# Stock index volatility prediction and VaR measurement based on machine learning

## Introduction

One of the lessons we learned form the repetitive financial crisis is that financial risk is real,and we should consider this through our investment protoil.VaR can be said the most common tools to measure financial risk.VaR (Value at Risk) is a risk measurement method developed by JP Morgan in the 1990s to measure the market risk faced by assets. Its specific definition is: the maximum loss faced by a financial asset or asset portfolio within a certain period of time and at a certain confidence level. More strictly speaking, VaR describes the quantile of the expected distribution of asset gains and losses within a certain time interval. It can quantify intangible risks into specific loss amounts, which helps investors and financial institutions to more intuitively monitor the size of risks and control risks, thereby achieving the purpose of rationally allocating capital and maximizing returns.

## problem statement

The volatility of financial asset returns is a determinant of VaR and a powerful tool for measuring VaR. However, volatility cannot be obtained directly from market return data and requires us to perform modeling calculations.In the past, most methods used the GARCH family model to estimate the variance of the rate of return, and then assumed the distribution of the rate of return to deduce VaR.

However, the existing problem is that the GARCH family models are linear models, and the stock market has too many noise transactions, and its volatility is not only affected by the country's macroeconomic policies, but also by many factors such as individual investors, institutional investors and corporate decisions. The multiple influencing factors have led to the volatility of financial market showing very complex and nonlinear characteristics, which has reduced the prediction performance of the traditional linear volatility prediction model. In recent years with the developmentIn recent years, the improvement of machine learning and artificial intelligence has achieved increasingly excellent results in simulating nonlinear structures, bringing new possibilities for accurately predicting volatility.So I attempt to combine the emerging deep learning algorithm with the traditional GARCH model to improve the accuracy of conditional variance prediction.

**Research objectives:**

The main objective of this research is to enhance the accuracy of financial volatility prediction by integrating deep learning techniques with the traditional GARCH model. The specific objectives can are lisited:

1,To develop a hybrid predictive framework combining deep learning algorithms with the GARCH model:

2,To compare the performance of the hybrid model with traditional GARCH models

**Scope**

This study focuses on the following two aspects:

1,Accurate prediction of stock index volatility.

2,VaR risk measurement based on volatility prediction results

**Literature review:**

With the development of machine learning technology, its applications in the financial field have become increasingly widespread. Many scholars have begun to explore the use of machine learning methods to study Value at Risk (VaR). This section will review the relevant literature in this area.

1, Review on direct VaR prediction using machine learning

Petneházi (2021))explored the use of convolutional neural networks (CNNs) for predicting Value at Risk (VaR). By making adjustments to the algorithm, the CNN was adapted to estimate arbitrary quantiles of a distribution rather than focusing solely on mean predictions, thereby enabling its application in VaR forecasting.Similarly, Pradeepkumar and Ravi (2017) developed a quantile regression neural network, enhanced with particle swarm optimization (SPOQRNN), to forecast financial time series volatility. Their study compared the performance of this model against three traditional forecasting approaches, including GARCH, multilayer perceptrons, general regression neural networks, and random forests. The results demonstrated that the SPOQRNN consistently outperformed the other methods.Blom et al. (2023) investigated the application of machine learning methods to predict Value at Risk (VaR) in the EURUSD currency market. Their study proposed a hybrid approach combining quantile regression with machine learning techniques, leveraging implied volatility metrics such as at-the-money volatilities and risk reversals as explanatory variables. The authors tested several models, including Light Gradient Boosting Machine (LightGBM), CatBoost, Random Forest, and neural networks like LSTMs and RNNs. Among these, the ensemble models especially LightGBM and CatBoost—outperformed the benchmark Quantile Regression Implied Moments (QR-IM) model in accuracy. However, neural networks were found to be less stable, requiring more data and refined architectures to achieve consistent performance.

2，Review on combine traditional model with machine learning

Kim et al.(2018) focused on the volatility of KOSPI 200 and developed a hybrid model, GEW-LSTM, combining LSTM with GARCH, EGARCH, and EWMA models. This hybrid model demonstrated better stock market volatility forecasting performance with lower Mean Squared Error (MSE) and Mean Absolute Error (MAE) compared to single models.Yan et al.(2020) proposed a novel hybrid approach to forecast copper price volatility. This method not only integrated deep neural networks with the classical GARCH model but also combined LSTM with traditional Artificial Neural Networks (ANN). Experimental results showed that the GARCH model could significantly enhance the predictive capacity of neural network models, and the fusion of LSTM and ANN proved to be an effective approach to improve neural network structures for enhanced forecasting performance.Cao Wei et al. (2020) developed a hybrid model combining LSTM and GARCH-family models to predict the volatility of the Chinese Renminbi exchange rate. Experimental results indicated that the hybrid model outperformed single models, with the standalone LSTM model performing better than the standalone GARCH model.Zeng et al. (2022) integrated deep learning methods with the Generalized Autoregressive Conditional Heteroskedasticity Mixed

Data Sampling (GARCH-MIDAS) model. The GARCH-MIDAS model was used to address the mixed-frequency issue between macroeconomic variables and stock market volatility for short-term volatility forecasting. The predicted short-term volatility was then used as an input variable for the deep learning model to predict realized stock market volatility.Ke and Yin (2024) proposed an improved method for optimizing the Conditional Autoregressive Value at Risk (CAViaR) model using Genetic Algorithm (GA) and Gradient Descent (GD). Their multivariate multilevel CAViaR model was employed to examine tail risk spillovers between the US stock market, foreign exchange market, and credit market. The study found that the Genetic Algorithm outperformed Gradient Descent in handling the complexities of the CAViaR loss function, offering superior performance in global optimization and robustness.Jakub et al. (2023) developed a hybrid approach combining GARCH-family models (GARCH, EGARCH, GJR-GARCH, APARCH) with Gated Recurrent Unit (GRU) neural networks to forecast financial volatility and Value at Risk (VaR). The study applied this method to S&P 500, Bitcoin, and Gold, finding that the hybrid models significantly outperformed traditional GARCH models. For VaR estimation, GJR-GARCH-GRU and APARCH-GRU models provided robust predictions at various confidence levels, especially under conditions with skewed or heavy-tailed distributions.Buczynski & Chlebus (2023) proposed the GARCHNet model, which combines GARCH models with Long Short-Term Memory (LSTM) networks. By replacing the GARCH conditional variance equation with LSTM layers, the hybrid model captured non-linear dependencies in financial data. Experimental results showed that GARCHNet significantly outperformed traditional GARCH models in both variance prediction and VaR forecasting, particularly during periods of high market volatility, such as the COVID-19 pandemic. The model effectively handled heavy tails and asymmetries, delivering robust performance under t-distribution and skewed t-distribution assumptions

**Research methodology**

The research object of this paper is the volatility and return of the S&P 500 Index. The data used include the four core indicators of the index's daily opening price, closing price, highest price and lowest price from 2014 to 2024. In order to further analyze the volatility of the return, this paper calculates the daily return series based on the closing price for volatility modeling and prediction.

**Research method for volitality:**

This study will first use traditional GARCH-family models for volatility modeling. Next, it will employ price data as input to an LSTM model to directly predict volatility. Finally, the parameters from the GARCH-family models, along with the volume and price data, will be used as inputs to the LSTM model for a second round of volatility prediction.

1,Standard GARCH(p,q) model:

$$\begin{cases} y_t = x_t\gamma + \varepsilon_t \\ \sigma_t^2 = \omega \sum_{i=1}^{q} a_i\epsilon_{i-i}^2 + \sum_{j=1}^{p} \beta_j\sigma_{t-j}^2 \end{cases}$$

where $y_t$ represents the observed value at time t, such as financial returns, while $x_t$ denotes any external variables influencing the observation, with γ being their corresponding coefficient. The $\varepsilon_t$ term refers to the residual. The conditional variance, $\sigma_t^2$, represents the volatility at time t and is modeled as a function of three components: a constant term ω, ensuring positivity; the past q-order squared residuals, $a_i$ which reflect the impact of previous shocks on current volatility; and the past p-order conditional $\beta_j$ variances, representing the persistence of historical volatility. Together, these parameters describe the dynamic behavior of volatility over time.

In actual financial market transactions, the lower the investor's expectations for stock returns, the greater the volatility of stock returns. Conversely, when the investor's expectations for stock returns are higher, the volatility of stock returns is smaller. This is called the leverage effect. Based on this problem, we introduce the EGARCH model proposed by Nelson (1991).

$$ln\sigma_i^2 = \omega + \sum_{i=1}^{q} \alpha_i(|\frac{\epsilon_{t-i}}{\sigma_{t-i}} - E(\frac{\varepsilon_{t-i}}{\sigma_{t-i}})| + \gamma_i\frac{\varepsilon_{t-i}}{\sigma_{t-i}}) + \sum_{i=1}^{p} \beta_i ln\sigma_{t-i}^2$$

2，LSTM

There are three gates in the LSTM neural network structure: input gate, forget gate, and output gate. The gate structure can control the updating and forgetting of historical information.

The forget gate is defined by the following formula:

$$f_t = \sigma (W_f \bullet [h_{t-1}, x_t] + b_f)$$

The forget gate refers to the processing of the state of the neuron at the previous node in the modeling process of a neural network. The gate will input vectors $x_t$ and $h_{t-1}$, perform operations through the Sigmoi activation function, and output a vector $f_t$ with all elements between 0 and 1, and assign it as a weight to each element in the unit state, thereby achieving control of the information state.

The input gate is used to determine the amount of information stored in the unit state at $c_t$ the current moment from the information input $x_t$ of the current time node in the modeling process of a neural network. The input gate is implemented by the following formula:

$$\tilde{c}_t = tanh(W_c \bullet [h_{t-1}, x_t] + b_c)$$

$$i_t = \sigma (W_i \bullet [h_{t-1}, x_t] + b_i)$$

$\tilde{c}_t$ indicates that the candidate memory extracts the potential features of the current input and historical state, and $i_t$ controls how much information in the candidate memory can be added to the current memory state $c_t$

The output gate is used to determine the amount of information that the current cell state $c_t$ outputs to the current output value $h_t$

$$c_t = f_t \bullet c_{t-1} + i_t \bullet \tilde{c}_t$$

$$o_t = \sigma (W_o \bullet [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t \bullet tanh(c_t)$$

In this study, the input variables of a single LSTM are only stock price data, including opening price, closing price, highest price, and lowest price.

3, LSTM-GARCH

The price data of S&P 500 Index, GARCH model parameters, and EGARCH model parameters are input into the LSTM model for training. The output of the LSTM model passes through a fully connected layer, and finally the volatility of the S&P 500 Index is obtained as the output variable.

**Research method for VaR:**

Koenker et al. (2006) proposed a quantile autoregressive model, which has the following general form:

$$Q_{y_t}(\tau \mid y_{t-1},\dots,y_{t-p},\sigma_t) = \alpha_0(\tau) + \alpha_1(\tau)y_{t-1} + \dots + \alpha_p(\tau)y_{t-p} + \beta(\tau)\sigma_t$$

Where $y_{t-1},\dots y_{t-p}$ are the lagged term of the return, $\sigma_t$ is the volatility, and $\alpha_i(\tau)$ and $\beta(\tau)$ are the quantile coefficients. The model estimates the parameters by minimizing the asymmetric loss function.

**Significance and revelance**:

This study introduces volatility prediction into the financial risk measurement framework, which enriches the research perspective of financial risk measurement methods to a certain extent and broadens the ideas of existing financial risk measurement methods. At the same time, it combines traditional measurement models with neural network models to provide a new research method for volatility prediction and VaR value measurement.

**Expected outcomes:**

In order to more comprehensively measure the training effects of different models, this paper introduces five error evaluation indicators, namely, mean square error (MSE), mean absolute error (MAE), root mean square error (RMSE), heteroscedasticity adjusted The MSE (HMSE) and the heteroskedasticity-adjusted MAE (HMAE).We are expected to show that the hybrid model has a significant reduction in error indicators compared to the single GARCH model and the single LSTM model.

**Proposed work schedule:**

Work schedule for semester 1:

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|---|---|
| week1-4 | Learn about machine learning and LSTM model |
| week 5-7 | Collect data,Clean and preprocess the datase |
| week 8-14 | Use the GARCH family model to predict volatility and calculate the corresponding VaR |

Work schedule for semester 2.

| | |
|---|---|
| week1-4 | Use single LSTM model to caculate volitality |
| week 5-10 | Experiment and try to use various GARCH-LSTM fusion methods to select the best one. |
| week 11-14 | Write article and complete research |

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