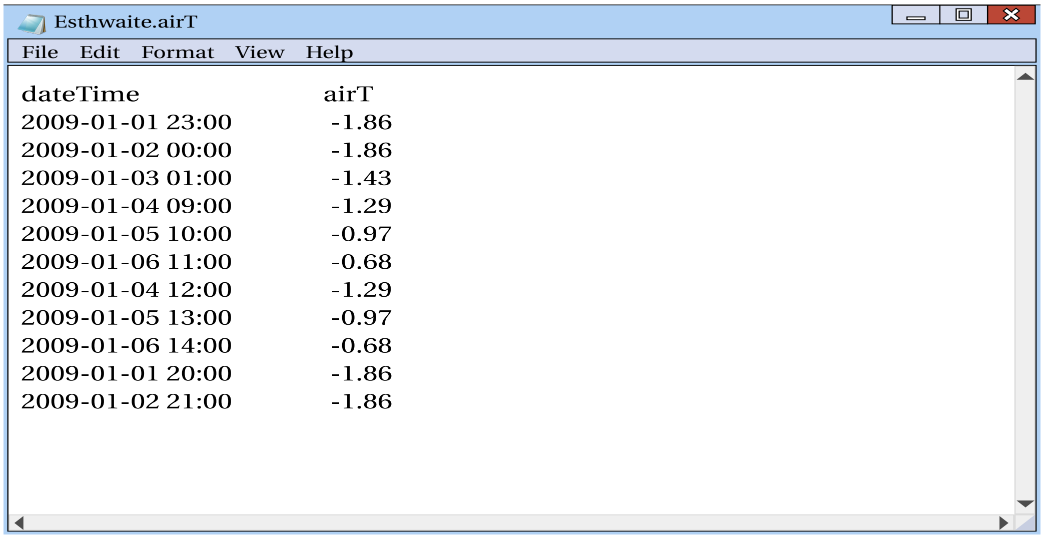
File Edit Format View Help

dateTime	wnd
2009-01-01 23:00	0.34
2009-01-02 00:00	0.24
2009-01-03 01:00	0.28
2009-01-04 09:00	0.19
2009-01-05 10:00	0.41
2009-01-06 11:00	0.58
2009-01-04 12:00	0.19
2009-01-05 13:00	0.41
2009-01-06 14:00	0.58
2009-01-01 20:00	0.34
2009-01-02 21:00	0.24

An example wind speed file used for Esthwaite Water.

1.3 Air temperature

The air temperature file is a tab-delimited file with a file extension of (.airT). The file should contain one header which starts with ‘dateTime’, followed by the air temperature ‘airT’. The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM], and air temperature data in °C should be described.



Esthwaite.airT

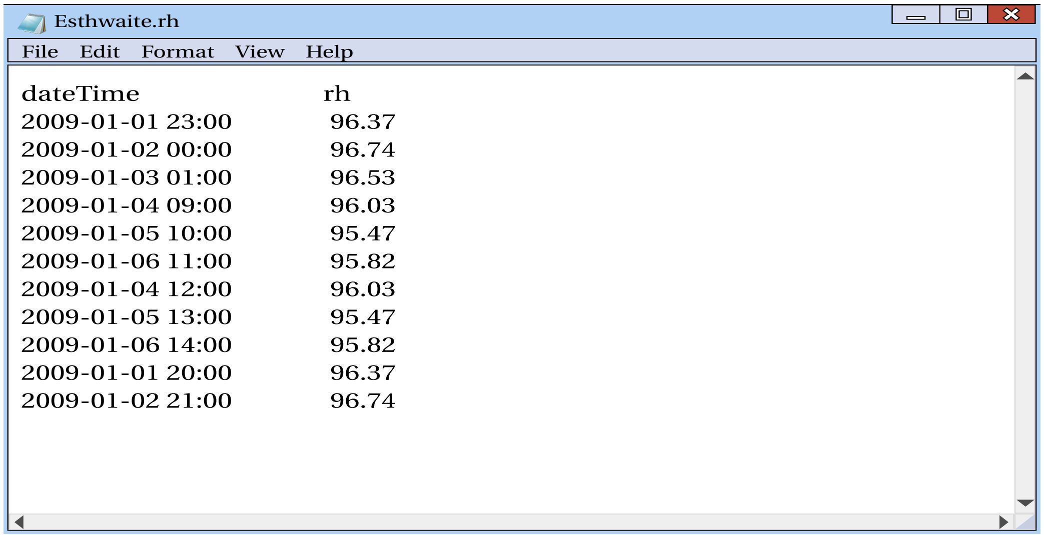
File Edit Format View Help

dateTime	airT
2009-01-01 23:00	-1.86
2009-01-02 00:00	-1.86
2009-01-03 01:00	-1.43
2009-01-04 09:00	-1.29
2009-01-05 10:00	-0.97
2009-01-06 11:00	-0.68
2009-01-04 12:00	-1.29
2009-01-05 13:00	-0.97
2009-01-06 14:00	-0.68
2009-01-01 20:00	-1.86
2009-01-02 21:00	-1.86

An example air temperature file used for Esthwaite Water.

1.4 Relative humidity

The relative humidity file is a tab-delimited file with a file extension of (.rh). The file should contain one header which starts with 'dateTime', followed by the relative humidity 'rh'. The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM], and relative humidity data in % should be described.



Esthwaite.rh

File Edit Format View Help

dateTime	rh
2009-01-01 23:00	96.37
2009-01-02 00:00	96.74
2009-01-03 01:00	96.53
2009-01-04 09:00	96.03
2009-01-05 10:00	95.47
2009-01-06 11:00	95.82
2009-01-04 12:00	96.03
2009-01-05 13:00	95.47
2009-01-06 14:00	95.82
2009-01-01 20:00	96.37
2009-01-02 21:00	96.74

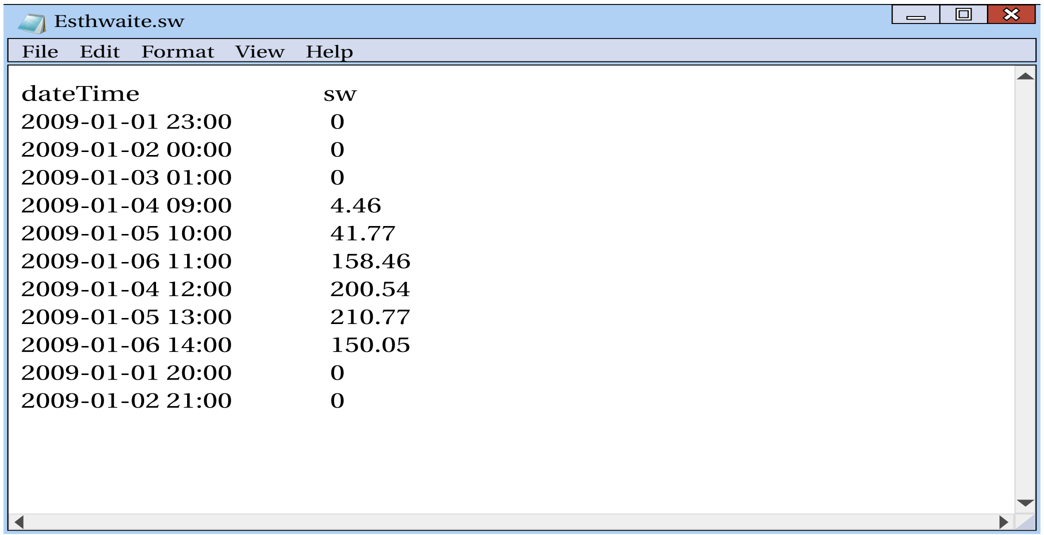
An example relative humidity file used for Esthwaite Water.

1.5 Short-wave radiation

The short-wave radiation file is a tab-delimited file with a file extension of (.sw). The file should contain one header which starts from 'dateTime', followed by the short-wave radiation 'sw'. The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM], and short-wave radiation data in W m^{-2} should be described.

1.6 Net long-wave radiation

The net long-wave radiation file is a tab-delimited file with a file extension of (.lwnet). The file should contain one header which starts from 'dateTime', followed by the net long-wave radiation 'lwnet'. The data starts from



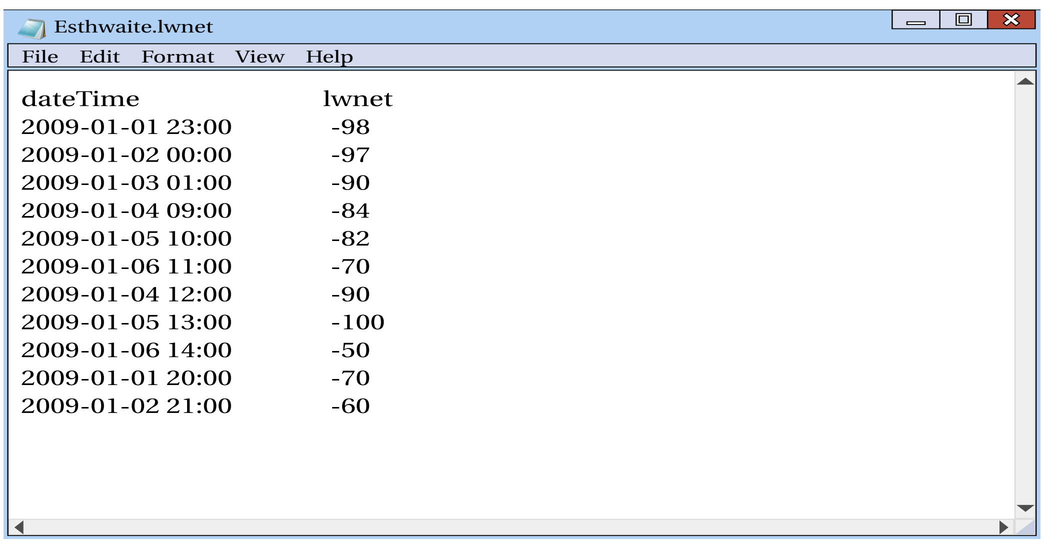
Esthwaite.sw

File Edit Format View Help

```
dateTime            sw
2009-01-01 23:00    0
2009-01-02 00:00    0
2009-01-03 01:00    0
2009-01-04 09:00    4.46
2009-01-05 10:00    41.77
2009-01-06 11:00    158.46
2009-01-04 12:00    200.54
2009-01-05 13:00    210.77
2009-01-06 14:00    150.05
2009-01-01 20:00    0
2009-01-02 21:00    0
```

An example short-wave radiation file used for Esthwaite Water.

date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM], and net long-wave radiation data in W m^{-2} should be described.



Esthwaite.lwnet

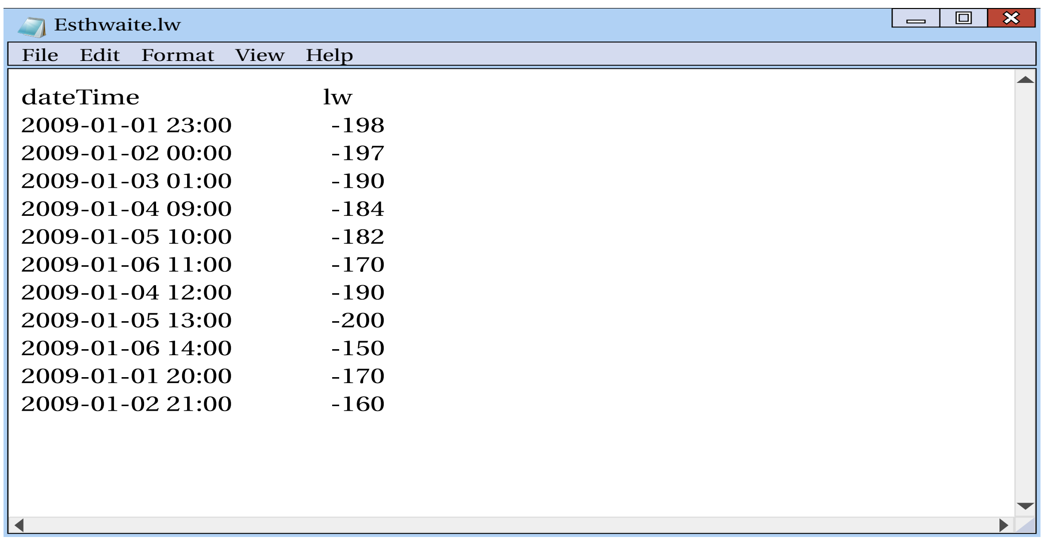
File Edit Format View Help

```
dateTime            lwnet
2009-01-01 23:00    -98
2009-01-02 00:00    -97
2009-01-03 01:00    -90
2009-01-04 09:00    -84
2009-01-05 10:00    -82
2009-01-06 11:00    -70
2009-01-04 12:00    -90
2009-01-05 13:00    -100
2009-01-06 14:00    -50
2009-01-01 20:00    -70
2009-01-02 21:00    -60
```

An example net long-wave radiation file used for Esthwaite Water.

1.7 Incoming long-wave radiation

The incoming long-wave radiation file is a tab-delimited file with a file extension of (.lw). The file should contain one header which starts from 'dateTime', followed by the incoming long-wave radiation 'lw'. The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM], and incoming long-wave radiation data in W m^{-2} should be described.

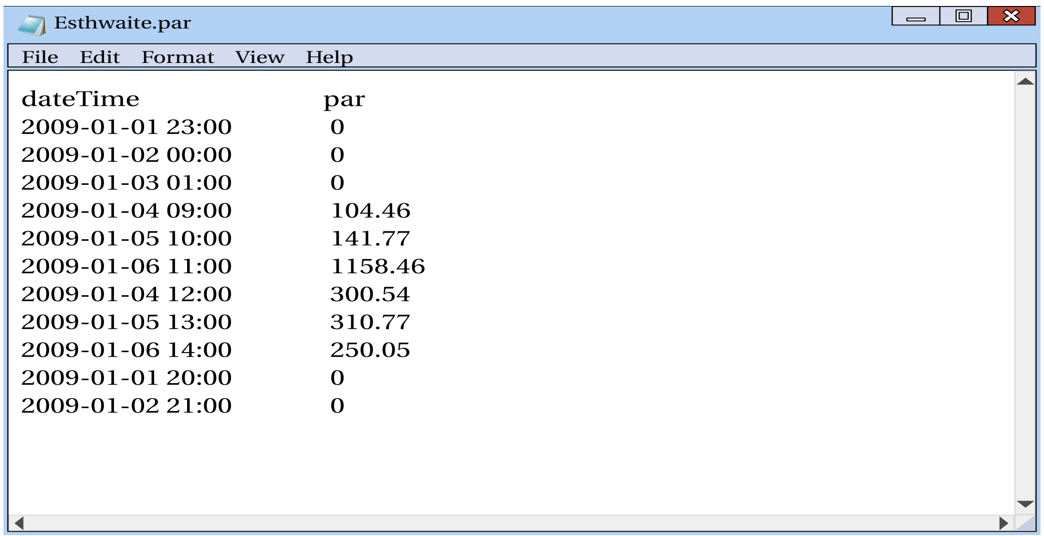


dateTime	lw
2009-01-01 23:00	-198
2009-01-02 00:00	-197
2009-01-03 01:00	-190
2009-01-04 09:00	-184
2009-01-05 10:00	-182
2009-01-06 11:00	-170
2009-01-04 12:00	-190
2009-01-05 13:00	-200
2009-01-06 14:00	-150
2009-01-01 20:00	-170
2009-01-02 21:00	-160

An example incoming long-wave radiation file used for Esthwaite Water.

1.8 Photosynthetically active radiation

The photosynthetically active radiation file is a tab-delimited file with a file extension of (.par). The file should contain one header which starts from 'dateTime', followed by the photosynthetically active radiation 'par'. The data starts from date/time inputs, which should be formatted as [yyyy-mm-dd HH:MM], and photosynthetically active radiation data in mmol m^{-2} s^{-1} should be described.

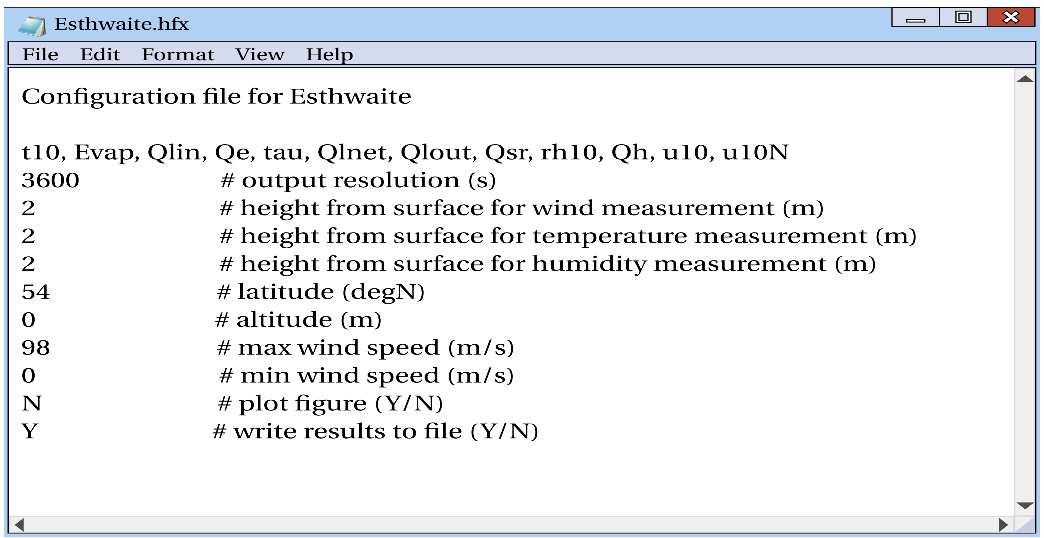

Esthwaite.par
File Edit Format View Help

dateTime	par
2009-01-01 23:00	0
2009-01-02 00:00	0
2009-01-03 01:00	0
2009-01-04 09:00	104.46
2009-01-05 10:00	141.77
2009-01-06 11:00	1158.46
2009-01-04 12:00	300.54
2009-01-05 13:00	310.77
2009-01-06 14:00	250.05
2009-01-01 20:00	0
2009-01-02 21:00	0

An example photosynthetically active radiation file used for Esthwaite Water.

1.9 Configuration file

The configuration file manages the operation of Lake Heat Flux Analyzer with an extension of (.hfx). The configuration file is automatically created by Lake Heat Flux Analyzer through the configuration window. A list of input files required for each output is provided in Woolway et al., (2014).



Esthwaite.hfx

File Edit Format View Help

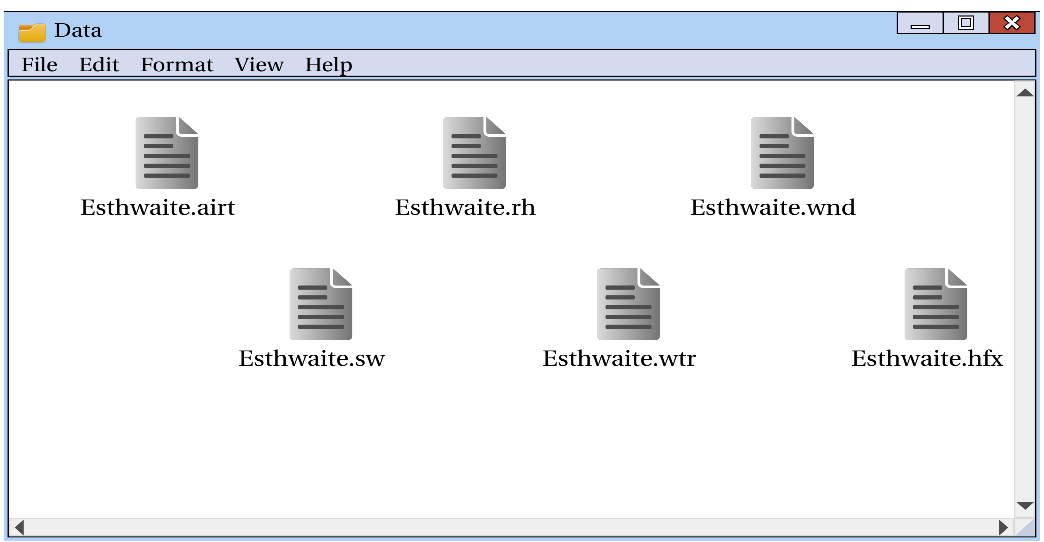
```
Configuration file for Esthwaite

t10, Evap, Qlin, Qe, tau, Qlnet, Qlout, Qsr, rh10, Qh, u10, u10N
3600          # output resolution (s)
2             # height from surface for wind measurement (m)
2             # height from surface for temperature measurement (m)
2             # height from surface for humidity measurement (m)
54            # latitude (degN)
0             # altitude (m)
98            # max wind speed (m/s)
0             # min wind speed (m/s)
N             # plot figure (Y/N)
Y             # write results to file (Y/N)
```

An example configuration file used for Esthwaite Water (not all output options are shown)

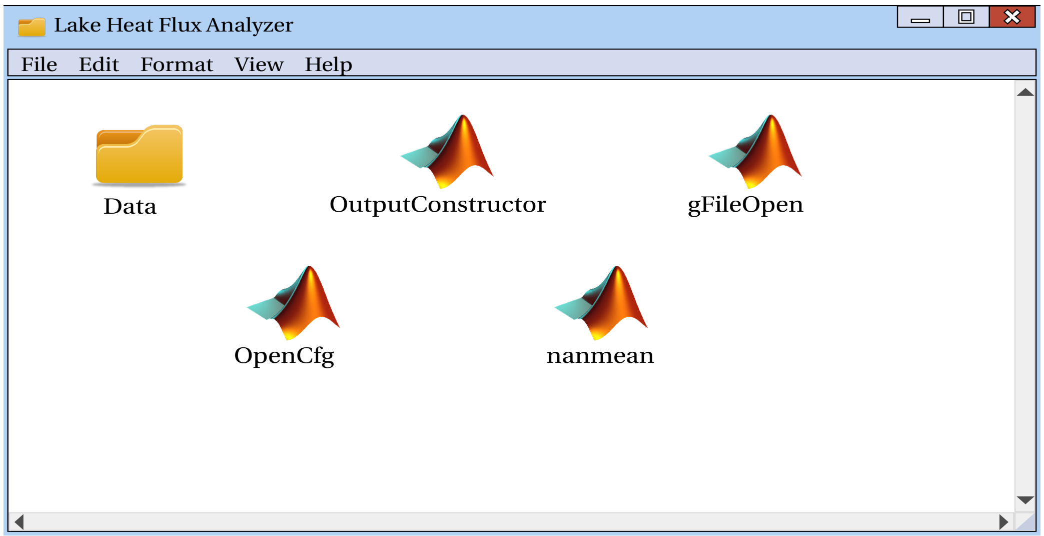
Chapter 2

Program operation

A step by step guide to Lake Heat Flux Analyzer is provided here.



Step 1: Set up the input files.



Step 2: Allocate the folder with inputs under the directory of Lake Heat Flux Analyzer.

Command Window

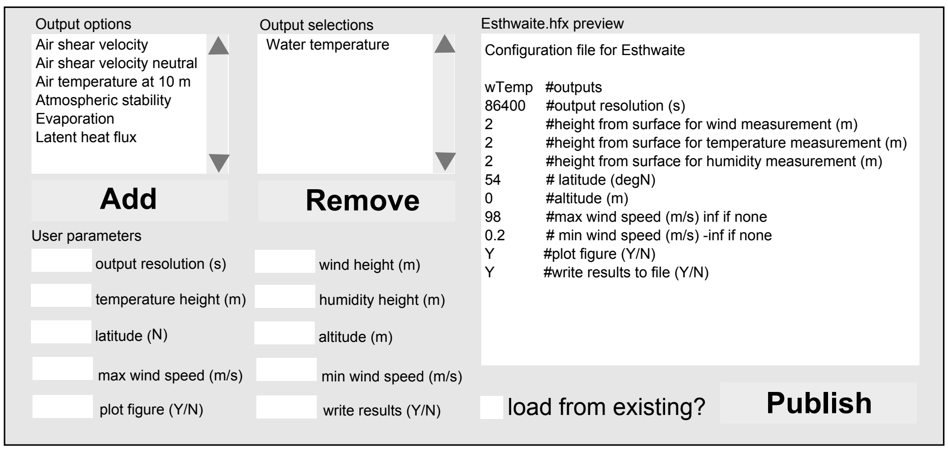
```
fx>> cd('C:\Lake Heat Flux Analyzer')
```

Step 3: Start MATLAB. Set the current directory to the folder where Lake Heat Flux Analyzer is located.

Command Window

```
fx>> Run_LHFA('LakeName','FolderName')
```

Step 4: Initiate Lake Heat Flux Analyzer. LakeName is the file name shared in the input files, and the FolderName is the name of the folder that contains the input files. Configuration window will appear.

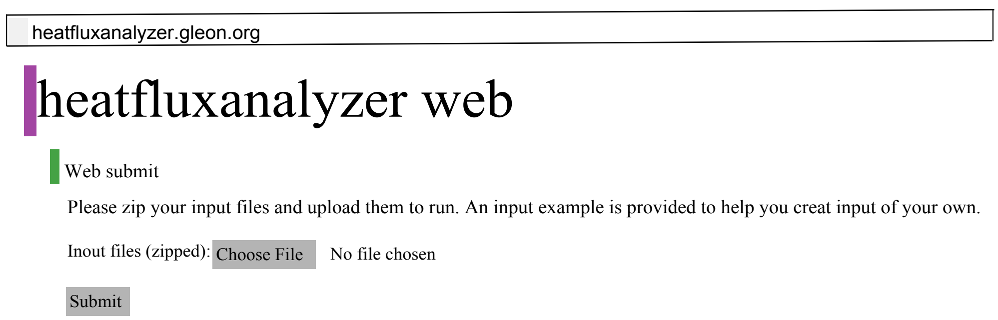


Step 5: The configuration window automatically creates the configuration file (.hfx). Select the desired outputs and click ‘Add’. The selected outputs will then appear in both the ‘output selections’ and the ‘LakeName.hfx preview’ sections of the gui. Provide specific lake characteristics and then click ‘Publish’ to operate the program. When the analysis has successfully finished, the folder which the input files are located should contain a new results file. The output file has tab delimited format, thus the files can be viewed by Microsoft Excel or text editors. The user can also select ‘load from existing’, which will allow a previously generated configuration file to be used by the Lake Heat Flux Analyzer program.

Command Window

```
>> cd('C:\Lake Heat Flux Analyzer')

>> Run_LHFA('Esthwaite','Data')

Reading Esthwaite.hfx file...completed

****Building program structure****

openWtr
openWnd
openRH
openAirT
openSW

****completed****

Reading Esthwaite.wtr file...completed

Reading Esthwaite.wnd file...completed

Reading Esthwaite.sw file...completed

Reading Esthwaite.airT file...completed

Reading Esthwaite.rh file...completed

Writing results to file...completed

Lake Heat Flux Analyzer is complete

fx>>
```

Example program run for Esthwaite Water, which will vary depending on the output selection. Lake Heat Flux Analyzer will only open the data files necessary for calculating the selected outputs.



Lake Heat Flux Analyzer has a web application heatfluxanalyzer.gleon.org which operates exactly the same way as the MATLAB application. The online application, however, does not show a configuration window, thus the user must prepare the configuration file (.hfx) prior to the program operation. Please follow the examples shown in Chapter 1 to create the configuration file. All the input files should have common names. The input files should be zipped into one file and the zip file must share its name with the other input files. An example file for Esthwaite Water can be found on the website. Once the input files are prepared, the file name and its location should be chosen by following the instructions on the website.

Chapter 3

Bulk parameterization of surface fluxes

The following section describes the algorithm developed for estimating the surface energy fluxes from lake buoy data. We describe the methods used for calculating the reflected short-wave radiation (Q_{sr}), the sensible (Q_h) and latent (Q_e) heat fluxes, the incoming long-wave (Q_{lin}) and the outgoing long-wave radiation (Q_{lout}), expressed in terms of the total surface heat flux (Q_{tot}) as

$$Q_{tot} = Q_s + Q_{sr} + Q_e + Q_h + Q_{lin} + Q_{lout} \tag{3.1}$$

where Q_s is short-wave radiation incident on the lake surface.

3.1 Incident and reflected short-wave radiation

The insolation (direct solar and diffuse sky radiation) reaching the lake surface is a large variable term in the heat budget of a lake and can be measured directly, using relatively inexpensive radiometers. The reflected short-wave radiation, however, is rarely measured by instrumented lake buoys and must, therefore, be estimated from empirical relationships, the most common of which is in terms of the albedo, α_{sw}, as $Q_{sr} = \alpha_{sw} Q_{sin}$. Here, we calculate α_{sw} according to Neumann and Pierson (1966) as

$$\alpha_{sw} = \frac{1}{2}\left[\frac{(sin^2 Z - R)}{(sin^2 Z + R)} + \frac{(tan^2 Z - R)}{(tan^2 Z + R)}\right] \tag{3.2}$$

where R is the angle of refraction which is calculated from Snells's law and Z is the solar zenith angle calculated as a function of latitude, solar declination and the hour angle (e.g. Neumann and Pierson, 1966).

3.2 Net long-wave radiation

The net long-wave heat flux (Q_{lnet}, W m^{-2}) across the air-water interface is comprised of two main components (i) outgoing long-wave radiation (Q_{lout}, W m^{-2}) and (ii) incoming atmospheric long-wave radiation (Q_{lin}, W m^{-2}).

The bulk formulae may be expressed as

$$Q_{lnet} = Q_{lout} - Q_{lin}, \tag{3.3}$$

where in the absence of direct measurements, we estimate these terms from frequently measured variables. Outgoing long-wave radiation, Q_{lout}, which acts to cool the water surface, is estimated by an emission law that is close to a black body

$$Q_{lout} = \varepsilon_w \sigma T_{0K}^4, \tag{3.4}$$

where $\varepsilon_w = 0.972$ is the emissivity of water, $\sigma = 5.67$ e^{-8} W m^{-2} K^{-4} is the Stefan-Boltzmann constant, and T_{0K} is the surface water temperature in Kelvin. Incoming long-wave radiation, Q_{lin}, which acts to increase the surface water temperature, is estimated from atmospheric emissivity, ε_a, and near-surface air temperature, T_{zK}, as

$$Q_{lin} = \varepsilon_a \sigma T_{zK}^4 \tag{3.5}$$

Following Crawford and Duchon (1999) we estimate atmospheric emissivity as $\varepsilon_a = clf + (1 - clf)\,\varepsilon_c$, where clf is the cloud cover fraction, estimated by $clf = (I_c - I_m)/I_c$, where I_c is the clear-sky short-wave radiation and I_m is the measured short-wave radiation, and ε_c is the clear-sky atmospheric emissivity. The clear-sky short-wave radiation is estimated as $I_c = I_0 (\cos Z)\, T_R T_{pg} T_w T_a$ where $I_0 = 1370\,(\bar{r}/r)^2$ is the effective solar constant (W m^{-2}) where $\bar{r}$ and r is the average daily distances between the sun and earth, respectively. The cosine of the solar zenith angle, $\cos Z$, is estimated as $\cos Z = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos H$ where φ is latitude, δ is the solar declination (Spencer, 1971) and $H = (\pi/12)\,(t_{noon} - t)$ is the hour angle where t_{noon} is the local solar noon and t is local solar time. The transmission coefficients, T_i, represent the transmission coefficients for Rayleigh scattering R, absorption by permanent gases pg and water vapour, w, and absorption and scattering by aerosols, a, estimated following Crawford and Duchon (1999).

3.3 Bulk algorithms for momentum, sensible and latent heat fluxes

The air-water fluxes of momentum (τ, N m^{-2}), sensible heat (Q_h, W m^{-2}) and latent heat (Q_e, W m^{-2}) are parameterized by bulk algorithms that relate surface layer data to surface fluxes using formulas based on similarity theory and empirical relationships. The standard procedure involves the calculation of roughness lengths for momentum, heat and moisture (z_o,z_{oh},z_{oq}) and the corresponding transfer coefficients (C_{dz},C_{hz},C_{ez}) from observed wind speed (u), temperature (T) and humidity (q) profiles, via an iteration routine involving a friction velocity term, u_{*a} (m s^{-1}), a scaling temperature term, T_* (K), and a scaling humidity term, q_* (g kg^{-1}). Using the above terms, surface fluxes for momentum, sensible heat and latent heat can be calculated as

$$\tau = C_{dz}\rho_z u_z^2 = \rho_z u_{*a}^2, \tag{3.6}$$

$$Q_h = \rho_z C_{pa} C_{hz} u_z \left(T_0 - T_z\right) = -\rho_z C_{pa} u_{*a} T_*, \tag{3.7}$$

$$Q_e = \rho_z L_v C_{ez} u_z \left(q_0 - q_z\right) = -\rho_z L_v u_{*a} q_*, \tag{3.8}$$

where $\rho_z = 100p/[R_a(T_z + 273.16)]$ is the density of the overlying air (kg m^{-3}, Verburg and Antenucci (2010)); u_z is the wind speed (m s^{-1}) at height z_u (m) above the water surface; $C_{pa} = 1005$ is the specific heat of air at constant pressure (J kg^{-1} °C^{-1}); T_0 is the surface water temperature (°C); T_z is air temperature (°C) at height z_t (m) above the water surface; $L_v = 2.501 \times 10^6 - 2370T_0$ is the latent heat of vaporization (J kg $^{-1}$); $q_0 = \left(\lambda e_{sat}\right) / \left(p + (\lambda - 1)\right) e_{sat}$ is the specific humidity at saturation pressure (kg kg^{-1}), where p is the surface air pressure (hPa), λ (= 0.622) is the ratio of the gas constants for dry and moist air; e_{sat} is the saturated vapour pressure (hPa) at T_0, calculated following Bolton (1980) as $e_{sat} = 6.11 \exp\left[\left(17.27T_0\right) / \left(237.3 + T_0\right)\right]$; $q_z = \left(\lambda e_z\right) / \left(p + (\lambda - 1)\right) e_z$ is the specific humidity of the air (kg kg^{-1}) at height z_q (m) above the water surface, where $e_z = \left(R_h e_s\right)/100$, where R_h is the relative humidity (%) and $e_s = 6.11 \exp\left[\left(17.27T_z\right) / \left(237.3 + T_z\right)\right]$ is the saturated vapour pressure (hPa) at T_z.

A variety of bulk flux algorithms are presently used in the literature (e.g. Fairall et al., 2003; Verburg and Antenucci, 2010; Zeng et al., 1998). While these algorithms all use equations 3.6 to 3.8 to calculate the surface fluxes, they differ in the parameterization of the transfer coefficients, the treatment of free convective conditions and surface layer gustiness. Following Zeng et al. (1998), we calculated these turbulent fluxes according to the Monin-Obukhov similarity theory applied to the surface of the atmospheric boundary layer. This theory states that wind, temperature and humidity profile gradients depend on unique functions of the stability parameter, ζ, where $\zeta = zL_w^{-1}$:

$$\frac{\kappa z_u}{u_{*a}}\frac{du}{dz} = \phi_m(\zeta), \qquad \frac{\kappa z_t}{T_*}\frac{dT}{dz} = \phi_h(\zeta), \qquad \frac{\kappa z_q}{q_*}\frac{dq}{dz} = \phi_e(\zeta), \tag{3.9}$$

where L_w is the Monin-Obukhov length scale (m), κ is the von Karman constant (= 0.41), and ϕ_m, ϕ_h and ϕ_e are the similarity functions that relate the fluxes of momentum, heat, and moisture to the mean profile gradients of wind, temperature, and humidity, respectively. According to Brutsaert (1982), the Monin-Obukhov length scale is a measure of the ratio of the reduction of potential energy due to wind mixing and the growth of atmospheric stratification due to surface fluxes and may be calculated following Monin and Obukhov (1954) as

$$L_w = \frac{-\rho_z u_{*a}^3 T_v}{\kappa g\left(\frac{Q_h}{C_{pa}} + 0.61 \times \frac{(T_z+273.16)Q_e}{L_v}\right)}, \tag{3.10}$$

where $T_v = (T_z + 273.16) \times (1 + 0.61q_z)$ is the virtual air temperature (K) and $g = 9.78033\left(1 + \left(0.0053\sin^2\varphi - 5.8 \times 10^{-6}\sin^2 2\varphi\right)\right)$ is the gravitational acceleration (m s^{-2}).

Following Panofsky and Dutton (1984), the differential equations for ϕ_m, ϕ_h and ϕ_e can be integrated between the roughness length (z_o, z_{oh}, z_{oq}) and the measurement height, z, to obtain the wind, temperature and humidity gradients in the atmospheric boundary layer and the corresponding scaling parameters used in calculating the surface fluxes. The roughness length of momentum (z_o) was calculated following Smith (1998) as

$$z_o = \left(\frac{\alpha_1 u_{*a}^2}{g}\right) + \left(\frac{\alpha_2 \nu_a}{u_{*a}}\right) \tag{3.11}$$

where α_1 is the Charnock constant (= 0.013), α_2 = 0.11, and ν_a is the kinematic viscosity of air (= 1.5 x 10^{-5} m^2 s^{-1}). The functional form of Brutsaert (1982) is then used to estimate the roughness length of heat (z_{ot}) and humidity (z_{oq}) as

$$z_{oq} = z_{ot} = z_o \exp\left(-b_1 Re^{0.25} - b_2\right) \tag{3.12}$$

where b_1 = 2.67, b_2 = -2.57, and Re = $u_{*a} z_o / \nu_a$ is the roughness Reynolds number.

Using the Monin-Obukhov similarity theory, the flux gradient relations for momentum, $\phi_m(\zeta)$, are

$$\phi_m(\zeta) = 5 + \zeta \qquad \text{for} \quad \zeta > 1 \,(\text{very stable}) \tag{3.13}$$

$$\phi_m(\zeta) = 1 + 5\zeta \qquad \text{for} \quad 0 \leq \zeta \leq 1 \,(\text{stable}) \tag{3.14}$$

$$\phi_m(\zeta) = (1 - 16\zeta)^{-1/4} \qquad \text{for} \qquad -1.574 \leq \zeta < 0 \,(\text{unstable}) \tag{3.15}$$

$$\phi_m(\zeta) = \left(0.7\kappa^{2/3}\right)(-\zeta)^{1/3} \qquad \text{for} \qquad \zeta < -1.574 \,(\text{very unstable}) \tag{3.16}$$

and for heat and humidity, where $\phi_e(\zeta) = \phi_h(\zeta)$

$$\phi_e(\zeta) = 5 + \zeta \qquad \text{for} \qquad \zeta > 1 \,(\text{very stable}) \tag{3.17}$$

$$\phi_e(\zeta) = 1 + 5\zeta \qquad \text{for} \qquad 0 \leq \zeta \leq 1 \,(\text{stable}) \tag{3.18}$$

$$\phi_e(\zeta) = (1 - 16\zeta)^{-1/2} \qquad \text{for} \qquad -0.465 \leq \zeta < 0 \,(\text{unstable}) \tag{3.19}$$

$$\phi_e(\zeta) = 0.9\kappa^{4/3}(-\zeta)^{1/3} \qquad \text{for} \qquad \zeta < -0.465 \,(\text{very unstable}) \tag{3.20}$$

where to ensure continuous functions of ϕ_m, ϕ_h and ϕ_e we match the relations for very unstable conditions at ζ_m = -1.574 for $\phi_m(\zeta)$ and $\zeta_h = \zeta_e$ = -0.465 for $\phi_h = \phi_e$. The flux gradient relations can then be integrated to yield wind profiles, and the corresponding friction velocity, u_{*a}, as

$$\phi_m = \frac{\kappa z_u}{u_{*a}}\frac{du}{dz} = 5 + \zeta, \tag{3.21}$$

$$\phi_m = \frac{\kappa z_u}{u_{*a}}\frac{du}{dz} = 5 + \left(\frac{z_u}{L_w}\right),$$

$$du = \frac{u_{*a}}{\kappa}\left\{\left(\ln\left(\frac{L_w}{z_o}\right) + 5\right) + \left[5\ln\left(\frac{z_u}{L_w}\right) + \left(\frac{z_u}{L_w}\right) - 1\right]\right\},$$

$$u_z - u_0 = \frac{u_{*a}}{\kappa}\left\{\left(\ln\left(\frac{L_w}{z_o}\right) + 5\right) + \left[5\ln\left(\frac{z_u}{L_w}\right) + \left(\frac{z_u}{L_w}\right) - 1\right]\right\},$$

$$u_z = \frac{u_{*a}}{\kappa}\left\{\left(\ln\left(\frac{L_w}{z_o}\right) + 5\right) + \left[5\ln\left(\frac{z_u}{L_w}\right) + \left(\frac{z_u}{L_w}\right) - 1\right]\right\}, \tag{3.22}$$

$$u_{*a} = u_z \kappa \left\{ \left(\ln\left(\frac{L_w}{z_o}\right) + 5 \right) + \left[5 \ln\left(\frac{z_u}{L_w}\right) + \left(\frac{z_u}{L_w}\right) - 1 \right] \right\}^{-1}, \quad (3.23)$$

$$u_{10} = \frac{u_{*a}}{\kappa} \left\{ \left(\ln\left(\frac{L_w}{z_o}\right) + 5 \right) + \left[5 \ln\left(\frac{10}{L_w}\right) + \left(\frac{10}{L_w}\right) - 1 \right] \right\}, \quad (3.24)$$

for very stable conditions ($\zeta > 1$),

$$\phi_m = \frac{\kappa z_u}{u_{*a}} \frac{du}{dz} = 1 + 5\zeta, \quad (3.25)$$

$$\phi_m = \frac{\kappa z_u}{u_{*a}} \frac{du}{dz} = 1 + 5\left(\frac{z_u}{L_w}\right),$$

$$du = \frac{u_{*a}}{\kappa} \left[\ln\left(\frac{z_u}{z_o}\right) + 5\left(\frac{z_u}{L_w}\right) \right],$$

$$u_z - u_0 = \frac{u_{*a}}{\kappa} \left[\ln\left(\frac{z_u}{z_o}\right) + 5\left(\frac{z_u}{L_w}\right) \right],$$

$$u_z = \frac{u_{*a}}{\kappa} \left[\ln\left(\frac{z_u}{z_o}\right) + 5\left(\frac{z_u}{L_w}\right) \right], \quad (3.26)$$

$$u_{*a} = u_z \kappa \left[\ln\left(\frac{z_u}{z_o}\right) + 5\left(\frac{z_u}{L_w}\right) \right]^{-1}, \quad (3.27)$$

$$u_{10} = \frac{u_{*a}}{\kappa} \left[\ln\left(\frac{10}{z_o}\right) + 5\left(\frac{10}{L_w}\right) \right], \quad (3.28)$$

for stable conditions ($0 < \zeta < 1$),

$$\phi_m = \frac{\kappa z_u}{u_{*a}} \frac{du}{dz} = \left[1 - 16\zeta \right]^{-1/4}, \quad (3.29)$$

$$\phi_m = \frac{\kappa z_u}{u_{*a}} \frac{du}{dz} = \left[1 - 16\left(\frac{z_u}{L_w}\right) \right]^{-1/4},$$

$$du = \frac{u_{*a}}{\kappa} \left[\ln\left(\frac{z_u}{z_o}\right) - \psi_m\left(\frac{z_u}{L_w}\right) \right],$$

$$u_z - u_0 = \frac{u_{*a}}{\kappa}\left[\ln\left(\frac{z_u}{z_o}\right) - \psi_m\left(\frac{z_u}{L_w}\right)\right],$$

$$u_z = \frac{u_{*a}}{\kappa}\left[\ln\left(\frac{z_u}{z_o}\right) - \psi_m\left(\frac{z_u}{L_w}\right)\right], \tag{3.30}$$

$$u_{*a} = u_z\kappa\left[\ln\left(\frac{z_u}{z_o}\right) - \psi_m\left(\frac{z_u}{L_w}\right)\right]^{-1}, \tag{3.31}$$

$$u_{10} = \frac{u_{*a}}{\kappa}\left[\ln\left(\frac{10}{z_o}\right) - \psi_m\left(\frac{10}{L_w}\right)\right], \tag{3.32}$$

for unstable conditions ($\zeta_m < \zeta < 0$), and

$$\phi_m = \frac{\kappa z_u}{u_{*a}}\frac{du}{dz} = \left(0.7\kappa^{2/3}\right)\left(-\zeta\right)^{1/3}, \tag{3.33}$$

$$\phi_m = \frac{\kappa z_u}{u_{*a}}\frac{du}{dz} = \left(0.7\kappa^{2/3}\right)\left(-\frac{z_u}{L_w}\right)^{1/3},$$

$$\begin{aligned} du = \frac{u_{*a}}{\kappa}\Bigg\{&\left[\ln\left(\frac{\zeta_m L_w}{z_o}\right) - \psi_m\left(\zeta_m\right)\right] \\ &+ 1.14\left[\left(-\frac{z_u}{L_w}\right)^{1/3} - \left(-\zeta_m\right)^{1/3}\right]\Bigg\}, \end{aligned}$$

$$\begin{aligned} u_z - u_0 = \frac{u_{*a}}{\kappa}\Bigg\{&\left[\ln\left(\frac{\zeta_m L_w}{z_o}\right) - \psi_m\left(\zeta_m\right)\right] \\ &+ 1.14\left[\left(-\zeta_m\right)^{1/3} - \left(-\zeta_m\right)^{1/3}\right]\Bigg\}, \end{aligned}$$

$$\begin{aligned} u_z = \frac{u_{*a}}{\kappa}\Bigg\{&\left[\ln\left(\frac{\zeta_m L_w}{z_o}\right) - \psi_m\left(\zeta_m\right)\right] \\ &+ 1.14\left[\left(-\zeta_m\right)^{1/3} - \left(-\zeta_m\right)^{1/3}\right]\Bigg\}, \end{aligned} \tag{3.34}$$

$$\begin{aligned} u_{*a} = u_z\kappa\Bigg\{&\left[\ln\left(\frac{\zeta_m L_w}{z_o}\right) - \psi_m\left(\zeta_m\right)\right] \\ &+ 1.14\left[\left(-\frac{z_u}{L_w}\right) - \left(-\zeta_m\right)^{1/3}\right]\Bigg\}^{-1}, \end{aligned} \tag{3.35}$$

$$u_{10} = \frac{u_{*a}}{\kappa}\left\{\left[\ln\left(\frac{\zeta_m L_w}{z_o}\right) - \psi_m(\zeta_m)\right] + 1.14\left[\left(-\frac{10}{L_w}\right)^{1/3} - (-\zeta_m)^{1/3}\right]\right\}, \tag{3.36}$$

for very unstable conditions ($\zeta < \zeta_m = -1.574$) where

$$\psi_m(\zeta) = 2\ln\left(\frac{1+\chi}{2}\right) + \ln\left(\frac{1+\chi^2}{2}\right) - 2\tan^{-1}\chi + \frac{\pi}{2}, \tag{3.37}$$

is the stability function for momentum under stable conditions and

$$\chi = (1 - 16\zeta)^{1/4}. \tag{3.38}$$

Under stable conditions ($\zeta > 0$), the wind speed u_z is defined as

$$u_z = \max[u_z, 0.1], \tag{3.39}$$

and for unstable cases ($\zeta < 0$), u_z is given by

$$u_z = \left[u_z^2 + (\beta w_{*a})^2\right]^{1/2}, \tag{3.40}$$

where w_{*a} is the convective velocity scale in the atmospheric boundary layer, defined as

$$w_{*a} = \left(-\frac{g}{T_v} T_{v*} u_{*a} z_i\right)^{1/3}, \tag{3.41}$$

where z_i is the convective boundary layer height (= 100 m), $\beta = 1$, and T_{v*} is the virtual potential temperature scaling parameter, calculated as

$$T_{v*} = T_* \left(1 + 0.61 q_z / 100\right) + 0.61 \times T_h \tag{3.42}$$

where $T_h = (T_z + 273.16) \times (100/p)^{(287.1/1004.67)}$ is the potential temperature, which is the temperature that a parcel of air would acquire if it was adiabatically brought to a standard reference pressure. The flux gradient relations for heat follow a similar manner to that of wind, but by integrating the flux gradients for heat we obtain

$$\phi_h = \frac{\kappa z_t}{T_*}\frac{dT}{dz} = 5 + \zeta, \tag{3.43}$$

$$\phi_h = \frac{\kappa z_t}{T_*}\frac{dT}{dz} = 5 + \left(\frac{z_t}{L_w}\right),$$

$$dT = \frac{T_*}{\kappa}\left\{\left[\ln\left(\frac{L_w}{z_{ot}}\right) + 5\right] + \left[5 + \ln\left(\frac{z_t}{L_w}\right) + \left(\frac{z_t}{L_w}\right) - 1\right]\right\},$$

$$T_z - T_0 = \frac{T_*}{\kappa}\left\{\left[\ln\left(\frac{L_w}{z_{ot}}\right) + 5\right] + \left[5 + \ln\left(\frac{z_t}{L_w}\right) + \left(\frac{z_t}{L_w}\right) - 1\right]\right\},$$

$$T_z = \frac{T_*}{\kappa}\left\{\left[\ln\left(\frac{L_w}{z_{ot}}\right) + 5\right] + \left[5 + \ln\left(\frac{z_t}{L_w}\right) + \left(\frac{z_t}{L_w}\right) - 1\right]\right\} + T_0, \tag{3.44}$$

$$T_* = T_z \kappa\left\{\left\{\left[\ln\left(\frac{L_w}{z_{ot}}\right) + 5\right] + \left[5 + \ln\left(\frac{z_t}{L_w}\right) + \left(\frac{z_t}{L_w}\right) - 1\right]\right\} + T_0\right\}^{-1}, \tag{3.45}$$

$$T_{10} = \frac{T_*}{\kappa}\left\{\left[\ln\left(\frac{L_w}{z_{ot}}\right) + 5\right] + \left[5 + \ln\left(\frac{10}{L_w}\right) + \left(\frac{10}{L_w}\right) - 1\right]\right\} + T_0, \tag{3.46}$$

for very stable conditions ($\zeta > 1$),

$$\phi_h = \frac{\kappa z_t}{T_*}\frac{dT}{dz} = 1 + 5\zeta, \tag{3.47}$$

$$\phi_h = \frac{\kappa z_t}{T_*}\frac{dT}{dz} = 1 + 5\left(\frac{z_t}{L_w}\right),$$

$$dT = \frac{T_*}{\kappa}\left[\ln\left(\frac{z_t}{z_{ot}}\right) + 5\left(\frac{z_t}{L_w}\right)\right],$$

$$T_z - T_0 = \frac{T_*}{\kappa}\left[\ln\left(\frac{z_t}{z_{ot}}\right) + 5\left(\frac{z_t}{L_w}\right)\right],$$

$$T_z = \frac{T_*}{\kappa}\left[\ln\left(\frac{z_t}{z_{ot}}\right) + 5\left(\frac{z_t}{L_w}\right)\right] + T_0, \tag{3.48}$$

$$T_* = T_z\kappa\left\{\left[\ln\left(\frac{z_t}{z_{ot}}\right) + 5\left(\frac{z_t}{L_w}\right)\right] + T_0\right\}^{-1}, \tag{3.49}$$

$$T_{10} = \frac{T_*}{\kappa}\left[\ln\left(\frac{10}{z_{ot}}\right) + 5\left(\frac{10}{L_w}\right)\right] + T_0, \tag{3.50}$$

for stable conditions ($0 < \zeta < 1$),

$$\phi_h = \frac{\kappa z_t}{T_*}\frac{dT}{dz} = (1 - 16\zeta)^{-1/2}, \tag{3.51}$$

$$\phi_h = \frac{\kappa z_t}{T_*}\frac{dT}{dz} = \left(1 - 16\left(\frac{z_t}{L_w}\right)\right)^{-1/2},$$

$$dT = \frac{T_*}{\kappa}\left[\ln\left(\frac{z_t}{z_{ot}}\right) - \psi_h\left(\frac{z_t}{L_w}\right)\right],$$

$$T_z - T_0 = \frac{T_*}{\kappa}\left[\ln\left(\frac{z_t}{z_{ot}}\right) - \psi_h\left(\frac{z_t}{L_w}\right)\right],$$

$$T_z = \frac{T_*}{\kappa}\left[\ln\left(\frac{z_t}{z_{ot}}\right) - \psi_h\left(\frac{z_t}{L_w}\right)\right] + T_0, \tag{3.52}$$

$$T_* = T_z \kappa \left\{ \left[\ln\left(\frac{z_t}{z_{ot}}\right) - \psi_h\left(\frac{z_t}{L_w}\right) \right] + T_0 \right\}^{-1}, \tag{3.53}$$

$$T_{10} = \frac{T_*}{\kappa} \left[\ln\left(\frac{10}{z_{ot}}\right) - \psi_h\left(\frac{10}{L_w}\right) \right] + T_0, \tag{3.54}$$

for unstable conditions ($\zeta_h < \zeta < 0$), and

$$\phi_h = \frac{\kappa z_t}{T_*} \frac{dT}{dz} = 0.9\kappa^{4/3} \left(-\zeta\right)^{-1/3}, \tag{3.55}$$

$$\phi_h = \frac{\kappa z_t}{T_*} \frac{dT}{dz} = 0.9\kappa^{4/3} \left(-\frac{z_t}{L_w}\right)^{-1/3}, \tag{3.56}$$

$$\begin{aligned} dT = \frac{T_*}{\kappa} \Bigg\{ & \left[\ln\left(\frac{\zeta_h L_w}{z_{ot}}\right) - \psi_h\left(\zeta_h\right) \right] \\ & +0.8 \left[\left(-\zeta_h\right)^{-1/3} - \left(-\frac{z_t}{L_w}\right)^{-1/3} \right] \Bigg\}, \end{aligned}$$

$$\begin{aligned} T_z - T_0 = \frac{T_*}{\kappa} \Bigg\{ & \left[\ln\left(\frac{\zeta_h L_w}{z_{ot}}\right) - \psi_h\left(\zeta_h\right) \right] \\ & +0.8 \left[\left(-\zeta_h\right)^{-1/3} - \left(-\frac{z_t}{L_w}\right)^{-1/3} \right] \Bigg\}, \end{aligned}$$

$$\begin{aligned} T_z = \frac{T_*}{\kappa} \Bigg\{ & \left[\ln\left(\frac{\zeta_h L_w}{z_{ot}}\right) - \psi_h\left(\zeta_h\right) \right] \\ & +0.8 \left[\left(-\zeta_h\right)^{-1/3} - \left(-\frac{z_t}{L_w}\right)^{-1/3} \right] \Bigg\} + T_0, \end{aligned} \tag{3.57}$$

$$\begin{aligned} T_* = T_z \kappa \Bigg\{ \Bigg\{ & \left[\ln\left(\frac{\zeta_h L_w}{z_{ot}}\right) - \psi_h\left(\zeta_h\right) \right] \\ & +0.8 \left[\left(-\zeta_h\right)^{-1/3} - \left(-\frac{z_t}{L_w}\right)^{-1/3} \right] \Bigg\} + T_0 \Bigg\}^{-1}, \end{aligned} \tag{3.58}$$

$$\begin{aligned} T_{10} = \frac{T_*}{\kappa} \Bigg\{ & \left[\ln\left(\frac{\zeta_h L_w}{z_{ot}}\right) - \psi_h\left(\zeta_h\right) \right] \\ & +0.8 \left[\left(-\zeta_h\right)^{-1/3} - \left(-\frac{10}{L_w}\right)^{-1/3} \right] \Bigg\} + T_0, \end{aligned} \tag{3.59}$$

for very unstable conditions ($\zeta < \zeta_h = -0.465$), where the stability function for temperature and humidity ($\psi_h = \psi_h$) is given by

$$\psi_h = \psi_e = 2\ln\left(\frac{1+\chi^2}{2}\right). \tag{3.60}$$

The calculation for q_* is the same as those for T_* except that $[T_z - T_0]$ and z_{ot} are replaced by $[q_z - q_0]$ and z_{oq} respectively:

$$\phi_e = \frac{\kappa z_q}{q_*}\frac{dq}{dz} = 5 + \zeta, \tag{3.61}$$

$$\phi_e = \frac{\kappa z_q}{q_*}\frac{dq}{dz} = 5 + \left(\frac{z_q}{L_w}\right),$$

$$dq = \frac{q_*}{\kappa}\left\{\left[\ln\left(\frac{L_w}{z_{oq}}\right) + 5\right] + \left[5 + \ln\left(\frac{z_q}{L_w}\right) + \left(\frac{z_q}{L_w}\right) - 1\right]\right\},$$

$$q_z - q_0 = \frac{q_*}{\kappa}\left\{\left[\ln\left(\frac{L_w}{z_{oq}}\right) + 5\right] + \left[5 + \ln\left(\frac{z_q}{L_w}\right) + \left(\frac{z_q}{L_w}\right) - 1\right]\right\},$$

$$q_z = \frac{q_*}{\kappa}\left\{\left[\ln\left(\frac{L_w}{z_{oq}}\right) + 5\right] + \left[5 + \ln\left(\frac{z_q}{L_w}\right) + \left(\frac{z_q}{L_w}\right) - 1\right]\right\} + q_0, \tag{3.62}$$

$$q_* = q_z\kappa\left\{\left\{\left[\ln\left(\frac{L_w}{z_{oq}}\right) + 5\right] + \left[5 + \ln\left(\frac{z_q}{L_w}\right) + \left(\frac{z_q}{L_w}\right) - 1\right]\right\} + q_0\right\}^{-1}, \tag{3.63}$$

$$q_{10} = \frac{q_*}{\kappa}\left\{\left[\ln\left(\frac{L_w}{z_{oq}}\right) + 5\right] + \left[5 + \ln\left(\frac{10}{L_w}\right) + \left(\frac{10}{L_w}\right) - 1\right]\right\} + q_0, \tag{3.64}$$

for very stable conditions ($\zeta > 1$),

$$\phi_e = \frac{\kappa z_q}{q_*}\frac{dq}{dz} = 1 + 5\zeta, \tag{3.65}$$

$$\phi_e = \frac{\kappa z_q}{q_*}\frac{dq}{dz} = 1 + 5\left(\frac{z_q}{L_w}\right),$$

$$dq = \frac{q_*}{\kappa}\left[\ln\left(\frac{z_q}{z_{oq}}\right) + 5\left(\frac{z_q}{L_w}\right)\right],$$

$$q_z - q_0 = \frac{q_*}{\kappa}\left[\ln\left(\frac{z_q}{z_{oq}}\right) + 5\left(\frac{z_q}{L_w}\right)\right],$$

$$q_z = \frac{q_*}{\kappa}\left[\ln\left(\frac{z_q}{z_{oq}}\right) + 5\left(\frac{z_q}{L_w}\right)\right] + q_0, \tag{3.66}$$

$$q_* = q_z\kappa\left\{\left[\ln\left(\frac{z_q}{z_{oq}}\right) + 5\left(\frac{z_q}{L_w}\right)\right] + q_0\right\}^{-1}, \tag{3.67}$$

$$q_{10} = \frac{q_*}{\kappa}\left[\ln\left(\frac{10}{z_{oq}}\right) + 5\left(\frac{10}{L_w}\right)\right] + q_0, \tag{3.68}$$

for stable conditions ($0 < \zeta < 1$),

$$\phi_e = \frac{\kappa z_q}{q_*}\frac{dq}{dz} = (1 - 16\zeta)^{-1/2}, \tag{3.69}$$

$$\phi_e = \frac{\kappa z_q}{q_*}\frac{dq}{dz} = \left(1 - 16\left(\frac{z_q}{L_w}\right)\right)^{-1/2},$$

$$dq = \frac{q_*}{\kappa}\left[\ln\left(\frac{z_q}{z_{oq}}\right) - \psi_e\left(\frac{z_q}{L_w}\right)\right],$$

$$q_z - q_0 = \frac{q_*}{\kappa}\left[\ln\left(\frac{z_q}{z_{oq}}\right) - \psi_e\left(\frac{z_q}{L_w}\right)\right],$$

$$q_z = \frac{q_*}{\kappa}\left[\ln\left(\frac{z_q}{z_{oq}}\right) - \psi_e\left(\frac{z_q}{L_w}\right)\right] + q_0, \tag{3.70}$$

$$q_* = q_z\kappa\left\{\left[\ln\left(\frac{z_q}{z_{oq}}\right) - \psi_e\left(\frac{z_q}{L_w}\right)\right] + q_0\right\}^{-1}, \tag{3.71}$$

$$q_{10} = \frac{q_*}{\kappa}\left[\ln\left(\frac{10}{z_{oq}}\right) - \psi_e\left(\frac{10}{L_w}\right)\right] + q_0, \tag{3.72}$$

for unstable conditions ($\zeta_e < \zeta < 0$), and

$$\phi_e = \frac{\kappa z_q}{q_*}\frac{dq}{dz} = 0.9\kappa^{4/3}\left(-\zeta\right)^{-1/3}, \tag{3.73}$$

$$\phi_e = \frac{\kappa z_q}{q_*}\frac{dq}{dz} = 0.9\kappa^{4/3}\left(-\frac{z_q}{L_w}\right)^{-1/3}, \tag{3.74}$$

$$\begin{aligned} dq = \frac{q_*}{\kappa}\Bigg\{&\left[\ln\left(\frac{\zeta_e L_w}{z_{oq}}\right) - \psi_e\left(\zeta_e\right)\right] \\ &+0.8\left[\left(-\zeta_e\right)^{-1/3} - \left(-\frac{z_q}{L_w}\right)^{-1/3}\right]\Bigg\}, \end{aligned}$$

$$\begin{aligned} q_z - q_0 = \frac{q_*}{\kappa}\Bigg\{&\left[\ln\left(\frac{\zeta_e L_w}{z_{oq}}\right) - \psi_e\left(\zeta_e\right)\right] \\ &+0.8\left[\left(-\zeta_e\right)^{-1/3} - \left(-\frac{z_q}{L_w}\right)^{-1/3}\right]\Bigg\}, \end{aligned}$$

$$\begin{aligned} q_z = \frac{q_*}{\kappa}\Bigg\{&\left[\ln\left(\frac{\zeta_e L_w}{z_{oq}}\right) - \psi_e\left(\zeta_e\right)\right] \\ &+0.8\left[\left(-\zeta_e\right)^{-1/3} - \left(-\frac{z_q}{L_w}\right)^{-1/3}\right]\Bigg\} + q_0, \end{aligned} \tag{3.75}$$

$$\begin{aligned} q_* = q_z\kappa\Bigg\{\Bigg\{&\left[\ln\left(\frac{\zeta_e L_w}{z_{oq}}\right) - \psi_e\left(\zeta_e\right)\right] \\ &+0.8\left[\left(-\zeta_e\right)^{-1/3} - \left(-\frac{z_q}{L_w}\right)^{-1/3}\right]\Bigg\} + q_0\Bigg\}^{-1}, \end{aligned} \tag{3.76}$$

$$\begin{aligned} q_{10} = \frac{q_*}{\kappa}\Bigg\{&\left[\ln\left(\frac{\zeta_e L_w}{z_{oq}}\right) - \psi_e\left(\zeta_e\right)\right] \\ &+0.8\left[\left(-\zeta_e\right)^{-1/3} - \left(-\frac{10}{L_w}\right)^{-1/3}\right]\Bigg\} + q_0, \end{aligned} \tag{3.77}$$

for very unstable conditions ($\zeta < \zeta_e = -0.465$).

The transfer coefficients for momentum(C_{dz}), sensible heat (C_{hz}) and latent heat (C_{ez}) can be calculated by re-arranging equations 3.6 to 3.8.

$$\tau = C_{dz}\rho_z u_z^2 = \rho_z u_{*a}^2, \tag{3.78}$$

$$C_{dz} = \rho_a u_{*a}^2 \Big/ \rho_z u_z^2, \tag{3.79}$$

$$C_{dz} = u_{*a}^2 \Big/ u_z^2, \tag{3.80}$$

$$Q_h = \rho_z C_{pa} C_{hz} u_z \left(T_0 - T_a\right) = \rho_z C_{pa} u_* T_*, \tag{3.81}$$

$$C_{hz} = \rho_z C_{pa} u_* T_* \Big/ \rho_z C_{pa} u_z \left(T_0 - T_a\right), \tag{3.82}$$

$$C_{hz} = -u_* T_* \Big/ u_z \left(T_0 - T_a\right). \tag{3.83}$$

$$Q_e = \rho_z L_v C_{ez} u_z \left(q_0 - q_z\right) = -\rho_z L_v u_{*a} q_*, \tag{3.84}$$

$$C_{ez} = \rho_z L_v u_z \left(q_0 - q_z\right) \Big/ - \rho_z L_v u_{*a} q_*, \tag{3.85}$$

$$C_{ez} = -u_* q_* \Big/ u_z \left(q_0 - q_z\right). \tag{3.86}$$

In order to compare the transfer coefficients among sites, however, we also need to calculate them at a reference height of 10 m ($C_{d10}, C_{h10}, C_{e10}$) as

$$C_{h10} = Q_h \Big/ \rho_z C_{pa} u_{10} \left(T_0 - T_{10}\right), \tag{3.87}$$

$$C_{e10} = Q_e \Big/ \rho_z L_v u_{10} \left(q_0 - q_{10}\right), \tag{3.88}$$

$$C_{d10} = u_*^2 \Big/ u_{10}^2. \tag{3.89}$$

We can also calculate the neutral transfer coefficient at a height of 10 m as

$$C_{d10N} = \left[\kappa \Big/ \ln\left(\frac{10}{z_o}\right)\right]^2, \tag{3.90}$$

$$C_{e10N} = C_{e10N} = \kappa^2 \Big/ \left[\ln\left(\frac{10}{z_o}\right)\ln\left(\frac{10}{z_{oq}}\right)\right], \tag{3.91}$$

where z_o and z_{oq} are calculated under neutral atmospheric conditions, that is $\zeta = 0$.

Evaporation is a key component in the water and energy budgets of lakes, where in most cases, represents major water loss. Information on lake evaporation, is therefore, essential for water resource management and for predicting future changes in water levels as a result of climate change. Here, evaporation (E) is estimated according to the latent heat flux (Q_e) as

$$E = \frac{Q_e}{\rho_0 \times L_v}, \tag{3.92}$$

where ρ_w is the density of the surface water.

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