Multi-Depot Croudsourced VRP with Capacity Constraint - ILP Formulation

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Given:

A set of depots $W = \{0, 1, ..., n\}$

A set of vehicles $V = \{0, 1, 2, ..., m\}$

And a set of packages $P = \{0, 1, 2, ..., q\}$

Where n is the number of depots, m is the number of vehicles available, and q is the number of packages needing to be delivered. And the following constants:

$c_{ij} \in C$ where C is a $q \times q$ matrix and c_{ij} represents the distance between the destination of package i and the destination of package j.

$c'_{ki} \in C'$ where C' is a $n \times q$ matrix and c'_{ki} represents the distance between depot k and the destination of package i.

$c''_{ih} \in C''$ where C'' is a $q \times m$ matrix and c''_{ih} represents the distance between package i destination and the final destination of car h.

$c'''_{hk} \in C'''$ where C''' is a $m \times n$ matrix and c'''_{hk} represents the distance from the starting location of car h to depot k.

$n_h \in n_1, n_2, ..., n_m$ represents the maximum number of packages car h can deliver.

We want to minimize total distance traveled by all the cars while ensuring the following constraints:

- All packages must be delivered
- A package can only delivered once and by one car
- A car cannot deliver more packages than its maximum number of packages n_h.

1 Integer Linear Formulation

Variables:

$x_{ij}^h \in \{0,1\}$: if $x_{ij}^h = 1$ then car h travels from package destination i directly to package destination j (i.e. edge $pkg_i \to pkg_j$ is used by car h).

$y_{ki}^h \in \{0,1\}$: if $y_{ki}^h = 1$ then car h travels from depot k directly to package destination i (i.e. edge $dpt_k \to pkg_i$ is used by car h).

$z_{ih} \in \{0,1\}$: if $z_{ih} = 1$ then car h delivers package i last and then goes to car h's final destination (i.e. edge $pkg_i \to dest_h$ is used by car h).

$\beta_{hk} \in \{0,1\}$: where $\beta_{hk} = 1$ means car h uses depot k (i.e. edge $start_h \to dpt_k$ is used by car h).

$t_i^h \in \mathbb{Z}^+$: order variable for MTZ constraint. [MTZ60]

Minimize:

$$\sum_{h\in V}(\sum_{i\in P}\sum_{j\in P} c_{ij}x_{ij}^h + \sum_{i\in P}\sum_{k\in W} c'_{ki}y_{ki}^h + \sum_{i\in P} c''_{ih}z_{ih} + \sum_{k\in W} c'''_{hk}\beta_{hk}) \tag{1}$$

Subject to:

$$\sum_{h\in V}\sum i \in P x_{ij}^h = 1 \qquad \forall j \in P \tag{2}$$

$$\sum_{h\in V}\sum j \in P x_{ij}^h = 1 \qquad \forall i \in P \tag{3}$$

$$\sum_{i\in P} x_{ij}^h \sum_{k\in W} y_{kj}^h = \sum_{l\in P} x_{jl}^h + z_{jh} \qquad \forall h \in V, \forall j \in P \tag{4}$$

$$\sum_{k\in W} \beta_{hk} \le 1 \qquad \forall h \in V \tag{5}$$

$$\sum_{i\in P} y_{ki}^h = \beta_{hk} \qquad \forall h \in V, \forall kinW \tag{6}$$

$$\sum_{j\in P} x_{ij}^h + z_{ih} \le \sum_{k\in W} \beta_{hk} \qquad \forall h \in V, \forall i \in P \tag{7}$$

$$t_j^h \ge t_i^h - B(1 - x_{ij}^h) + 1 \qquad \forall h \in V, \forall i \in P, \forall j \in P \tag{8}$$

Constraint 2 - 3: Each package must be delivered exactly once.

Constraint 2: Exactly one car enters each package location.

Constraint 3: Exactly one car leaves each package location.

Constraint 4: If a car a enters a package location then that car must also leave that same location.

Constraint 5: Each car goes first to at most one depot.

Constraint 6: If a car enters a depot then it also leaves that depot.

Constraint 7: A car visits at least one depot when it delivers at least one package.

Constraint 8: MTZ constraint to eliminate cycles [MTZ60].

References

[MTZ60] C. E. Miller, A. W. Tucker, and R. A. Zemlin. Integer programming formulation of traveling salesman problems. *J. ACM*, 7(4):326329, oct 1960.